Introduction to the Standard Model
Problem sheet 9

Deadline: Monday 22 June 2015 (12 am)
at Dr. Giudice's office (KP 301) and Dr. Piemonte's office (KP 412)

Topics covered: theory of weak interactions

1. a) (2 P) Show that a free massless neutrino can be described by a two-component spinor.

   b) (2 P) Starting from the Dirac equation derive the two-component wave equation for a free massless neutrino (Weyl equation).

   c) (2 P) The helicity of a particle is defined as

   $$\lambda = \vec{S} \cdot \frac{\vec{p}}{|\vec{p}|},$$

   where $\vec{S}$ is spin and $\vec{p}$ is momentum. Derive the relation between helicity and chirality for massless neutrinos.

2. Consider scattering of leptons in the framework of Fermi's theory, e. g. $\nu_\mu \, e^- \to \mu^- \, \nu_e$.

   a) (1 P) Refresh your knowledge of partial wave analysis of quantum mechanical scattering theory. What is the upper bound on the s-wave contribution to the total cross section?

   b) (1 P) Bring forward an argument that in Fermi's theory only s-wave scattering occurs.

3. The Lagrangian for a real vector boson field is given by

   $$\mathscr{L} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{m^2}{2} B_\mu B^\mu \,,$$

   where

   $$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,.$$

   a) (2 P) Calculate the propagator $D_{F,\mu\nu}(k)$ of the vector boson in momentum space.

   b) (2 P) Calculate its transversal and longitudinal parts, $D^{(T)}_{F,\mu\nu}(k)$ and $D^{(L)}_{F,\mu\nu}(k)$, and compare their behaviour for large k with that of the photon propagator.

4. a) (2 P) The semileptonic decays of strange hadrons, like $K^- \to \pi^0 \, e^- \, \bar{\nu}_e$ or $K^+ \to \pi^0 \, e^+ \, \nu_e$ or $\Sigma^- \to n \, e^- \, \bar{\nu}_e$, obey the rule $\Delta S = \Delta Q$, where ΔS and ΔQ are the changes of strangeness S and charge Q of the hadron. Explain this rule in the framework of V-A-theory in the quark picture.

   b) (1 P) Strangeness-changing weak decays, like the above ones, are suppressed relative to strangeness-conserving ones, like $n \to p \, e^- \, \bar{\nu}_e$ or $\pi^- \to \pi^0 \, e^- \, \bar{\nu}_e$, by a factor of about 20. Explain this.