Introduction to Quantum Field Theory
Problem sheet 5

Deadline: Wednesday 9 December 2015 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Canonical quantisation of a scalar field and Green's functions.

1. (2 P) For a complex scalar field the charge operator is given by

$$Q = \mathrm{i} \int d^3x \, \left(\varphi^\dagger \dot{\varphi} - \varphi \dot{\varphi}^\dagger \right).$$

   Show that

$$Q = \int \frac{d^3k}{(2\pi)^3 \, 2\omega_k} \left[a^\dagger(k) a(k) - b^\dagger(k) b(k) \right].$$

2. (2 P) Let $\varphi(x)$ be a free real scalar field. Show that

$$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^4k}{(2\pi)^3} \, \delta(k^2 - m^2)\Theta(k^0) \, \mathrm{e}^{-\mathrm{i}k\cdot(x-y)}.$$

3. (2 P) Verify that

$$\Delta_R(x-y) = -\mathrm{i} \int \frac{d^3k}{(2\pi)^3 \, 2\omega_k} \, \mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}-\vec{y})} \left(\mathrm{e}^{-\mathrm{i}\omega_k(x^0-y^0)} - \mathrm{e}^{\mathrm{i}\omega_k(x^0-y^0)} \right) \theta(x^0 - y^0)$$

   is a Green function of the Klein-Gordon operator.

4. (2 P) Solve the equation

$$(\nabla^2 - m^2) G(\vec{r}) = -\delta^{(3)}(\vec{r}).$$

   You should get $G(\vec{r}) = \mathrm{e}^{-mr}/4\pi r$. *Hint:* use Fourier transform.