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Reliability of associative recall based on data manipulations in phase encoded volume holographic storage systems

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Abstract
We investigate the characteristics of correlation signals accomplished by content addressing in a phase encoded volume holographic storage system under different realistic conditions. In particular, we explore two crucial cases with respect to the structure of the data. The first one deals with a scenario where only partial or defective data are available for content addressing. The second case takes similarities among the stored data sets into account, which significantly differ from their statistical correlation. For both the cases we provide, for the first time, a theoretical approach and present experimental results when employing phase-code multiplexing. Finally, we discuss the reliability of the employed methods.

Keywords: holographic data storage, optical correlators, phase-code multiplexing, associative search

Introduction

In the past volume holographic data storage has demonstrated its outstanding potential to become the basis of next generation storage devices [1]. Particularly storage density and data readout speed are exceptional [1, 2]. These features are achieved by the page-oriented storage principle and the superposition of many holograms in one location of the storage material. The latter is accomplished by exploiting the Bragg condition [3]. Hence, the basic methods for superimposing holograms are angular and wavelength multiplexing (e.g. [4–6]). Other superimposing techniques are created by combination or variation of these basic methods and spatial multiplexing (e.g. [7, 8]). Among the different methods phase-code multiplexing [9], which is a variant of angular multiplexing, offers several advantages [10]. Most notably it avoids moving components in the set-up and provides a two orders of magnitude higher signal-to-noise ratio [11, 12]. Additionally it offers the opportunities to perform optical arithmetic operations during readout [13, 14] and to easily implement a very powerful address based data encryption method [15–17]. However, the steadily enormous growing storage demands also necessitate effective and fast database search routines, especially in high capacity systems. Volume holographic storage systems allow content addressing, i.e. associative recall, which can be exploited in order to simultaneously search data by optical correlation. This has been demonstrated in systems based on angular or phase-code multiplexing [18–21].

The implementation of associative data search in phase encoded systems is not straightforward, since the addresses of particular data pages are only comprised in the phase pattern of the corresponding reference wave. Therefore, in order to realize associative data search one usually would have to measure the particular phase pattern of reconstructed reference waves. However, it is possible to transform this information into characteristic intensity patterns by intentionally distorting the composition of the correlation peaks and hence avoiding the usually required complicated phase measurements [21].

The main objective in this paper is to investigate the reliability of the introduced approaches under different realistic conditions. We explore the behaviour of associative recall for two crucial cases. The first one is that erroneous data sets are used for addressing (section 3.1). In the second case data sets are recorded that possess different degrees of similarity, which significantly differ from the typical statistical correlation (section 3.2). For both cases a detailed theoretical description and corresponding experiments are presented. At the end of each section we thoroughly discuss the reliability of associative recall when employing the presented methods. It turns out that under certain conditions an unambiguous interpretation of the intensity pattern, obtained during associative recall using the introduced techniques, cannot be guaranteed at any time.

In the following we briefly recapitulate the principles of phase-code multiplexing (section 1) and explain the ideas of the methods used to transform the phase pattern into an amplitude pattern in order to perform associative recall (section 2). Thereafter we turn to the main subject (section 3), the performance of the introduced approaches under realistic conditions.

phase code	reference beam 1	2	3	4	5	6	7	8
Φ_1:	0	0	0	0	0	0	0	0
Φ_2:	0	π	0	π	0	π	0	π
Φ_3:	0	0	π	π	0	0	π	π
Φ_4:	0	π	π	0	0	π	π	0
Φ_5:	0	0	0	0	π	π	π	π
Φ_6:	0	π	0	π	π	0	π	0
Φ_7:	0	0	π	π	π	π	0	0
Φ_8:	0	π	π	0	π	0	0	π

Figure 1. Binary phase code matrix generated by the recursive Hadamard algorithm, for the case of $N = 8$. Each phase code $\Phi_{1\ldots N}$ contains the appropriate phase shifts for the N reference beams in order to record one data page.

1. Associative recall in phase encoded systems

In order to record data a signal beam S containing the data to be stored and a coherent reference beam R interfere in the storage material. The data are imprinted onto the signal beam as a 2D amplitude pattern, the data page. The reference wave consists of N separated reference beams, whose phases can be individually adjusted by use of a phase-only modulator. These reference beams possess a discrete angular spectrum and hence obey the Bragg condition. In contrast to angular multiplexing, all reference beams interfere simultaneously with the signal wave. In order to store several data pages in one location the phases of the reference beams are specifically changed between subsequent recordings. The set of phase shifts corresponding to recording a particular data page is called its phase code or address in the storage material. In the reconstruction process, the material is illuminated with the N reference beams while the appropriate phase code is readjusted. In this step actually all stored data pages are reconstructed, since they have been recorded by the same reference beams. However, due to the use of orthogonal phase codes, which accomplished binary phase shifts of 0 or π during the storage process, all parts of unaddressed pages interfere destructively. In this manner data pages can be recorded and reconstructed independently with minimal noise [11, 12]. Appropriate orthogonal phase codes can for example be constructed by use of standard, Paley or Williamson type Hadamard matrices [22]. If not explicitly stated standard Hadamard matrices (whose orders are powers of 2) have been used throughout the investigations and experiments presented in this article. An example of phase codes corresponding to the Hadamard matrix for $N = 8$ is given in figure 1.

In order to perform associative recall the storage material is addressed with any data page D' displayed on the amplitude modulator. This page correlates with all stored data pages in Fourier space. Applying the convolution theorem and performing a Fourier transform (see below), the amplitudes of the correlation peaks in a phase encoded system are given by [21]

$$\mathcal{R} \propto \sum_{i=1}^{N}\sum_{j=1}^{N} \left(\delta(x_j) \otimes \left[e^{i\Phi_{ij}} \cdot (D_i * D')\right]\right), \qquad (1)$$

where the indices i and j indicate the data page number and the reference beam number, respectively. N denotes the total number of reference beams, which equals the number of data pages that can be multiplexed in one location by orthogonal phase-code multiplexing. $e^{\Phi_{i,(1\ldots N)}}$ represents the phase code used to record data page D_i and the $\delta(x_j)$ correspond to the Fourier transformed plane reference beams. Due to this Fourier transform, which is performed by the lens in figure 2(b), the convolution with the reference beams has no effect. Hence, a pure correlation signal, consisting of N spatially separated peaks at the positions $x_{1\ldots N}$ on the detector, is obtained, as indicated in the magnified cut-out in figure 2.

1.1. Experimental specifications

All experimental results are obtained in a typical 90° set-up, as depicted in figure 2. The beam of a frequency doubled Nd:YAG laser ($\lambda = 532$ nm, cw) is split into a reference and a signal beam. In the signal arm a transmissive twisted nematic liquid crystal display (TN LCD) with a resolution of 800×600 pixels is used to imprint data onto the beam. In the reference arm a 1D TN LCD with a resolution of 128×1 is used to represent the binary storage addresses (the relative phase shifts of each beam are performed with an accuracy better than $\pm 0.5\%$). That is, in this case up to 128 data pages can be overlaid in one location by use of orthogonal phase-code multiplexing. In the specific arrangement used to perform the experiments presented here, an expanded collimated laser beam is incident on the phase modulator. After passing the modulator the beam is focused to overlap with the signal beam inside the storage material, which is an iron doped $LiNbO_3$ crystal. Due to this simple arrangement, unmodulated light, which passes through the inter-pixel spacing of the phase modulator, will always be incident on the storage material. Since it is not modulated, this light gives rise to increased system noise. Additionally, light gets diffracted at the phase modulator and overlays the actual reference wave. It is another shortcoming that the employed type of TN LCD used to modulate the phase distribution of the reference wave always slightly modulates the amplitudes of the reference beams. In the realized configuration the maximal amplitude variation is $\leqslant \pm 5\%$. Eventually, these effects reduce the contrast of the correlation peaks obtained during associative readout (cf sections 2.1, 3.1 and 3.2). The data pages used to obtain the presented results (simulations and experiments) are generated by randomly distributing a constant number of ON bits over each page. Therefore,

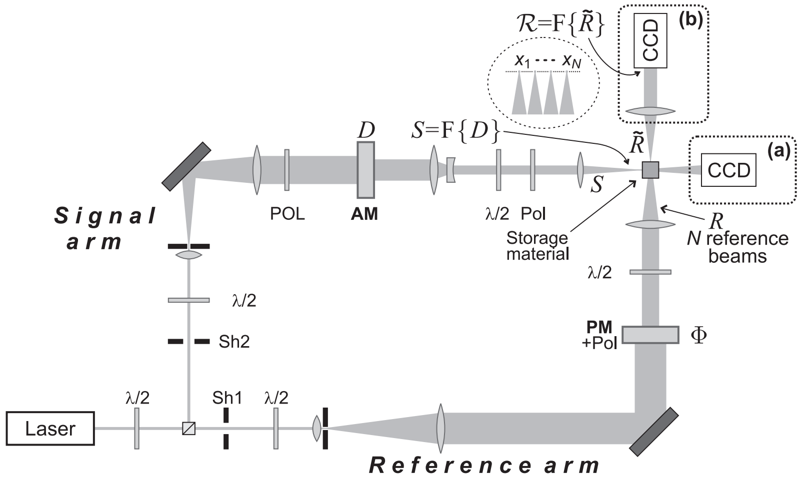

Figure 2. Set-up in the 90° geometry (AM = amplitude modulator; PM = phase modulator; Sh1, Sh2 = shutter; Pol = polarizer). (a) Camera used for data retrieval (shutter 2 is closed). (b) Camera used to detect correlation signals during associative data search (shutter 1 is closed).

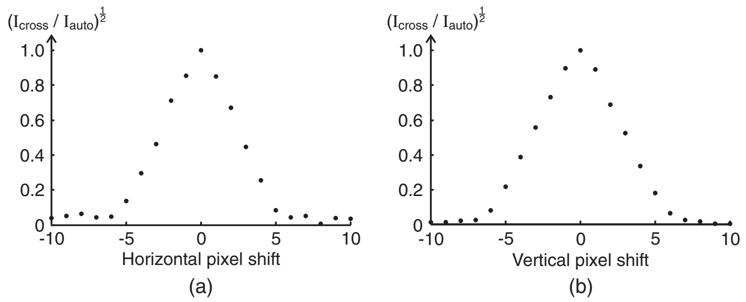

Figure 3. The shift invariance, which is typically present in optical correlators, is controlled by moving the recording plane away from the Fourier plane. In this experiment bits are (a) horizontally and (b) vertically shifted, whose oversampling has been adjusted to 6 × 6 pixels.

all data pages possess the same sparseness h, which is defined as the quotient of the number of ON bits by the total number of bits per page. Holograms of equal diffraction efficiency are subsequently recorded using an incremental recording schedule [23]. During the reconstruction process CCD cameras are used to detect either the reconstructed data pages (figure 2(a)) or the resulting correlation signals when performing content addressing (figure 2(b)). By use of oversampling the set-up was adjusted in a way that reconstructed data pages typically bear bit error rates of $\leqslant 10^{-3}$ without any further data encoding scheme.

It is the goal of associative recall to generate correlation signals for each ON bit in the addressing page that meets an ON bit at the same position in any previously recorded data page. However, the correlation signals may be distorted due to the shift invariance phenomenon known from typical van der Lugt correlators [24]. In volume holography horizontal shift-invariance practically does not occur due to the sharpness of the Bragg condition. On the other hand, vertical shift invariance is typically present and may yield unintended cross-correlation signals [20]. This means that an ON bit in any row of the addressing page also produces a correlation signal if data pages have been recorded that possess ON bits in the same column but in a neighbouring row. In order to avoid misleading correlation signals it is appropriate not to record exactly the Fourier transforms of data pages but to record in the Fresnel region [25]. That is, in the experiment as sketched in figure 2, the signal wave is focused to a point a few millimetres in front of the storage material. In this manner it is achieved that any ON bit in the addressing data page only produces a correlation signal if data pages have been recorded with ON bits at the same position. This was experimentally verified, as shown in figure 3. The correlation signal was detected while the ON bit in the addressing page was shifted horizontally (figure 3(a)) and vertically (figure 3(b)). If the addressing bit is shifted one bit length in any direction, no correlation signal can be detected any more. Hence, in this case the correlation of two data pages is equal to their congruence with regard to the ON bits. All discussions in this paper are based on this pre-condition. Additionally, using the appropriate data encoding scheme, this arrangement facilitates 'fuzzy searching' as discussed in [19].

2. Appearance of correlation signals in a phase encoded system

When performing content addressing in a phase encoded storage system, the intensity distribution of the correlation signal is composed by a phase sensitive addition of one auto- and several cross-correlation signals. If the ON bits are statistically distributed over each data page, the peak intensities corresponding to the cross-correlation of the input page with any of the other pages is constant. Due to the orthogonality of the phase codes the intensity distribution of the compound correlation peaks is identical no matter which page was used for addressing. The requested information is only contained in the relative phases of the correlation peaks. In order to avoid the necessity of phase sensitive detection two easily implementable methods, which transform the phase information into a characteristic intensity distribution, are explained in the following [21].

2.1. Recording a data page with sparseness zero or unity

In order to force the system answer to yield a distinctive intensity pattern, either a page with sparseness unity or zero (i.e. a complete bright or dark page) is recorded among the actual data pages that are stored in the same location. The capacity loss of one storage address, i.e. one data page per location, becomes negligible for large numbers of multiplexed data pages. In the following discussions the correlation peaks are normalized by an ideal auto-correlation signal. Due to this normalization, the constant sparseness h of the data pages and the statistical distribution of the bits, the absolute value of the amplitude of any cross-correlation signal is approximately h if $M \gg (1-h)/h$ (M = number of bits per page) [21]. Additionally, the normalization allows a dimensionless notation of the correlation amplitudes. The actual amplitude of the jth peak is determined by equation (1). The indices b and a denote the addressing data page and the additional page with sparseness unity or zero respectively.

- A page with unit sparseness has been recorded using phase code Φ_a. Two cases have to be distinguished.
  (i) $\Phi_b j = \Phi_{aj}$
  During the storage process the reference beam R_j used to record data page b has had the same relative phase shift as used to record the bright page a. That is, the amplitudes of these two auto-correlation signals interfere constructively. Since in each column (and row), except the first one, of the Hadamard matrix the numbers of 0 and π phase shifts are equal to $N/2$, the number of cross-correlation signals which interfere destructively is clearly $N-4$. The remaining two cross-correlation signals must be of opposite phase than the two reconstructed auto-correlation signals. Hence, the compound signal intensity of the correlation peaks obeying $\Phi_{bj} = \Phi_{aj}$ can be written as
  $$P_{\mathrm{I(br)}} : I_j^{(\Phi_{bj}=\Phi_{aj})} = [2-2h]^2 \qquad (j \neq 1) \qquad (2)$$
  and for the first peak $I_1 = [2+(N-2)h]^2$ due to constructive interference of all auto-and cross-correlation signals.
  (ii) $\Phi_{bj} \neq \Phi_{aj}$
  The two auto-correlation signals interfere destructively and so do all cross-correlation signals. Therefore, the intensity of these peaks is
  $$P_{\mathrm{II(br)}} : I_j^{(\Phi_{bj}\neq\Phi_{aj})} = 0. \qquad (3)$$
  In real systems the actual heights of these peaks depend on the system noise (cf discussions at the end of this section and sections 3.1 and 3.2).
- A page with sparseness zero has been recorded, i.e. the storage address Φ_{aj} has not been used to record any signal wave. Two cases can occur.
  (i) $\Phi_{bj} = \Phi_{aj}$
  One auto-correlation signal is reconstructed. The cross-correlation signals interfere destructively, except for two, which interfere destructively with the auto-correlation signal. Therefore, the intensity of the corresponding correlation peaks can be written as
  $$P_{\mathrm{I(da)}} : I_j^{(\Phi_{bj}=\Phi_{aj})} = [1-2h]^2 \qquad (j \neq 1). \qquad (4)$$
  The intensity of the first peak is given by $I_1 = [1+(N-2)h]^2$.
  (ii) $\Phi_{bj} \neq \Phi_{aj}$
  One auto-correlation signal is reconstructed. All $N-2$ cross-correlation signals interfere destructively, since the numbers of 0 and π phase shifts in each column (row) of the Hadamard matrices are equal ($N/2$). The intensity of the correlation peaks is
  $$P_{\mathrm{II(da)}} : I_j^{(\Phi_{bj}\neq\Phi_{aj})} = |1|\,. \qquad (5)$$

Due to the orthogonality of the phase codes, in both cases the numbers H_N of the peaks that appear with a certain intensity, P_{I} or P_{II}, are given by

$$H_N(P_{\mathrm{I}}) = \frac{N}{2} - 1 \qquad \text{and} \qquad H_N(P_{\mathrm{II}}) = \frac{N}{2}. \qquad (6)$$

Now an interpretation of the total correlation signal and assignment of an addressing page to an actually stored data page is straightforward. The distribution of peaks with an intensity of P_{I} and P_{II} uniquely depends on the input information. For the cases where seven data pages and an additional page with sparseness unity or zero are stored, figure 4 shows a simulation of the subsequently detected correlation signals when addressing with an exact match of one of the recorded pages. In this simulation Φ_b of the addressing page and Φ_a of the page with sparseness unity or zero correspond to the phase codes Φ_3 and Φ_1, cf figure 1, respectively. The ON bits in the seven data pages with $h = 0.4$ are assumed to be statistically distributed. Addressing with one of the other pages recorded with phase code $\Phi_{2,4,\ldots,8}$ yields the same numbers of peaks P_{I} and P_{II}, but with a different unique distribution.

Figure 5 shows the results of associative recall in an experiment in which 31 data pages and a page with unit sparseness have been recorded, employing phase codes generated by the Hadamard algorithm for $N = 32$. The sparseness of the data pages is adjusted to $h = 0.25$ and the ON bits are statistically distributed. The plots show the detected correlation peak intensity distributions when addressing with exact matches of the data pages recorded with the phase codes Φ_{17} (a) and Φ_{23} (b). Though the uniqueness of the distributions is obvious, the difference of the peak heights in each category is conspicuous. However, taking the noise sources as mentioned in section 1.1 into account, i.e. phase errors, amplitude variations of the reference beams (due to the phase modulator and the Gaussian beam profile) and the transmission of unmodulated light, the values and heights of

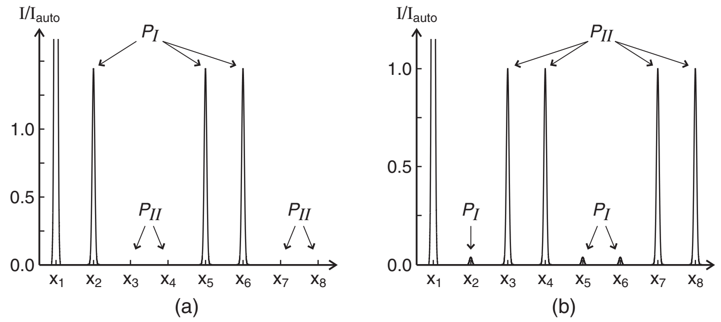

Figure 4. The intensity distribution of the correlation peaks is unique for the addressing page during content addressed readout, if one of the recorded pages possesses a sparseness of unity (a) or zero (b).

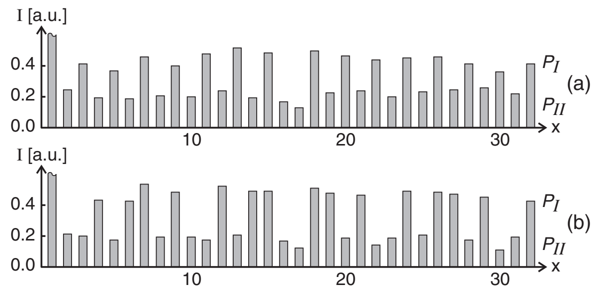

Figure 5. In this experiment 31 data pages ($h = 0.25$) and a page with unit sparseness are recorded. The plots show the detected correlation peak intensity distributions when addressing with exact matches of (a) the 17th or (b) the 23rd data page.

the experimentally obtained peaks are concordant with the simulations within the range of statistical errors.

The two introduced techniques are based on the assumption that basically two categories of correlation peak intensities occur. It is the goal of the methods to assign an input information to the best matching page among the recorded pages, just by simple classification of the detected correlation peaks into two categories and relating their distribution to the employed phase codes. Hence, these methods do not inherently yield information about the cross-correlation of the input page and the other previously recorded data pages. In comparison to systems based on angular multiplexing, such an approach is equivalent to only considering the position of the correlation peak that possesses the highest intensity, which should indicate the address of the best matching data page.

3. Content addressing in phase encoded systems under realistic conditions

So far it has been assumed that the ON bits are statistically distributed over data pages of equal sparseness and that associative recall is performed by addressing with an exact facsimile of a recorded page. In a real system such situations are rather unlikely. Typically, the correlation between data pages is not given *a priori*. Furthermore, the correlation of actual user information is usually not equal to the correlation of the data pages, unless special data encoding schemes, like fuzzy data encoding [20], are used to display the data. Additionally, it is more realistic to assume that, due to bit errors or incomplete advance information input by the user, only a certain fraction of a whole page of information can be provided. However, in any case, when performing content addressing it is most desirable to find the page that possesses the highest correlation to the input information. In this section these two crucial cases, that are most relevant in real systems, will be studied. It turns out that in both cases the contrast of the correlation peaks to be detected reduces if the defectiveness of the input data or the congruence of recorded data pages is increased.

3.1. Addressing with defective data pages

In the following investigation associative recall is performed by addressing with a data page that matches one of the recorded pages, index b, just to a certain degree k, with $k \in [0 \ldots 1]$. Additionally, the cross-correlation of the input page and any other recorded data page is controlled to be always equal to h. Therefore, the amplitudes of an auto- or a cross-correlation signal are given by $\pm k$ or $\pm h$ respectively, if the peaks are normalized by the absolute value of a pure auto-correlation signal. Hence, the relative intensities of the correlation peaks can be computed for the two methods.

- If a page with unit sparseness has been recorded by use of phase code Φ_a, two cases can be distinguished.
  (i) $\Phi_{bj} = \Phi_{aj}$
  The auto-correlation signal and the signal with amplitude k, produced by the correlation of the addressing page and the page that has been recorded with Φ_{bj}, interfere constructively. Due to the orthogonality of the phase codes additionally two cross-correlation amplitudes of opposite sign are generated. Therefore, the intensities of these peaks can be computed to be

$$P_{\mathrm{I(err,br)}} : I_j^{(\Phi_{bj}=\Phi_{aj})} = [1 + k - 2h]^2 \qquad (j \neq 1) \quad (7)$$

  and for the first peak $I_1 = [1 + k + (N - 2)\,h]^2$.
  (ii) $\Phi_{bj} \neq \Phi_{aj}$
  The auto-correlation signals and the signal with amplitude k interfere destructively and so do all cross-correlation signals. Therefore, the intensity of these peaks is given by

$$P_{\mathrm{II(err,br)}} : I_1^{(\Phi_{bj}\neq\Phi_{aj})} = [1 - k]^2 \,. \quad (8)$$

  Theoretically, the detected intensity of these peaks is zero if $k = 1$, i.e. the addressing page is an exact match of a stored data page.

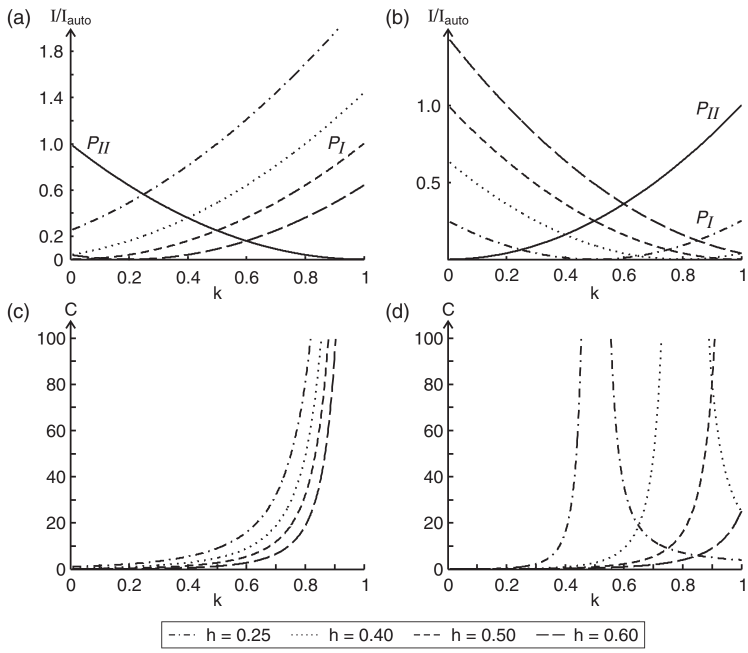

Figure 6. Addressing with defective data pages: (a) recording a page with unit sparseness and (b) skipping one phase code show the characteristics of the peak intensities when addressing with a data page that matches one stored page only to a certain degree k. The resulting contrast functions, (c) and (d) respectively, determine the unambiguousness of the identification of the addressing page.

- A page with sparseness zero has been recorded, i.e. the storage address Φ_{aj} has not been used to record any signal wave. Two cases can occur.
  (i) $\Phi_{bj} = \Phi_{aj}$
  One auto-correlation signal is reconstructed. The cross-correlation signals interfere destructively, except for two, which interfere destructively with the auto-correlation signal. Therefore, the intensity of the corresponding correlation peaks is given by

$$P_{\mathrm{I(err,da)}} : I_j^{(\Phi_{bj}=\Phi aj)} = [k - 2h]^2 \qquad (j \neq 1). \qquad (9)$$

  (ii) $\Phi_{bj} \neq \Phi_{aj}$
  One auto-correlation signal is reconstructed. All $N-2$ cross-correlation signals interfere destructively, since the numbers of 0 and π phase shifts in each column (row) (except the first ones) of the Hadamard matrices are equal ($N/2$). The intensity of the correlation peaks is

$$P_{\mathrm{II(err,da)}} : I_j^{(\Phi_{bj}\neq\Phi_{aj})} = [k]^2 . \qquad (10)$$

The characteristics of the correlation peaks and the corresponding contrasts C in dependence on the matching degree k are shown in figure 6 for different sparseness values h. The contrast C is defined by the ratio of the two peak heights P_{I} and P_{II}. Since their roles are exchanged in the two methods C is either given by $P_{\mathrm{II}}/P_{\mathrm{I}}$, when recording a page with unit sparseness among the data pages, or by $P_{\mathrm{I}}/P_{\mathrm{II}}$, when skipping one phase code. As expected the curves in figures 6(a) and (b) follow a quadratic behaviour. In both methods, i.e. recording a page with sparseness unity or zero among the data pages, the slope of the P_{I} peaks depend on k and the sparseness h of the data pages, whereas the characteristics of the P_{II} peaks do not change for different values of h. If a bright page has been recorded, it is obvious that a low sparseness yields a high contrast of the peaks that have to be classified. If one phase code is skipped during the recording process, it turns out that only for $h = 0.5$ the contrast grows continuously from zero to infinity for $k = 0 \to 1$. If $h < 0.5$, the contrast function grows between 0 and $2h$ from 0 to ∞ and drops down again between $k = 2h \to 1$. If $h > 0.5$ the contrast grows continuously with k and arrives at a finite value for $k = 1$. For $h \neq 1$ and $k = 1$, the contrast only reaches values greater than 100 (which is assumed to be trustworthy) if $h \in [0.45 \ldots 0.55]$.

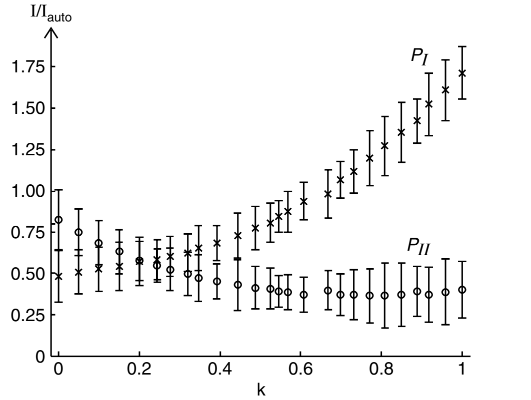

Figure 7. Experimental study of the peak intensity characteristics when addressing with a defective data page, whose correlation degree with one of the recorded pages is equal to k.

Figure 7 shows an experimental result when addressing with a not exactly matching data page. In this experiment seven data pages ($h = 0.25$) and a page with unit sparseness are recorded in the storage material. Content addressed readout is performed using a page whose correlation degree with one of the recorded pages is equal to k. The cross-correlation with all other pages is controlled to be equal to h (ON bits are statistically distributed). Though the qualitative accordance with the simulations is eminent, it is obvious that the minima of the experimental curves at $k = 0$ or 1 do not reach the simulated values. This discrepancy is due to the fact that the simulations illustrate idealistic conditions, omitting any losses or noise sources. As already indicated in section 1.1, it turns out that this error is mainly caused by unmodulated light passing the phase modulator, amplitude variations in the reference beam and inexact phases. When incorporating unmodulated light with an intensity of 3% of the total reference wave intensity, amplitude variations in the reference wave of up to ±5% and maximal phase errors of $\leqslant \pm 0.5\%$, the simulation agrees with the experimental data within the range of statistical errors.

In comparison to the described cases the peak contrast characteristics when employing angular multiplexing are different. If erroneous data are used for content addressing, the contrast is given by $(k/h)^2$. Qualitatively, the contrast characteristic is similar to the case of phase-code multiplexing, when recording an additional page with unit sparseness. However, the contrast functions of these two multiplexing techniques intersect at $k = h$. For $k = h \to 1$ the contrast goes from unity to infinity for phase-code multiplexing. In the case of angular multiplexing it stays finite. In particular, its maximum at $k = 1$ will not reach values greater than 100 as long as $h > 0.1$. Appropriate digital data pages with a sparseness below 0.1 could be easily generated, e.g. by use of modulation coding, but at the cost of poor code rates [17].

3.2. Non-statistical correlation between data pages

The objective in the following analysis is to control the ON bit distribution of one data page, recorded with phase code Φ_c, in such a way that its cross-correlation with the addressing page, recorded with phase code Φ_b, equals k. Since the ON bits in all other pages are statistically distributed, the cross-correlation of the input page with one of these can be assumed to be equal to their sparseness h. The special case $k = h$ yields the same description as discussed in section 2.1.

- If a page with unit sparseness has been recorded using phase code Φ_a, three different peak intensities can occur (when omitting the first peak).
  (i) $\Phi_{bj} = \Phi_{aj} = \Phi_{cj}$
  The two auto-correlation signals and the cross-correlation signal k with the second page interfere constructively. The remaining cross-correlation signals interfere destructively except for three, which possess opposite sign to the auto-correlation signals. Therefore the compound intensity of these correlation peaks is given by

$$P_{\mathrm{I,1(corr,br)}} : I_j^{(\Phi_{bj}=\Phi_{aj}=\Phi_{cj})} = [2 + k - 3h]^2 \qquad (j \neq 1) \tag{11}$$

  and for the first peak $I_1 = [2 + k + (N - 3)\,h]^2$.
  (ii) $\Phi_{bj} = \Phi_{aj} \neq \Phi_{cj}$
  The two auto-correlation signals interfere constructively. All cross-correlation signals interfere destructively except for one, which possesses the opposite sign of the auto-correlation amplitudes and the cross-correlation signal generated with the second page. Therefore, the intensity of these peaks is

$$P_{\mathrm{I,2(corr,br)}} : I_1^{(\Phi_{cj}\neq\Phi_{bj}=\Phi_{aj})} = [2 - k - h]^2 \,. \tag{12}$$

  (ii) $\Phi_{bj} \neq \Phi_{aj}$
  The two cross-correlation signals interfere destructively. All cross-correlation signals interfere destructively except for one, which is of opposite sign to the cross-correlation signal due to the second page. Hence the intensity of the corresponding peaks is given by

$$P_{\mathrm{II(corr,br)}} : I_j^{(\Phi_{bj}=\Phi_{aj})} = [k - h]^2 \,. \tag{13}$$

  If $k = h$ these peaks have zero intensity as discussed in section 2.1.
- The storage address Φ_{aj} has not been used to record any data page. In this case three different peak intensities, which are relevant for classification, also occur.
  (i) $\Phi_{bj} = \Phi_{cj} = \Phi_{aj}$
  An auto-correlation signal and the cross-correlation signal with amplitude $|k|$ of the same sign are reconstructed. $N-6$ cross-correlation signals interfere pairwise destructively and the three remaining cross-correlation signals interfere destructively with the auto-correlation signal. Therefore, the resulting correlation peak intensity can be written as

$$P_{\mathrm{I,1(corr,da)}} : I_j^{(\Phi_{bj}=\Phi_{cj}=\Phi_{aj})} = [1 + k - 3h]^2 \tag{14}$$

  and for the first peak $I_1 = [1 + k + (N - 2)h]^2$.
  (ii) $\Phi_{bj} = \Phi_{cj} \neq \Phi_{aj}$
  One auto-correlation signal and the cross-correlation signal with amplitude $|k|$ and the same sign is reconstructed. $N - 4$ cross-correlation signals interfere pairwise destructively and the one remaining cross-correlation signal interferes destructively with the auto-correlation signal. Therefore, the intensities of these correlation peaks are given by

$$P_{\mathrm{II,1(corr,da)}} : I_j^{(\Phi_{bj}=\Phi_{cj}\neq\Phi_{aj})} = [1 + k - h]^2 \,. \tag{15}$$

  (iii) $\Phi_{bj} \neq \Phi_{cj}$
  The cross-correlation signal k interferes destructively with the auto-correlation signal. All other cross-correlation signals interfere destructively, except for one. Depending on the sign of the skipped phase code Φ_{aj} the amplitude of this remaining cross-correlation signal has to be added or subtracted. Therefore two cases can occur:

$$P_{\mathrm{II,2(corr,da)}} : I_j^{(\Phi_{bj}\neq\Phi_{cj}=\Phi_{aj})} = [1 - k + h]^2 \qquad (j \neq 1) \tag{16}$$

  or

$$P_{\mathrm{I,2(corr,da)}} : I_j^{(\Phi_{bj}\neq\Phi_{cj}\neq\Phi_{aj})} = [1 - k - h]^2 \qquad (j \neq 1). \tag{17}$$

For the two methods the graphs in figure 8 illustrate the dependence of the peak intensities of k for different sparseness values. In figure 9 this dependence was experimentally verified when recording seven data pages ($h = 0.25$) and a page with unit sparseness. In all graphs the correlation of the page recorded with Φ_b (addressing page) and that recorded with

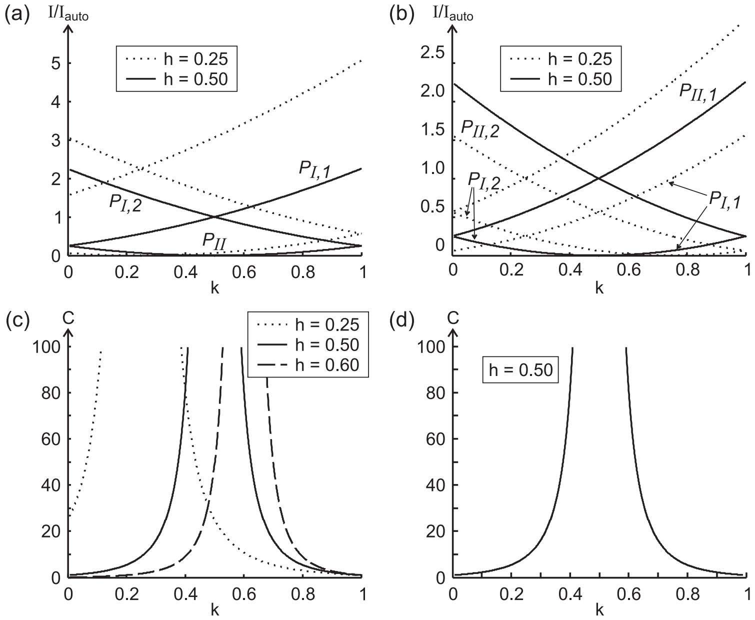

Figure 8. Non-statistical correlation between two data pages: (a) recording a page with unit sparseness and (b) skipping one phase code show the characteristics of the correlation peak intensities for different values of the cross-correlation k of two recorded data pages, when addressing with an exact match of one of them. (c) and (d) illustrate the resulting contrast values, respectively.

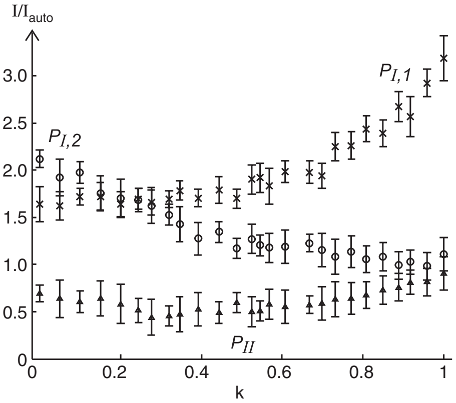

Figure 9. Experimental study of the correlation signals if content addressing is performed with an exact facsimile of one of the recorded pages, whose correlation with one of the other recorded pages is controlled to be equal to k.

Φ_c is controlled to vary from $k = 0$ to 1. At the same time the cross-correlation of the addressing page and any other page is kept constant, $k_{db} = k_{eb} = k_{fb} = \cdots = h$. However, this means that the cross-correlation values of these pages, $k_{dc}, k_{eb}, \ldots, k_{ed}, \ldots$, may vary. Since this effect is of no relevance for the present analysis, it is not taken into further consideration. The difference of the actual peak heights between the experiment and the presented simulations is again caused by the main noise sources identified in section 1.1. Allowing for the same errors as discussed in section 3.1, the simulated and the experimental curves agree within the range of statistical errors.

In order to be able to unambiguously identify the addressing page the two peak heights P_{I} and P_{II} need to be distinguished. That is, the peaks P_{I} need to be detected with greater intensity than the peaks P_{II} when recording a page with unit sparseness among the data pages, or vice versa when skipping one phase code. In the case when a page with unit sparseness has been recorded along with the actual data pages, theoretically this condition is guaranteed for all values of k. If $k = 1$ the peaks $P_{\mathrm{I,2(corr,br)}}$ and $P_{\mathrm{II,1(corr,br)}}$ possess equal intensities, indicating that the addressing page has been recorded with phase code Φ_b as well as with code Φ_c. The contrast of the relevant peaks that have to be classified is plotted in figure 8(c). Starting at $k = h$ the contrast of these peaks drops continuously from infinity down to a finite value (minimum unity) for $k \to 1$ and $k \to 0$. Ideally the contrast should drop from infinity (or at least values e.g. >100) at $k = 0$ to unity at $k = 1$. In angular multiplexed systems the corresponding peak contrasts show almost such a behaviour. In these systems the contrast is given by $1/k^2$, $k \in [h, 1]$, and $1/h^2$, $k \in [0, h]$ (auto-correlation signal divided by the value of a cross-correlation signal). Hence, the contrast first stays constant and then drops continuously from $1/h^2$ to 1 for $k = h \to 1$. In phase encoded systems, in which a page with unit sparseness has been recorded, the graphs suggest similar idealistic characteristics. For $k \in [h_c, h]$ the contrast will always yield values greater than 100, if h_c denotes the solution of $P_{\mathrm{I,1(corr,br)}}/P_{\mathrm{II(corr,br)}} > 100$. If $h < \frac{2}{9}$ this is guaranteed *a priori* for $k < h$.

If one phase code has been skipped during the recording process the situation is more complicated. For $h \leqslant 0.5$ and $k \leqslant 2h$ the peaks $P_{\mathrm{I(corr,da)}}$ can be unambiguously distinguished from the peaks $P_{\mathrm{II(corr,da)}}$. However, if $k > 2h$ the intensity of the peaks $P_{\mathrm{I,1(corr,da)}}$ becomes smaller than the intensity of the peaks $P_{\mathrm{II,1(corr,da)}}$ and the contrast is less than unity. That is, an unambiguous assignment of the addressing page to one of the recorded pages is not possible any more. The characteristics for $h > 0.5$ are the same as for $h < 0.5$, except that the graphs are mirrored at $k = 0.5$ ($I(k, h > 0.5) =$

$|I(k, h < 0.5) - 1|$). Hence, only a sparseness of $h = 0.5$ allows an unambiguous assignment of the input page to the appropriate recorded page just by the separation of the peaks into two intensity categories. However, in this case the contrast function, as depicted in figure 8(d), drops from infinity at $k = h$ to unity for $k \rightarrow 1$ and $k \rightarrow 0$. This behaviour suggests that skipping one phase code is only a feasible method if the correlation between recorded data pages can be guaranteed to be greater than $k_c = 0.45$. That is, the contrast is greater than 100 if $k \in [k_c, h]$ and shows an ideal behaviour for $k \in [h, 1]$. Bearing in mind the characteristics of this technique in cases when addressing with not exactly matching pages, associative recall realized by skipping phase codes is only reliable for a sparseness of $h = 0.5$.

4. Conclusion

Associative recall in phase encoded volume holographic storage systems can easily be accomplished either by recording a complete bright page along with the actual data pages or by skipping one phase code during the recording process. The goal of these straightforward techniques is to point out the storage address of the data page that best matches the input information. Beyond this the techniques do not inherently yield the cross-correlation values of the input page with any of the other recorded pages. Additionally, the characteristics ascertained here suggest that for a reliable performance the techniques should be constrained to sparseness values of 0.5 or less than 0.22 when skipping one phase code or recording a complete bright page respectively. In comparison to angular multiplexing the latter method suggests a better performance when addressing with defective data pages and higher contrast values if correlated pages are recorded. In order to additionally generate the cross-correlation values of the input information with all of the other recorded pages, the individual intensity values of each correlation peak need to be taken into account, which is beyond the scope of the present investigation. However, the straightforward implementation of the two techniques along with the contrast values attained for certain sparseness values make them worthwhile for accomplishing associative recall in phase encoded volume holographic storage systems.

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