Einführung in das Standardmodell der Teilchenphysik

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Sheet 12

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Problem 1: Adjoint Higgs mechanism **[9 Points]**

Let us discuss the Higgs mechanism with a real, scalar field in the adjoint representation of the gauge group $\Phi = \phi^a T^a$. In particular let us regard a SU(3) gauge theory coupled to an adjoint Higgs field

$$\mathcal{L} = -\frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{2}(D_\mu \phi^a)(D^\mu \phi^a) - V(\Phi)\,,$$

with the non-Abelian gauge field-strength tensor $G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu$ and the covariant derivative $D_\mu \phi^a = \partial_\mu \phi^a - g f^{abc} W^b_\mu \phi^c$. We further choose $V(\Phi)$ to be the most general super-renormalizable 'λ^8-like' potential

$$V(\Phi) = \frac{1}{2}\kappa(\phi^a\phi^a - v^2)^2 + \frac{1}{2}\xi(v\phi^a + \sqrt{3}d^{abc}\phi^b\phi^c)^2\,.$$

'λ^8-like' just means that the potential is minimized by $\Phi = 2vT^8 = v\lambda^8$, where the Gell-Mann matrices read:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \ \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \ \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

(a) (3 Points) Which symmetry group remains unbroken by $\Phi = v\lambda^8$ and why? *Hint: Which group is generated by T^1, T^2, T^3?*

(b) (1 Point) Neglecting the gauge fields, how many Goldstone bosons arise due to the spontaneous symmetry breaking in the scalar part of the Lagranian?

Since the Higgs field is coupled to the SU(3) gauge field there will not be any Goldstone bosons. Instead the Higgs mechanism will give masses to some of the gauge bosons. Use the Higgs field in unitary gauge

$$\phi = (\chi_1, \chi_2, \chi_3, 0, 0, 0, 0, v + \varphi)\,,$$

to determine which gauge bosons aquire a mass term (the χ's and φ are the quantum fluctuations not connected to Goldstone modes).

(c) (5 Points) Determine the masses of the gauge bosons.

Problem 2: Standard Model Higgs mechanism [6 Points]

The electro-weak sector of the Standard Model is described by the Glashow-Weinberg-Salam (GWS) model. The gauge group of the GWS model is $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. The Langrangian (without fermions) is given by

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}G^{a\mu\nu}G^{a}_{\mu\nu} + \frac{1}{2}(D_\mu\phi)^\dagger(D^\mu\phi) + V(\phi) \; .$$

$F^{\mu\nu}$ is the Abelian field-strength tensor and ϕ is now a complex scalar field in the fundamental representation of the gauge group

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \; ,$$

with hypercharge $Y = +\frac{1}{2}$.

(a) (2 Points) Show that the charges of the Higgs components are given by $+1$ and 0.
Hint: The electric charge operator is given by $Q = T^3 + Y$.

The covariant derivative is given by

$$D_\mu\phi = \left(\partial_\mu + igT^iW^i_\mu + i\frac{g'}{2}B_\mu\right)\phi,$$

and the Higgs potential is

$$V(\phi) = -\frac{m^2}{2}\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \; .$$

The scalar field develops a nonzero VEV $\phi^\dagger\phi$ for $m^2 > 0$. Due to the symmetry of $V(\phi)$ there is an infinite number of degenerate states with minimum energy satisfying $\phi^\dagger\phi = v^2/2$. Since the potential depends only on the combination $\phi^\dagger\phi$, we arbitrarily choose

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \; .$$

Recall from earlier considerations that in the unitary gauge the spectrum is obvious and there are no Goldstone bosons, but only the physical Higgs. For convenience, we thus write the scalar doublet in the unitary gauge as follows:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \; . \tag{1}$$

(b) (4) Calculate all the masses of the gauge bosons, thus show that three of the four gauge bosons aquire a mass through the Higgs mechanism.

Problem 3: Elitzur's theorem and the Higgs mechanism [5 Points]

Most textbooks and lectures about the Standard Model describe the Higgs mechanism as spontaneous symmetry breaking of the electroweak gauge group. We also found massive gauge bosons after applying Higgs mechanism, apparently violating gauge invariance. However there is a very general theorem by Elitzur that states:

A local gauge symmetry can not be spontaneously broken.

Therefore the Higgs mechanism can not break gauge symmetry spontaneously. Still, everything predicted by Higgs mechanism has been observed experimentally. So, what is happening here? The answer is, that Elitzur's theorem is correct and all the calculations above are actually ill-defined. Yet, we have also been very lucky: Frhlich, Morchio and Strocchi were able to show that there is a well-defined way to construct the Glashow-Weinberg-Salam model and to have the Higgs

mechanism without violating Elitzur's theorem. And even better, if you repeat all calculations with the Frhlich-mechanism we recover exactly the same results as with the naive and ill-defined way of treating the Higgs mechanism.
Identify the step in our calculations where we did something ill-defined!
Hint: Think about gauge invariance.