Introduction to Quantum Field Theory
Problem sheet 2

Deadline: Wednesday 11 November 2015 (12 am)
at Dr. Giudice's office (KP 301)

Topics covered: Dirac equation and γ-matrices.

1. (3 P) Show that if $\psi(x)$ is a solution of the Dirac equation in an electromagnetic field, then it satisfies
$$\left[(\partial_\mu + \mathrm{i}eA_\mu)(\partial^\mu + \mathrm{i}eA^\mu) + \frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu} + m^2\right]\psi(x) = 0,$$
which is a generalized form of the Klein-Gordon equation. In this equation e is the charge of the particle, $\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma_\mu, \gamma_\nu]$ and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

2. (4 P) The Weyl representation of the γ-matrices is
$$\gamma^0 = -\begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \quad \gamma^\kappa = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}.$$
   a) Show that they obey the Clifford algebra;
   b) Calculate γ_5 in this representation.

3. (4 P) Show that:
   a) $\bar{u}^{(r)}(\vec{k})u^{(s)}(\vec{k}) = 2mc^2\delta_{rs}$,
   b) $\bar{v}^{(r)}(\vec{k})v^{(s)}(\vec{k}) = -2mc^2\delta_{rs}$,
   c) $\bar{u}^{(r)}(\vec{k})v^{(s)}(\vec{k}) = 0$,
   d) $\bar{v}^{(r)}(\vec{k})u^{(s)}(\vec{k}) = 0$.

   Hint: use the fact that $\bar{u}^{(r)}(\vec{k})u^{(s)}(\vec{k}), \ldots,$ are Lorentz invariant.

4. (3 P) Show that:
$$\omega^{\nu\mu}\gamma_\mu = -\frac{\mathrm{i}}{4}\omega^{\alpha\beta}[\gamma^\nu, \sigma_{\alpha\beta}].$$