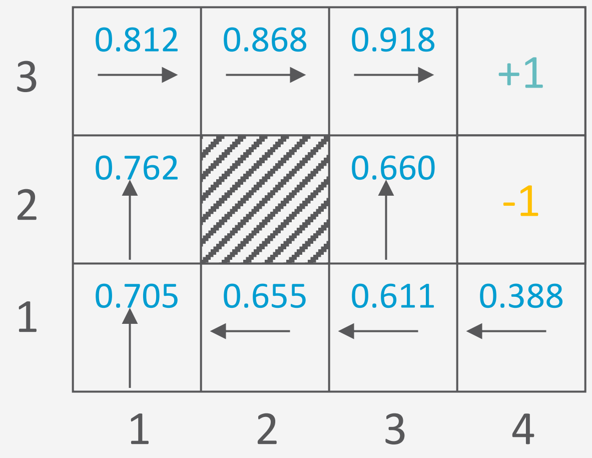
Universität Münster

Automated Planning and Acting

Decision Making: Foundations



living.knowledge

Tanya Braun
Research Group Data Science, Computer Science Department

Content: Planning and Acting

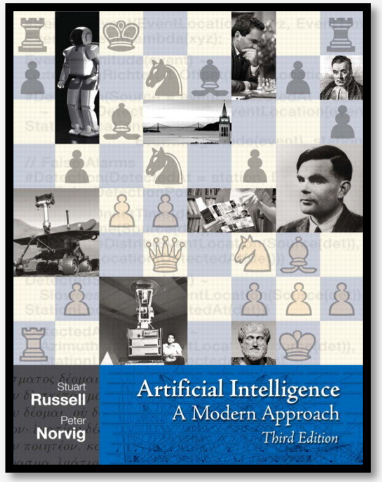
1. With **Deterministic** Models
2. With **Refinement** Methods
3. With **Temporal** Models
4. With **Nondeterministic** Models
5. With **Probabilistic** Models
6. By **Decision Making**
   A. Foundations
      - Utility theory
      - Markov decision processes
      - Reinforcement learning

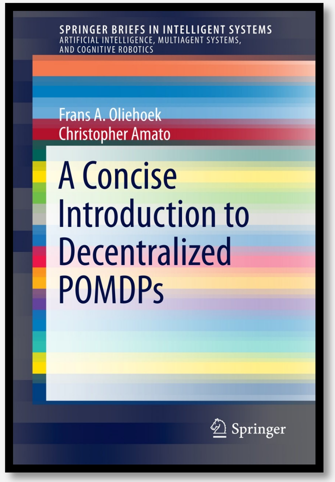
   B. Extensions

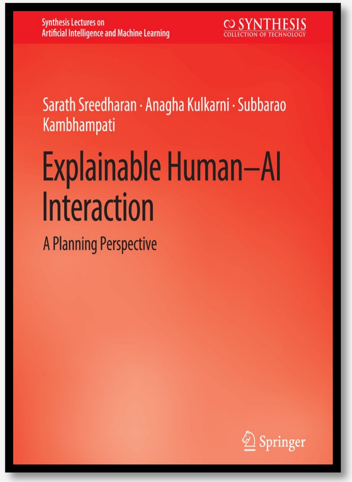
   C. Structure
7. With **Human-awareness**

Literature

- We leave behind the planning book…
- Content based on
  - Artificial Intelligence: A Modern Approach (3rd ed.; abbreviation: AIMA)
    - Stuart Russell, Peter Norvig
    - Decision making (Chs. 16 + 17), reinforcement learning (Ch. 21)
  - A Concise Introduction to Decentralized POMDPs
    - Frans A. Oliehoek, Christopher Amato
  - Explainable Human-AI Interaction: A Planning Perspective
    - Sarath Sreedharan, Anagha Kulkarni, Subbarao Kambhampati
  - Further research papers announced in lectures







- I do not expect you to read all the books!

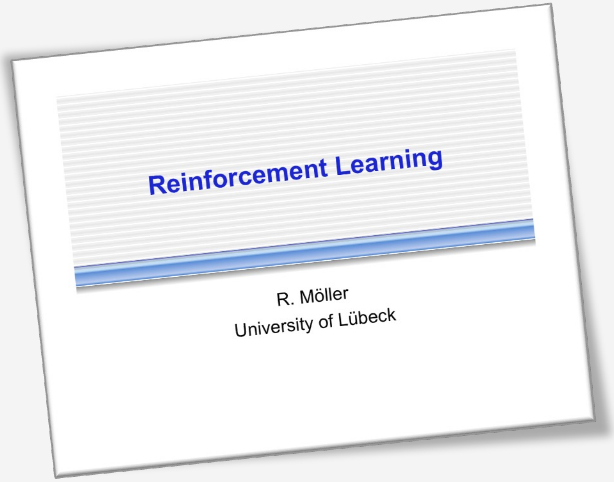
http://aima.cs.berkeley.edu

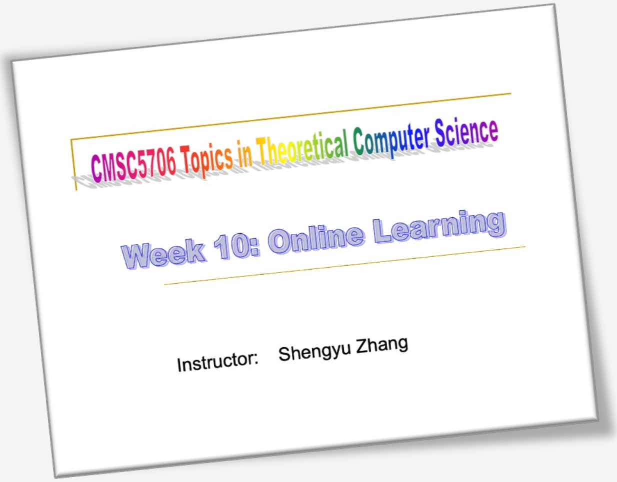
https://link.springer.com/book/10.1007/978-3-319-28929-8

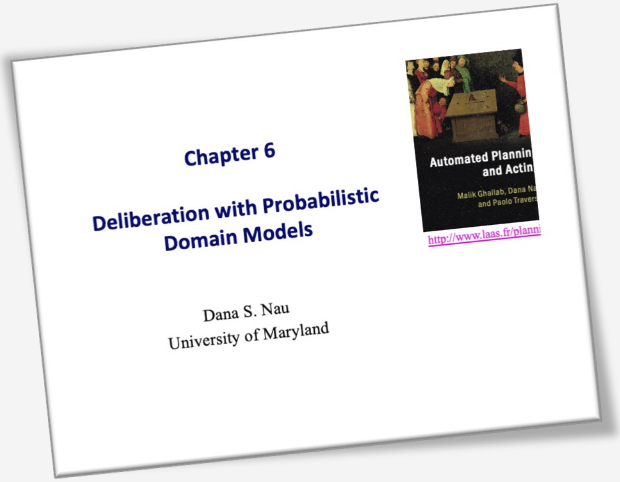
https://link.springer.com/book/10.1007/978-3-031-03767-2

Acknowledgements

- Slides based on material provided by Dana Nau, Ralf Möller, and Shengyu Zhang
  - In part based on *AIMA Book, Chapters 16, 17, 21*







http://people.eecs.berkeley.edu/~russell/talks/2020/russell-aaai20-hntdtwwai-4x3.pptx
http://rbr.cs.umass.edu/camato/decpomdp/overview.html

Decision Making under Uncertainty

- Goal-based: binary distinction between *happy* and *unhappy*
- Utility as a distribution over possible states
  - Essentially an internalisation of a performance measure
    - If internal utility function *agrees with* external performance measure:
    - Agent that chooses actions to maximize its utility will be *rational* according to the external performance measure
      - Rationality as a measure of intelligence

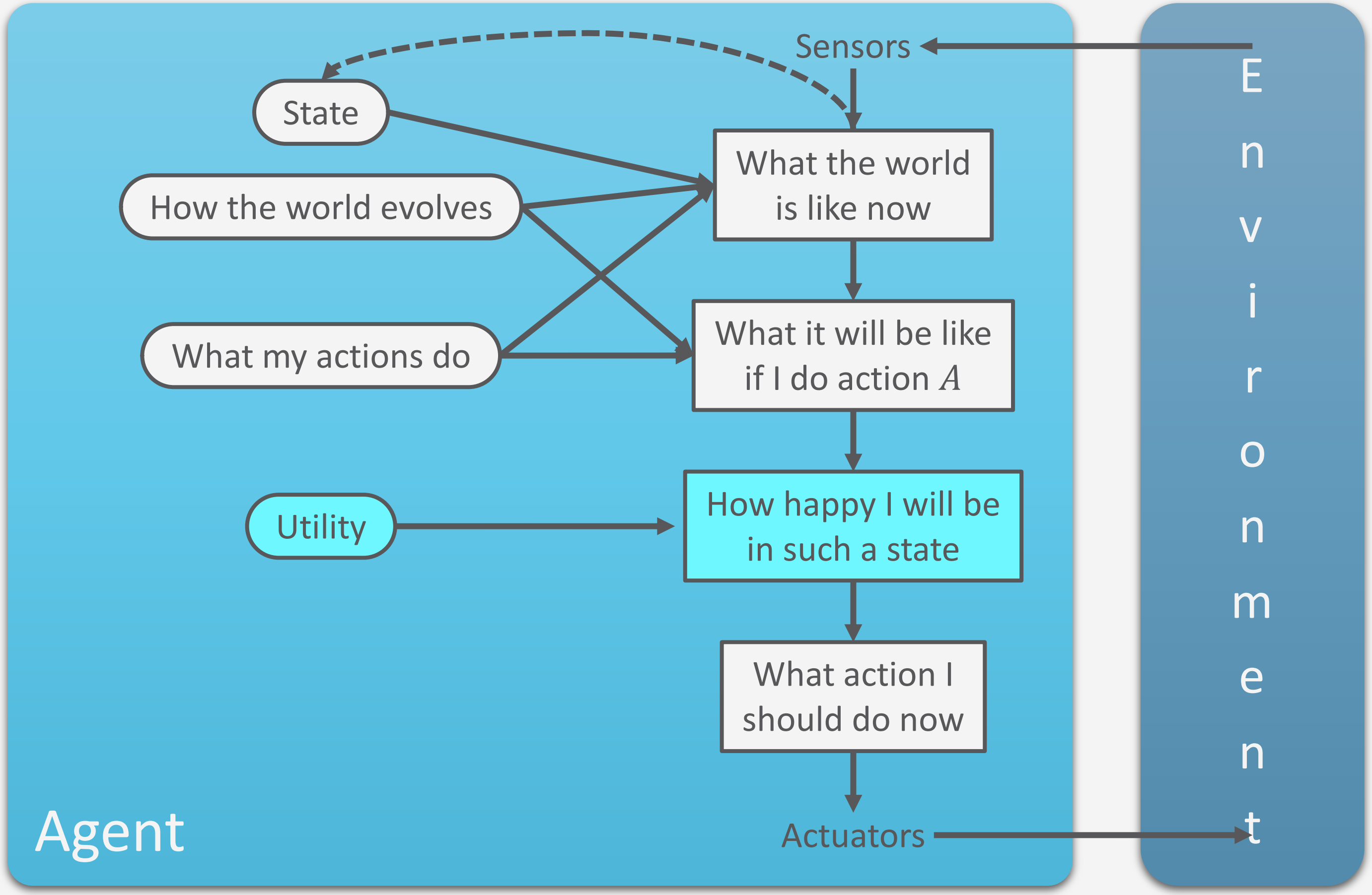


Figure: AIMA, Russell/Norvig

Setting

- Agent can perform actions in an environment
  - Environment
    - Outcomes of actions not unique
    - Associated with probabilities (→ probabilistic model)
  - Agent has preferences over states/action outcomes
    - Encoded in utility or utility function → Utility theory
- "Decision theory = Utility theory + Probability theory"
  - Model the world with a probabilistic model
  - Model preferences with a utility (function)
  - Find action that leads to the maximum expected utility, also called decision making

Outline: Decision Making – Foundations

Utility Theory

- Preferences
- Utilities
- Preference structure

Markov Decision Process / Problem (MDP)

- Sequence of actions, history, policy
- Value iteration, policy iteration

Reinforcement Learning (RL)

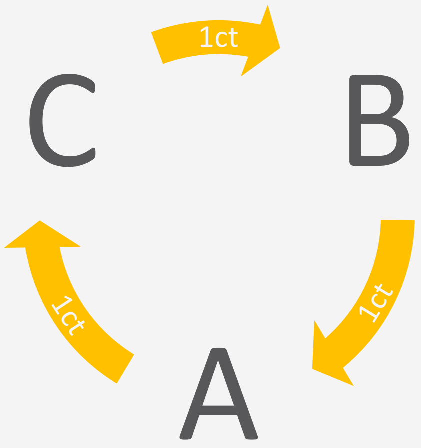
- Passive and active, model-free and model-based RL
- Multi-armed bandit

Preferences

- An agent chooses among prizes (A, B, etc.) and lotteries, i.e., situations with uncertain prizes
  - Outcome of a nondeterministic action is a lottery
- Lottery $L = [p, A; (1-p), B]$
  - A and B can be lotteries again
  - Prizes are special lotteries: $[1, R; 0, \text{not } R]$
  - More than two outcomes:
    - $L = [p_1, S_1; p_2, S_2; \cdots; p_M, S_M], \sum_{i=1}^{M} p_i = 1$
- Notation
  - $A \succ B$ $\quad$ A preferred to B
  - $A \sim B$ $\quad$ indifference between A and B
  - $A \succsim B$ $\quad$ B not preferred to A

Rational Preferences

- Idea: preferences of a rational agent must obey constraints
  - As prerequisite for reasonable preference relations
- Rational preferences → behaviour describable as maximisation of expected utility
- Violating constraints leads to self-evident irrationality
  - Example
    - An agent with intransitive preferences can be induced to give away all its money
      - If $B \succ C$, then an agent who has C would pay (say) 1 cent to get B
      - If $A \succ B$, then an agent who has B would pay (say) 1 cent to get A
      - If $C \succ A$, then an agent who has A would pay (say) 1 cent to get C

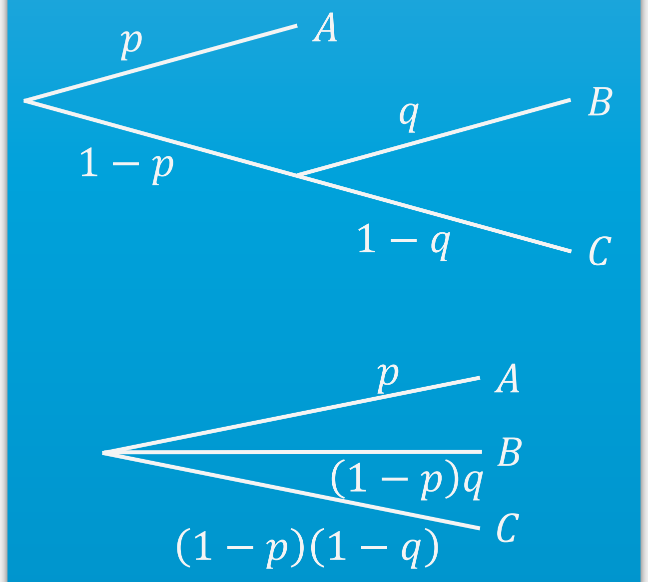


Axioms of Utility Theory

1. Orderability
   - $(A \succ B) \vee (A \prec B) \vee (A \sim B)$
     - $\{\prec, \succ, \sim\}$ jointly exhaustive, pairwise disjoint
2. Transitivity
   - $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$
3. Continuity
   - $A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$
4. Substitutability
   - $A \sim B \Rightarrow [p, A; 1-p, C] \sim [p, B; 1-p, C]$
     - Also holds if replacing $\sim$ with $\succ$
5. Monotonicity
   - $A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; 1-p, B] \gtrsim [q, A; 1-q, B])$
6. Decomposability
   - $\left[p, A;\ 1-p, [q, B;\ 1-q, C]\right] \sim [p, A;\ (1-p)q, B; (1-p)(1-q), C]$

Decomposability:
There is no fun in gambling.

Equivalent lotteries:



And Then There Was Utility

- Theorem (Ramsey, 1931; von Neumann and Morgenstern, 1944):
  - Given preferences satisfying the constraints, there exists a real-valued function U such that

$$U(A) \geq U(B) \Leftrightarrow A \gtrsim B$$

    - Existence of a utility function
  - Expected utility of a lottery:

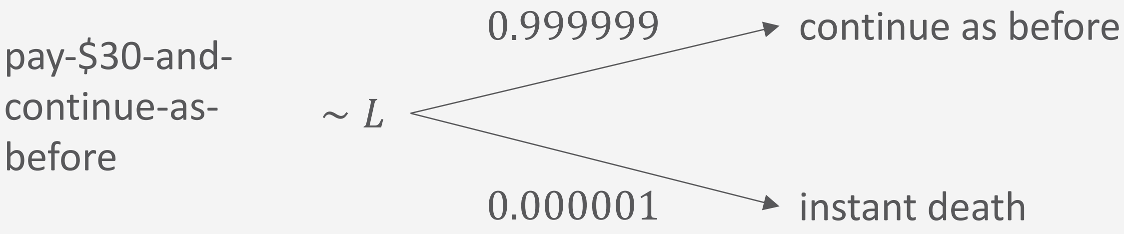
$$U([p_1, S_1; \dots; p_M, S_M]) = \sum_{i=1}^{M} p_i U(S_i)$$

- MEU principle
  - Choose the action that maximises expected utility

Utilities

- Utilities map states to real numbers.
  Which numbers?

- Standard approach to assessment of human utilities:
  - Compare a given state A to a standard lottery L_p that has
    - "best possible outcome" ⊤ with probability p
    - "worst possible catastrophe" ⊥ with probability $(1-p)$
  - Adjust lottery probability p until $A \sim L_p$



Utility Scales

- Normalised utilities: $u_\top = 1.0, u_\bot = 0.0$
  - Utility of lottery $L \sim$ (pay-\$30-and-continue-as-before): $U(L) = u_\top \cdot 0.999999 + u_\bot \cdot 0.000001 = 0.999999$
- Micromorts: one-millionth chance of death
  - Useful for Russian roulette, paying to reduce product risks, etc.
  - Example for low risk
    - Drive a car for 370km ≈ 1 micromort → lifespan of a car: 150,000km ≈ 400 micromorts
    - Studies showed that many people appear to be willing to pay US$10,000 for a safer car that halves the risk of death → US$50/micromort
- QALYs: quality-adjusted life years
  - Useful for medical decisions involving substantial risk
- In planning: task becomes minimisation of cost instead of maximisation of utility

Money

- Money does not behave as a utility function
- Given a lottery L with expected monetary value $EMV(L)$, usually $U(L) < U\left(S_{EMV(L)}\right)$, i.e., people are risk-averse
  - S_M: state of possessing total wealth $\$M$
  - Utility curve
    - For what probability p am I indifferent between a prize x and a lottery $[p, \$M; (1-p), \$0]$ for large M?
    - Right: Typical empirical data, extrapolated with risk-prone behaviour for negative wealth

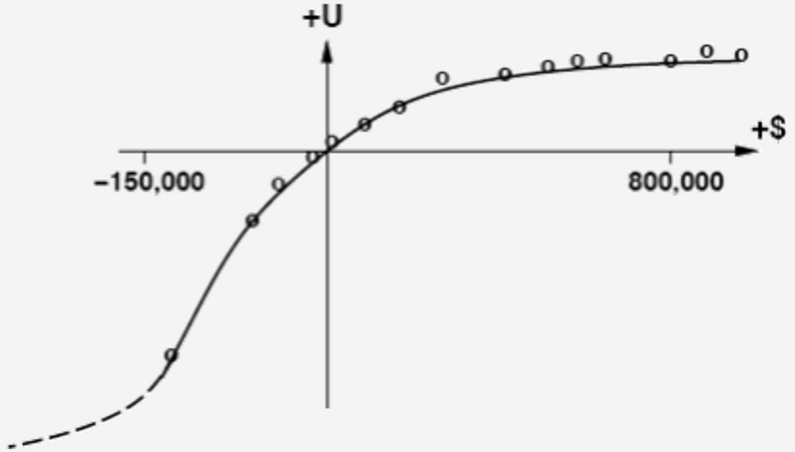


Figure: AIMA, Russell/Norvig

Money Versus Utility

- Money ≠ Utility
  - More money is better, but not always in a linear relationship to the amount of money
- Expected Monetary Value
  - Risk-averse
    - $U(L) < U\left(S_{EMV(L)}\right)$
  - Risk-seeking
    - $U(L) > U\left(S_{EMV(L)}\right)$
  - Risk-neutral
    - $U(L) = U\left(S_{EMV(L)}\right)$
    - Linear curve
    - For small changes in wealth relative to current wealth

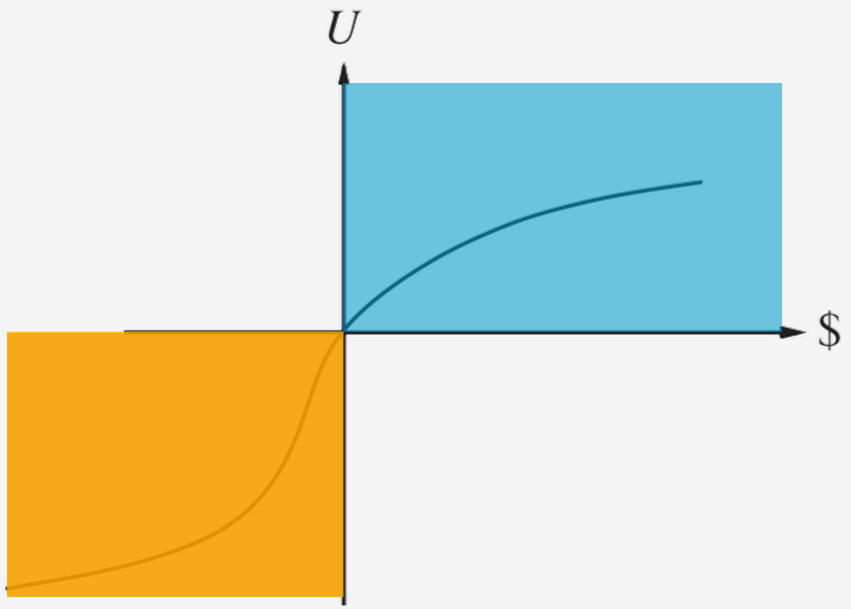


Figure: AIMA, Russell/Norvig

Utility Scales

- Behaviour is invariant w.r.t. positive linear transformation
$$U'(r) = k_1 U(r) + k_2$$
  - No unique utility function; $U'(r)$ and $U(r)$ yield same behaviour
- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes
  - Ordinal utility function also called value function
  - Provides a ranking of alternatives (states), but not a meaningful metric scale (numbers do not matter)
- Note:
An agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe

Multi-attribute Utility Theory

- A given state may have multiple utilities
  - ...because of multiple evaluation criteria
  - ...because of multiple agents (interested parties) with different utility functions

- There are:
  - Cases in which decisions can be made *without* combining the attribute values into a single utility value
    - Strict dominance
  - Cases in which the utilities of attribute combinations can be specified very concisely
    - Preference structure

Preference Structure

- To specify the complete utility function $U(r_1, \dots, r_M)$, we need d^M values in the worst case
  - M attributes
  - each attribute with d distinct possible values
  - Worst case meaning: Agent's preferences have no regularity at all
- Supposition in multi-attribute utility theory
  - Preferences of typical agents have much more structure
- Approach
  - Identify regularities in the preference behaviour
  - Use so-called representation theorems to show that an agent with a certain kind of preference structure has a utility function
$$U(r_1, \dots, r_M) = \Xi[f_1(r_1), \dots, f_M(r_M)]$$
    - where Ξ is hopefully a simple function such as *addition*

Preference Independence

- R_1 and R_2 preferentially independent (PI) of R_3 iff
  - Preference between $\langle r_1, r_2, r_3 \rangle$ and $\langle r_1', r_2', r_3 \rangle$ does not depend on r_3
  - E.g., $\langle Noise, Cost, Safety \rangle$
    - $\langle 20{,}000\ suffer, \$4.6\ billion, 0.06\ deaths/month \rangle$
    - $\langle 70{,}000\ suffer, \$4.2\ billion, 0.06\ deaths/month \rangle$

- Theorem (Leontief, 1947)
  - If every pair of attributes is PI of its complement, then every subset of attributes is PI of its complement
    - Called mutual PI (MPI)

Preference Independence

- Theorem (Debreu, 1960):
  - MPI ⇒ ∃ *additive* value function
$$V(r_1, \dots, r_M) = \sum_{i=1}^{M} V_i(r_i)$$
  - Hence assess M single-attribute functions
    - Decomposition of V into a set of summands (additive semantics)

    similar to
    - Decomposition of $P_{\boldsymbol{R}}$ into a set of factors (multiplicative semantics)
  - Often a good approximation
  - Example:
$$V(Noise, Cost, Deaths) = -Noise \cdot 10^4 - Cost - Deaths \cdot 10^{12}$$

Interim Summary

- Preferences
  - Preferences of a rational agent must obey constraints
- Utilities
  - Rational preferences = describable as maximisation of expected utility
  - Utility axioms
  - MEU principle
- Multi-attribute utility theory
  - Preference structure
  - (Mutual) preferential independence

Outline: Decision Making – Foundations

Utility Theory

- Preferences
- Utilities
- Preference structure

Markov Decision Process / Problem (MDP)

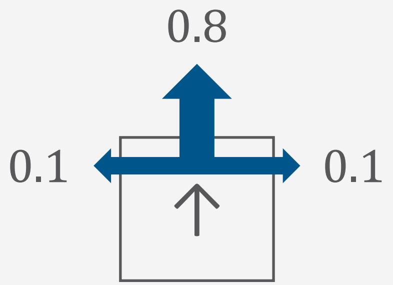
- Sequence of actions, history, policy
- Value iteration, policy iteration

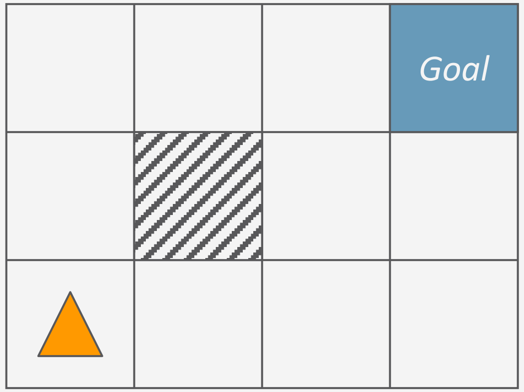
Reinforcement Learning (RL)

- Passive and active, model-free and model-based RL
- Multi-armed bandit

Simple Robot Navigation Problem

- In each state, the possible actions are U, D, R, and L
- The effect of action U is as follows (transition model):
  - With probability 0.8, move up one square
    - If already in top row or blocked, no move
  - With probability 0.1, move right one square
    - If already in rightmost row or blocked, no move
  - With probability 0.1, move left one square
    - If already in leftmost row or blocked, no move
- Same transition model holds for D, R, and L and their respective directions





Markov Property

The transition properties depend only on the current state, not on previous history (how that state was reached).

- Also known as Markov-k with $k = 1$
  - $k \leq t$

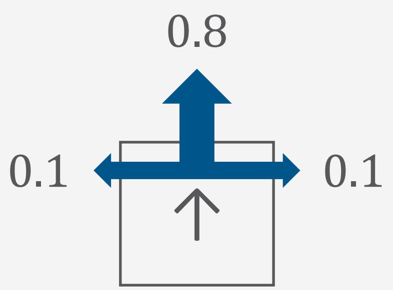
$$P\left(x^{(t+1)} \middle| x^{(t)}, \dots, x^{(0)}\right) = P\left(x^{(t+1)} \middle| x^{(t)}, \dots, x^{(t-k+1)}\right)$$

  - $k = 1$

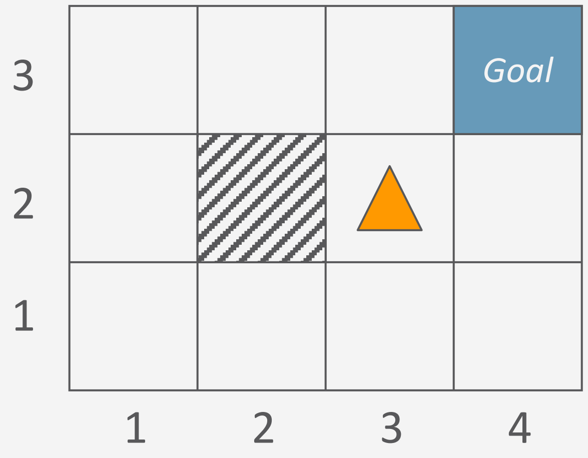
$$P\left(x^{(t+1)} \middle| x^{(t)}, \dots, x^{(0)}\right) = P\left(x^{(t+1)} \middle| x^{(t)}\right)$$

Sequence of Actions

- In each state, the possible actions are U, D, R, and L; the transition model for each action is (pictured):
- Current position: [3,2]
- Planned sequence of actions: (U, R)

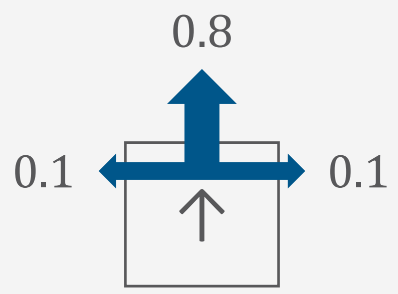


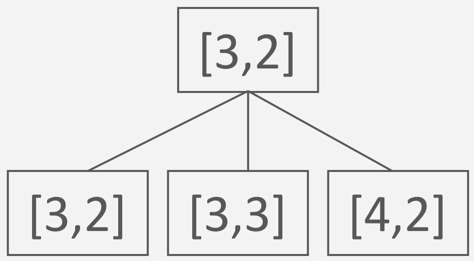
[3,2]

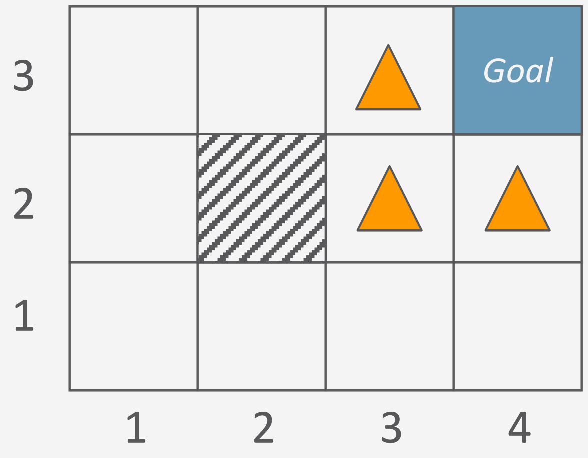


Sequence of Actions

- In each state, the possible actions are U, D, R, and L; the transition model for each action is (pictured):
- Current position: [3,2]
- Planned sequence of actions: (U, R)
  - U is executed

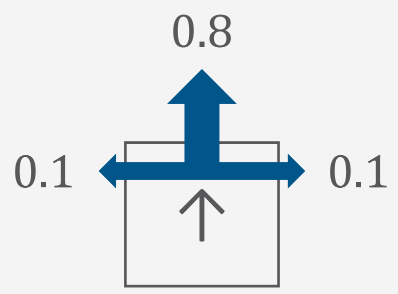


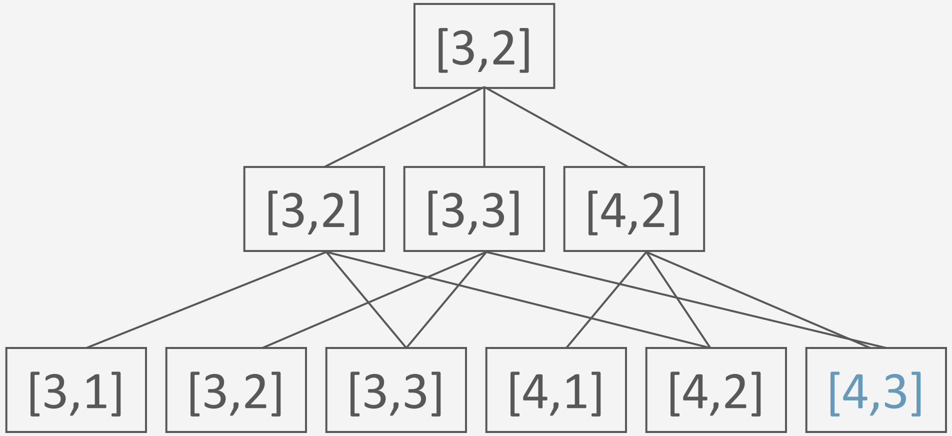


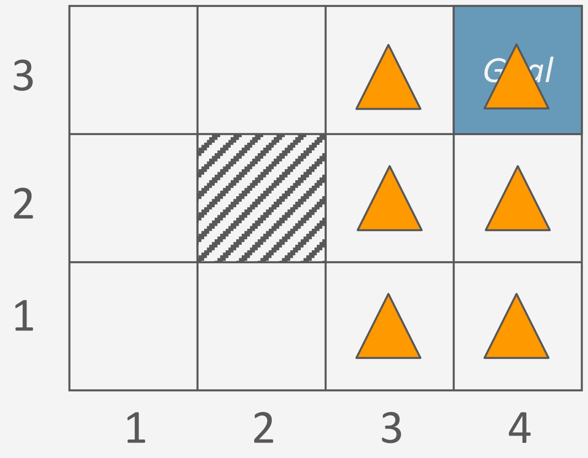


Sequence of Actions

- In each state, the possible actions are U, D, R, and L; the transition model for each action is (pictured):
- Current position: [3,2]
- Planned sequence of actions: (U, R)
  - U has been executed
  - R is executed

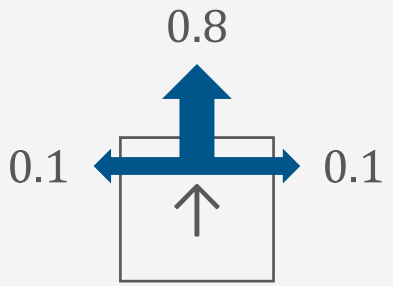


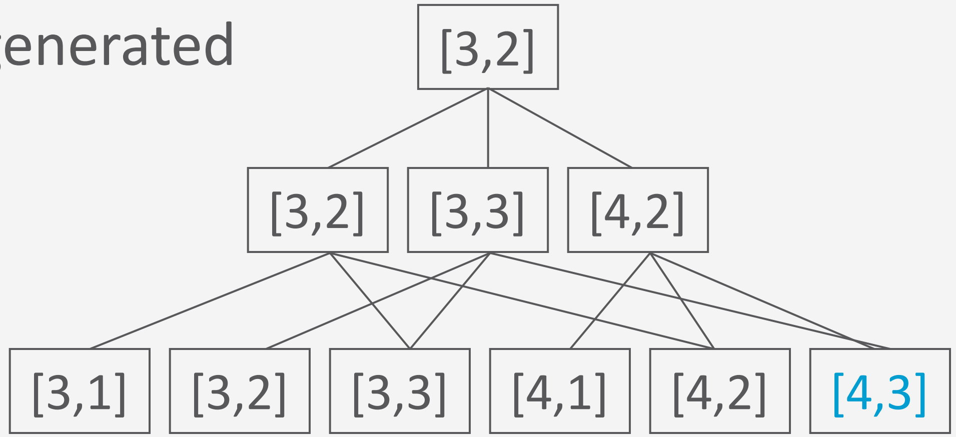


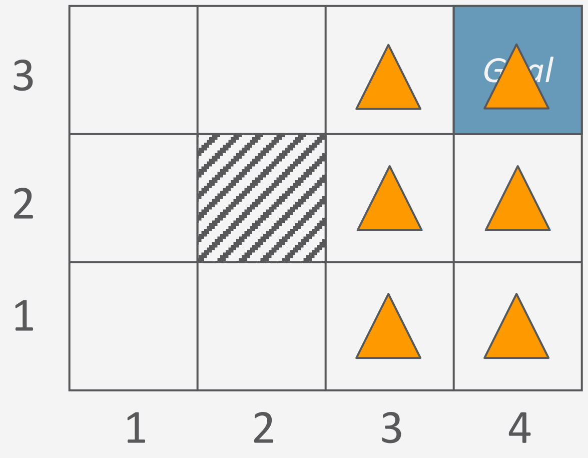


Histories

- In each state, the possible actions are U, D, R, and L; the transition model for each action is (pictured):
- Current position: [3,2]
- Planned sequence of actions: (U, R)
  - U has been executed
  - R is executed
- History: sequence of states generated by sequence of actions
  - 9 possible sequences with 6 possible final states, only 1 of which is a goal state







Probability of Reaching the Goal

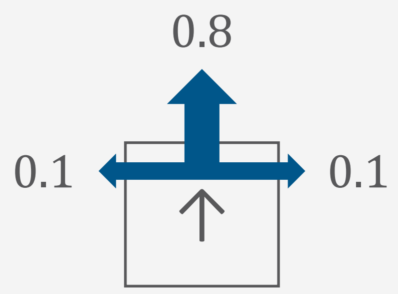
- In each state: possible actions U, D, R, L; trans. model:

$$P([4{,}3] \mid (\mathrm{U},\mathrm{R}), [3{,}2]) = P([4{,}3] \mid \mathrm{R}, [3{,}3]) \cdot P([3{,}3] \mid \mathrm{U}, [3{,}2]) + P([4{,}3] \mid \mathrm{R}, [4{,}2]) \cdot P([4{,}2] \mid \mathrm{U}, [3{,}2])$$

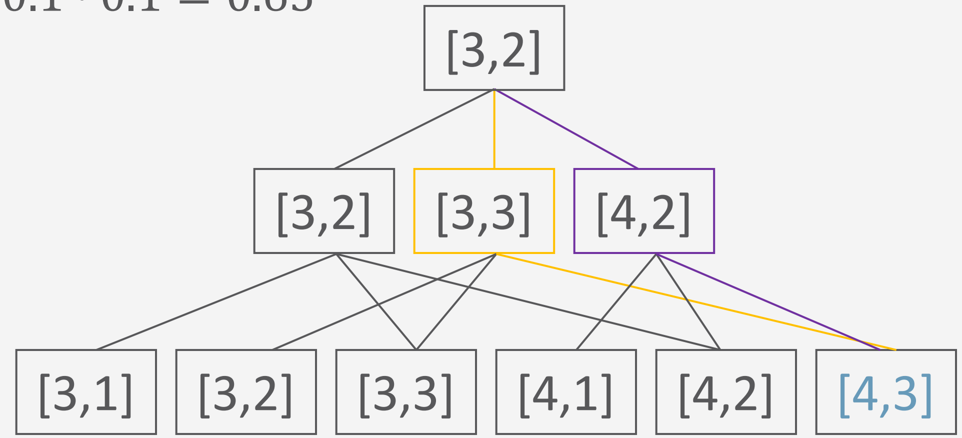
$P([4{,}3] \mid \mathrm{R}, [3{,}3]) = 0.8$ $P([3{,}3] \mid \mathrm{U}, [3{,}2]) = 0.8$

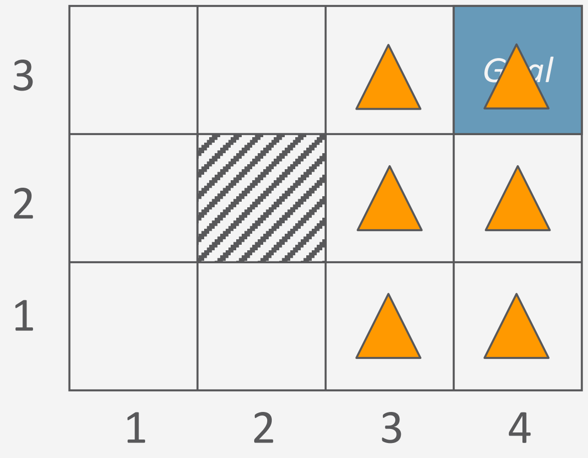
$P([4{,}3] \mid \mathrm{R}, [4{,}2]) = 0.1$ $P([4{,}2] \mid \mathrm{U}, [3{,}2]) = 0.1$

$$P([4{,}3] \mid (\mathrm{U},\mathrm{R}), [3{,}2]) = 0.8 \cdot 0.8 + 0.1 \cdot 0.1 = 0.65$$



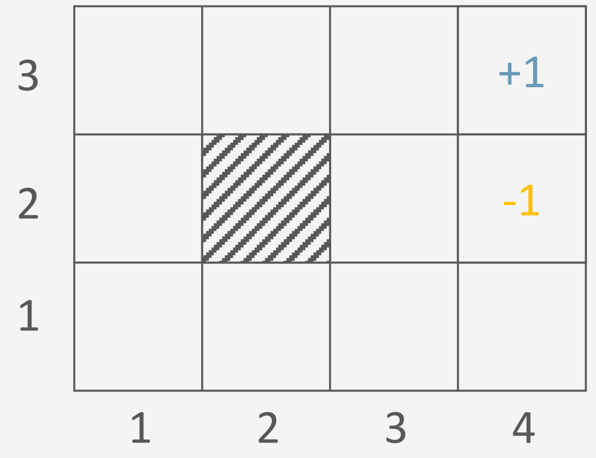
Note importance of Markov property in this derivation





Utility Function

- [4,3] : power supply (stops the run)
- [4,2] : sand area the robot cannot escape (stops the run)
- Goal: robot needs to recharge its batteries
- [4,3] and [4,2] are terminal states
- In this example, we define the utility of a history by
  - The utility of the last state (+1 or –1) minus $0.04 \cdot n$
    - n is the number of moves
    - I.e., each move costs 0.04, which provides an incentive to reach the goal fast



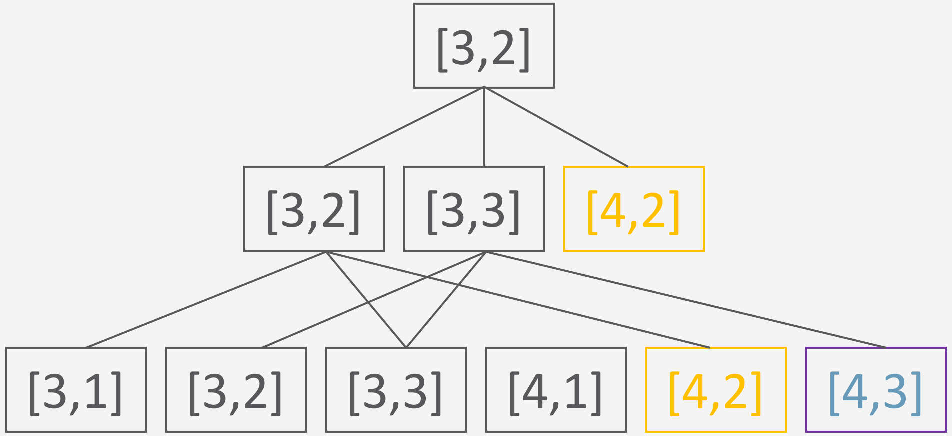
Utility of an Action Sequence

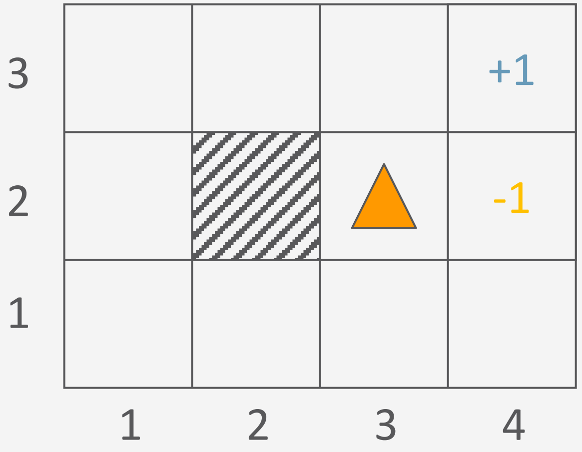
- Consider the action sequence $\boldsymbol{a} =$ (U,R) from [3,2]
- A run produces one of 7 possible histories, each with a probability
- Utility of the sequence is the expected utility of histories h:

$$U(\boldsymbol{a}) = \sum_h U_h P(h)$$

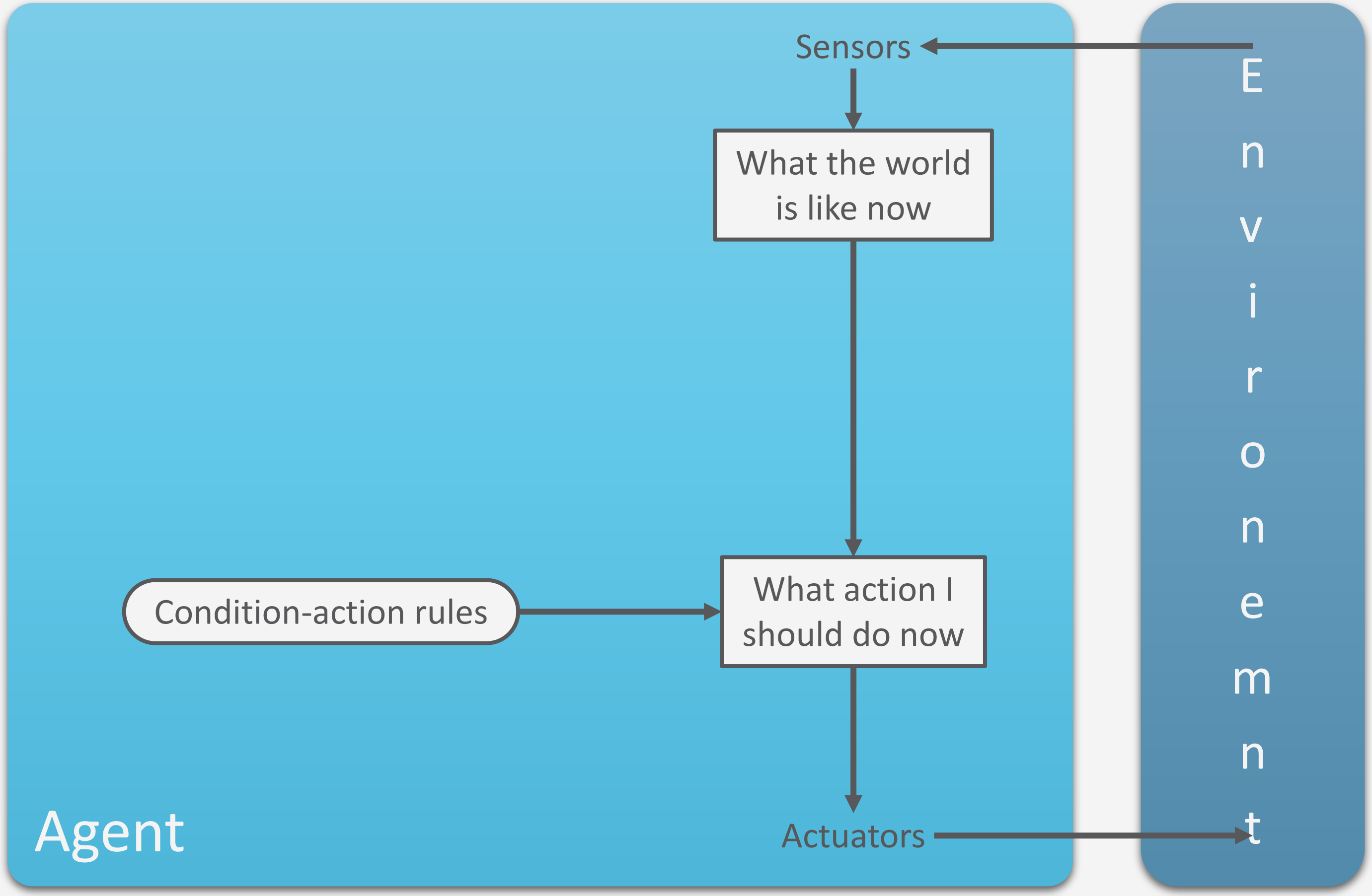
- Optimal sequence = the one with maximum utility

Is the optimal sequence what we want?





Reactive Agent Algorithm




```
Act()
    repeat
        s ← sensed state
        if s is terminal then
            exit
        a ← choose action (given s)
        perform a
```

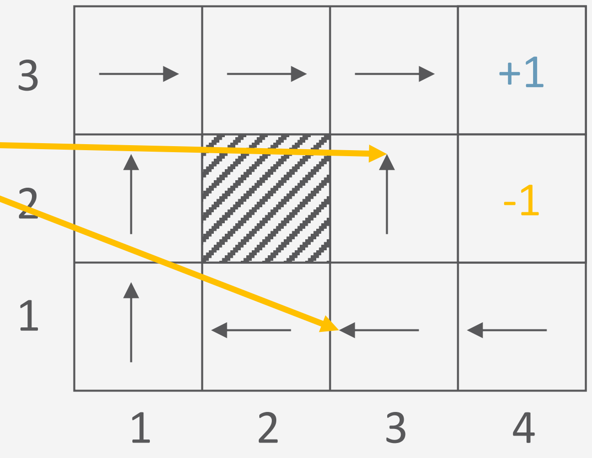
Figure: AIMA, Russell/Norvig

Policy (Reactive/Closed-loop Strategy)

- Policy π
  - *Complete* mapping from states to actions
- Optimal policy π^*
  - Always yields a history (ending at terminal state) with maximum expected utility
    - Due to Markov property

```
Act()
    repeat
        s ← sensed state
        if s is terminal then
            exit
        a ← π(s)
        perform a
```

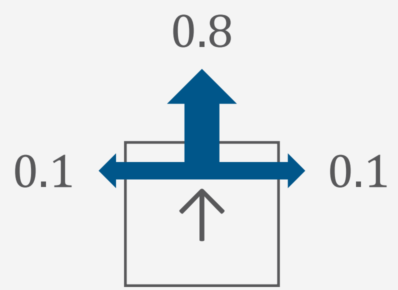
Note that [3,2] is a "dangerous" state that the optimal policy tries to avoid



How to compute π^*?
Solving a Markov Decision Process

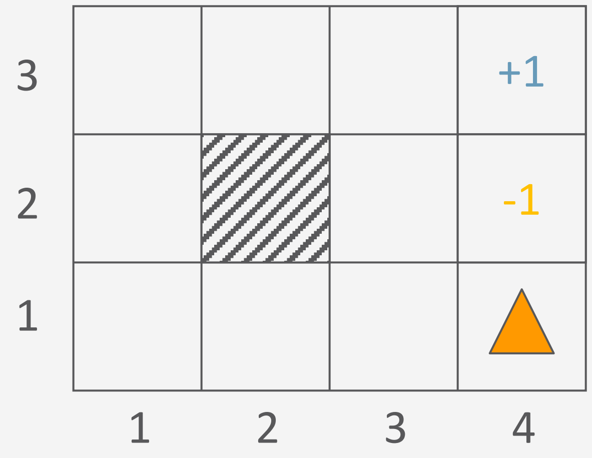
Markov Decision Process / Problem (MDP)

- *Sequential* decision problem
  for a fully observable, stochastic environment
  with a Markovian transition model
  and additive rewards (next slide)
- MDP is a four-tuple (S, A, T, R) with
  - S a random variable whose domain is a set of states (with an initial state s_0)
  - For each $s \in \mathrm{dom}(S)$
    - a set $A(s)$ of actions
    - a transition model $T(s', s, a) = P(s'|s, a)$
    - a reward function $R(s)$ (also with a possible)
- Robot navigation example to the right



U, D, L, R

each move costs 0.04



Additive Utility

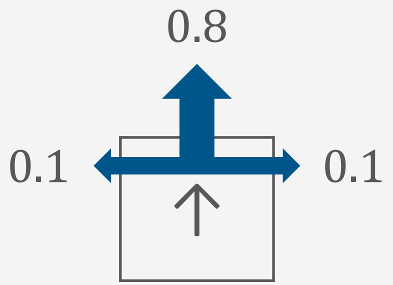
- History $h = \left(s^{(0)} = s_0, s^{(1)}, \dots, s^{(T)}\right)$
- In each state s, agent receives reward $R(s)$
- Utility of h is additive iff

$$U\left(s^{(0)}, s^{(1)}, \dots, s^{(T)}\right) = R\left(s^{(0)}\right) + U\left(s^{(1)}, \dots, s^{(T)}\right)$$
$$= \sum_{t=0}^{T} R\left(s^{(t)}\right)$$

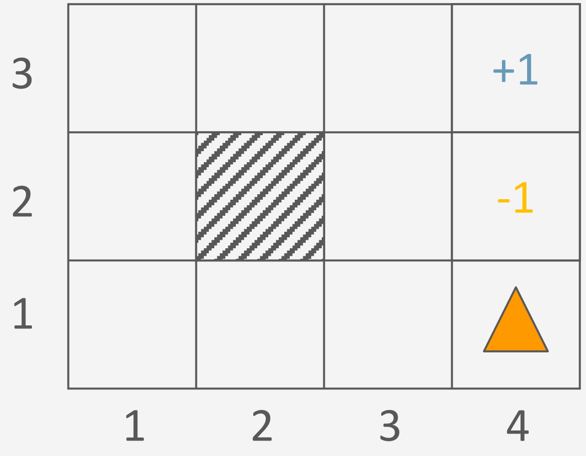
  - Discount factor $\gamma \in]0,1]$:

$$U\left(s^{(0)}, s^{(1)}, \dots, s^{(T)}\right) = \sum_{t=0}^{T} \gamma^t R\left(s^{(t)}\right)$$

    - Close to 0: future rewards insignificant
    - Corresponds to interest rate ${}^{1-\gamma}/_{\gamma}$



U, D, L, R each move costs 0.04



Principle of MEU

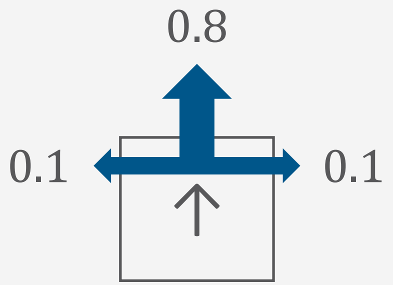
- Bellman equation:

$$U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s)U(s')$$

- Optimal policy:

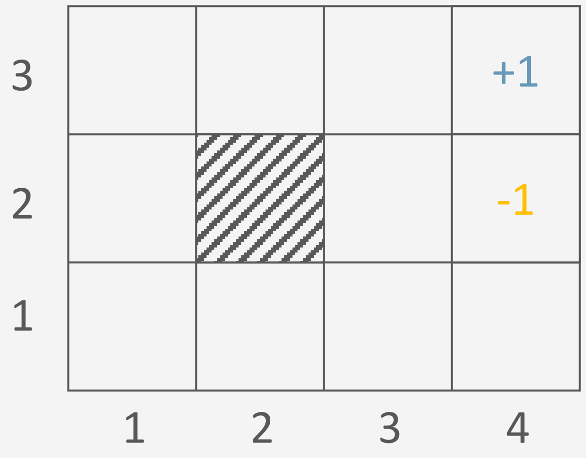
$$\pi^*(s) = \operatorname*{argmax}_{a \in A(s)} \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s)U(s')$$

  - Bellman equation for $[1,1]$ with $\gamma = 1$ as discount factor
    - $U(1,1) = -0.04 + \gamma \max_{\mathrm{U,L,D,R}} \{ 0.8U(1,2) + 0.1U(2,1) + 0.1U(1,1),$ (U)
      $0.8U(1,1) + 0.1U(1,1) + 0.1U(1,2),$ (L)
      $0.8U(1,1) + 0.1U(2,1) + 0.1U(1,1),$ (D)
      $0.8U(2,1) + 0.1U(1,2) + 0.1U(1,1) \}$ (R)



U, D, L, R

each move costs 0.04

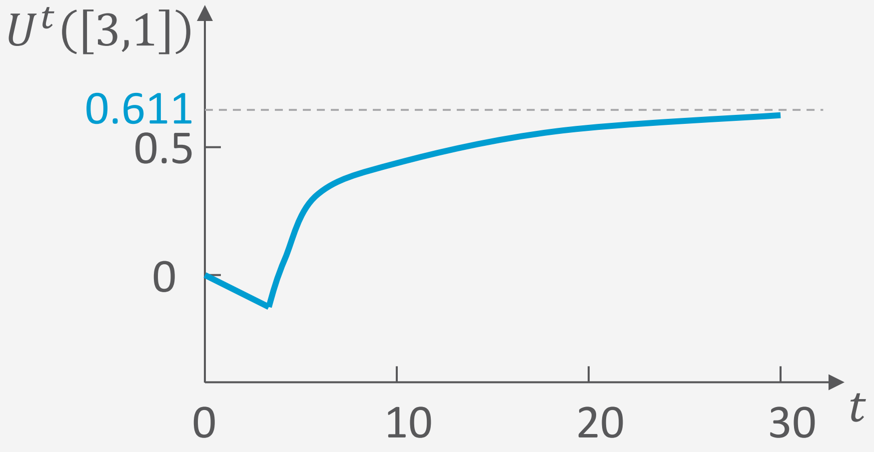


Value Iteration

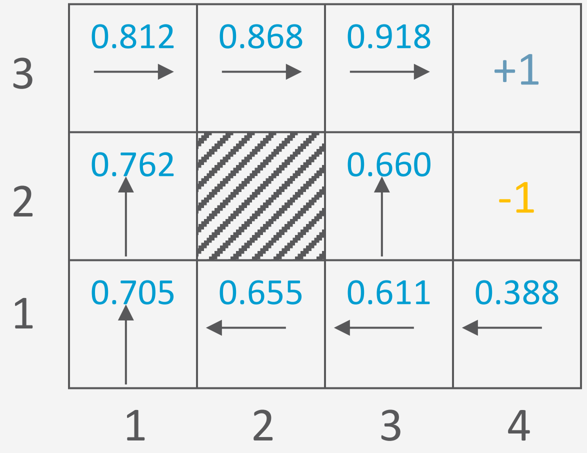
- Initialise the utility of each non-terminal state s to $U^{(0)}(s) = 0$
- For $t = 0, 1, 2, \ldots$, do

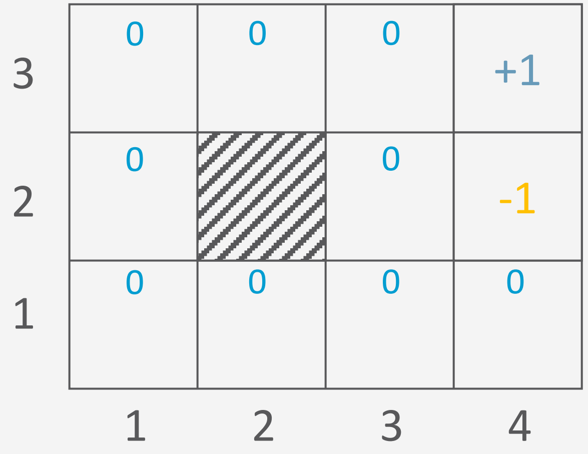
$$U^{(t+1)}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s) U^{(t)}(s')$$

  - So called Bellman update



Note the importance of terminal states and connectivity of the state-transition graph





Value Iteration: Algorithm

- Returns a policy π that is optimal
- Inputs
  - MDP $mpd = (S, A, T, R)$
    - Set of states S
    - For each $s \in S$
      - Set $A(s)$ of applicable actions
      - Transition model $T = P(s'|s, a)$
      - Reward function $R(s)$
  - Maximum error allowed ϵ

```
function value-iteration(mdp,ε)
    U' ← 0, π ← ⟨⟩
    repeat
        U ← U'
        δ ← 0
        for each state s ∈ S do
            U'[s] ← R(s) + γ max_{a∈A(s)} Σ_{s'} P(s'|a.s) U[s']
            if |U'[s] - U[s]| > δ then
                δ ← |U'[s] - U[s]|
    until δ < ε(1-γ)/γ
    for each state s ∈ S do
        π(s) ← argmax_{a∈A(s)} Σ_{s'} P(s'|a.s) U[s']
    return π
```

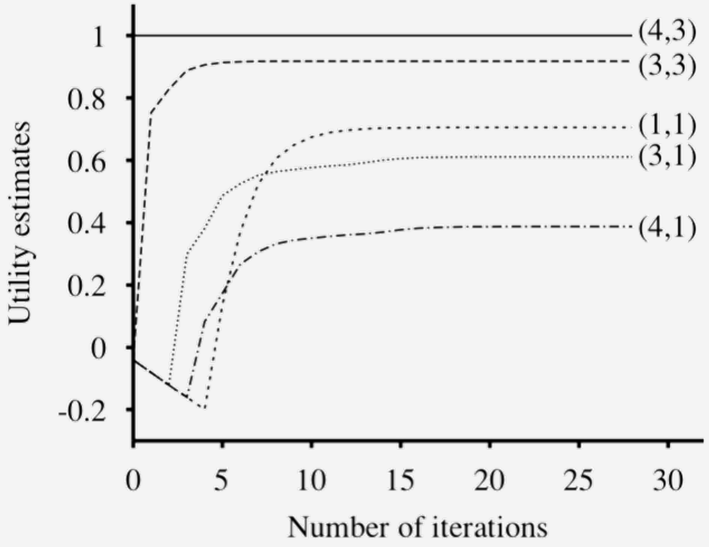
- Local variables
  - U, U' vectors of utilities for states in S
  - δ maximum change in utility of any state in an iteration

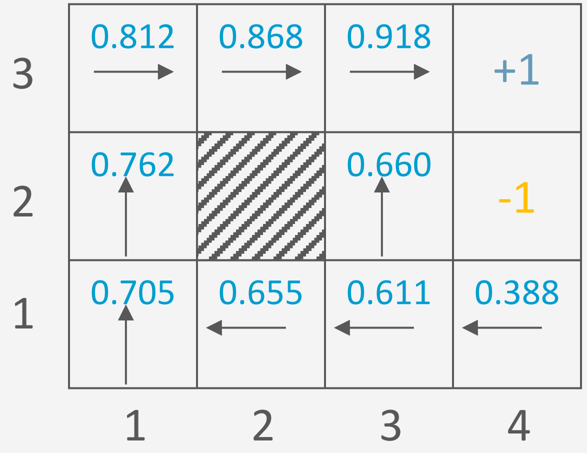
Evolution of Utilities

- For $t = 0, 1, 2, \ldots,$ do

$$U^{(t+1)}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s' \in \mathrm{dom}(S)} P(s'|a, s) U^{(t)}(s')$$

- Value iteration ≈ information propagation
  - Argmax action may change over time due to utilities changing





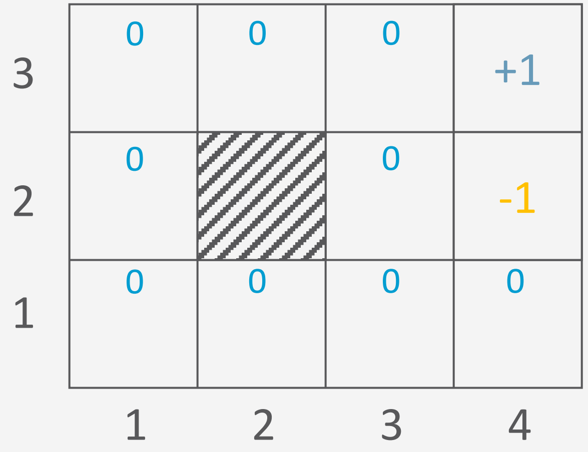


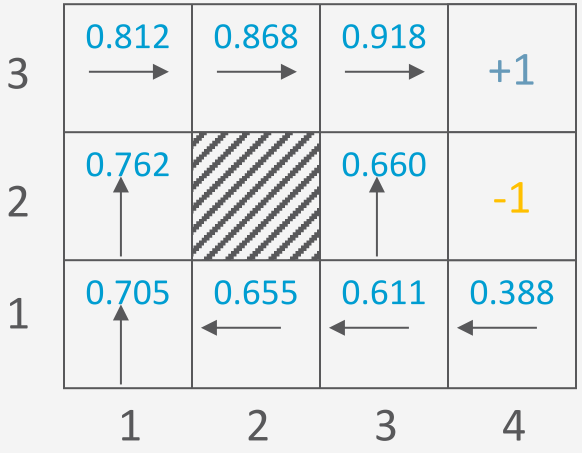
Figure left: AIMA, Russell/Norvig

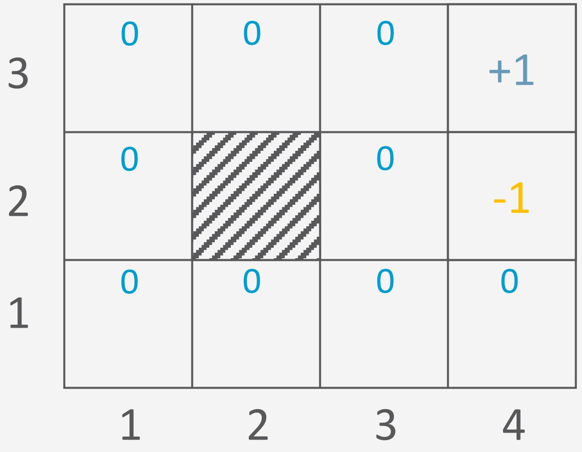
Effect of Rewards

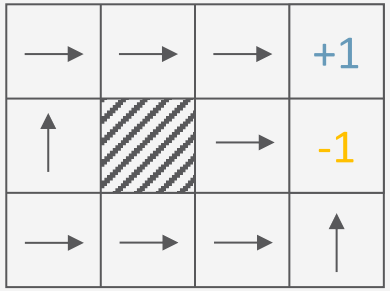
- For $t = 0, 1, 2, \ldots$, do

$$U^{(t+1)}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s) U^{(t)}(s')$$

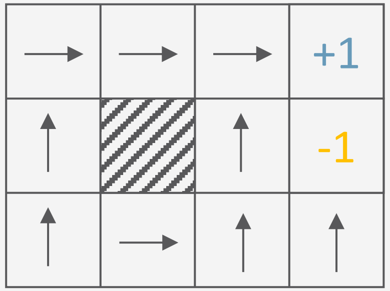
- Optimal policies for different rewards:
  - For $R(s) = -0.04$, see right ⇢



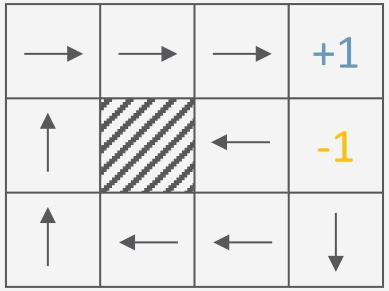




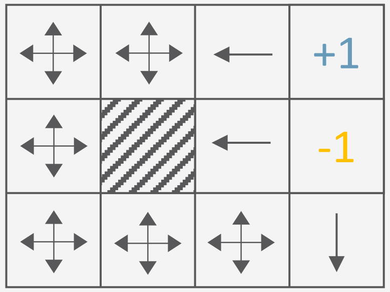
$R(s) < -1.6284$



$-0.4278 < R(s) < -0.0850$



$-0.0221 < R(s) \leq 0$



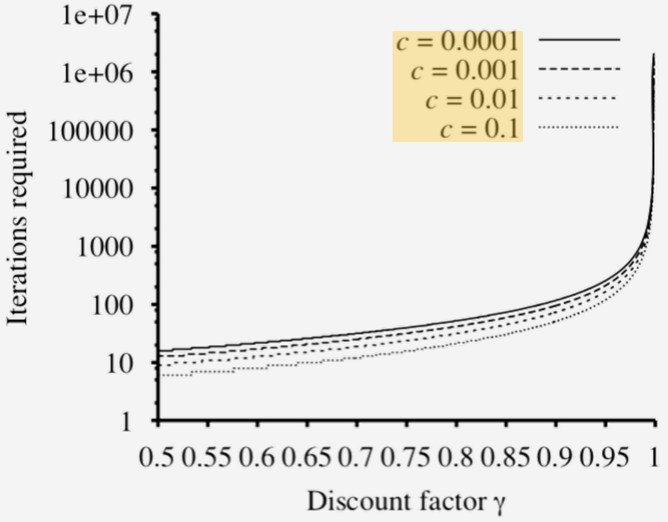
$R(s) > 0$

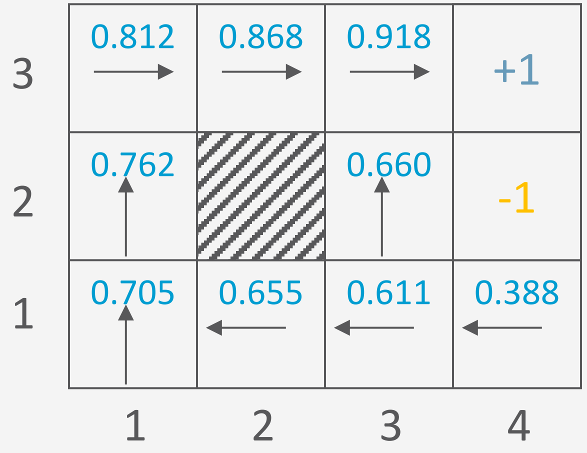
Effect of Allowed Error & Discount

- For $t = 0, 1, 2, \ldots,$ do

$$U^{(t+1)}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s) U^{(t)}(s')$$

- Iterations required to ensure a maximum error of $\varepsilon = c \cdot R_{max}$
  - R_{max} maximum reward





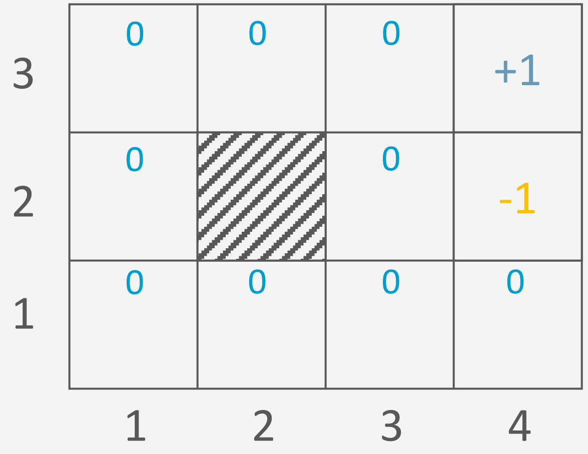


Figure right: AIMA, Russell/Norvig

Policy Iteration

- Pick a policy π_0 at random
- Repeat:
  - Policy evaluation: Compute the utility of each state for π_t
    - $\boxed{U^{(t)}(s) = R(s) + \gamma \sum_{s' \in \text{dom}(S)} P(s'|a,s) U^{(t)}(s')}$
      - No longer involves a max operation as action is determined by π_t
  - Policy improvement: Compute the policy π_{t+1} given U_t
    - $\pi^{(t+1)}(s) = \underset{a \in A(s)}{\text{argmax}} \sum_{s' \in \text{dom}(S)} P(s'|a,s) U^{(t)}(s')$
  - If $\pi^{(t+1)} = \pi^{(t)}$, then return $\pi^{(t)}$

Solve the set of linear equations:

$$U(s) = R(s) + \gamma \sum_{s' \in \text{dom}(S)} P(s'|a,s) U(s')$$

(often a sparse system)

Policy Iteration: Algorithm

- Returns a policy π that is optimal
  - Inputs: MDP $mpd = (S, A, T, R)$
    - Set of states S
    - For each $s \in S$
      - Set $A(s)$ of applicable actions
      - Transition model $T = P(s'|s, a)$
      - Reward function $R(s)$

```
function policy-iteration(mdp)
    repeat
        U ← policy-evaluation(π,U,mdp)
        unchanged ← true
        for each state s ∈ S do
            if max_{a∈A(s)} Σ_{s'} P(s'|a.s) U[s'] > Σ_{s'} P(s'|π[s].s) U[s'] then
                π[s] ← argmax_{a∈A(s)} Σ_{s'} P(s'|a.s) U[s']
                unchanged ← false
    until unchanged
    return π
```

- Local variables
  - U vectors of utilities for states in S, initially 0
  - π a policy vector indexed by state, initially random

Policy Evaluation

- Compute the utility of each state for π
  - $U^{(t)}(s) = R(s) + \gamma \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s) U^{(t)}(s')$
- Complexity of policy evaluation: $O(n^3)$, $n = |\mathrm{dom}(S)|$
  - For n states, n linear equations with n unknowns
  - Prohibitive for large n
- Approximation of utilities
  - Perform k value iteration steps with fixed policy π_t, return utilities
    - Simplified Bellman update: $U^{(t+1)}(s) = R(s) + \gamma \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s) U^{(t)}(s')$
  - Asynchronous policy iteration (next slide)
    - Pick any subset of states

Asynchronous Policy Iteration

- Further approximation of policy iteration
  - Pick any subset of states and do one of the following
    - Update utilities
      - Using simplified value iteration as described on previous slide
    - Update the policy
      - Policy improvement as before
- Is not guaranteed to converge to an optimal policy
  - Possible if each state is still visited infinitely often, knowledge about unimportant states, etc.
- Freedom to work on any states allows for design of domain-specific heuristics
  - Update states that are likely to be reached by a good policy

Intermediate Summary

- Markov property
  - Current state depends only on previous state
- Sequence of actions, history, policy
  - Sequence of actions may yield multiple histories, i.e., sequences of states, with a utility
  - Policy: complete mapping of states to actions
  - Optimal policy: policy with maximum expected utility
- MDP
  - State space, actions, transition model, reward function
- Value iteration, policy iteration
  - Algorithms for calculating an optimal policy for an MDP

Outline: Decision Making – Foundations

Utility Theory

- Preferences
- Utilities
- Preference structure

Markov Decision Process / Problem (MDP)

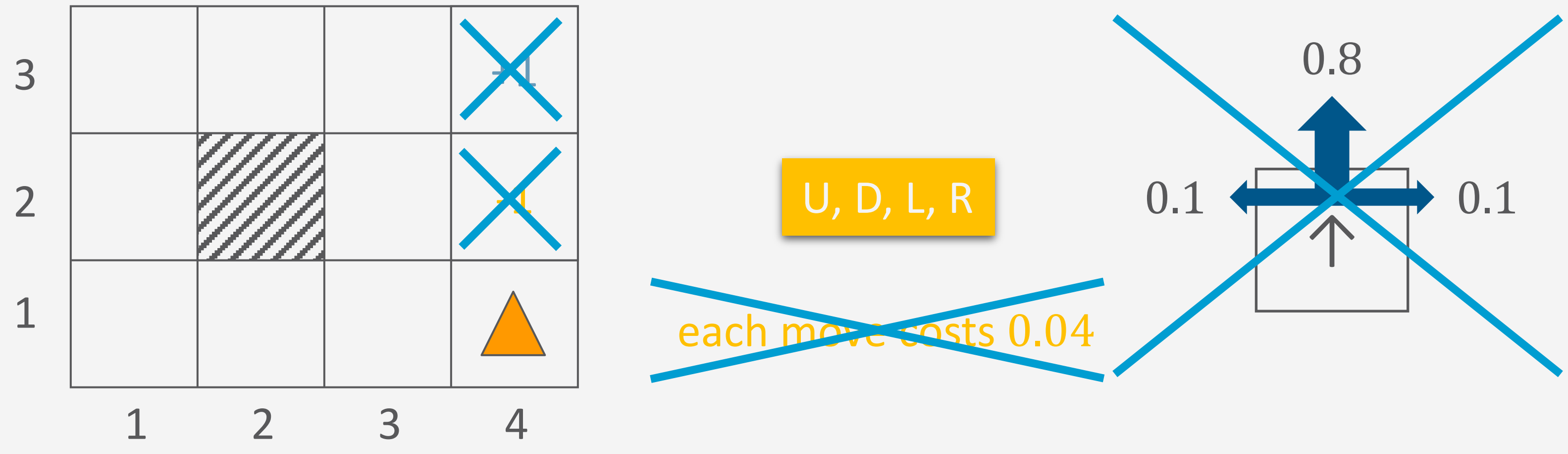
- Sequence of actions, history, policy
- Value iteration, policy iteration

Reinforcement Learning (RL)

- Passive and active, model-free and model-based RL
- Multi-armed bandit

Acting as Reinforcement Learning (RL)

- Agent, placed in an environment, must learn to act optimally in it
- Assume that the world behaves like an MDP, except
  - Agent can act but does not know the transition model
  - Agent observes its current state and its reward but does not know the reward function
- Goal: learn an optimal policy



Factors That Make RL Hard

- Actions have non-deterministic <u>effects</u>
  - which are initially <u>unknown</u> and must be learned
- Rewards / punishments can be infrequent
  - Often at the end of long sequences of actions
  - How does an agent determine what action(s) were really responsible for reward or punishment?
    - Credit assignment problem
  - World is large and complex

Passive vs. Active Learning

- Passive learning
  - Agent acts based on a fixed policy π and tries to learn how good the policy is by observing the world go by
  - Analogous to policy iteration (without the optimisation part)
- Active learning
  - Agent attempts to find an optimal (or at least good) policy by exploring different actions in the world
  - Analogous to solving the underlying MDP

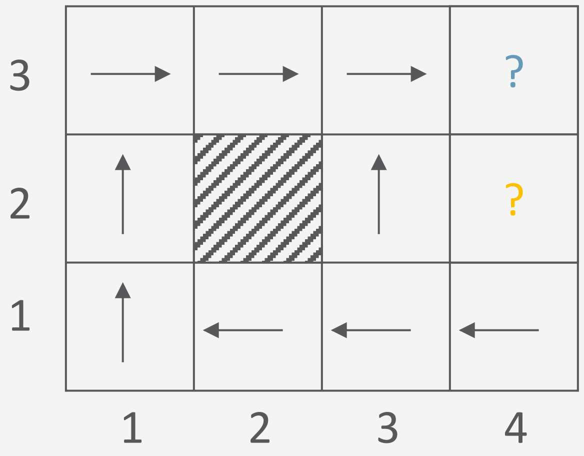
Model-based vs. Model-free RL

- Model-based approach to RL
  - Learn the MDP model ($P(s'|s, a)$ and R), or an approximation of it
  - Use it to find the optimal policy
- Model-free approach to RL
  - Derive the optimal policy without explicitly learning the model

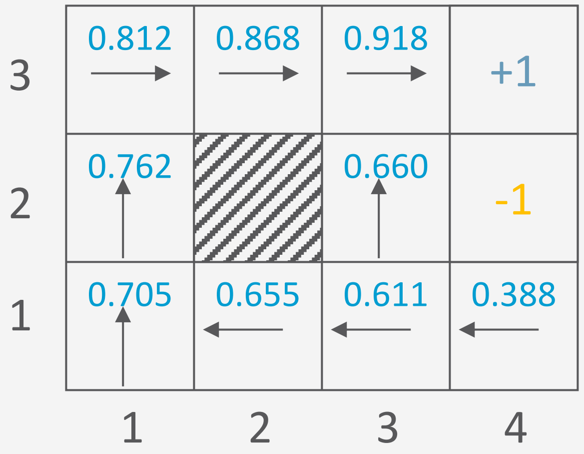
Passive RL

- Suppose the agent is given a policy
- Wants to determine how good it is

- Given π:

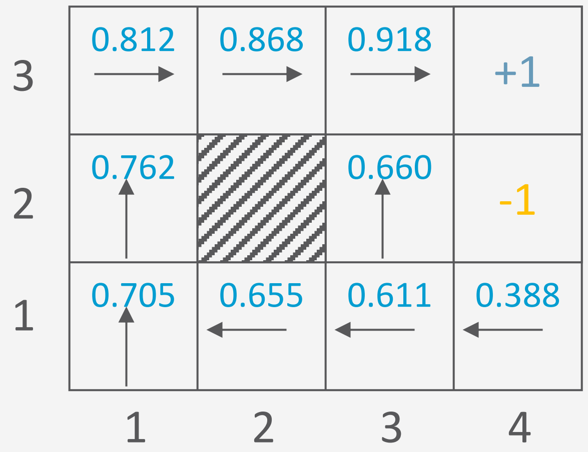


Need to learn $U^{\pi}(s)$:



Passive RL

- Given policy π:
  - Estimate $U^{\pi}(s)$
- Not given
  - Transition model $P(s'|s,a)$
  - Reward function $R(s)$
- Simply follow the policy for many epochs
  - Epochs: training sequences / trials



$$(1,1) \to (1,2) \to (1,3) \to (1,2) \to (1,3) \to (2,3) \to (3,3) \to (4,3) + 1$$
$$(1,1) \to (1,2) \to (1,3) \to (2,3) \to (3,3) \to (3,2) \to (3,3) \to (4,3) + 1$$
$$(1,1) \to (2,1) \to (3,1) \to (3,2) \to (4,2) - 1$$

  - Assumption: restart or reset possible (or no terminal states with the end of an epoch given by the receipt of a reward)

Direct Utility Estimation (DUE)

- Model-free approach
  - Estimate $U^{\pi}(s)$ as average total reward of epochs containing s
    - Calculating from s to end of epoch
- Reward-to-go of a state s
  - The sum of the (discounted) rewards from that state until a terminal state is reached
- Key: use observed reward-to-go of the state as the direct evidence of the actual expected utility of that state

DUE: Example

- Suppose the agent observes the following trial:
  - $(1,1)_{-0.04} \rightarrow (1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (2,3)_{-0.04} \rightarrow (3,3)_{-0.04} \rightarrow (4,3)_{+1}$
- The total reward starting at $(1,1)$ is $7 \cdot -0.04 + 1 = 0.72$
  - I.e., a sample of the observed-reward-to-go for $(1,1)$
- For $(1,2)$, there are two samples of the observed-reward-to-go
  - Assuming $\gamma = 1$

1. $(1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (2,3)_{-0.04} \rightarrow (3,3)_{-0.04} \rightarrow (4,3)_{+1}$
   [Total: 0.76]
2. $(1,2)_{-0.04} \rightarrow (1,3)_{-0.04} \rightarrow (2,3)_{-0.04} \rightarrow (3,3)_{-0.04} \rightarrow (4,3)_{+1}$
   [Total: 0.84]

DUE: Convergence

- Keep a running average of the observed reward-to-go for each state
  - E.g., for state $(1,2)$, it stores $\frac{(0.76+0.84)}{2} = 0.8$
- As the number of trials goes to infinity, the sample average converges to the true utility

DUE: Problem

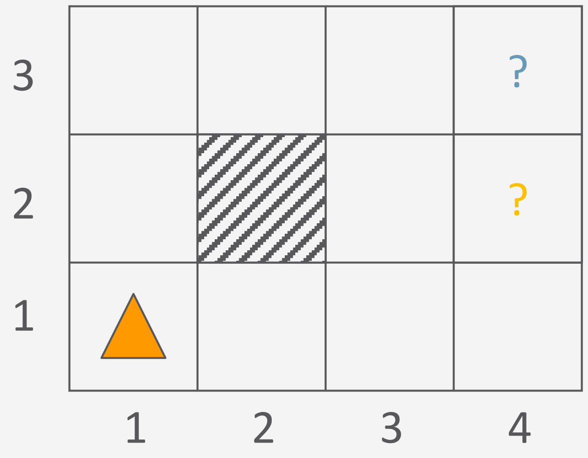
- Big problem: it converges very slowly!
- Why?
  - Does not exploit the fact that utilities of states are not independent
  - Utilities follow the Bellman equation

$$U^\pi(s) = R(s) + \gamma \sum_{s' \in \mathrm{dom}(S)} P(s'|\pi(s), s) U^\pi(s')$$

Dependence on neighbouring states

DUE: Problem

- Using the dependence to your advantage
  - Suppose you know that state $(3,3)$ has a high utility
  - Suppose you are now at $(3,2)$
  - Bellman equation would be able to tell you that $(3,2)$ is likely to have a high utility because $(3,3)$ is a neighbour
- DUE cannot tell you that until the end of the trial



Adaptive Dynamic Programming (ADP)

- Model-based approach
- Given policy π:
  - Estimate $U^{\pi}(s)$
  - All while acting in the environment

How?

- Basically learns the transition model $P(s'|s, a)$ and the reward function $R(s)$
  - Takes advantage of constraints in the Bellman equation
- Based on $P(s'|s, a)$ and $R(s)$, performs policy evaluation (part of policy iteration)

Recap: Policy Iteration

- Pick a policy π_0 at random
- Repeat:
  - Policy evaluation: Compute the utility of each state for π_t
    - $U^{(t)}(s) = R(s) + \gamma \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s) U^{(t)}(s')$
      - No longer involves a max operation as action is determined by π_t
  - Policy improvement: Compute the policy π_{t+1} given U_t
    - $\pi^{(t+1)}(s) = \underset{a \in A(s)}{\operatorname{argmax}} \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s) U^{(t)}(s')$
  - If $\pi^{(t+1)} = \pi^{(t)}$, then return $\pi^{(t)}$

Solve the set of linear equations:

$$U(s) = R(s) + \gamma \sum_{s' \in \mathrm{dom}(S)} P(s'|a,s) U(s')$$

(often a sparse system)

Can be solved in $O(n^3)$, where $n = |S|$

ADP: Estimate the Utilities

- Make use of policy evaluation to estimate the utilities of states
- To use policy equation

$$U^{(t+1)}(s) = R(s) + \gamma \sum_{s' \in \mathrm{dom}(S)} P(s'|\pi(s), s) U^{(t)}(s')$$

  agent needs to learn $P(s'|s, a)$ and $R(s)$
- How?

ADP: Learn the Model

- Learning $R(s)$
  - Easy because it is deterministic
  - Whenever you see a new state, store the observed reward value as $R(s)$
- Learning $P(s'|s, a)$
  - Keep track of how often you get to state s' given that you are in state s and do action a
  - E.g., if you are in $s = (1,3)$ and you execute R three times and you end up in $s' = (2,3)$ twice, then $P(s'|\mathrm{R}, s) = \frac{2}{3}$

ADP: Algorithm

- Learning the MDP while acting according to a fixed policy π

Update reward function

Update transition model

```
function passive-ADP-agent(percept)
   returns an action
   input: percept, indicating current state s′, reward r′
   static:
       π, fixed policy
       mdp, MDP with P[s′|s,a], R(s), γ
       U, table of utilities, initially empty
       N_sa, table of freq. for s-a pairs, initially 0
       N_sas′, table of freq. for s-a-s′ triples, initially 0
       s,a, previous state and action, initially null
   if s′ is new then
       U[s′] ← r′
       R[s′] ← r′
   if s is not null then
       increment N_sa[s,a] and N_sas′[s,a,s′]
       for each t s.t. N_sas′[s,a,t] ≠ 0 do
           P[t|s,a] ← N_sas′[s,a,t] / N_sa[s,a]
   U ← Policy-evaluation(π,U,mdp)
   if Terminal?(s′) then
       s,a ← null
   else
       s,a ← s′,π[s′]
   return a
```

ADP: Problem

- Need to solve a system of simultaneous equations – costs $O(n^3)$
  - Very hard to do if you have 10^{50} states like in Backgammon
  - Could make things a little easier with modified policy iteration
- Can the agent avoid the computational expense of full policy evaluation?

Temporal Difference Learning (TD)

- Instead of calculating the exact utility for a state, can the agent approximate it and possibly make it less computationally expensive?
- Yes, it can! Using TD:

$$U^{\pi}(s) = R(s) + \gamma \sum_{s' \in \mathrm{dom}(S)} P(s' | \pi(s), s) U^{\pi}(s')$$

  - Instead of doing the sum over all successors, only adjust the utility of the state based on the successor observed in the trial
  - Does not estimate the transition model – model-free

TD: Example

- Suppose you see that $U^{\pi}(1,3) = 0.84$ and $U^{\pi}(2,3) = 0.92$
- If the transition $(1,3) \rightarrow (2,3)$ happens all the time, you would expect to see:

$$U^{\pi}(1,3) = R(1,3) + U^{\pi}(2,3)$$
$$\Rightarrow U^{\pi}(1,3) = -0.04 + U^{\pi}(2,3)$$
$$\Rightarrow U^{\pi}(1,3) = -0.04 + 0.92 = 0.88$$

- Since you observe $U^{\pi}(1,3) = 0.84$ in the first trial and it is a little lower than 0.88, so you might want to “bump” it towards 0.88

Aside: Online Mean Estimation

- Suppose that we want to incrementally compute the mean of a sequence of numbers
  - E.g., to estimate the mean of a random variable from a sequence of samples

$$\underbrace{\hat{X}_{n+1}}_{\text{average of } n+1 \text{ samples}} = \frac{1}{n+1}\sum_{i=1}^{n+1} x_i = \left(\frac{1}{n+1}\sum_{i=1}^{n} x_i\right) + \frac{1}{n+1}x_{n+1} = \left(\frac{n}{n(n+1)}\sum_{i=1}^{n} x_i\right) + \frac{1}{n+1}x_{n+1}$$

$$= \left(\frac{n+1-1}{n(n+1)}\sum_{i=1}^{n} x_i\right) + \frac{1}{n+1}x_{n+1} = \left(\frac{n+1}{n(n+1)}\sum_{i=1}^{n} x_i\right) - \left(\frac{1}{n(n+1)}\sum_{i=1}^{n} x_i\right) + \frac{1}{n+1}x_{n+1}$$

$$= \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) - \left(\frac{1}{(n+1)} \cdot \frac{1}{n}\sum_{i=1}^{n} x_i\right) + \frac{1}{n+1}x_{n+1} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) + \frac{1}{n+1}\left(x_{n+1} - \frac{1}{n}\sum_{i=1}^{n} x_i\right)$$

$$= \hat{X}_n + \underbrace{\frac{1}{n+1}}_{\text{learning rate}}\left(\underbrace{x_{n+1}}_{\text{sample } n+1} - \hat{X}_n\right)$$

Given a new sample x_{n+1}, the new mean is the old estimate (for n samples) plus the weighted difference between the new sample and old estimate

TD Update

- TD update for transition from s to s'

$$U^\pi(s) = U^\pi(s) + \underset{\text{learning rate}}{\alpha}\underbrace{\left(R(s) + \gamma U^\pi(s') - U^\pi(s)\right)}_{\text{new (noisy) sample of utility based on next state}}$$

  - Similar to one step of value iteration
  - Equation called backup
- So, the update is maintaining a "mean" of the (noisy) utility samples
- If the learning rate decreases with the number of samples (e.g., $1/n$), then the utility estimates will eventually converge to true values

$$U^\pi(s) = R(s) + \gamma \sum_{s' \in \mathrm{dom}(S)} P(s'|\pi(s), s) U^\pi(s')$$

TD: Convergence

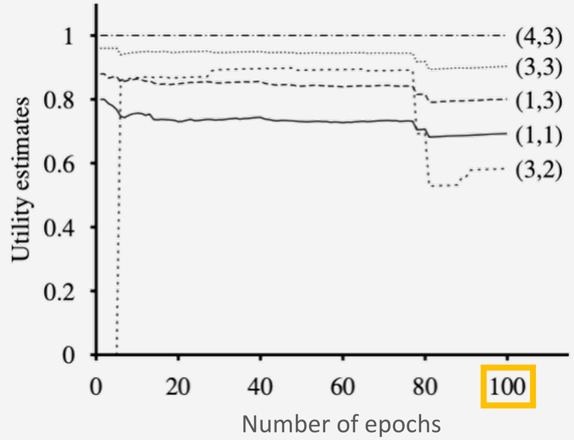
- Since TD uses the observed successor s' instead of all the successors, what happens if the transition $s \longrightarrow s'$ is very rare and there is a big jump in utilities from s to s'?
  - How can $U^\pi(s)$ converge to the true equilibrium value?
- Answer:
  The average value of $U^\pi(s)$ will converge to the correct value
  - This means the agent needs to observe enough trials that have transitions from s to its successors
  - Essentially, the effects of the TD backups will be averaged over a large number of transitions
  - Rare transitions will be rare in the set of transitions observed

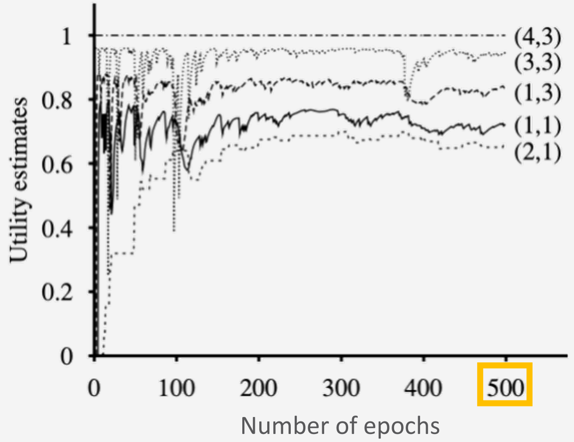
Comparison between ADP and TD

- Advantages of ADP
  - Converges to true utilities in fewer iterations
  - Utility estimates do not vary as much from the true utilities
- Advantages of TD
  - Simpler, less computation per observation
  - Crude but efficient first approximation to ADP
  - Do not need to build a transition model to perform its updates

ADP and TD

- Utility estimates for 4x3 grid
  - ADP, given optimal policy (above)
    - Notice the large changes occurring around the 78th trial—this is the first time that the agent falls into the −1 terminal state at (4,2)
  - TD (below)
    - More epochs required
    - Faster runtime per epoch





Figures: AIMA, Russell/Norvig

Overall comparisons

- DUE (model-free)
  - Simple to implement
  - Each update is fast
  - Does not exploit Bellman constraints and converges slowly
- ADP (model-based)
  - Harder to implement
  - Each update is a full policy evaluation (expensive)
  - Fully exploits Bellman constraints
  - Fast convergence (in terms of epochs)
- TD (model-free)
  - Update speed and implementation similar to direct estimation
  - Partially exploits Bellman constraints – adjusts state to “agree” with observed successor
    - Not all possible successors
  - Convergence in between DUE and ADP

Passive Learning: Disadvantage

- Learning $U^{\pi}(s)$ does not lead to an optimal policy, why?
  - Only evaluated π (no optimisation)
  - Models are incomplete/inaccurate
  - Agent has only tried limited actions, cannot gain a good overall understanding of $P(s'|s, a)$
- Solution: Active learning

Goal of Active Learning

- Assume that the agent still has access to some sequence of trials performed by the agent
  - Agent is not following any specific policy
  - Assume for now that the sequences should include a thorough exploration of the space
  - We will talk about how to get such sequences later

- The goal is to learn an optimal policy from such sequences
  - Active RL agents
    - Active ADP agent
    - Q-learner (based on TD algorithm)

Active ADP Agent

- Model-based approach
- Using the data from its trials, agent estimates a transition model $\hat{T}$ and a reward function $\hat{R}$
  - With $\hat{T}(s, a, s')$ and $\hat{R}(s)$, it has an estimate of the underlying MDP
  - Like passive ADP using policy evaluation
- Given estimate of the MDP, it can compute the optimal policy by solving the Bellman equations using value or policy iteration

$$U(s) = \hat{R}(s) + \gamma \max_{a \in A(s)} \sum_{s' \in \mathrm{dom}(S)} \hat{T}(s, a, s') U(s')$$

- If $\hat{T}$ and $\hat{R}$ are accurate estimations of the underlying MDP model, agent can find the optimal policy this way

Issues with ADP Approach

- Need to maintain MDP model
- T can be very large, $O(|S|^2 \cdot |A|)$
- Also, finding the optimal action requires solving the Bellman equation – time consuming

- Can the agent avoid this large computational complexity both in terms of time and space?

Q-learning

- So far, focus on utilities for states
  - $U(s)$ = utility of state s = expected maximum future rewards
- Alternative: store Q-values
  - $Q(a, s)$ = utility of taking action a at state s
= expected maximum future reward if action a taken at state s

- Relationship between $U(s)$ and $Q(a, s)$?

$$\boxed{U(s) = \max_{a \in A(s)} Q(a, s)}$$

Q-learning can be model-free

- Note that after computing $U(s)$, to obtain the optimal policy, the agent needs to compute
$$\pi(s) = \underset{a \in A(s)}{\operatorname{argmax}} \sum_{s' \in \operatorname{dom}(S)} T(s, a, s')U(s')$$
  - Requires T, model of the world
  - Even if it uses TD learning (model-free), it still needs the model to get the optimal policy
- However, if the agent successfully estimates $Q(a, s)$ for all a and s, it can compute the optimal policy without using the model
$$\pi(s) = \underset{a \in A(s)}{\operatorname{argmax}} Q(a, s)$$

Q-learning

- At equilibrium when Q-values are correct, we can write the constraint equation:

$$\underbrace{Q(a,s)}_{\text{Expected value for action-state pair } (a,s)} = \underbrace{R(s)}_{\text{Reward at state } s} + \gamma \underbrace{\sum_{s' \in \mathrm{dom}(S)} T(s,a,s')U(s')}_{\text{Expected value averaged over all possible states } s' \text{ that can be reached from } s \text{ after executing action } a}$$

Q-learning

- At equilibrium when Q-values are correct, we can write the constraint equation:

$$\underbrace{Q(a,s)}_{\text{Expected value for action-state pair } (a,s)} = \underbrace{R(s)}_{\text{Reward at state } s} + \gamma \sum_{s' \in \mathrm{dom}(S)} T(s,a,s') \max_{a' \in A(s')} Q(a',s')$$

Expected value averaged over all possible states s' that can be reached from s after executing action a

Best value at the next state = max over all actions in state s'

Q-learning without a Model

- Q-update: after moving from s to state s' using action a

$$\underbrace{Q(a,s)}_{\substack{\text{New estimate}\\ \text{of } Q(a,s)}} \leftarrow \underbrace{Q(a,s)}_{\substack{\text{Old estimate}\\ \text{of } Q(a,s)}} + \underbrace{\alpha}_{\substack{\text{Learning rate}\\ 0<\alpha<1}} \Big(\underbrace{R(s) + \gamma \max_{a'\in A(s')} Q(a',s') - Q(a,s)}_{\substack{\text{Difference between old}\\ \text{estimate } Q(a,s) \text{ and the}\\ \text{new noisy sample after}\\ \text{taking action } a}}\Big)$$

  - TD approach
  - Transition model does not appear anywhere!
  - Once converged, optimal policy can be computed without transition model
    - Completely model-free learning algorithm

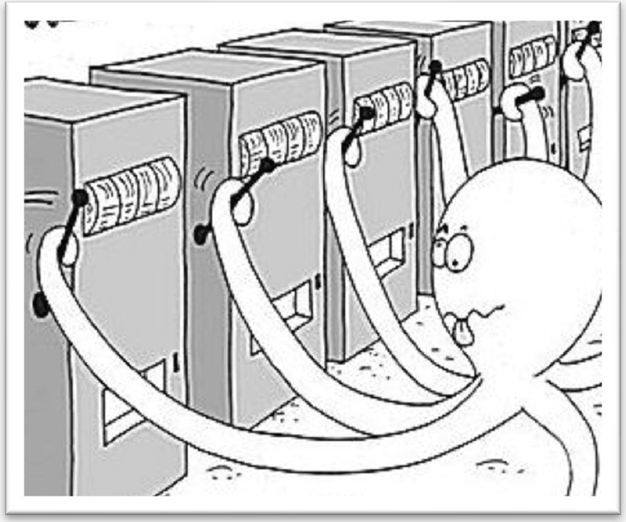
Q-learning: Convergence

- Guaranteed to converge to true Q-values given enough exploration
- Very general procedure
  - Because it is model-free
- Converges slower than ADP agent
  - Because it is completely model-free and it does not enforce consistency among values through the model

Exploitation vs. Exploration

- Actions are always taken for one of the two following purposes
  - Exploitation: Execute the current optimal policy to get high payoff
  - Exploration: Try new sequences of (possibly random) actions to improve the agent’s knowledge of the environment even though current model does not show they have a high payoff
- Pure exploitation: gets stuck in a rut
- Pure exploration: not much use if you do not put that knowledge into practice

Multi-Arm Bandit Problem

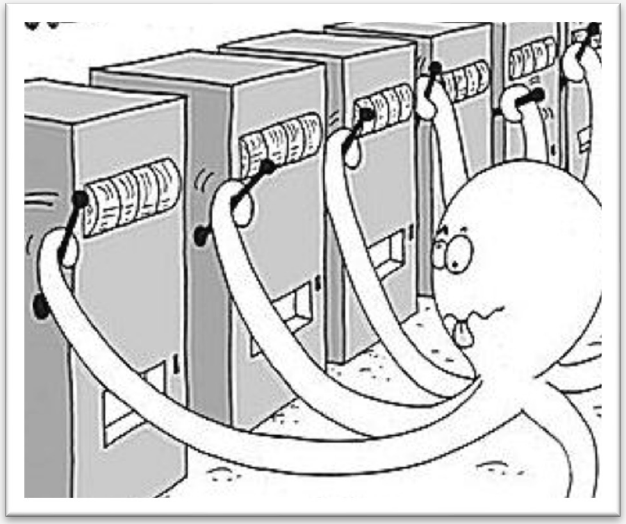


- So far, we assumed that the agent has a set of epochs of sufficient exploration
  - Problem: How to get a set of epochs that sufficiently explores the state space?
- Multi-arm bandit problem:
  Statistical model of sequential experiments
  - Name comes from a traditional slot machine (one-armed bandit)
- Question:
  Which machine to play?

Actions

- n arms, each with a fixed but unknown distribution of reward
  - In terms of actions: Multiple actions $a_1, a_2, \dots, a_n$
    - Each a_i provides a reward from an unknown (but stationary) probability distribution p_i
    - Specifically, expectation μ_i of machine i's reward unknown
      - If all μ_i's were known, then the task is easy: just pick $\operatorname*{argmax}_i \mu_i$

- With μ_i's unknown, question is which arm to pull

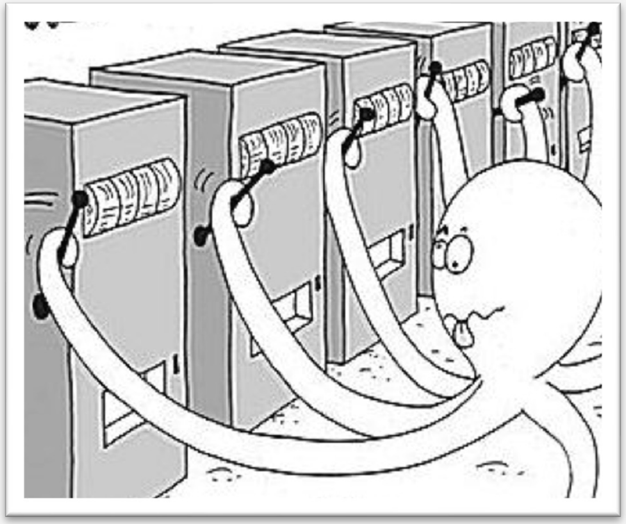


Formal Model

- At each time step $t = 1, 2, \ldots, T$:
  - Each machine i has a random reward $X_i^{(t)}$
    - $E\left[X_i^{(t)}\right] = \mu_i$ independent of the past (Markov property again)
  - Pick a machine I_t and get reward $X_{I_t}^{(t)}$
  - Other machines' rewards hidden
- Over T time steps, the agent has a total reward of $\sum_{t=1}^{T} X_{I_t}^{(t)}$
  - If all μ_i's known, it would have selected $\operatorname{argmax}_i \mu_i$ at each time t
    - Expected total reward $T \cdot \max_i \mu_i$
- Agent's *regret*: $\underbrace{T \cdot \max_i \mu_i}_{\text{best machine's reward (in expectation)}} - \underbrace{\sum_{t=1}^{T} X_{I_t}^{(t)}}_{\text{agent's reward}}$

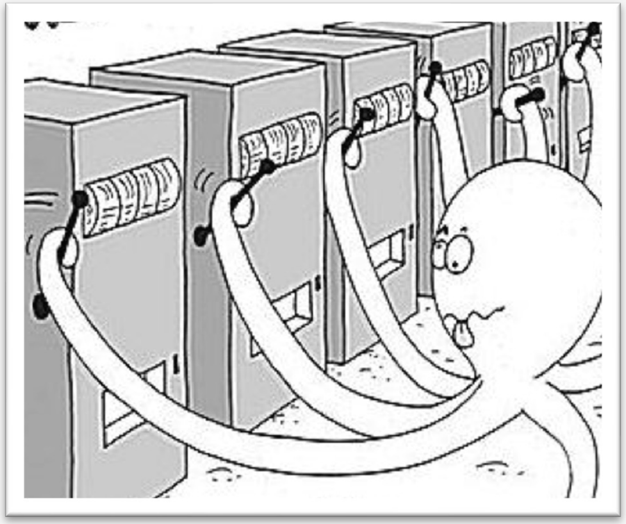
Exploitation vs. Exploration Reprise

- Exploration: to find the best
  - Overhead: big loss when trying bad arms
- Exploitation: to exploit what the agent has discovered
  - Weakness: there may be better arms that it has not explored and identified
- *Question:*
  With a fixed budget, how to balance exploration and exploitation such that the total loss (or regret) is small?



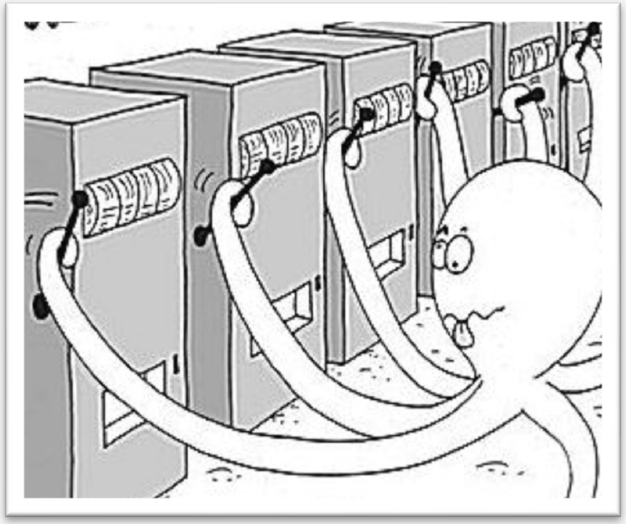
Where Does the Loss Come from?

- If μ_i is small, trying this arm too many times makes a big loss
  - So, the agent should try it less if it finds the previous samples from it are bad
- But how to know whether an arm is good?
- The more the agent tries an arm i, the more information it gets about its distribution
  - In particular, the better estimate to its mean μ_i



Where Does the Loss Come from?

- So, the agent wants to estimate each μ_i precisely, and at the same time, it does not want to try bad arms too often
  - Two competing tasks
    - Exploration vs. exploitation dilemma
- Rough idea: the agent tries an arm if
  - Either
    it has not tried it often enough
  - Or
    its estimate of μ_i so far is high



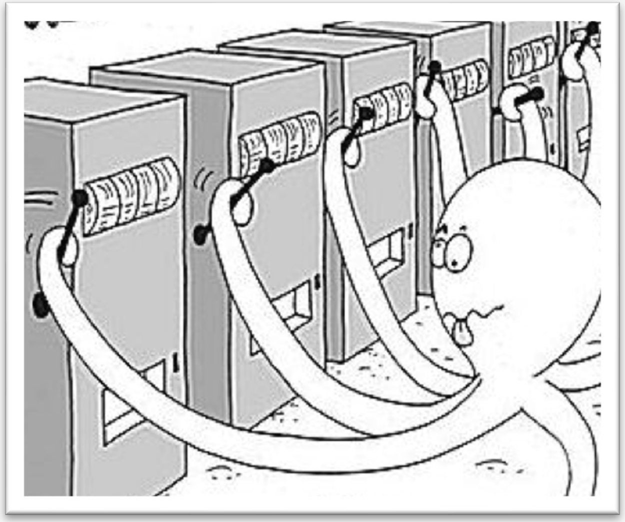
UCB (Upper Confidence Bound) Algorithm

- Input: Set of actions A
- Assume rewards between 0 and 1
  - If they are not, normalise them
- For each action a_i , let
  - r_i = average reward from a_i
  - t_i = number of times a_i tried
- $t = \sum_i t_i$
- Confidence interval around r_i

r_i

$r_i + \sqrt{\frac{2 \ln t}{t_i}}$

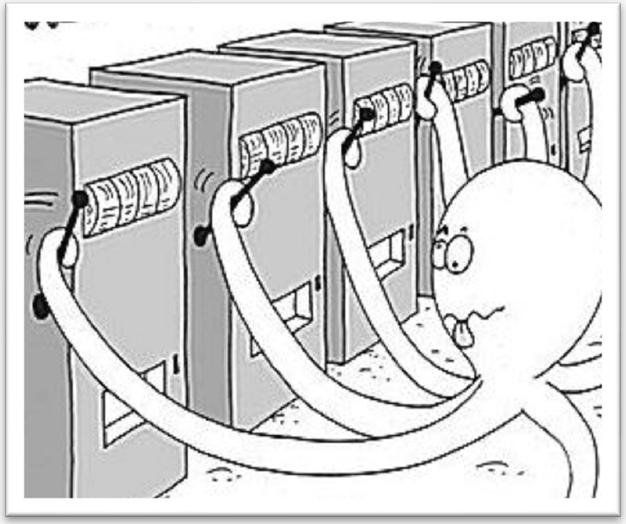
```
UCB(A)
    Try each action a_i once
    loop
        choose an action a_i that has
            the highest value of r_i + √(2·ln(t)/t_i)
        perform a_i
        update r_i , t_i , t
```



UCB: Performance

- Uses principle of optimism in face of uncertainty
  - Agent does not have a good estimate $\hat{\mu}_i$ of μ_i before trying it many times
    - Thus, give a big confidence interval $[-c_i, c_i]$ for such i
      - $c_i = \sqrt{\frac{2 \ln t}{t_i}}$
    - And select an i with maximum $\mu_i + c_i$

$$r_i \qquad r_i + \sqrt{\frac{2 \ln t}{t_i}}$$



  - If an action has not been tried many times, then the big confidence interval makes it still possible to be tried
  - I.e., in face of uncertainty (of μ_i), the agent acts optimistically by giving chances to those that have not been tried enough

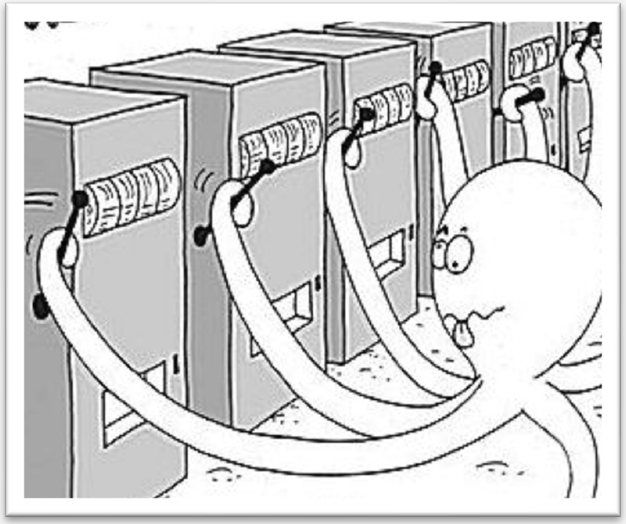
UCB: Performance

- Theorem: If each distribution of reward has support in $[0,1]$, i.e., rewards are normalised, then the regret of the UCB algorithm is at most

$$O\left(\sum_{i:\mu_i<\mu^*} \frac{\ln T}{\Delta_i} + \sum_{j\in\{1,\dots,n\}} \Delta_j\right)$$

  - $\mu^* = \max_{\mathrm{i}} \mu_i$
  - $\Delta_i = \mu^* - \mu_i$
    - Expected loss of choosing a_i once
  - [without proof]

- Loss grows very slowly with T



Universität Münster

UCT Algorithm for Cost-based Planning

- Recursive UCB computation to compute $Q(s,a)$ for *cost*
  - Min ops instead of max
  - Planning domain Σ, state s
  - Horizon h (steps into the future)
  - Constant C:
    - Relative weight of exploration of less sampled actions (C high) to exploitation of promising actions (C low)
    - Empirical tuning significantly affects performance of UCT
- Anytime algorithm:
  - Call repeatedly until time runs out
  - Then choose action $\text{argmin}_a\, Q(s,a)$

```
UCT(Σ,s,h)
    if s ∈ S then
        return 0
    if h = 0 then
        return V_0(s)
    if s ∉ Envelope then
        add s to Envelope
        n(s) ← 0
        for all a ∈ Applicable(s) do
            Q(s,a) ← 0
            n(s,a) ← 0
    Untried ← {a ∈ Applicable(s)| n(s,a)=0}
    if Untried ≠ ∅ then
        ã ← Choose(Untried)
    else
        ã ← argmin_{a∈Applicable(s)}
            {Q(s,a)-C·[log(n(s))/n(s,a)]^½}
    s' ← Sample(Σ,s,ã)
    cost-rollout ← cost(s,ã) + UCT(s',h-1)
    Q(s,ã) ← [n(s,ã)·Q(s,ã)+cost-rollout]
             /(1+n(s,ã))
    n(s) ← n(s) + 1
    n(s,ã) ← n(s,ã) + 1
    return cost-rollout
```

UCT as an Acting Procedure

- Suppose probabilities and costs unknown
- Suppose you can restart your actor as many times as you want
- Can modify UCT to be an acting procedure
  - Use it to explore the environment

```
UCT(Σ,s,h)
    if s ∈ S then
        return 0
    if h = 0 then
        return V0(s)
    if s ∉ Envelope then
        add s to Envelope
        n(s) ← 0
        for all a ∈ Applicable(s) do
            Q(s,a) ← 0
            n(s,a) ← 0
    Untried ← {a ∈ Applicable(s)| n(s,a)=0}
    if Untried ≠ ∅ then
        ã ← Choose(Untried)
    else
        ã ← argmin a∈Applicable(s)
              {Q(s,a)−C·[log(n(s))/n(s,a)]^½}
    s' ← Sample(Σ,s,ã)
    cost−rollout ← cost(s,ã) + UCT(s',h-1)
    Q(s,ã) ← [n(s,ã)·Q(s,ã)+cost-rollout]
                /(1+n(s,ã))
    n(s) ← n(s) + 1
    n(s,ã) ← n(s,ã) + 1
    return cost-rollout
```

perform $\tilde{a}$; observe s'

UCT as a Learning Procedure

- Suppose probabilities and costs unknown
  - But you have an accurate simulator for the environment
- Run UCT multiple times in the simulated environment
  - Learn what actions work best

```
UCT(Σ,s,h)
    if s ∈ S then
        return 0
    if h = 0 then
        return V0(s)
    if s ∉ Envelope then
        add s to Envelope
        n(s) ← 0
        for all a ∈ Applicable(s) do
            Q(s,a) ← 0
            n(s,a) ← 0
    Untried ← {a ∈ Applicable(s)| n(s,a)=0}
    if Untried ≠ ∅ then
        ã ← Choose(Untried)
    else
        ã ← argmin_{a∈Applicable(s)}
              {Q(s,a)-C·[log(n(s))/n(s,a)]^½}
    s' ← Sample(Σ,s,ã)
    cost-rollout ← cost(s,ã) + UCT(s',h-1)
    Q(s,ã) ← [n(s,ã)·Q(s,ã)+cost-rollout]
                /(1+n(s,ã))
    n(s) ← n(s) + 1
    n(s,ã) ← n(s,ã) + 1
    return cost-rollout
```

simulate $\tilde{a}$; observe s'

Intermediate Summary

- Passive learning
  - DUE
  - ADP
  - TD
- Active learning
  - Active ADP
  - Q-learning
- Multi-armed bandit problem
  - UCB, UCT

Outline: Decision Making – Foundations

Utility Theory

- Preferences
- Utilities
- Preference structure

Markov Decision Process / Problem (MDP)

- Sequence of actions, history, policy
- Value iteration, policy iteration

Reinforcement Learning (RL)

- Passive and active, model-free and model-based RL
- Multi-armed bandit

$\Longrightarrow$ Next: Decision Making – Extensions