Introduction to Monte Carlo event generators
part II

Korinna Zapp

LIP (Lisbon), CERN

Münster 19. 10.–2. 11. 2016

Graduiertenkolleg 2149
Research Training Group

LIP

FCT
Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

CERN

MCnet

Outline

Outline

Gluon radiation - 1

- Radiation of a gluon in $e^+ + e^- \to q + \bar{q}$

$$\frac{\mathrm{d}^2\sigma_{q\bar{q}g}}{\mathrm{d}z\,\mathrm{d}(\cos\theta)} = \sigma_{q\bar{q}} \frac{C_F \alpha_s}{2\pi} \left(\frac{2}{\sin^2\theta} \cdot \frac{1+(1-z)^2}{z} - z \right)$$

θ: angle between gluon and quark
z: gluon energy fraction $z = 2E_g/E_{cm}$

- divergent in
  - collinear limit: $\theta \to 0, \pi$
  - soft limit: $z \to 0$
- rewrite

$$\frac{2\mathrm{d}(\cos\theta)}{\sin^2\theta} = \frac{\mathrm{d}(\cos\theta)}{1-\cos\theta} + \frac{\mathrm{d}(\cos\theta)}{1+\cos\theta} = \frac{\mathrm{d}(\cos\theta)}{1-\cos\theta} + \frac{\mathrm{d}(\cos\bar{\theta})}{1-\cos\bar{\theta}} \approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

$\bar{\theta} = \pi - \theta$: angle between gluon and anti-quark

Gluon radiation - 2

- in collinear limit both legs radiate independently

$$\mathrm{d}\sigma_{\mathrm{q\bar{q}g}} \approx \sigma_{\mathrm{q\bar{q}}} \sum_{i\in\{q,\bar{q}\}} \frac{C_{\mathrm{F}}\alpha_{\mathrm{s}}}{2\pi} \frac{1+(1-z)^2}{z} \frac{\mathrm{d}\theta_{ig}^2}{\theta_{ig}^2} \mathrm{d}z = \sigma_{\mathrm{q\bar{q}}} \sum_{i\in\{q,\bar{q}\}} \frac{\alpha_{\mathrm{s}}}{2\pi} P_{gi}(z) \frac{\mathrm{d}\theta_{ig}^2}{\theta_{ig}^2} \mathrm{d}z$$

- same form for all quantities $\propto \theta^2$, e.g.
  - transverse momentum: $k_\perp^2 \approx z^2(1-z)^2\theta^2 E^2$
  - invariant mass: $Q^2 \approx z(1-z)\theta^2 E^2$

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}k_\perp^2}{k_\perp^2} = \frac{\mathrm{d}Q^2}{Q^2}$$

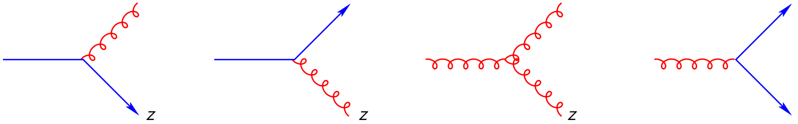
  - use $t \in \{\theta^2, k_\perp^2, Q^2\}$
- factorisation in collinear limit fundamental property of QCD

Factorisation in collinear limit

- to all orders and for all processes

$$d\sigma_{n+1} \approx d\sigma_n \frac{dt}{t} \frac{d\phi}{2\pi} dz \frac{\alpha_s}{2\pi} P_{ab}(z)$$

- Altarelli-Parisi splitting functions:



$$P_{qq} = C_F \frac{1+z^2}{1-z} \qquad P_{gq} = C_F \frac{1+(1-z)^2}{z} \qquad P_{gg} = C_A \frac{(1-z(1-z))^2}{z(1-z)} \qquad P_{qg} = T_R(z^2+(1-z)^2)$$

Quasi-collinear limit

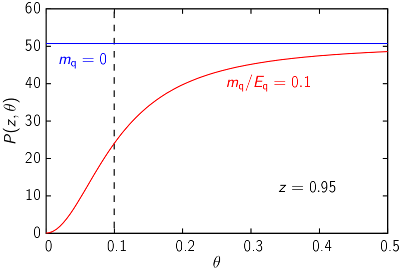
Catani, Dittmaier, Trócsányi, Phys. Lett. B500 (2001) 149

- gluon radiation off massive quark
- for $k_\perp$, $m_q \ll E_q$:

$$P_{gQ}(z,\theta) \approx \frac{C_F}{1-z}\left(1+z^2-\frac{2z}{1+z^2(\theta E_q/m_q)^2}\right)$$

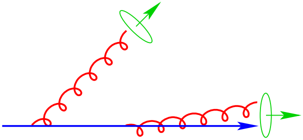
- emission suppressed for $\theta \lesssim m_q/E_q$

  $\rightarrow$ "dead cone"



Infra-red cut-off

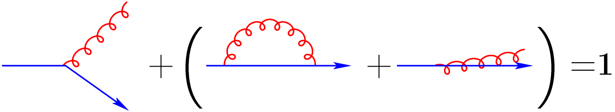
- nearly collinear emissions not separately resolvable



- classify emissions with $t < t_0$ as unresolvable
- combine unresolved emissions with virtual corrections
  $\rightarrow$ divergences cancel

Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems

- unitarity: probabilities add up to unity



Kinematics

- use $t = Q^2$
- in splitting $a \to b + c$: $k_\perp^2 \simeq z(1-z)p_a^2 - (1-z)p_b^2 - zp_c^2$
- $p_a^2 = Q^2$, $p_b^2, p_c^2 > Q_0^2$ and $k_\perp^2 > 0$ leads to

$$z, 1-z > \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4Q_0^2}{Q^2}} \approx \frac{Q_0^2}{Q^2}$$

- t_0: infra-red scale $\mathcal{O}(1\,\mathrm{GeV}^2)$
- t_{max}: characteristic scale of hard process, e.g.
  - s in e^+e^-
  - boson mass in DY
  - $p_\perp$ in di-jet production
  - finding a good scale can be nontrivial
  - nonsensical choices lead to nonsensical results

Naive gluon emission probability

- naive gluon emission probability off a quark:

$$\Pi_1^{(q)} = \frac{1}{2}\frac{\sigma_{q\bar{q}g}}{\sigma_{q\bar{q}}} \approx \frac{\alpha_s}{2\pi}\int \frac{dQ^2}{Q^2}\int dz\, P_{gq}(z)$$

$$\approx \frac{C_F\alpha_s}{2\pi}\int_{Q_0^2}^{Q_{max}^2} \frac{dQ^2}{Q^2}\int_{z_{min}}^{1} dz\,\frac{2}{z} = \frac{C_F\alpha_s}{2\pi}\int_{Q_0^2}^{Q_{max}^2}\frac{dQ^2}{Q^2} 2\ln\left(\frac{Q^2}{Q_0^2}\right)$$

$$= \frac{C_F\alpha_s}{2\pi}\ln^2\left(\frac{Q_{max}^2}{Q_0^2}\right)$$

- for sufficiently hard processes $\Pi_1^{(q)} \gtrsim 1$
- $\Pi_1^{(q)}$ is not a probability
- multiple emissions

Need for resummation

$$\Pi_n^{(q)} \simeq \frac{1}{n!} \frac{C_F^n}{(2\pi)^n} \alpha_s^n \ln^{2n} \left(\frac{Q_{max}^2}{Q_0^2} \right)$$

- contributions comparable at all orders
- have to resum the entire stack of $\alpha_s^n \ln^{2n}$ terms
- analytic resummation observable by observable or
- Monte Carlo $\rightarrow$ parton shower

Parton shower: overview

- parton shower generates extra gluon emissions to all orders
- systematic approximation to multi-leg matrix elements
- independent of hard process — except for starting scale t_{max}
- the parton shower is unitary
  $\rightarrow$ does not affect integrated cross section
  can affect fiducial cross sections by modifying phase space distributions
- parton shower has leading log accuracy, i.e. it resums the $\alpha_s^n \ln^{2n}$ terms to all orders
- contains some sub-leading pieces
- leading contribution from planar diagrams
- leading colour only

The Sudakov form factor

- have to take survival probability into account

  generalised radioactive decay & Poisson distribution discussed last week
- differential probability for first splitting at scale t

$$\mathcal{P}_{\text{first}}(t_{\text{max}}, t) = -\frac{\mathrm{d}\Delta(t_{\text{max}}, t)}{\mathrm{d}t} = \mathcal{P}_{\text{em}}(t)\Delta(t_{\text{max}}, t)$$

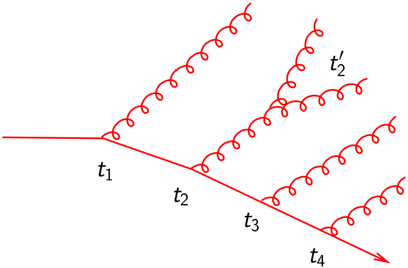
$$\mathcal{P}_{\text{em}}(t) = \frac{1}{t}\sum_b \int_{z_{\text{min}}(t)}^{z_{\text{max}}(t)} \mathrm{d}z\, \frac{\alpha_s}{2\pi} P_{ba}(z)$$

- Sudakov form factor (no-splitting probability)

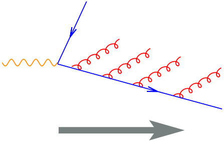
$$\Delta(t_{\text{max}}, t) = \exp\left(-\int_t^{t_{\text{max}}} \frac{\mathrm{d}t}{t}\, \mathcal{P}_{\text{em}}(t)\right)$$

Iteration: the shower

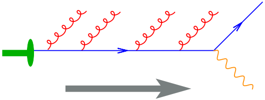
- event generation: veto algorithm
- splitting probability $\mathcal{P}_{\text{first}}$ depends only on starting scale
- splitting process can be iterated — Markov chain
- leading contribution from strongly ordered histories
  $t_1 \gg t_2 \gg t_3 \gg t_4$ and $t_2 \gg t_2'$
- in MC $t_1 > t_2 > t_3 > t_4$ and $t_2 > t_2'$ to fill entire phase space



Initial state evolution



t decreasing



t increasing

- in principle initial state evolution the same as in final state
- but: both ends of evolution fixed
- must account for probability to resolve parton at larger $x = zx'$

$$\mathcal{P}_{\text{em}}^{(\text{is})}(x,t) = \frac{1}{t}\sum_a \int \text{d}z\, \frac{\alpha_s}{2\pi} P_{ba}(z) \frac{x' f_a(x',t)}{x f_b(x,t)}$$

- hard to implement in forward evolution

  have to reach at flavour and t_{max} set by hard process
- evolve backwards from hard process towards incoming hadron

Initial state evolution: equivalence to DGLAP

- DGLAP evolution equation:

$$t\frac{\mathrm{d}f_b(x,t)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}P_{ba}(z)f_a\left(\frac{x}{z},t\right)$$

- splitting probability of individual parton related to change in parton distribution

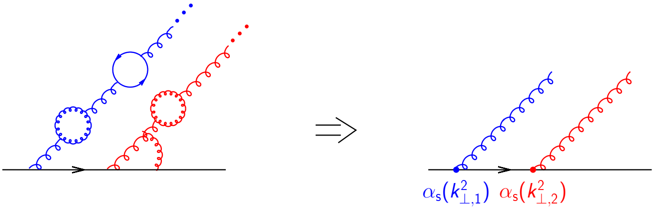
$$\mathcal{P}^{(\mathrm{is})}_{\mathrm{em}}(x,t)\mathrm{d}t = \frac{\mathrm{d}f_b(x,t)}{f_b(x,t)} = \frac{\mathrm{d}t}{t}\sum_a \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}P_{ba}(z)\frac{f_a(x/z,t)}{f_b(x,t)}$$

$$= \frac{\mathrm{d}t}{t}\sum_a \int_x^1 \mathrm{d}z\frac{\alpha_s}{2\pi}P_{ba}(z)\frac{x'f_a(x',t)}{xf_b(x,t)}$$

Sub-leading corrections

Formally sub-leading terms can be numerically relevant.
Parton showers typically include

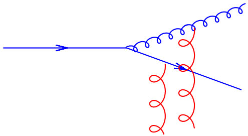
- $g \to q\bar{q}$ splitting
- summing loop corrections → running coupling: $\alpha_s(k_\perp^2)$



faster parton multiplication, especially at low $k_\perp^2$

- 4-momentum conservation

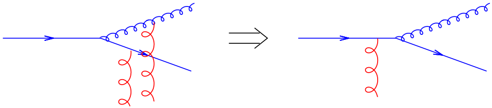
Soft limit



- soft limit also universal
- soft gluons come from everywhere in the event

⇒ quantum interference – independent evolution picture still valid?

Angular ordering



- outside cone soft gluons sum coherently
- don't resolve two partons, but see only combined charge
- angular ordering

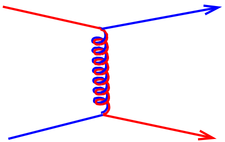
  automatically incorporated when using θ as evolution variable
- analogue of Chudakov effect in QED

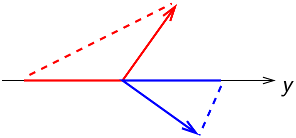
  suppression of soft bremsstrahlung from e^+e^- pairs

Interference between initial and final state: colour coherence

- initial conditions for showers set by colour structure of hard process
- ISR+FSR add coherently in regions of colour flow and destructively else

→ emission from each parton confined to cone extending to its colour partner

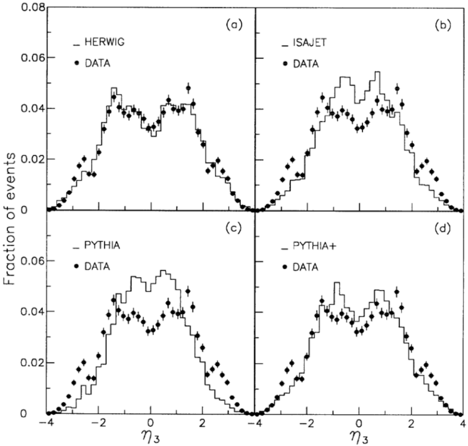




Interference between initial and final state: colour coherence

rapidity of third hardest jet in jet events

CDF, Phys. Rev. D **50** (1994) 5562.

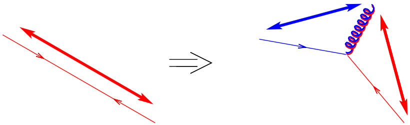


- HERWIG: full colour coherence
- ISAJET: no CC
- PYTHIA: no CC
- PYTHIA+: partial CC
- modern generators: full CC

Dipole picture

first implemented in ARIADNE (Lonnblad, Comput. Phys. Commun. 71, 15 (1992))

- can formulate parton shower based on colour dipoles
- gluon emission: split a dipole into two



- better description of soft emissions
- colour coherence automatically accounted for
- simpler kinematics: everything stays on-shell
- close relation to subtraction kernels used in NLO calculations

Dipole picture: the splitting function

- look at $e^+ + e^- \to q + \bar{q} + g$ again:

$$\mathrm{d}\sigma_{\mathrm{q\bar{q}g}} = \sigma_{\mathrm{q\bar{q}}} \frac{C_\mathrm{F}\alpha_\mathrm{s}}{2\pi} \left(\frac{2}{\sin^2\theta} \cdot \frac{1+(1-z)^2}{z} - z \right) \mathrm{d}z\, \mathrm{d}(\cos\theta)$$

Dipole picture: the splitting function

- look at $e^+ + e^- \to q + \bar{q} + g$ again:

$$
\begin{aligned}
\mathrm{d}\sigma_{\mathrm{q\bar{q}g}} &= \sigma_{\mathrm{q\bar{q}}} \frac{C_\mathrm{F}\alpha_\mathrm{s}}{2\pi} \frac{x_\mathrm{q}^2 + x_{\bar{\mathrm{q}}}^2}{(1-x_\mathrm{q})(1-x_{\bar{\mathrm{q}}})} \mathrm{d}x_\mathrm{q}\,\mathrm{d}x_{\bar{\mathrm{q}}} \\
&= \sigma_{\mathrm{q\bar{q}}} \frac{C_\mathrm{F}\alpha_\mathrm{s}}{2\pi} [x_\mathrm{q}^2(p_\perp, y) + x_{\bar{\mathrm{q}}}^2(p_\perp, y)] \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \mathrm{d}y \\
&= \sigma_{\mathrm{q\bar{q}}} \frac{C_\mathrm{F}\alpha_\mathrm{s}}{2\pi} D(p_\perp, y) \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \mathrm{d}y
\end{aligned}
$$

Dipole picture: the splitting function

- look at $e^+ + e^- \to q + \bar{q} + g$ again:

$$\begin{aligned} d\sigma_{q\bar{q}g} &= \sigma_{q\bar{q}} \frac{C_F \alpha_s}{2\pi} \frac{x_q^2 + x_{\bar{q}}^2}{(1 - x_q)(1 - x_{\bar{q}})} dx_q dx_{\bar{q}} \\ &= \sigma_{q\bar{q}} \frac{C_F \alpha_s}{2\pi} [x_q^2(p_\perp, y) + x_{\bar{q}}^2(p_\perp, y)] \frac{dp_\perp^2}{p_\perp^2} dy \\ &= \sigma_{q\bar{q}} \frac{C_F \alpha_s}{2\pi} D(p_\perp, y) \frac{dp_\perp^2}{p_\perp^2} dy \end{aligned}$$

- evolution variable: $p_\perp$, splitting variable: rapidity y
- exact reproduction of matrix element for final state $q\bar{q}$ dipoles
- analogous splitting functions for qg, $\bar{q}g$ and gg dipoles

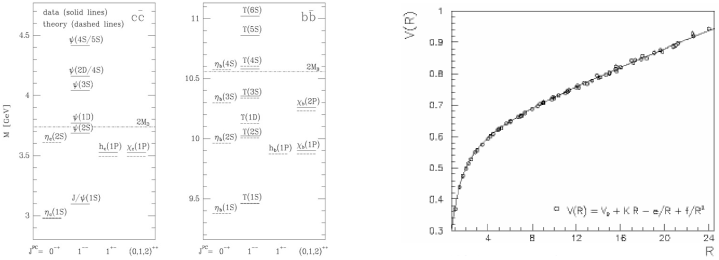
  no exact factorisation for these dipoles
- Ds for initial-final and final-final dipoles from crossing relations

Outline

Confinement and interquark potential

- Hadronisation is QCD at low scales where α_s is $\mathcal{O}(1)$

$\Rightarrow$ non-perturbative dynamics, not easily calculable from first principles



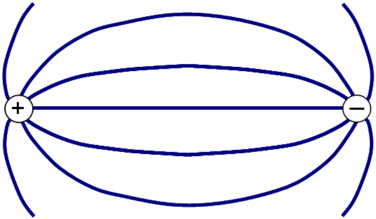
- measure QCD potential from quarkonia masses
- or calculate using lattice QCD

$\Rightarrow$ approximately linear potential

Confinement and interquark potential

- Hadronisation is QCD at low scales where α_s is $\mathcal{O}(1)$

$\Rightarrow$ non-perturbative dynamics, not easily calculable from first principles



QED dipole

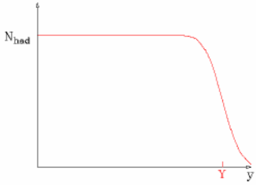
QCD dipole

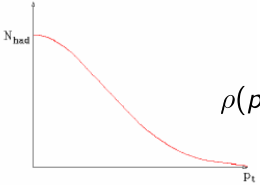
$\Rightarrow$ formation of flux tubes in QCD

Feynman-Field model

Experimental findings:

Feynman, Field NPB136(1978)1





$$\rho(p_\perp^2) = \exp\left(-p_\perp^2/\sigma^2\right)$$

Realisation:

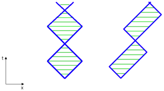
- recursively split $q \to q' +$ hadron
  - transverse momentum from fitted Gaussian
  - longitudinal momentum arbitrary (fitted to measurements)
  - flavour from symmetry arguments+measurements
- **problems:** frame dependent, "last quark", infrared safety, no link to perturbation theory

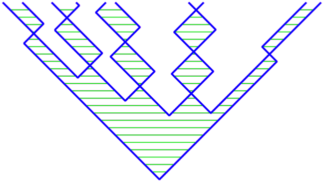
Lund string model

Andersson, Gustafson, Ingelman, Sjöstrand PR97(1983)31

- start with $e^+e^- \to q\bar{q}$
- QCD flux tube with constant energy per unit rapidity 
- new $q\bar{q}$-pairs by pair creation in the flux tube (κ-string tension)

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}x\mathrm{d}t} = \exp\left\{-\frac{\pi^2 m_q^2}{\kappa}\right\}$$

- expanding string breaks into hadrons, then yoyo modes 
- mesons as quark-antiquark pairs, baryons as quark-diquark pairs



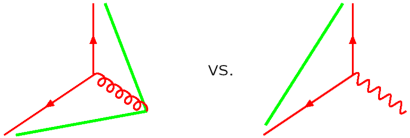
Lund string model

Andersson, Gustafson, Ingelman, Sjöstrand PR97(1983)31

- Lund string model very well motivated, but many parameters

$\Rightarrow$ gives genuine prediction of “string effect”

- strings span between quarks and anti-quarks, gluons form kinks in string
  $\rightarrow$ string accelerated in direction of gluon
- infrared safe matching to parton showers
  gluons with $k_\perp \lesssim 1/\kappa$ irrelevant



Cluster model

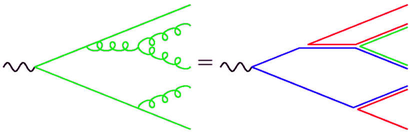
Webber NPB238(1984)492

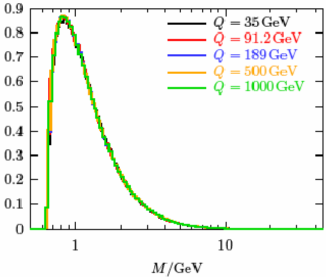
- underlying idea: preconfinement

⇒ follow colour structure of parton showers, colour singlets end up close in phase space

- singlet mass $\mathcal{O}(t_c)$

⇒ primordial clusters independent of collider energy

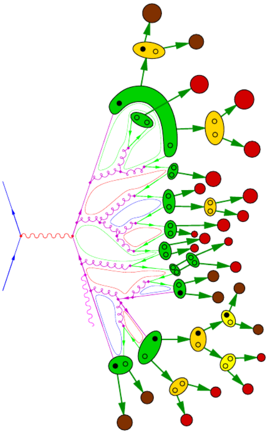




Cluster model

Naïve model:

- split gluons non-perturbatively into $q\bar{q}$-pairs
- colour-adjacent pairs form primordial clusters
- clusters decay into hadrons according to phase space
  $\rightarrow$ diquark & heavy quark production suppressed

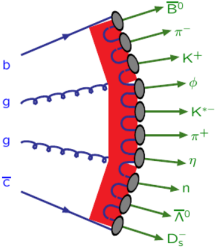


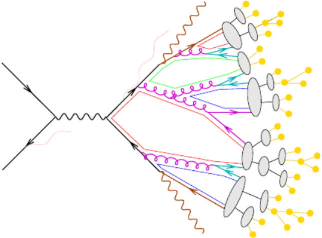
Improved model:

- heavy cluster decay first into lighter cluster, or radiate a hadron
  C$\rightarrow$CC, C$\rightarrow$CH, C$\rightarrow$HH
- leading particle effects incorporated naturally

String vs cluster

Sjöstrand, Durham '09





program	PYTHIA	HERWIG
model	string	cluster
energy–momentum picture	powerful	simple
	predictive	unpredictive
parameters	few	many
flavour composition	messy	simple
	unpredictive	in-between
parameters	many	few

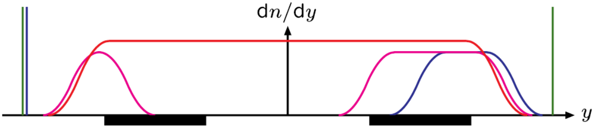
"There ain't no such thing as a parameter-free *good* description"

Outline

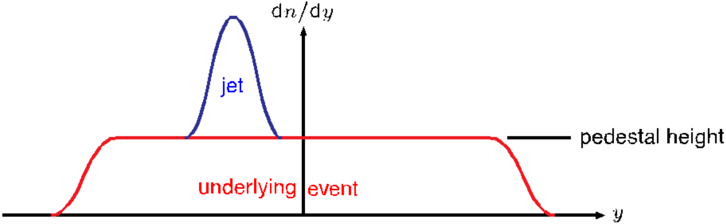
Classification

- Soft inclusive collision

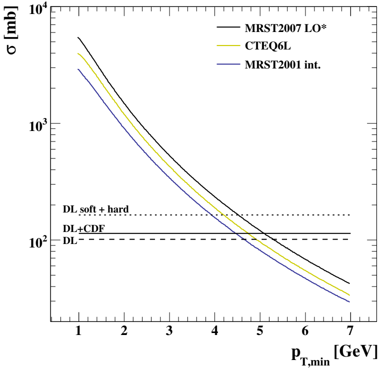
$$\sigma_{\text{tot}} = \sigma_{\text{elastic}} + \sigma_{\text{single diffractive}} + \sigma_{\text{double diffractive}} + \sigma_{\text{non-diffractive}}$$



- underlying event



Modelling the pedestal



Sjöstrand, van Zijl Phys.Rev.D36(1987)2019

- partonic cross sections diverge like $\mathrm{d}p_\perp^2/p_\perp^4$

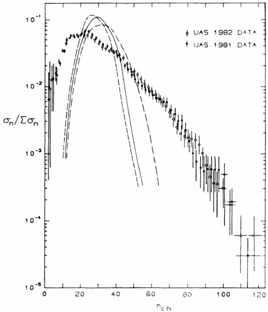
$\Rightarrow$ for small $p_\perp \approx 2-5$ GeV $\sigma_{\text{partonic}} > \sigma_{\text{non-diffractive}}$

- interprete as multiple hard scatters with

$$\langle n \rangle = \frac{\sigma_{\text{partonic}}(p_{\perp,\text{min}})}{\sigma_{\text{non-diffractive}}}$$

- main parameter is $p_{\perp,\text{min}}$, determines multiplicity $\langle n \rangle$

Modelling the pedestal



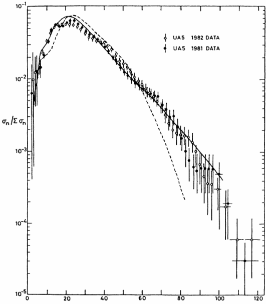
- simple model with

$$\langle n \rangle = \frac{\sigma_{\text{partonic}}}{\sigma_{\text{non-diffractive}}}$$

gives wrong charged multi distribution

- incorporate hadron shape into prediction

$$\Downarrow$$

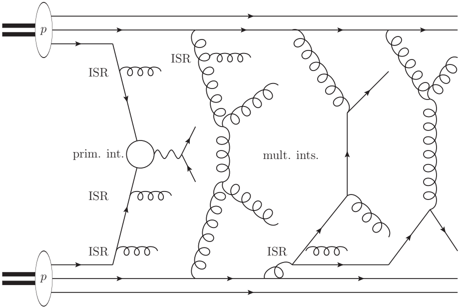


- various shape models to determine hadron-hadron overlap

$$\langle n(b) \rangle = f_c f(b) \frac{\sigma_{\text{partonic}}}{\sigma_{\text{non-diffractive}}}$$

- hardness of the collision determines overlap

Combination with parton showers



- **naïve**: $\langle n(b) \rangle$ independent secondary interactions

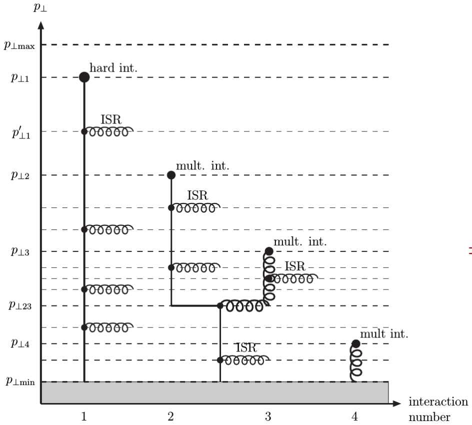
$\Rightarrow$ separation of perturbative picture of hard interaction

– no way to include rescattering

– completely seperate colour and momentum evolution

Combination with parton showers

Sjöstrand, Skands hep-ph/0408302



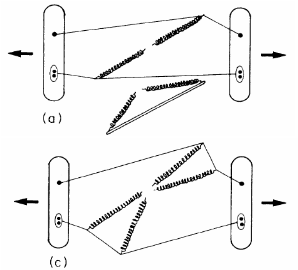
- **improvement**: interleaving
  - $\mathcal{P}_{\mathrm{MPI}}$ evolution kernel, combine w/ pert. IS evol.

$$\mathcal{P} = \mathcal{P}_{\mathrm{ISR}} + \mathcal{P}_{\mathrm{MPI}}$$

⇒ interleaved colour and momentum structure, rescattering effects

– IS evolution not completely perturbative anymore

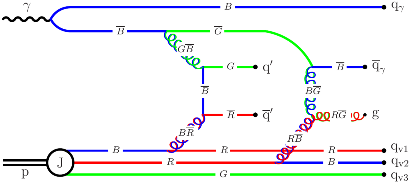
Colour connections and beam remnants



- embed scatters into existing topologies
- three options:
  - at random
  - rapidity ordered
  - minimal string length

Sjöstrand, Skands hep-ph/0402078

- secondary scatterings need to be colour-connected to something
- simplest model would decouple them from proton remnants
- next-to-simplest model would put all scatters on one colour string



A model for minimum bias collisions

Butterworth, Forshaw, Seymour hep-ph/9601371

Borozan, Seymour hep-ph/0207283

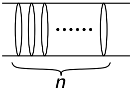
- Assume parton distribution within proton is
$$\frac{\mathrm{d}n_a(x,\mathbf{b})}{\mathrm{d}^2\mathbf{b}\,\mathrm{d}x} = f_a(x)\,G(\mathbf{b})$$
- Use electromagnetic form factor
$$G(\mathbf{b}) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\,\frac{\exp(\mathbf{k}\cdot\mathbf{b})}{(1+\mathbf{k}^2/\mu^2)^2}$$
- EM measurements indicate $\mu = 0.71$ GeV, but left as free model parameter
- continue model below $p_{\perp,\mathrm{min}}$ with same b-space parametrisation, but cross section as Gaussian in $p_\perp$ $\rightarrow$ inclusive non-diffractive events

Minimum bias as multiple pomeron scatterings

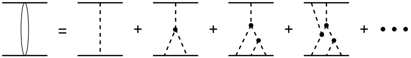
exploits optical theorem, eikonal ansatz:

$$A_{\mathrm{el}}(s, b) = i\left(1 - e^{-\Omega(s,b)/2}\right)$$

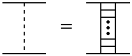
$$= i\sum_{n=1}^{\infty}$$



Khoze-Martin-Ryskin model:

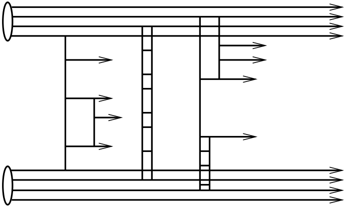


where



'gluon' ladder with effective vertices and propagators

- cut KMR diagrams to obtain differential total cross section



- allow for parton showering of final state legs
- hadronise