Universität Münster

Probability, Dynamics, and the Geometry of Groups

September 9–13, 2024
Münster

Organizing Committee:

David Kerr
Chiranjib Mukherjee

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MM Mathematics Münster
Cluster of Excellence

General information

You can find the latest information on the homepage of the conference.

Venue. The Seminarraumzentrum SRZ, Orléans-Ring 12 (cf. Map A). This is one of the two four-story buildings with corrugated metal cladding, distinguished by its square footprint. The entrance is located at the southeast corner under the bridge that connects it to the neighbouring building.
Registration: Second floor of the SRZ on Monday at 8.30 a.m.
Lecture room: The talks will take place in room SRZ 216/217.

Wi-Fi access. If you are part of the eduroam community, you may connect to the network "eduroam" as usual. Otherwise you can connect to the SSID "GuestOnCampus" by starting any web browser. You will automatically be redirected to the login page. Confirm the terms of use and click on "log in for free". There is 1 GB data volume available per device and day. Please note that the connection is not encrypted.

Talks. If your presentation uses **slides**, please try to send them to luzie.kupffer@uni-muenster.de ideally by the day before your talk.

Coffee break/Lunch. We provide coffee and snacks during the coffee breaks. There are a couple of restaurants for lunch in the vicinity:

- Canteen - Mensa am Ring, Domagkstraße 61,
- Ristorante Milano (Italian), Wilhelmstraße 26 (closed on Monday),
- Il Gondoliere (Italian), Von-Esmarch-Straße 28 (closed on Monday),
- Buddha Palace (Indian), Von-Esmarch-Straße 18,
- La Gondola D'oro (Italian), Hüfferstraße 34,
- A2 am See (German), Annette-Allee 3,

- Gustav Grün (Green Fast Food), Wilhelmstraße 1,
- Áro (Green Fast Food), Neutor 3,
- Krimphove (Bakery), Horstmarer Landweg 101.

Conference dinner. The conference dinner takes place on Wednesday at 6.30 p.m. at the Mövenpick Hotel (Kardinal-von-Galen-Ring 65).

Public transportation. You can check the bus schedule on the website of Stadtwerke Münster or using Google maps.

Free afternoon on Wednesday/City tour. There will be a free afternoon on Wednesday. Some suggestions: You may want to go and see the castle, its surroundings and the botanic garden which is right next to it. You can also visit a museum, e.g. the LWL Museum of Art and Cultural History, or the Picasso-Museum. You may also enjoy a walk around the lake "Aasee" or visit the City Hall, a centerpiece of European history, where the "Westphalian peace" terminating the Thirty Years' War was signed in 1648.

Questions. In case of further questions, please contact David Kerr:
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or Chiranjib Mukherjee:
Email: chiranjib.mukherjee@uni-muenster.de.

Acknowledgements

The conference is funded by the Cluster of Excellence "Mathematics Münster" as well as by the Collaborative Research Centre 1442 Geometry: Deformations and Rigidity.

Schedule

Monday, September 9

08:30-09:10	Registration
09:10-09:50	**Miklós Abért**: Growth and walks on unimodular random trees
10:00-10:30	Coffee
10:30-11:10	**Damien Gaboriau**: Dynamics on the space of subgroups
11:20-12:00	**Anush Tserunyan**: Weak mixing of Markov measures on the boundary of free groups
12:00-14:00	Lunch
14:00-14:40	**Indira Chatterji**: The Rapid Decay property and 3-manifold groups
14:50-15:20	Coffee
15:20-16:00	**Gábor Pete**: Nonamenable Poisson Zoo
16:10-16:50	**Konstantin Recke**: Percolation and the Geometry of Groups and Graphs
17:00	Wine and cheese reception, open end

Tuesday, September 10

09:10-09:50	**Tatiana Nagnibeda**: Benjamini-Schramm and spectral convergence of Rauzy graphs
10:00-10:30	Coffee

10:30-11:10 **Robin Tucker-Drob**: Measure Equivalence of Baumslag-Solitar Groups

11:20-12:00 **Pierre-François Rodriguez**: Critical phenomena for percolation models with long-range dependence

12:00-14:00 Lunch

14:00-14:40 **Sam Mellick**: The Ideal Poisson Voronoi Tessellation of $\mathbb{H}^2 \times \mathbb{H}^2$

14:50-15:20 Coffee

15:20-16:00 **Andreas Thom**: On non-isomorphic universal sofic groups

16:10-16:50 **Russell Lyons**: Ideal Voronoi Tessellations in Groups and in Hyperbolic Space

Wednesday, September 11

09:10-09:50 **Eduardo Silva**: The Poisson boundary of wreath products

10:00-10:30 Coffee

10:30-11:10 **Mahan Mj.**: Ising model on Hyperbolic groups

11:20-12:00 **Luzie Kupffer**: Bi-infinite random walk paths and geodesic flow on hyperbolic groups

12:50-14:00 Lunch

14:00 Free afternoon

18:30 Conference dinner (Mövenpick Hotel)

Thursday, September 12

09:10-09:50 **Romain Tessera**: Classification of nilpotent groups up to integrable measure equivalence

10:00-10:30 Coffee

10:30-11:10 **Jenna Zomback**: Cayley spherical Polish groups

11:20-12:00 **Rostislav Grigorchuk**: The Collatz map, Self-Similar Groups, Automorphisms of Rooted Trees, and Ergodic Decomposition

12:00-14:00 Lunch

14:00-14:40 **Amandine Escalier**: Measure and orbit equivalence of graph products

14:50-15:20 Coffee

15:20-16:00 **Ben Hayes**: Coamenability and cospectral radius for equivalence relations

16:10-16:50 **Cornelia Drutu**: Combings, local-to-global properties and divergence

Friday, September 13

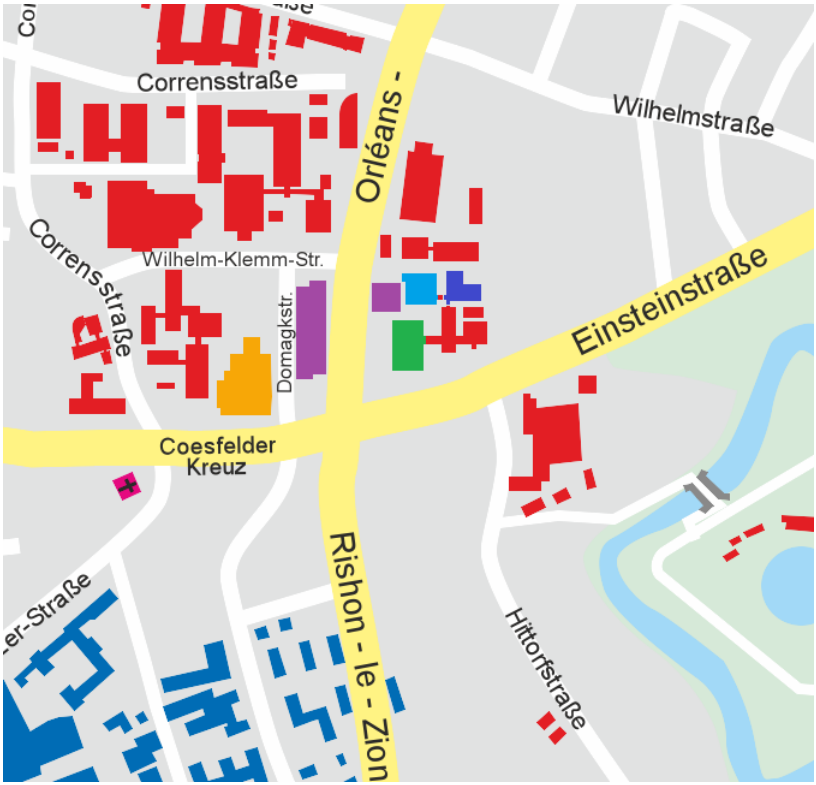
09:10-09:50 **Nathanaël Berestycki**: On the cover time of random walks on graphs

10:00-10:30 Coffee

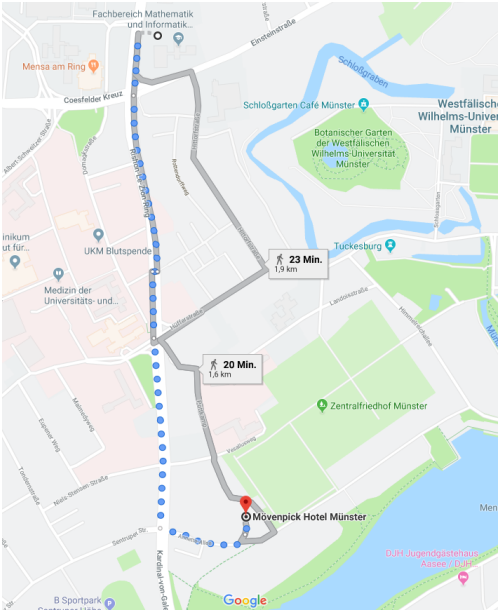
10:30-11:10 **Subhajit Goswami**: Roughness of geodesics in Liouville quantum gravity

11:20-12:00 **Gady Kozma**: Some applications of the Olashanskii-Osin-Sapir construction

Maps and locations



Map A: Lecture building, canteen, SRZ, MM building, parking lot.



Map B: Route to Mövenpick Hotel.

Book of abstracts

Growth and walks on unimodular random trees

Miklós Abért (Rényi)

Kaimanovich asked whether every unimodular random rooted graph admits a growth exponent. Timar answered this question on the negative, producing a one-ended unimodular random rooted tree with different upper and lower growth. Such trees are always hyperfinite, in particular, when the degrees are bounded by d, their upper growth is bounded by $\sqrt{d-1}$. We show that once the upper growth supersedes this bound, the growth will exist. The proof uses invariant percolation theory and a new result on the existence of random walk sampling exponent for subrelations of countable p.m.p. relations. We conjecture that more is true and every unimodular random rooted tree of bounded degree and infinitely many ends will admit a growth exponent. This is joint work with Mikolaj Fraczyk and Ben Hayes. This initial work opens up the possibility of a Patterson-Sullivan theory for unimodular random rooted trees and Riemannian surfaces. I will discuss possible directions and some initial results with various research groups.

On the cover time of random walks on graphs

Nathanaël Berestycki (Vienna)

How long does it take for a random walk to cover all the vertices of a graph?

And what is the structure of the uncovered set (the set of points not yet visited by the walk) close to the cover time? We completely characterize the vertex-transitive graphs of bounded degree for which the cover time is in the Gumbel universality class. Surprisingly our characterization is in terms of a simple global geometric condition; this is furthermore equivalent to the decorrelation of the uncovered set at a time (in the sense that it is close to a product measure).

To prove this result we rely on recent breakthroughs in geometric group theory which give a quantitative form of Gromov's theorem on groups of polynomial growth. We also prove along the way optimal quantitative estimates giving an exponential approximation for hitting time of sets of vertices (irrespective of their geometry).

Joint work with Jonathan Hermon (UBC) and Lucas Teyssier (UBC).

The Rapid Decay property and 3-manifold groups

Indira Chatterji (Côte d'Azur)

I will recall basic facts about the Rapid Decay property, and explain how we prove it for polynomially distorted graphs of groups, which describe 3-manifold groups without sol components. This is joint work with François Gautero.

Combings, local-to-global properties and divergence

Cornelia Drutu (Oxford)

The study of groups from the geometric, respectively from the dynamical/probabilistic viewpoint, have several points of encounter. Two among them are the investigations of actions of groups on their various types of boundaries, and on their asymptotic cones. In this setting, several notions encoding non-positive curvature (e.g. combings, globalizations of local properties) are relevant, as well as various coarse invariants (such as divergence). In this talk, I shall overview these topics and explain some recent advances. Part of the talk is based on joint work with Davide Spriano and Stefanie Zbinden.

Measure and orbit equivalence of graph products

Amandine Escalier (Orsay)

Measure equivalence was introduced by Gromov as a measure-theoretic analogue of quasi-isometry.

In this talk we will study the behaviour of graph products under measure equivalence and its ergodic counterpart: orbit equivalence. We will also highlight the links with right-angled Artin groups and geometric group theory. This is joint work with Camille Horbez.

Dynamics on the space of subgroups

Damien Gaboriau (ENS Lyon)

Every countable discrete group G admits a canonical topological dynamics on a 0-dimensional space, namely the action by conjugation on its space $Sub(G)$ of subgroups. The process of successively removing the isolated points produces the perfect kernel $K(G)$ in $Sub(G)$ and the Cantor-Bendixson rank $rk(G)$. I will survey some results ranging from computations of the perfect kernels and Cantor-Bendixson ranks to topological transitivity properties of the action of G, illustrated with such examples as the Baumslag-Solitar groups, hyperbolic-like groups, right-angled-Artin groups including direct product of free groups.

These are joint works with P. Azuelos, S. Bontemps, A. Carderi, F. Le Maître and Y. Stalder.

Roughness of geodesics in Liouville quantum gravity

Subhajit Goswami (Tata Mumbai)

The metric associated with the Liouville quantum gravity (LQG) surface has been constructed through a series of recent works and several properties of its associated geodesics have been studied. In this talk we will discuss a proof of the folklore conjecture that the Euclidean Hausdorff dimension of LQG geodesics is strictly greater than 1 for all values of the so-called Liouville first passage percolation (LFPP) parameter ξ. This is deduced from an adaptation of a general criterion due to Aizenman and Burchard which in our case amounts to near-geometric bounds on the probabilities of certain crossing events for LQG geodesics in the number of crossings. We obtain such bounds using the axiomatic characterization of the LQG metric after proving a special regularity property for the Gaussian free field (GFF). If time permits, we will also discuss an analogous result for the LFPP geodesics.

The Collatz map, Self-Similar Groups, Automorphisms of Rooted Trees, and Ergodic Decomposition

Rostislav Grigorchuk (Texas A&M)

In my talk I will try to connect seemingly unrelated topics from my title.

Coamenability and cospectral radius for equivalence relations

Ben Hayes (U Virgina)

We consider inclusions of discrete, probability measure-preserving orbit equivalence relations. In previous work with Abért-Fraçzyk, we established the pointwise almost sure existence of the cospectral radius relative to the subrelation of a random walk on the equivalence classes in the bigger relation. In this paper, we investigate the connections of this cospectral radius to the coamenability of the inclusion, analogous to the group situation. We also undertake a systematic study of coamenability for inclusions of relations, establishing several equivalence formulations of this notion. Time permitting, I will discuss connections to problems in percolation theory. Previous knowledge of cospectral radius and coamenability for relations will not be assumed.

Some applications of the Olashanskii-Osin-Sapir construction

Gady Kozma (Weizmann)

Olashanskii, Osin and Sapir constructed a lacunary hyperbolic group which is amenable using some kind of 'non-commutative lamplighter'. We will show that the same basic idea allows to construct all kinds of interesting groups.

Bi-infinite random walk paths and geodesic flow on hyperbolic groups

Luzie Kupffer (Münster)

It is well-established that Patterson-Sullivan measures on the boundary of a hyperbolic space, along with the associated Bowen-Margulis-Sullivan measure, provide valuable insights into the action of a group of isometries on the space's boundary through analysis of the geodesic flow. Given that paths of a random walk on a hyperbolic groups lie close to the group's quasi-geodesics, it is natural to ask whether similar behaviour can also be seen in the flow along bi-infinite random walk paths.

In this talk, I will show how studying bi-infinite random walks on a discrete hyperbolic groups G leads to an analogue of the Patterson-Sullivan measure on $\partial^2 G$. This measure can be constructed in multiple measure-equivalent ways, each giving distinct perspectives on its intrinsic structure. Moreover, as in the classical case, the action $G \curvearrowright \partial^2 G$ is ergodic with respect to this measure.

This talk is based on joint work in progress with Mahan Mj and Chiranjib Mukherjee.

Ideal Voronoi Tessellations in Groups and in Hyperbolic Space

Russell Lyons (Indiana University)

Given a discrete set of points in a metric space, called nuclei, one associates to each such nucleus its Voronoi cell, which consists of all points closer to it than to other nuclei. In Euclidean space, one commonly uses a homogeneous Poisson point process to assign the locations of the nuclei. As the intensity of the point process tends to 0, the nuclei spread out and disappear in the limit, with each pair of space points eventually belonging to the same cell. Surprisingly, this does not happen in other settings such as hyperbolic space. We will describe properties of such a limiting tessellation, as well as analogous behavior on Cayley graphs of finitely generated groups. In particular, we discuss various ergodic properties. We will illustrate results with many pictures and videos. The talk is based on work of Sandeep Bhupatiraju and joint work with Matteo d'Achille, Nicolas Curien, Nathanael Enriquez, and Meltem Ünel.

Ising model on Hyperbolic groups

Mahan Mj. (Tata Mumbai)

We study Ising models and Gibbs states on general hyperbolic groups.

For large values of the inverse temperature, Series and Sinai (1990) constructed an uncountable number of mutually singular Gibbs states for Fuchsian groups, i.e. lattices in $\mathrm{SL}(2,\mathbb{R})$. We extend this result to all non-elementary hyperbolic groups.

This is joint work with Ritwik Chakraborty.

The Ideal Poisson Voronoi Tessellation of $\mathbb{H}^2 \times \mathbb{H}^2$

Sam Mellick (Jagiellonian U)

The Poisson Voronoi Tessellation (PVT) is a natural model from stochastic geometry. There has been recent interest in its behaviour on nonamenable spaces as the intensity of the Poisson process goes to zero. It turns out that there is a limiting tessellation (the IPVT). The existence of this object can be used to prove fixed price one for some groups, resolving some conjectures of Gaboriau and Abert-Nikolov-Gelander. In this talk, I will discuss the IPVT in detail for the special case of $\mathbb{H}^2 \times \mathbb{H}^2$, and sketch the proof of the key property responsible for fixed price one – in this tessellation, every pair of cells has a metrically unbounded intersection.

Benjamini-Schramm and spectral convergence of Rauzy graphs

Tatiana Nagnibeda (Geneva)

Many dynamical systems admit a natural symbolic representation as subshifts over a finite alphabet. Gérard Rauzy suggested to study their complexity by associating to the subshift an infinite family of finite graphs that describe the local structure of its orbits; such graphs are known as Rauzy graphs. In this talk we will be interested in convergence properties of these sequences of graphs and of their spectra.

Nonamenable Poisson Zoo

Gábor Pete (Rényi)

In this model introduced by Ráth and Rokob, we have an infinite Cayley graph Γ, a probability measure ν on rooted finite connected subsets, called lattice animals, and we place iid $Poisson(\lambda)$ independent copies of them at each vertex. If the expected volume of the animals wrt ν is infinite, then the whole Γ is covered for any $\lambda > 0$. If the second moment of the volume is finite, then it is easy to see that for small enough λ the union of the animals has only finite clusters, while for λ large enough there are also infinite clusters. In joint work with Sándor Rokob, we show that:

1. If Γ is a free product, then for ANY ν with an infinite second but finite first moment and any $\lambda > 0$, there will be infinite clusters, despite having arbitrarily low density.

2. The same result holds for ANY nonamenable Γ, when the lattice animals are worms: random walk trajectories of random length.

Then, how does a low density infinite cluster in a factor of iid percolation on a nonamenable Cayley graph look like? In joint work with Endre Csóka and Péter Mester, we show that, on trees, they must be just barely nonamenable, and use this to prove a quantitative strengthening for free groups of the Chifan-Ioana indistinguishability theorem.

Does this strong indistinguishability theorem hold for every nonamenable group? The Poisson Zoo results suggest that at least our strategy (maybe also the result) might break down for Gromov monsters.

Percolation and the Geometry of Groups and Graphs

Konstantin Recke (Münster)

Given a locally finite, connected graph, a bond percolation is a random subgraph obtained by keeping or deleting edges. It is a fundamental question how geometric features of the graph are reflected in the behavior of bond percolation models and vice versa.

In this talk, we will relate several large-scale geometric properties to the connectivity decay of bond percolation models with large marginals. In the setting of Cayley graphs of finitely generated groups, we will describe characterizations of the Haagerup property and Kazhdan's property (T) through invariant bond percolations, making progress on a program suggested by Russell Lyons. We will also provide a characterization of the non-equivariant L^1-compression exponent of graphs in the possible absence of a non-trivial group action. We will highlight a common thread, which is a new construction of bond percolation models using Poisson point processes on so-called spaces with measured walls, which yields explicit bounds on the two-point function. This talk is based on joint work with Chiranjib Mukherjee.

Critical phenomena for percolation models with long-range dependence

Pierre-François Rodriguez (Imperial)

The talk will survey recent developments regarding the (near-)critical behaviour of certain percolation models in dimensions larger than 2. Unlike the classical "long-range percolation" model obtained by adding bonds to arbitrary distances, these models involve different probabilistic objects, such as random walks and the Gaussian free field. One specific model in this class allows the rigorous derivation of various critical exponents below the upper-critical dimension. These results are proved for a large class of transient base graphs G but depend only on a small number of key parameters associated to G, a manifestation of universality.

The Poisson boundary of wreath products

Eduardo Silva (ENS Paris)

The Poisson boundary of a random walk on a countable group is a probability space that encodes the asymptotic behavior of sample paths. Given a group G endowed with a probability measure μ, it is a natural problem to identify an explicit model of the associated Poisson boundary, described in terms of the geometry of G. In this talk I will speak about the identification problem for wreath products, the main example being the lamplighter groups $Z/2Z \wr Z^d$, for $d \geq 3$. I will explain results that describe the Poisson boundary for probability measures with finite entropy and that satisfy a stabilization condition that naturally arises in this context. This is joint work with Joshua Frisch.

Classification of nilpotent groups up to integrable measure equivalence

Romain Tessera (Paris)

Measure equivalence (ME) was introduced by Gromov as a measurable analogue of quasi-isometries. The emblematic case of two ME groups are pairs of lattices in a same locally compact group. Another important source of examples comes from ergodic theory: two groups admitting orbit-equivalent free pmp actions are ME. By a spectacular result of Orsntein-Weiss, any two countable amenable groups are ME. In a way, this implies that ME is too flexible to remember geometric properties of the groups. A natural way to gradually restore rigidity consists in imposing an L^p-moment condition on the ME coupling, for various values of p. For instance, Bowen proved that L^1-ME remembers the volume growth. In particular, virtually nilpotent groups form a rigid class up to L^1-ME. Austin proved that two nilpotent groups that are L^1-ME have isomorphic Carnot associated Lie groups. In a joint work with Claudio Llosa Isenrich, we prove that the converse holds in a strong sense: two nilpotent groups with isomorphic associated Carnot are L^p-ME for some explicit $p > 1$ (so in particular for $p = 1$). Understanding the largest p for which this holds is an important problem, which is closely connected to the conjectural classification of nilpotent groups up to quasi-isometries.

On non-isomorphic universal sofic groups

Andreas Thom (Dresden)

We show that there are $2^{\aleph_0}$ non-isomorphic universal sofic groups. This proves a conjecture of Simon Thomas. This is joint work with Vadim Alekseev.

Weak mixing of Markov measures on the boundary of free groups

Anush Tserunyan (McGill)

Several recent methods for proving pointwise ergodic theorems for pmp actions of free groups critically use weak mixing properties of Markov measures on the boundary of a free group of finite rank. However, it was not known exactly which Markov measures are weak mixing. In joint work with Jenna Zomback, we give a complete characterization of such measures. It turns out that, under mild non-degeneracy assumptions, they are exactly the Markov measures arising from strictly irreducible transition matrices – a condition introduced by Bufetov in 2000 for a different purpose. The proof of this characterization goes through proving equivalences with a new combinatorial condition on the action that we call chaining, which is interesting in its own right.

Measure Equivalence of Baumslag-Solitar Groups

Robin Tucker-Drob (U Florida)

We show that all non-amenable non-unimodular Baumslag-Solitar groups $BS(r,s)$, $2 \leq |r| < s$, are measure equivalent to each other, thereby completing the measure equivalence classification of Baumslag-Solitar groups. Consequently, each $BS(r,s)$ belongs to one of three measure equivalence classes according to whether it is amenable ($|r| = 1$ or $|s| = 1$), virtually isomorphic to $F_n \times Z$ ($2 \leq |r| = |s|$), or non-amenable and non-unimodular ($2 \leq |r| < s$). This is joint work with Damien Gaboriau, Antoine Poulin, Anush Tserunyan, and Konrad Wrobel.

Cayley spherical Polish groups

Jenna Zomback (U Maryland)

A length function ℓ on a group G is a function from G to the nonnegative real numbers satisfying the following for all group elements x and y: $\ell(x) = 0$ if and only if $x = 1_G$, $\ell(x^{-1}) = \ell(x)$, and $\ell(xy) \leq \ell(x) + \ell(y)$.

In this talk, we will investigate the asymptotic behavior of compatible length functions on Polish groups, and in particular, the extent to which a sphere of large radius with respect to one length function looks spherical with respect to another.

This is joint work with Christian Rosendal.