Introduction to Quantum Field Theory
Problem sheet 10

Deadline: Wednesday 03 February 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Perturbation theory, Dirac propagator.

1. (6 P) Show that the propagator in $(1+1)$ dimensions

$$\Delta_F(x) = \int \frac{d^2k}{(2\pi)^2} \frac{\mathrm{e}^{-\mathrm{i}kx}}{k^2 - m^2 + \mathrm{i}\epsilon}$$

   is, for space-like x (set $x^0 = 0$), given by:

$$\Delta_F(x) = -\frac{\mathrm{i}}{2\pi} K_0(m|x|),$$

   where $K_n(y)$ is the modified Bessel function of second kind. What is the behaviour for large x ?

2. (3 P) In the φ^4 theory with a real scalar field the amputated 4-point function is defined by

$$\widetilde{G}_A^{(4)}(p_1,p_2,p_3,p_4) = \left[\widetilde{G}^{(2)}(p_1)\widetilde{G}^{(2)}(p_2)\widetilde{G}^{(2)}(p_3)\widetilde{G}^{(2)}(p_4)\right]^{-1} \widetilde{G}_c^{(4)}(p_1,p_2,p_3,p_4).$$

   (We neglect factors of Z_3 here.) A renormalised coupling can be defined through

$$g_R = \mathrm{i}\widetilde{G}_A^{(4)}(0,0,0,0).$$

   Calculate g_R to lowest order in perturbation theory.

3. (2 P) Show that

$$S_F(x) = (\mathrm{i}\gamma^\mu \partial_\mu + m)\Delta_F(x)$$

   is Green's function to the Dirac operator $\mathrm{i}\gamma^\mu\partial_\mu - m$.