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# EXPLOITING STRUCTURE IN DECISION MAKING UNDER THE LENS OF RECENT ADVANCES IN STARAI

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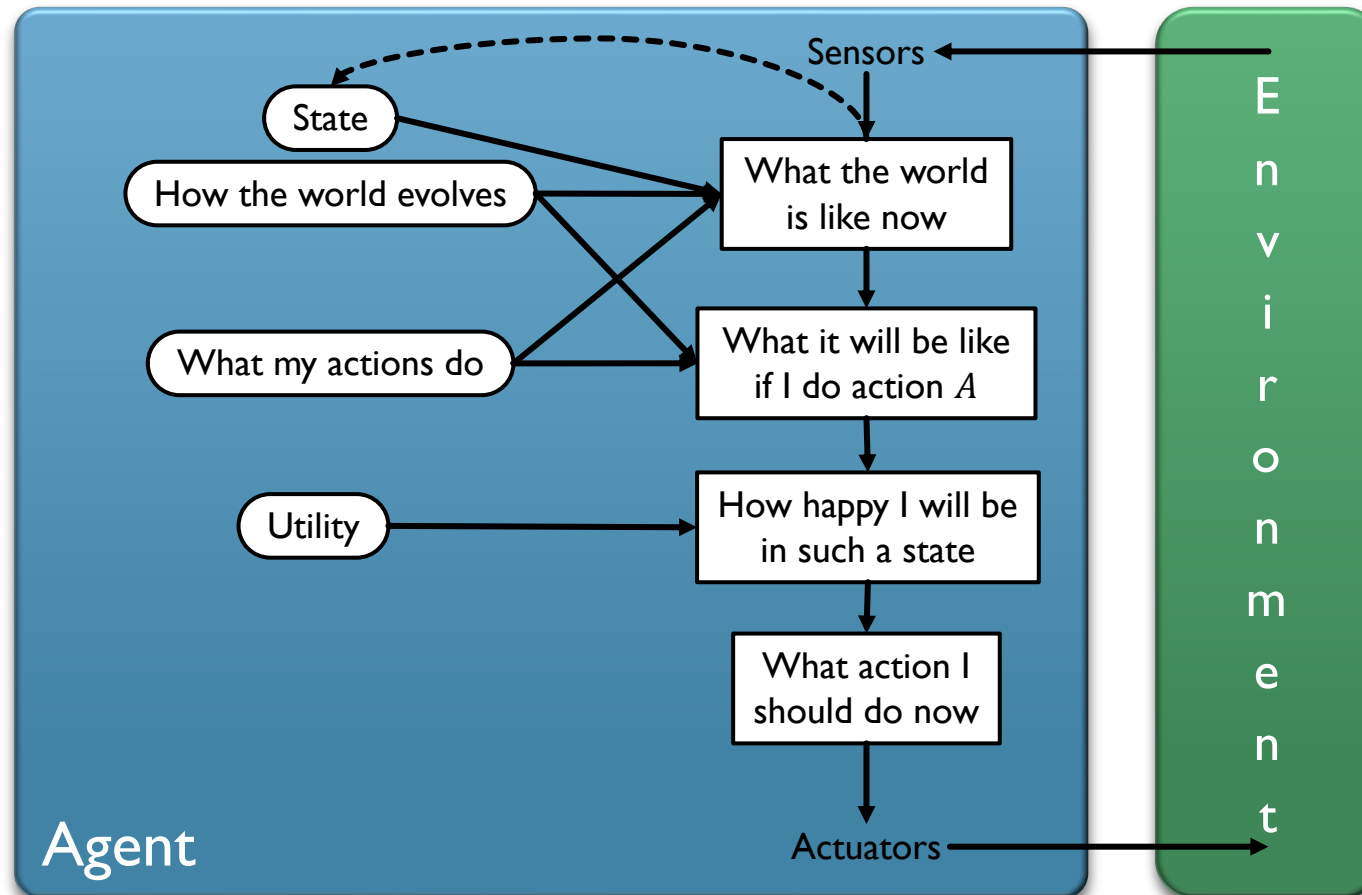
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# AGENDA

1. Introduction to Relational Models and Online Decision Making [Marcel]
  - Relational models under uncertainty
  - Lifted inference in decision-theoretic models (online decision making)
  - Markov Decision Process (Offline decision making)
2. Lifting Offline Decision Making [Flo]
3. Lifting Multi-Agent Decision Making [Nazlı Nur]
4. Summary [Marcel]

# GENERAL AGENT SETTING

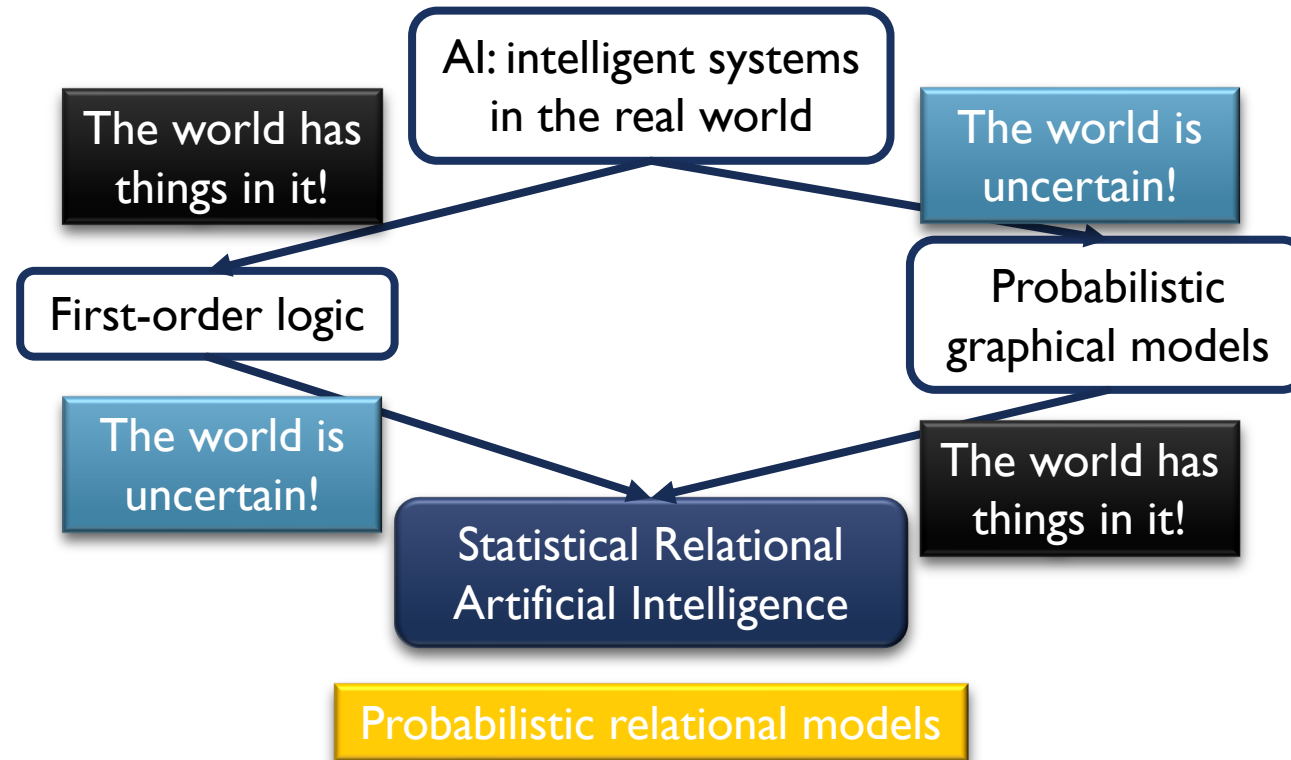




# RELATIONAL MODELS UNDER UNCERTAINTY

## INTRODUCTION

# WHY RELATIONAL MODELS?



# LOGICAL VARIABLES IN RANDOM VARIABLES

- Atoms: Parameterised random variables = PRVs

- With logical variables

- E.g.,  $X, M$
    - Possible values (domain):

$dom(X) = \{alice, eve, bob\}$   
 $dom(M) = \{injection, tablet\}$

$Nat(D) = \text{natural disaster } D$   
 $Acc(A) = \text{accident } A$

- With range

- E.g., Boolean, but any discrete, finite set possible
    - $ran(Travel(X)) = \{true, false\}$

- Represent sets of *indistinguishable* random variables

$Nat(D)$

$Acc(A)$

$Epid$

$Travel(X)$

$Treat(X, M)$

$Sick(X)$

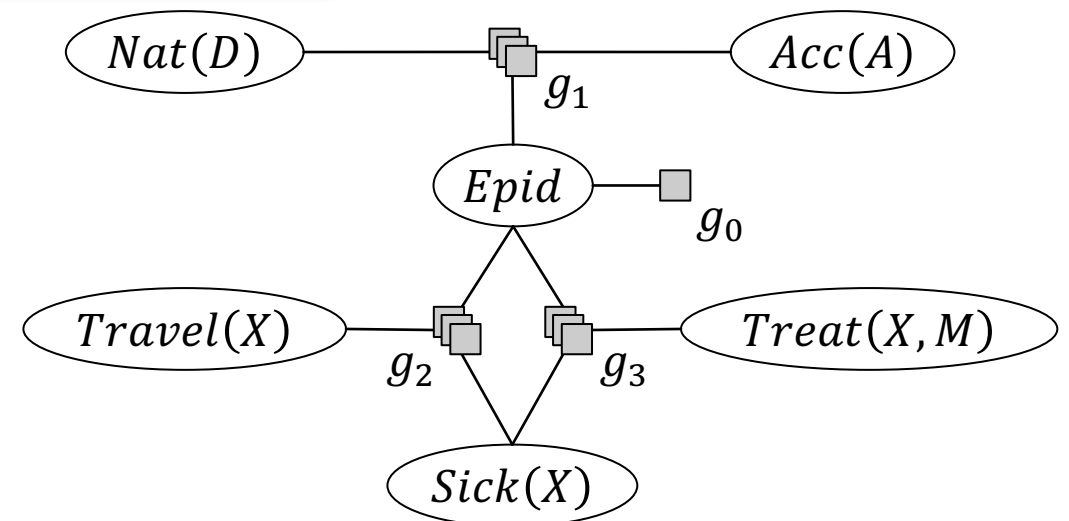
# PARFACTORS

- Factors with PRVs = parfactors

<i>Travel(X)</i>	<i>Epid</i>	<i>Sick(X)</i>	$g_2$
<i>false</i>	<i>false</i>	<i>false</i>	5
<i>false</i>	<i>false</i>	<i>true</i>	0
<i>false</i>	<i>true</i>	<i>false</i>	4
<i>false</i>	<i>true</i>	<i>true</i>	6
<i>true</i>	<i>false</i>	<i>false</i>	4
<i>true</i>	<i>false</i>	<i>true</i>	6
<i>true</i>	<i>true</i>	<i>false</i>	2
<i>true</i>	<i>true</i>	<i>true</i>	9

## Potentials

- In parfactors, just like in factors, no probability distribution as factors required



# FACTORS

- Grounding

<i>Travel(X)</i>	<i>Epid</i>	<i>Sick(X)</i>	$g_2$
<i>false</i>	<i>false</i>	<i>false</i>	5
<i>false</i>	<i>false</i>	<i>true</i>	0
<i>false</i>	<i>true</i>	<i>false</i>	4
<i>false</i>	<i>true</i>	<i>true</i>	6
<i>true</i>	<i>false</i>	<i>false</i>	4
<i>true</i>	<i>false</i>	<i>true</i>	6
<i>true</i>	<i>true</i>	<i>false</i>	2
<i>true</i>	<i>true</i>	<i>true</i>	9

<i>Travel(eve)</i>	<i>Epid</i>	<i>Sick(eve)</i>	$g_2$
<i>false</i>	<i>false</i>	<i>false</i>	5
<i>false</i>	<i>false</i>	<i>true</i>	0
<i>false</i>	<i>true</i>	<i>false</i>	4
<i>false</i>	<i>true</i>	<i>true</i>	6
<i>true</i>	<i>false</i>	<i>false</i>	4
<i>true</i>	<i>false</i>	<i>true</i>	6
<i>true</i>	<i>true</i>	<i>false</i>	2
<i>true</i>	<i>true</i>	<i>true</i>	9

<i>Travel(bob)</i>	<i>Epid</i>	<i>Sick(bob)</i>	$g_2$
<i>false</i>	<i>false</i>	<i>false</i>	5
<i>false</i>	<i>false</i>	<i>true</i>	0
<i>false</i>	<i>true</i>	<i>false</i>	4
<i>false</i>	<i>true</i>	<i>true</i>	6
<i>true</i>	<i>false</i>	<i>false</i>	4
<i>true</i>	<i>false</i>	<i>true</i>	6
<i>true</i>	<i>true</i>	<i>false</i>	2
<i>true</i>	<i>true</i>	<i>true</i>	9

<i>Travel(alice)</i>	<i>Epid</i>	<i>Sick(alice)</i>	$g_2$
<i>false</i>	<i>false</i>	<i>false</i>	5
<i>false</i>	<i>false</i>	<i>true</i>	0
<i>false</i>	<i>true</i>	<i>false</i>	4
<i>false</i>	<i>true</i>	<i>true</i>	6
<i>true</i>	<i>false</i>	<i>false</i>	4
<i>true</i>	<i>false</i>	<i>true</i>	6
<i>true</i>	<i>true</i>	<i>false</i>	2
<i>true</i>	<i>true</i>	<i>true</i>	9

*creat(X, M)*



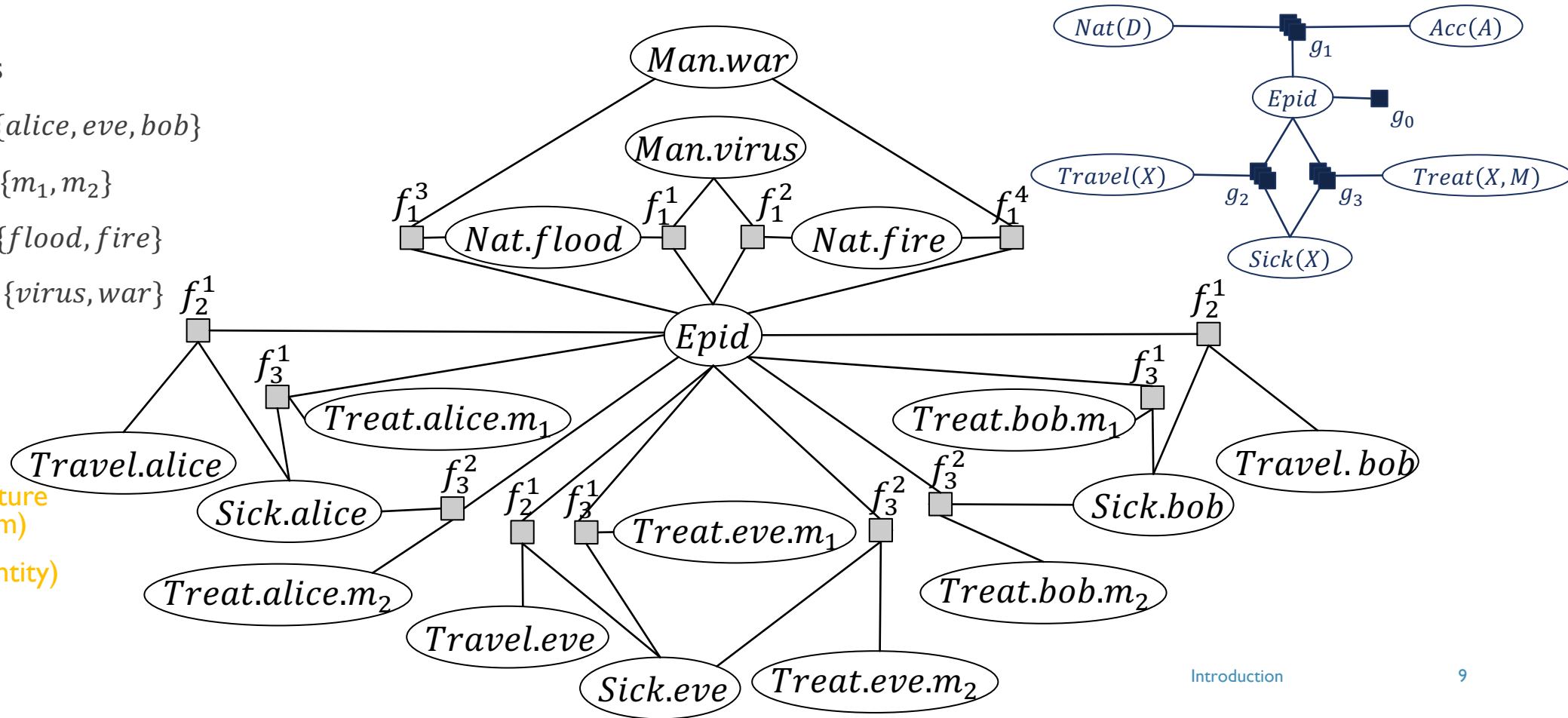
# GROUNDED MODEL

- Given domains

- $dom(X) = \{alice, eve, bob\}$
- $dom(M) = \{m_1, m_2\}$
- $dom(D) = \{flood, fire\}$
- $dom(W) = \{virus, war\}$

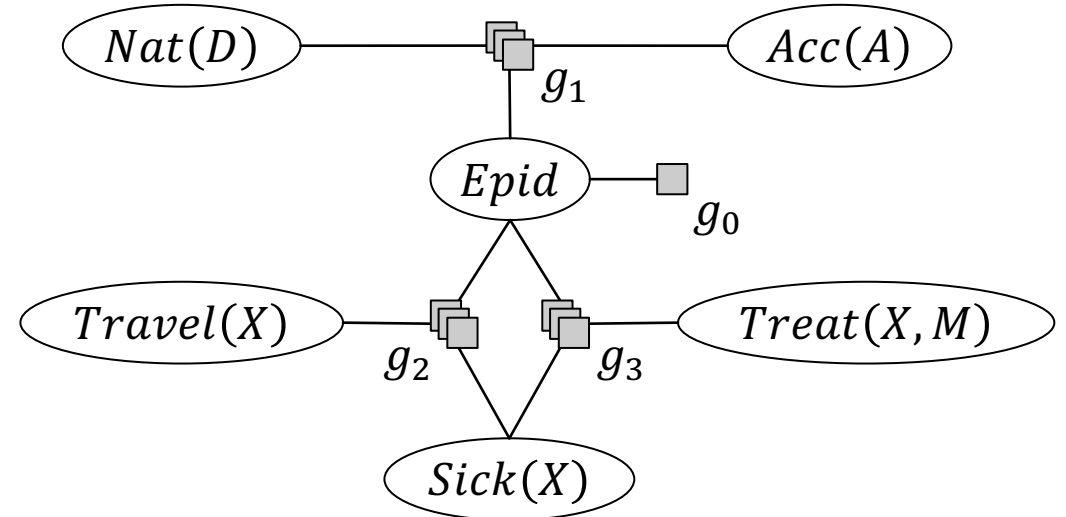
- Symmetry in

- Graph structure (isomorphism)
- Factors (identity)



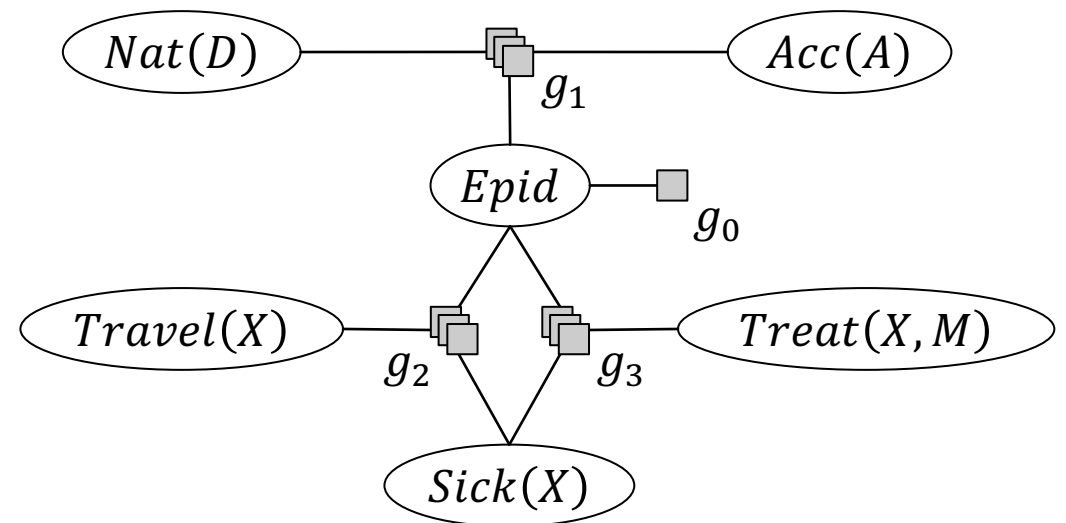
# INFERENCE PROBLEMS WITH AND WITHOUT EVIDENCE

- Query answering problem given a model:
  - Probability of events
    - E.g.,  $P(\text{Att}(\text{eve}, \text{ki}) = \text{true}), P(\text{Epid} = \text{true})$
  - Conditional (marginal) probability distributions
    - E.g.,  $P(\text{Att}(\text{ev}, \text{ki}) | \text{FarAway}(\text{ki})), P(\text{Epid} | \text{sick}(\text{alice}), \text{sick}(\text{eve}))$
  - Assignment queries:
    - Most probable states of random variables
    - Most-probable explanation (MPE), Maximum a posteriori (MAP)
- **Lifted inference:**  
Work with representatives for exchangeable random variables
  - Avoid grounding for as long as possible



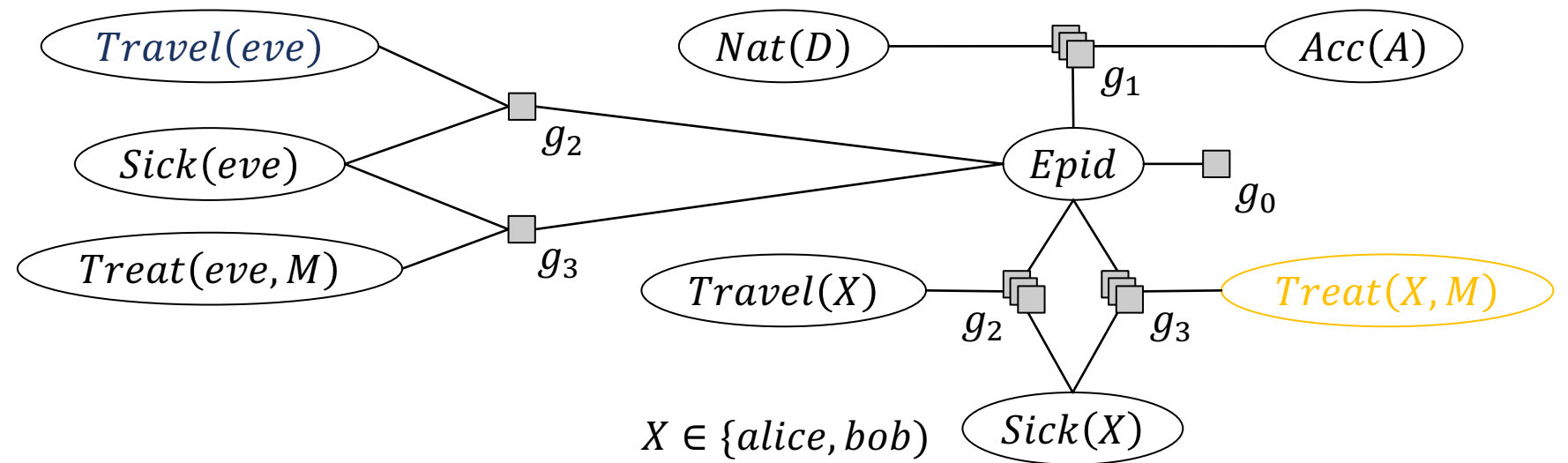
# QA IN PARFACTOR MODELS: LIFTED VARIABLE ELIMINATION (LVE)

- Eliminate all variables not appearing in query
  - [Poole 03, de Salvo Braz et al. 05, 06, Milch et al. 08, Taghipour et al. 13, 13a, B & Möller 18]
- Lifted summing out
  - Sum out *representative* instance as in propositional variable elimination
  - Exponentiate result for exchangeable instances
- Correctness: Equivalent ground operation
  - Each instance is summed out
  - Result: factor  $f$  that is identical for all instance
  - Multiplying indistinguishable results  
→ exponentiation of one representative  $f$



# QA: LVE IN DETAIL

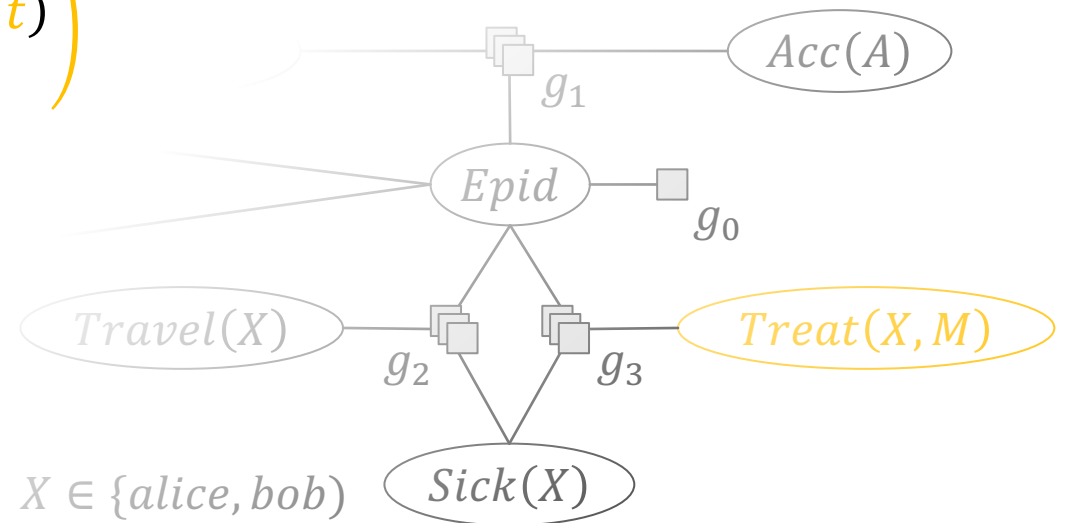
- Eliminate  $Treat(X, M)$ 
  - Appears in only one  $g: g_3$
  - Contains all logical variables of  $g_3: X, M$
  - For each  $X$  constant: the same number of  $M$  constants
- ✓ Preconditions of lifted summing out fulfilled, lifted summing out possible



# LVE IN DETAIL: LIFTED SUMMING OUT

- Eliminate  $Treat(X, M)$  by lifted summing out
  - Sum out representative

$$\left( \sum_{t \in r(Treat(X, M))} g_3(Epid = e, Sick(X) = s, Treat(X, M) = t) \right)^{\#M|X}$$



# LVE IN DETAIL: LIFTED SUMMING OUT

$$\left( \sum_{t \in r(\text{Treat}(X,M))} g_3(\text{Epid} = e, \text{Sick}(X) = s, \text{Treat}(X,M) = t) \right)^{\#M|X}$$

<i>Epid</i>	<i>Sick(X)</i>	<i>Treat(X,M)</i>	$g_3$
false	false	false	9
false	false	true	1
false	true	false	6
false	true	true	3
true	false	false	7
true	false	true	5
true	true	false	4
true	true	true	8

<i>Epid</i>	<i>Sick(X)</i>	$\Sigma$
false	false	10
false	true	9
true	false	12
true	true	12

<i>Epid</i>	<i>Sick(X)</i>	$\wedge$
false	false	$10^2$
false	true	$9^2$
true	false	$12^2$
true	true	$12^2$

# SYMMETRIES WITHIN

- Assume four epidemics with identical characteristics
  - $Epid_1, Epid_2, Epid_3, Epid_4$
  - Reasonable to model the epidemics such that it does not matter which  $Epid$  variables specifically are *true* or *false*, i.e., they are interchangeable

- All *false* maps to 8
- 1 *true*, 3 *false* maps to 6
- 2 *true*, 2 *false* maps to 4
- 3 *true*, 1 *false* maps to 2
- All *true* maps to 0
  - Five lines enough to describe

	# <i>true</i>	# <i>false</i>	
[0,4]	0	4	8
[1,3]	1	3	6
[2,2]	2	2	4
[3,1]	3	1	2
[4,0]	4	0	0

$Epid_1$	$Epid_2$	$Epid_3$	$Epid_4$	$\phi$
false	false	false	false	8
false	false	false	true	6
false	false	true	false	6
false	false	true	true	4
false	true	false	false	6
false	true	false	true	4
false	true	true	false	4
false	true	true	true	2
true	false	false	false	6
true	false	false	true	4
true	false	true	false	4
true	false	true	true	2
true	true	false	false	4
true	true	false	true	2
true	true	true	false	2
true	true	true	true	0

# COUNTING RANDOM VARIABLE

- New PRV type:  
(Parameterised) counting random variable ((P)CRV)

$$\#_X[A|C]$$

- $A|C$  a PRV under constraint  $C$
- $X \in lv(A)$
- Range values: Histogram  $h = \{(v_i, n_i)\}_{i=1}^m$ 
  - $m = |ran(A)|$  (number of buckets)
  - $n = \sum_{i=1}^m n_i = |gr(A|_{\pi_X(C)})|$  (number of instances to distribute into buckets)
  - $v_i \in ran(A)$  (buckets)
  - $n_i \in \mathbb{N}$  (number of instances in bucket  $v_i$ )
  - Shorthand:  $[n_1, \dots, n_m]$

- Range of a (P)CRV = space of histograms fulfilling the conditions on the histograms

- (All possible ways of distributing  $n$  interchangeable instances into  $m$  buckets)

- Single histogram encodes several interchangeable assignments at once

- Given by multinomial coefficient  $Mul(h)$

$$Mul(h) = \frac{n!}{\prod_{i=1}^m n_i!}$$

- If  $m = 2$ , binomial coefficient:

$$\binom{n}{n_1} = \frac{n!}{(n - n_1)! n_1!} = \frac{n!}{n_2! n_1!}$$



# CRV: EXAMPLE

- (P)CRV  $\#_X[A|C]$ 
  - Range values: Histogram  $h = \{(v_i, n_i)\}_{i=1}^m$ 
    - $m = |\text{ran}(A)|$  (number of buckets)
    - $n = \sum_{i=1}^m n_i = |\text{gr}(A|C)|$  (number of instances to distribute into buckets)
    - $v_i \in \text{ran}(A)$  (buckets)
    - $n_i \in \mathbb{N}$  (number of instances in bucket  $v_i$ )
    - Shorthand:  $[n_1, \dots, n_m]$
  - Single histogram encodes several interchangeable assignments at once:

$$\text{Mul}(h) = \frac{n!}{\prod_{i=1}^m n_i!}$$

- E.g., CRV:  $\#_E[\text{Epid}(E)]$ 
  - $\text{ran}(\text{Epid}(E)) = \{\text{true}, \text{false}\} \rightarrow m = 2$
  - $\text{dom}(E) = \{e_1, e_2, e_3, e_4\} \rightarrow n = 4$
  - Range values and multiplicities

$$\begin{aligned} \{(\text{true}, 0), (\text{false}, 4)\} &= [0,4] & \text{Mul}([0,4]) &= \frac{4!}{0! \cdot 4!} = 1 \\ \{(\text{true}, 1), (\text{false}, 3)\} &= [1,3] & \text{Mul}([1,3]) &= \frac{4!}{1! \cdot 3!} = 4 \\ \{(\text{true}, 2), (\text{false}, 2)\} &= [2,2] & \text{Mul}([2,2]) &= \frac{4!}{2! \cdot 2!} = 6 \\ \{(\text{true}, 3), (\text{false}, 1)\} &= [3,1] & \text{Mul}([3,1]) &= \frac{4!}{3! \cdot 1!} = 4 \\ \{(\text{true}, 4), (\text{false}, 0)\} &= [4,0] & \text{Mul}([4,0]) &= \frac{4!}{4! \cdot 0!} = 1 \end{aligned}$$

# CRV: EXAMPLE

- E.g., (continued)
  - CRV:  $\#_E[Epid(E)]$ 
    - Range values  
 [0,4], [1,3], [2,2], [3,1], [4,0]  
 1    4    6    4    1  
 how many assignments encoded
  - $g' = \phi(\#_E[Epid(E)])$

$\#_E[Epid(E)]$	$\phi'$
[0,4]	8
[1,3]	6
[2,2]	4
[3,1]	2
[4,0]	0

$Epid_1$	$Epid_2$	$Epid_3$	$Epid_4$	$\phi$
false	false	false	false	8
false	false	false	true	6
false	false	true	false	6
false	false	true	true	4
false	true	false	false	6
false	true	false	true	4
false	true	true	false	4
false	true	true	true	2
true	false	false	false	6
true	false	false	true	4
true	false	true	false	4
true	false	true	true	2
true	true	false	false	4
true	true	false	true	2
true	true	true	false	2
true	true	true	true	0

# CRVS CONTINUED

- (P)CRV  $\#_X[A|C]$  with
  - $m = |\text{ran}(A)|$  (number of buckets)
  - $n = \sum_{i=1}^m n_i = |\text{gr}(A|_{\pi_X(C)})|$  (number of instances to distribute into buckets)
- Instead of  $m^n$  mappings in the ground factor, the counted factor has

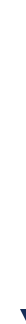
$$\binom{n+m-1}{n-1}$$

mappings

- Upper bound of range size of a CRV:

$$\binom{n+m-1}{n-1} \leq n^m$$

Exponential in number of  
random variables  $n$



Polynomial in number of  
random variables  $n$



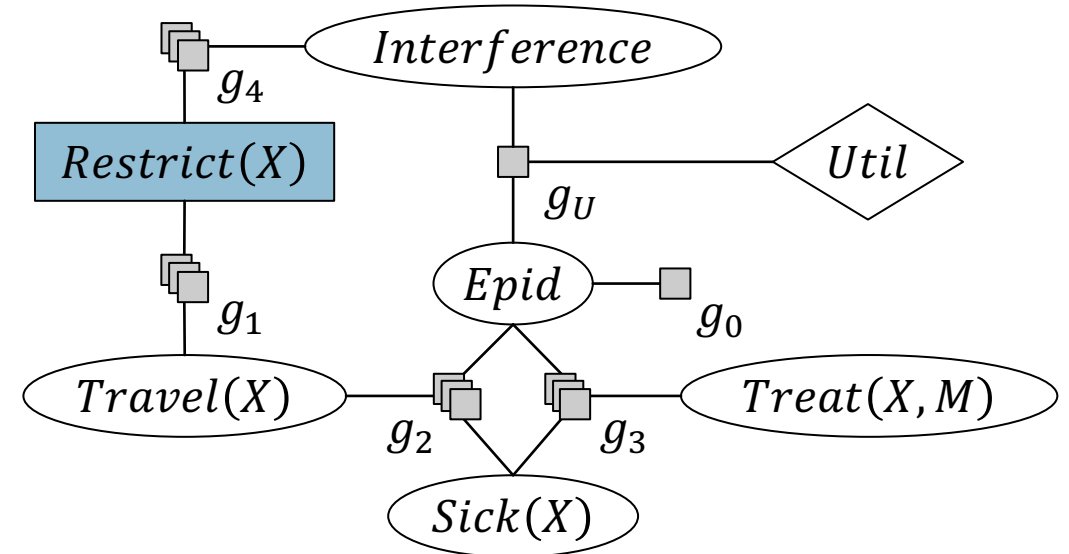
# LIFTED INFERENCE IN DECISION-THEORETIC MODELS (ONLINE DECISION MAKING)

INTRODUCTION

# DECISION PRVS

- Decision PRV  $D$ 
  - Range  $\text{ran}(D) = \{a_i\}_{i=1}^K$  set of possible actions
    - Actions  $a_i$  mutually exclusive (consistent with range definition)
    - Always have to get a value assigned
      - Cannot not make a decision!
  - Depicted by a rectangle in a graphical representation
  - E.g., travel restrictions for people  $X$ :  $\text{Restrict}(X)$ 
    - Range values:  $\text{ban}, \text{free}$
- Set of decision PRVs  $D$  in a model, i.e.,  $R = D \cup V$ 
  - $D$  can occur as arguments to any parfactor
  - Example:
    - $\phi_1(\text{Restrict}(X), \text{Travel}(X)), \phi_4(\text{Restrict}(X), \text{Interference})$

$R(X)$	$I$	$\phi_4$	$R(X)$	$Tl(X)$	$\phi_1$
<i>free</i>	<i>false</i>	1	<i>free</i>	<i>false</i>	1
<i>free</i>	<i>true</i>	0	<i>free</i>	<i>true</i>	1
<i>ban</i>	<i>false</i>	0	<i>ban</i>	<i>false</i>	1
<i>ban</i>	<i>true</i>	1	<i>ban</i>	<i>true</i>	0



# EXPECTED UTILITY QUERIES

- Given a decision model  $G = \{g_i\}_{i=1}^n \cup \{g_U\}$ 
  - One can ask queries for (conditional) marginal distributions or events as before given an action assignment  $\mathbf{d}$  based on the semantics,  $P_G[\mathbf{d}]$
  - New query type: query for an expected utility (EU)
    - What is the expected utility of making decisions  $\mathbf{d}$  in  $G$ ?

$$eu(\mathbf{e}, \mathbf{d}) = \sum_{r \in \text{ran}(\text{gr}(\text{rv}(g_U) \setminus E \setminus \mathbf{D}))} P(\mathbf{r}|\mathbf{e}, \mathbf{d}) \cdot \phi_U(\mathbf{r}, \mathbf{e}, \mathbf{d})$$

- $P(\mathbf{r}|\mathbf{e}, \mathbf{d})$  means that the PRVs not occurring in this expression need to be eliminated accordingly
  - I.e.,  $V = \text{rv}(G) \setminus \mathbf{D} \setminus E \setminus \text{rv}(g_U)$

# MEU PROBLEM

- Given a decision model  $G$  and evidence  $e$ , maximum Expected Utility (MEU) problem:
  - Find the action assignment that yields the highest expected utility in  $G$
  - Formally,  $\text{meu}(G|e) = (d^*, eu(E, d^*))$

$$d^* = \arg \max_{d \in \text{ran}(D)} eu(e, d)$$

Additive semantics with inner sum and outer max: Sum up utilities, then pick maximum  
→ Max-sum algorithms

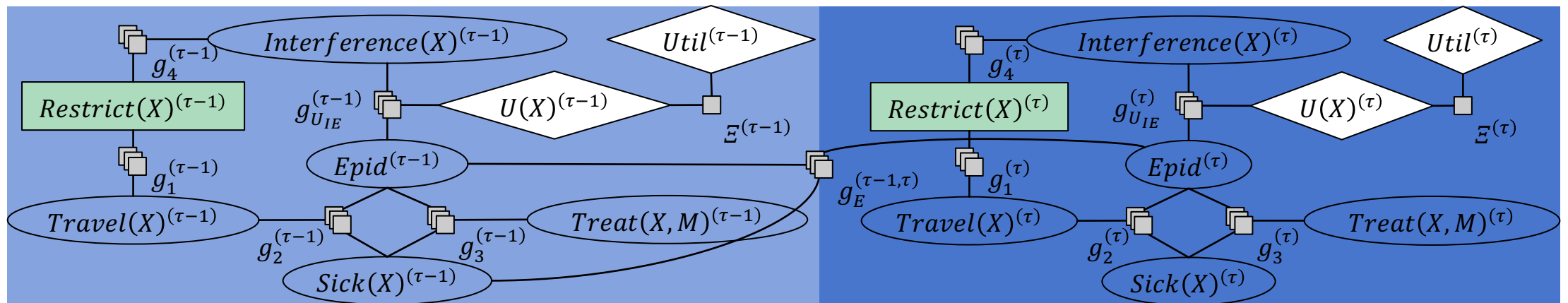
- For an exact solution,  $\text{meu}(G|e)$  requires an algorithm to go through **all**  $d \in \text{ran}(D)$ 
  - Size of  $\text{ran}(D)$  **exponential** in  $|D|$

Alternative specification

$$\text{meu}(G|e) = \left( \arg \max_{d \in \text{ran}(D)} eu(e, d), \max_{d \in \text{ran}(D)} eu(e, d) \right)$$

# DECISION MAKING OVER TIME

- Basis: a sequential model ( $G^0, G^\rightarrow$ )
  - Describe behaviour over time using interslice parafactors
  - Within a slice, describe intra-slice (episodic) behaviour
- Extend intra-slice parts with decision + utility PRVs
  - Intra-slice behaviour described using a decision model
  - Inter-slice behaviour allows for predicting effect of decision on next step





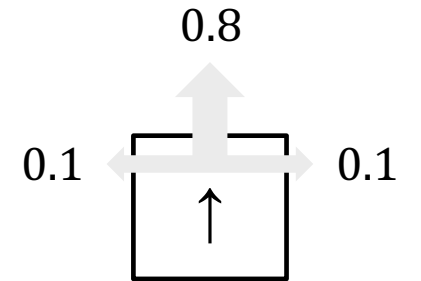


# MARKOV DECISION PROCESS (OFFLINE DECISION MAKING)

## INTRODUCTION

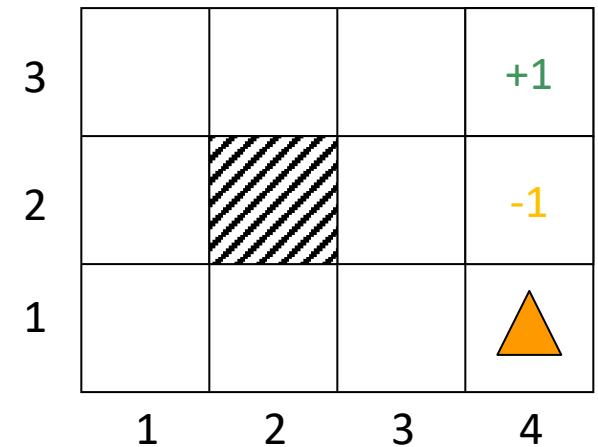
# MARKOV DECISION PROCESS / PROBLEM (MDP)

- *Sequential* decision problem for a fully observable, stochastic environment with a Markovian transition model and additive rewards (next slide)
- MDP is a four-tuple  $(S, A, T, R)$  with
  - $S$  a random variable whose domain is a set of states (with an initial state  $s^0$ )
  - For each  $s \in \text{dom}(S)$ 
    - a set  $A(s)$  of actions
    - a transition model  $T(s', s, a) = P(s' | s, a)$
    - a reward function  $R(s)$  (also with  $a$  possible)
- Robot navigation example to the right



U, D, L, R

each move costs 0.04



# ADDITIVE UTILITY

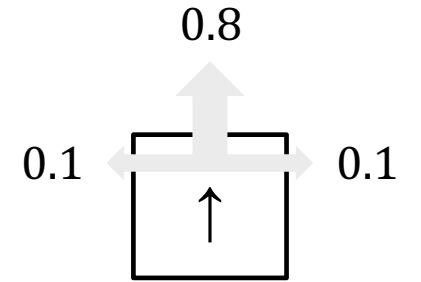
- History  $h = (s^{(0)}, s^{(1)}, \dots, s^{(T)})$
- In each state  $s$ , agent receives reward  $R(s)$
- Utility of  $h$  is additive iff

$$\begin{aligned}
 U(s^{(0)}, s^{(1)}, \dots, s^{(T)}) &= R(s^{(0)}) + U(s^{(1)}, \dots, s^{(T)}) \\
 &= \sum_{t=0}^T R(s^{(t)})
 \end{aligned}$$

- Discount factor  $\gamma \in ]0,1]$ :


$$U(s^{(0)}, s^{(1)}, \dots, s^{(T)}) = \sum_{t=0}^T \gamma^t R(s^{(t)})$$

- Close to 0: future rewards insignificant
- Corresponds to interest rate  $1-\gamma/\gamma$



U, D, L, R

each move costs 0.04

3				+1
2				-1
1				
	1	2	3	4

# PRINCIPLE OF MEU

- Bellman equation:

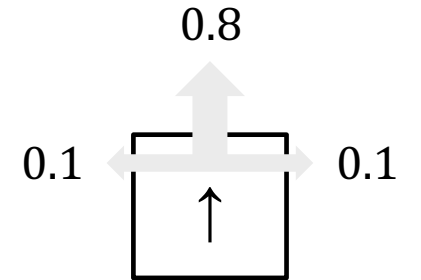
$$U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s' \in \text{dom}(s)} P(s'|a, s)U(s')$$

- Optimal policy:

$$\pi^*(s) = \operatorname{argmax}_{a \in A(s)} \sum_{s' \in \text{dom}(s)} P(s'|a, s)U(s')$$

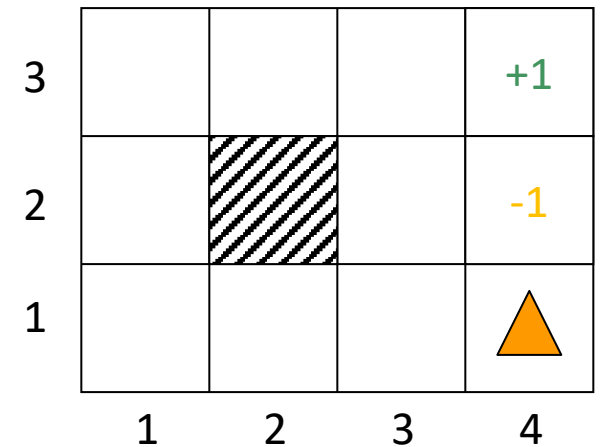
- Bellman equation for  $[1,1]$  with  $\gamma = 1$  as discount factor

$$U(1,1) = -0.04 + \gamma \max_{U,L,D,R} \left\{ \begin{array}{ll} 0.8U(1,2) + 0.1U(2,1) + 0.1U(1,1), & \text{(U)} \\ 0.8U(1,1) + 0.1U(1,1) + 0.1U(1,2), & \text{(L)} \\ 0.8U(1,1) + 0.1U(2,1) + 0.1U(1,1), & \text{(D)} \\ 0.8U(2,1) + 0.1U(1,2) + 0.1U(1,1) \} & \text{(R)} \end{array} \right.$$



U, D, L, R

each move costs 0.04

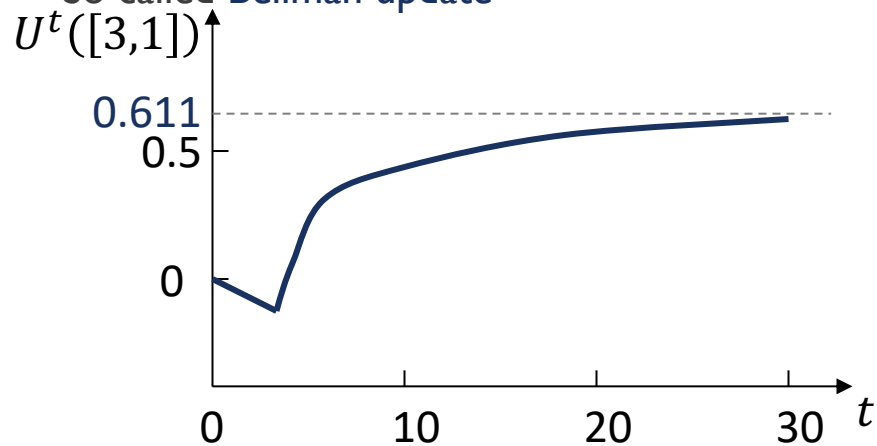


# VALUE ITERATION

- Initialise the utility of each non-terminal state  $s$  to  $U^{(0)}(s) = 0$
- For  $t = 0, 1, 2, \dots$ , do

$$U^{(t+1)}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s' \in \text{dom}(s)} P(s'|a, s) U^{(t)}(s')$$

- So called Bellman update



Note the importance of terminal states and connectivity of the state-transition graph

3	0.812 →	0.868 →	0.918 →	+1
2	0.762 		0.660 	-1
1	0.705 	0.655 ←	0.611 ←	0.388 ←
	1	2	3	4

3	0	0	0	+1
2	0		0	-1
1	0	0	0	0
	1	2	3	4

# VALUE ITERATION: ALGORITHM

- Returns a policy  $\pi$  that is optimal
- Inputs
  - MDP  $mdp$ 
    - Set of states  $S$
    - For each  $s \in S$ 
      - Set  $A(s)$  of applicable actions
      - Transition model  $P(s'|s, a)$
      - Reward function  $R(s)$
  - Maximum error allowed  $\epsilon$

```
function value-iteration( $mdp, \epsilon$ )
   $U' \leftarrow 0, \pi \leftarrow \langle \rangle$ 
  repeat
     $U \leftarrow U'$ 
     $\delta \leftarrow 0$ 
    for each state  $s \in S$  do
       $U'[s] \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s', P(s'|a, s)} U[s']$ 
      if  $|U'[s] - U[s]| > \delta$  then
         $\delta \leftarrow |U'[s] - U[s]|$ 
  until  $\delta < \epsilon(1-\gamma)/\gamma$ 
  for each state  $s \in S$  do
     $\pi(s) \leftarrow \operatorname{argmax}_{a \in A(s)} \sum_{s', P(s'|a, s)} U[s']$ 
  return  $\pi$ 
```

- Local variables
  - $U, U'$  vectors of utilities for states in  $S$
  - $\delta$  maximum change in utility of any state in an iteration

# POMDP

- POMDP = Partially Observable MDP

- Sensing operation returns multiple states, with a probability distribution

- Sensor model  $\Omega$  that encodes  $P(o|s)$  (or  $P(o|s, a)$ )
  - Probability of observing  $o$  given state  $s$  (and action  $a$ )

- Example:

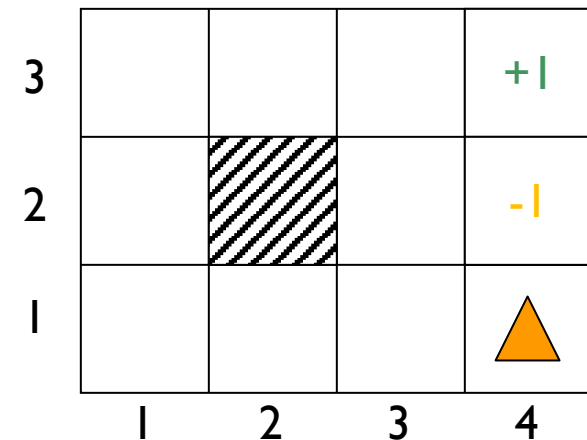
- Sensing number of adjacent walls (1 or 2)
- Return correct value with probability 0.9

- Formally, POMDP is a six-tuple  $(S, A, T, R, O, \Omega)$

- MDP  $(S, A, T, R)$  extended with a set of observations  $O$  and a sensor model  $\Omega$

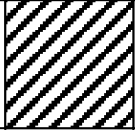
- Choosing action that maximizes expected utility of state distribution assuming “state utilities” computed as before not good enough  
→ Does not make sense (not rational)

- POMDP agent: Constructing a new MDP in which the current probability distribution over states plays the role of the state variable



# BELIEF STATE & UPDATE

- Belief state  $b(s)$  is the probability assigned to the actual state  $s$  by belief state  $b$
- Initial belief state
  - Probability of 0 for terminal states
  - Uniform distribution for rest
  - Robot navigation example:
    - $b = \left(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, 0, 0\right)$

3	$0.\bar{1}$	$0.\bar{1}$	$0.\bar{1}$	0.0
2	$0.\bar{1}$		$0.\bar{1}$	0.0
1	$0.\bar{1}$	$0.\bar{1}$	$0.\bar{1}$	$0.\bar{1}$
	1	2	3	4

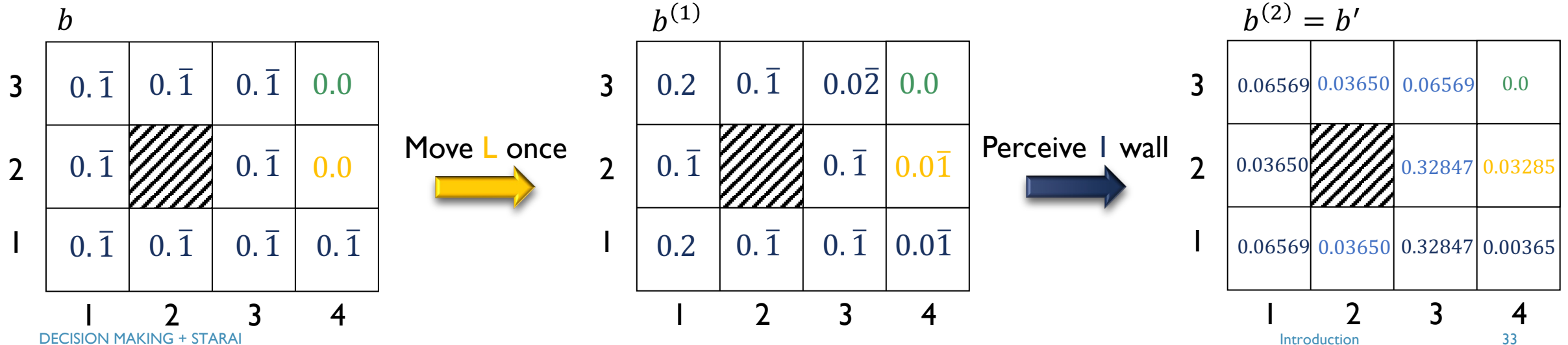


# BELIEF STATE & UPDATE

- Update  $b' = SE(b, a, o)$

$$b'(s') = P(s'|o, a, b) = \frac{P(o|s', a) \sum_{s \in \text{dom}(s)} P(s'|s, a) b(s)}{\sum_{s'' \in \text{dom}(s)} P(o|s'', a) \sum_{s \in \text{dom}(s)} P(s''|s, a) b(s)}$$

- Consider as two stage-update: (1) Update for the **action** (2) Update for the **observation**



# AGENDA

1. Introduction to Relational Models and Online Decision Making [Marcel]
  - Relational models under uncertainty
  - Lifted inference in decision-theoretic models (online decision making)
  - Markov Decision Process (Offline decision making)
2. Lifting Offline Decision Making [Flo]
3. Lifting Multi-Agent Decision Making [Nazlı Nur]
4. Summary [Marcel]

# AGENDA

1. Introduction to Relational Models and Online Decision Making [Marcel]
2. Lifting Offline Decision Making [Flo]
  - Factored Markov Decision Processes
  - First-order Markov Decision Processes
  - Lifted Factored Markov Decision Processes
3. Lifting Multi-Agent Decision Making [Nazlı Nur]
4. Summary [Marcel]



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