Introduction to QFT
Assignment 11

Due on 10.02.16

Compton Scattering

In this exercise we are going to compute the cross section for Compton scattering $e^-(p)+\gamma(k)\to e^-(p')+\gamma(k')$ step by step.

1. Draw the two Feynman diagrams that contribute to this process and write down the expression for $\mathcal{M}$.
2. Simplify the denominators (using $p^2=m^2$ and $k^2=0$) and the numerators (using Dirac algebra).
3. Obtain the analytic expression for $|\mathcal{M}|^2$ after averaging over initial states and summing over final states.
4. At this point, the result should be written as the sum of four terms, each of them including a complicated trace. Show that one of those traces can be obtained from another one with the replacement $k\to -k'$ and that the remaining two are equal to each other. Hence, only two traces have to be evaluated explicitly.
5. Compute those traces. Here you have to use the known properties of the γ matrices (e.g. $\gamma^\mu\gamma_\nu\gamma_\mu = -2\gamma_n u$, $\gamma_\mu\gamma^\mu=4$, etc.) and Dirac algebra such that $\not p \not p = p^2 = m^2$. At the end, all terms can be reduced to a trace of no more than two γ matrices.
6. Simplify the trace results by first writing everything in terms of the Mandelstam variables (s,t,u) and then using momentum conservation $(s+t+u=2m^2)$ to eliminate t.
7. Putting together the different pieces (and rewriting s and u in terms of scalar products) you should get
$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = 2e^4\left[\frac{p\cdot k'}{p\cdot k}+\frac{p\cdot k}{p\cdot k'}+2m^2\left(\frac{1}{p\cdot k}-\frac{1}{p\cdot k'}\right)+m^4\left(\frac{1}{p\cdot k}-\frac{1}{p\cdot k'}\right)^2\right].$$
8. Now, you have to choose a frame to analyze the kinematics. Compton scattering is usually analyzed in the lab frame, in which the electron is initially at rest. Draw a picture of the kinematics (before and after the collision). Choose $k=(\omega,\omega e_z)$, $p=(m,0)$, $k'=(\omega',\omega'\sin\theta,0,\omega'\cos\theta)$, $p'=(E',\boldsymbol{p}')$ and derive Compton's formula:
$$\omega' = \frac{\omega}{1+\frac{\omega}{m}(1-\cos\theta)}.$$
9. Use the general cross-section formula for two-by-two scattering, developed in the lecture, and calculate the differential cross section $\frac{d\sigma}{d\cos\theta}$.
10. take the low energy limit $\omega\to 0$ and show that the total cross section in this limit is the Thomson cross section for classical photon-electron scattering
$$\sigma_{\text{tot}} = \frac{8\pi\alpha^2}{3m^2}, \quad \text{with: } \alpha=\frac{e^2}{4\pi}\approx\frac{1}{137}.$$