ANALYTIC TORSION AND INTERACTIONS
ABSTRACTS

2-6 SEPTEMBER 2024, MÜNSTER

Amina Abdurrahman (IHES)
Hyperbolic homology 3-spheres, spectral gaps and torsion homology growth
Abstract: When does a sequence of hyperbolic 3-manifolds with volume going to infinity have exponentially growing torsion homology? For arithmetic towers, the work of Bergeron-Sengun-Venkatesh suggests a set of conditions that conjecturally imply exponential growth of torsion homology. Their work relies on Cheeger-Mueller's theorem, linking torsion homology and analytic torsion. For nice sequences of hyperbolic 3-manifolds we use a different approach to find a condition implying exponential torsion homology growth. The size of the first torsion homology group is a topological quantification of being a homology 3-sphere for a hyperbolic manifold. A geometric quantification is given by the spectral gap for the Laplacian on coclosed 1-forms. Our condition is that the sequence has a uniform spectral gap on coclosed 1-forms. We also construct concrete examples of such sequences, which answers a question asked by Lin and Lipnowski.
This is based on joint work with Anshul Adve, Vikram Giri, Ben Lowe and Jonathan Zung.

Pierre Albin (Illinois Urbana-Champaign)
Analytic torsion and the sub-Riemannian limit of a contact manifold
Abstract: Contact manifolds, which arise naturally in mechanics, dynamics, and geometry, carry natural Riemannian and sub-Riemannian structures and it was shown by Gromov that the latter can be obtained as a limit of the former. Subsequently, Rumin found a complex of differential forms reflecting the contact structure that computes the singular cohomology of the manifold. He used this complex to describe the behavior of the spectra of the Riemannian Hodge Lapacians in the sub-Riemannian limit. As many of the eigenvalues diverge, a refined analysis is necessary to determine the behavior of global spectral invariants. I will report on joint work with Hadrian Quan in which we determine the global behavior of the spectrum by explaining the structure of the heat kernel along this limit in a uniform way.

Léo Bénard (Marseille)
Reidemeister torsion and character variety
Abstract: In this talk I will explain that the Reidemeister torsion defines an algebraic function on the SL(2,C) character variety of knots and links exteriors. I will describe some results related to the divisor of this function, and some multiplicity phenomenon.

Jean-Michel Bismut (Institut de Mathématique d'Orsay)
A conversation with analytic torsion (Connecting Mathematical Fields Talk)
In this survey talk, we will review aspects of analytic torsion in real and complex geometry. Analytic torsion was introduced by Ray and Singer as a spectral invariant (a determinant) of the Hodge Laplacian in real and complex geometry. It has been used to define metrics on certain line bundles, the determinants of the cohomology. In real geometry, Ray and Singer conjectured that analytic torsion co- incides with Reidemeister torsion, a combinatorial invariant. In 1978, the conjecture was established by Cheeger and Müller. In complex geometry, Ray and Singer computed explicitly the analytic torsion of elliptic curves. Quillen used analytic torsion to define a metric on the determinant of the cohomology of a holomorphic line bundle on a Riemann surface, and proved the relevant curvature theorem. These were the starting points of a considerable body of work, whose scope is to make analytic torsion a natural object in suitable theories of secondary invariants, and also to find applications of analytic torsion in topology, number theory and dynamical systems. The purpose of our talk will be to address some of the above questions.

Xianzhe Dai (Santa Barbara)
Analytic torsion for Witten deformation on noncompact manifolds
Abstract: Motivated by the Landau-Ginzburg (LG) B-model in mirror symmetry, we develop the notion of analytic torsion for Witten deformation on noncompact manifolds following our previous work on cohomological theory and heat kernel expansions (for Witten deformation on noncompact manifolds). We establish the anomaly formula and Cheeger-Müller/Bismut-Zhang theorem for the analytic torsion in this setting. Moreover, we find an interesting connection between the analytic torsion of the Witten deformation on a noncompact manifold and the analytic torsion of its compact core with absolute/relative boundary conditions. This is joint work with Junrong Yan. If time permits we will also discuss a joint work with my student Debin Liu on an application of the theory.

Sebastian Goette (Freiburg)
Family Thom-Smale complexes and Igusa-Klein torsion
Abstract: There are various ways to generalise topological or analytic torsion to fibre bundles of closed manifolds. After a quick overview, we explain how Morse theory generalises to fibre bundles, following Igusa and Eliashberg-Mishachev. We will then describe the resulting family Thom-Smale complexes and give a generalisation of the finite-dimensional Bismut-Lott torsion. This turns out to agree with Igusa-Klein torsion, up to normalisation.

Ksenia Fedosova (Münster)
Inhomogeneous Laplace equation on modular surface and its application to the string theory
Abstract: Low-energy expansion of scattering amplitudes of elementary particles arising in string theory are often linked to an inhomogeneous Laplace equation. For example, for the IIB string theory in 10+1 dimensions and scattering of 4-gravitons, the equation is the inhomogeneous Laplace equation on the modular surface, where the inhomogeneous part contains products of Eisenstein series. In this talk, we discuss how to solve this partial differential equation. Additionally, from asymptotic behavior of its solutions we obtain exact identities for infinite convolution sums of even divisor functions weighted by Laurent polynomials with logarithms.

Siarhei Finski (Paris)
Analytic torsion on Riemann surfaces with hyperbolic cusps
Abstract: We define the analytic torsion on non-compact Riemann surfaces with hyperbolic cusp singularities and show that it enjoys properties similar to those satisfied by the analytic torsion on compact Riemann surfaces. More specifically, we prove a local version of the Riemann-Roch-Grothendieck theorem on surfaces with cusps, generalizing a formula of Takhtajan-Zograf. Our approach is based on establishing an analogue of the anomaly formula, which compares the newly defined analytic torsion on a non-compact surface with the classical analytic torsion of a compactified Riemann surface. The local version of the Riemann-Roch-Grothendieck theorem then follows from this formula and the curvature formula of Bismut-Gillet-Soulé.

Bingxiao Liu (Cologne)
Asymptotic analytic torsions for compact locally symmetric orbifolds
Abstract: In this talk, I would like to present a result on the full asymptotic expansions of Ray-Singer real analytic torsions associated with a sequence of flat vector bundles on a compact locally symmetric space, especially an orbifold. By applying Selberg's trace formula to compute the heat trace, we transform the problem into evaluating semi-simple orbital integrals. Then a key step is to evaluate the orbital integrals associated with nontrivial elliptic elements, that ultimately lead to exponential polynomials in the expansions of analytic torsions. To achieve this, we established a geometric localization formula which allows us to rewrite an elliptic orbital integral as a sum of several identity orbital integrals associated with the centralizer of the elliptic element. The explicit geometric formula of Bismut for semi-simple orbital integrals is crucial to these computations.

Bo Liu (Shanghai)
Eta invariant, differential K-theory and localization
Abstract: In 1968, Atiyah and Segal established a localization formula for the equivariant index which computes the equivariant index via the contribution near the fixed point sets

of the group action. It is natural to ask whether such localization property holds for more complicated spectral invariants, e.g. eta invariant. In this talk, we will provide a version of localization formula for equivariant eta invariant by using differential K-theory and discuss some further generalizations. This is joint work with Xiaonan Ma.

Gerard Freixas i Montplet (Paris)
Analytic torsion of holomorphic differentials and hypersurface singularities
Abstract: In this talk, I will discuss the behavior of the analytic torsion of bundles of holomorphic differentials for one-parameter families of complex projective manifolds that degenerate and acquire at most isolated singularities. I will explain the relationship between these and the topological and Hodge theoretic invariants in the theory of the Milnor fiber, and I will formulate some numerical conjectures that suggest the vanishing of analytic torsion in the limit. These conjectures turn out to generalize older conjectures by Durfee and K. Saito, extending them to the recently developed notions of higher rational singularities by Friedman-Laza, Mustata-Popa, and others. I will also present several cases where these conjectures are known to hold, thus yielding new results on the degeneration of the analytic torsion of holomorphic differentials. This is joint work in progress with Dennis Eriksson.

Wolfgang Lück (Bonn)
A survey on L^2-torsion
Abstract: Analytic and topological L^2-torsion were introduced by Mathai, Lott, Lück, and Rothenberg. They are L^2-analogues of analytic and topological torsion in the sense of Ray-Singer. We give a survey about the basic properties, main applications and interesting open conjectures. Despite the more complicated definition, there are many interesting cases, where one can compute L^2-torsion, e.g., for instance there is a closed formulas for prime 3-manifolds and locally symmetric spaces. We discuss applications, e.g., to 3-manifolds and to S^1-actions on hyperbolic manifolds and open problems, e.g., about homological growth, simplical volume, and varieties given by finite-dimensional (not necessarily unitary) representations of fundamental groups.

Werner Müller (Bonn)
Analytic torsion of locally symmetric spaces and cohomology of arithmetic groups
Abstract: The first part is joint work with Jasmin Matz. For a locally symmetric space of finite volume, we introduce a regularized version of the analytic torsion. Then we consider towers of finite coverings of a given locally symmetric space and prove that the regularized analytic torsion, renormalized by the volume, converges to the L^2-torsion of the initial manifold. The second part is joint work with Frederic Rochon. We consider locally symmetric spaces of $\mathbb{Q}$-rank one. These are manifolds with fibred cusps. For such a manifold, we compare the regularized analytic torsion with the Reidemeister torsion of the Borel-Serre compactification of this manifold. There is no equality, but the defect

term can be described explicitly. Then we apply this result to study the growth of torsion in the cohomology of congruence subgroups of arithmetic groups. An arithmetic group is a discrete subgroup of a semisimple Lie group, which is defined by arithmetic conditions such as the modular group $SL(2,\mathbb{Z})$. This generalizes results of Bergeron and Venkatesh who treated the case of cocompact arithmetic groups. There is a deep connection between cohomology of locally symmetric spaces defined by arithmetic groups, the theory of automorphic forms and number theory, which I will briefly discuss, if time permits.

Anna-Maria Pippich (Konstanz)
Modular curves and regularized determinants
Abstract: In this talk we present an overview of our joint work with G. Freixas on developing an arithmetic Riemann–Roch theorem for the line bundle of modular forms on modular curves, equipped with the logarithmically singular hyperbolic metric. The focus will be on the necessary modifications to the left-hand side of the theorem, which amounts to a suitable regularization of the determinant of the hyperbolic Laplacian. Additionally, we will discuss ongoing work with M. Dutour, where we are extending these ideas to the case of unitary vector bundles with finite monodromies at the cusps.

Martin Puchol (Paris)
Higher torsions and Witten-type deformations
Abstract: Consider a smooth fibration with compact fiber together with a flat complex vector bundle over the total space. The associated analytic torsion form is an even differential form on the base manifold, which arises from a transgression of Bismut and Lott's Riemann-Roch-Grothendieck formula. Igusa and Klein constructed the topological counterpart of the analytic torsion form, known as the higher topological torsion. The relation between the analytic torsion form and the higher topological torsion is a natural and important problem in the theory of higher torsion invariants, and a higher version of the Cheeger-Müller/Bismut-Zhang theorem is expected. In this talk, after reviewing some known results on this topic, I will present a relative version of this problem, where torsion invariants are renormalized by their value for trivial bundles. Our approach is based on the work of Bismut-Goette, in which they obtained a comparison formula performing a Witten deformation using a fiberwise Morse function. However, as such function does not always exist, we use instead a fiberwise generalized Morse function. In this context, our renormalization is key to overcoming the difficulties posed by singular critical points. This is a joint work with Junrong Yan.

Jean Raimbault (Marseille)
Torsion homology growth after 15 years
Abstract: Fifteen years ago Nicolas Bergeron and Akshay Venkatesh made a very precise conjecture on the asymptotic behaviour of torsion subgroups in the homology groups of

arithmetic lattices in semisimple Lie groups. So far all cases remain open, the simplest being the following :
*Conjecture (Bergeron–Venkatesh) *: Let M be an arithmetic 3–orbifold and

$$M < M_1 < ... < M_n < ...$$

a tower of principal congruence covers of M. Then the size of the torsion subgroup in $H_1(M_n)$ grows exponentially with respect to the degree of the covering map $M_n \to M$, with rate equal to the hyperbolic volume of M over 6π.
In this talk I will explain the context for the general version of this conjecture, and survey the proof approach introduced by Bergeron–Venkatesh, which uses the Cheeger–Müller theorem as a crucial ingredient. I will also discuss some limited numerical evidence for the conjecture.

Damian Rössler (Oxford)
Toward the functorial arithmetic Riemann-Roch theorem
Abstract: We shall present a functorial version of the Grothendieck-Riemann-Roch theorem in degree one. This theorem asserts that two different line bundles which can be associated with a smooth fibration are canonically isomorphic, compatibly with any base-change. We shall then explain what problems one is faced with when one tries to compute the norm of this isomorphism, when both line bundles are endowed with natural metrics.

Julie Rowlett (Chalmers)
Spectral invariants of integrable polygons
Abstract: An integrable polygon is one whose interior angles are of the form π/n for positive integers n. As an exercise, can you determine all integrable polygons? I'll discuss joint work with Gustav Mårdby in which we obtain new heat trace invariants for these polygons. It turns out that integrable polygons are also precisely those polygons which strictly tessellate the plane. I will also discuss joint work with M. Blom, H. Nordell, O. Thim, and J. Vahnberg in which we explore the generalization of integrable polygons to higher dimensions. This includes an equivalence between the geometric characterization of strict tessellation, an analytic characterization of Dirichlet Laplace eigenfunctions, and an algebraic characterization of crystallographic Coxeter groups.

Shu Shen (Paris)
Analytic torsion and Anosov flow
Abstract : In this talk, I will explain a construction of a torsion-like invariant for Anosov flows. This invariant is an element in the determinant line of the de Rham cohomology. In family, our invariant is shown to be flat with respect to the Gauss-Manin connection. This is a joint work with J.-M. Bismut arxiv:202405.14583

Boris Vertman (Oldenburg)
Microlocal Analysis on manifolds with fibered boundaries
Abstract: We discuss spectral geometric questions on on some non-compact manifolds with fibered boundaries. Main examples include non-compact complete hyper-Kähler 4-manifolds and scattering spaces. We discuss how such spaces naturally arise in the analysis of analytic torsion under degeneration of a smooth compact manifold to a space with conical singularities

Ken-Ichi Yoshikawa (Kyoto)
Degenerations of Riemann surfaces and small eigenvalues of Laplacian
Abstract: We consider a degeneration of compact Riemann surfaces over a complex curve, whose total space is a smooth Kähler surface. Then the fibers are endowed with the metric induced by the Kähler metric on the total space. In this situation, it is known that the k-th eigenvalue of the Laplacian, viewed as a function on the base curve, extends to a continuous function. In particular, for a reducible singular fiber, some eigenvalues of the Laplacian of the regular fiber converge to zero as the regular fiber approaches the singular fiber. Such eigenvalues are called small eigenvalues. In this talk, I report on work in progress with X. Dai on the asymptotic behavior of the small eigenvalues of the Laplacian when the singular fiber is reduced.

Tobias Weich (Paderborn)
Sharp bounds on the joint spectrum on locally symmetric spaces of higher rank and infinite volume
Abstract: It is well known that the spectral theory of locally symmetric spaces changes fundamentally if one passes from the case of lattices subgroups (i.e. finite volume quotients) to the class of thin subgroups (i.e. infinite volume quotients) simply du to the fact that the constant function is not an L^2 eigenfunction anymore.
For hyperbolic surfaces Elstrodt and Patterson gave a sharp lower bound on the bottom of the Laplace spectrum on hyperbolic surfaces in terms of the critical exponent. In this talk I will explain how these results can be generalized to higher rank using polyhedral norms on the joint spectrum as well as Quint's growth indicator function.
This is joint work with Chris Lutsko and Lasse Wolf.

Weiping Zhang (Chern Institute)
L^2-Alexander polynomial revisited
Abstract: Weiping Li and I introduced around 2006 an L^2-analogue of the classical Alexander polynomial for knots. In this talk we will describe a possible extension to the higher dimensional case, which is a joint work with Xiaonan Ma.