Bakterienwachstum: Hydrodynamische Modelle

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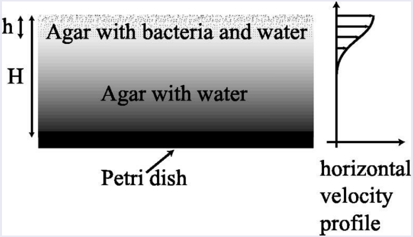
Outline

Introduction

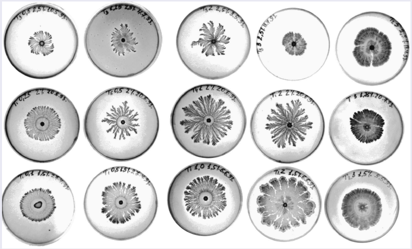
Experimental background

Experiments on the growth of bacterial colonies are very easy

- In a petri dish with a thin layer of agar bacteria are injected in a small inoculum.
- Closer look to the colony via light microscopic methods.



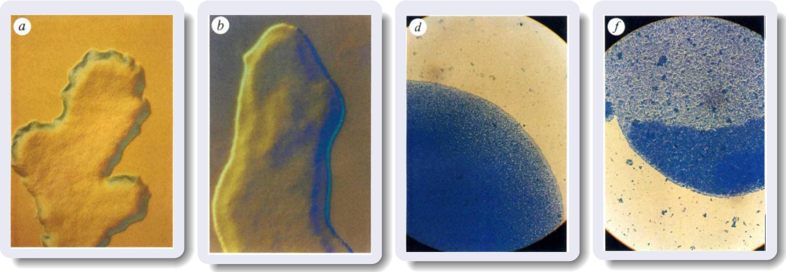
Experimental results



- The concentrations of nutrients are $0.1, 0.5, 1.0, 2.0$ and $3.0 g l^{-1}$ from left to right
- Structures become denser for higher concentration of nutrients. For low concentration, dense patterns are onserved.

[0]Y. Kozlovsky et al (1999)

Envelopes



- Sharp envelopes down to microscopic scales are observed.
- In "dry" regions bacteria produce extracellular liquid for being abel to swim.

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[0] Y. Kozlovsky et al (1999)
[0] E. Ben-Jacob et al. (1994)

Dynamics and Vortices

Vortices in motion of the bacteria in water.

Derivation of a hydrodynamic model

Basics

The hydrodynamics of bacterial colonies can be discribed by four coupled partial differential equations for the densities of:

- nutrients (S)
- water (W)
- bacteria(N)

and the dynamics of the velocity field (v)

Dynamics of nutrients

Dynamics of nutrients can be discribed by a reaction-diffusion equation:

$$\frac{\partial S}{\partial t} = R_S(S,N,W) + D_S \nabla^2 S$$

- $R_S(S,N,W)$: consumption of nutrients by the bacteria.

Dynamics of the bacteria

Continuity equation for dynamics of bacteria:

$$\frac{\partial N}{\partial t} + \nabla \cdot (Nv) = R_N - \nabla j^N$$

With $j^N = -D^N \nabla N$ one obtains

$$\frac{\partial N}{\partial t} + \nabla \cdot (Nv) = R_N + \nabla (D^N(S,N,W) \nabla N).$$

- R_N represents growth of the bacterial colonie.

The bacterial liquid

The behavior of the bacterial liquid and the interaction between bacteria is still relativly unknown. Assumptions are:

- The bacteria constitute a compressible Newtonian liquid.
- Bacteria are living species. They show an individuell activity represented by noise.

Dynamics of water

Continuity equation for density of water:

$$\frac{\partial W}{\partial t} + \nabla \cdot (Wv) = R_W + \nabla(D^W(S,N,W)\nabla N) - \nabla j^W$$

$j^N = -j^W$ can be assumed.

$$\frac{\partial W}{\partial t} + \nabla \cdot (Wv) = R_W + \nabla(D^W(S,N,W)\nabla N) - \nabla(D^N(S,N,W)\nabla N)$$

Conservation of momentum

The conservation equation of linear momentum of the mixture is the Navier-Stokes equation:

$$\rho \frac{\partial v}{\partial t} + \rho(v \cdot \nabla v) = \nabla T + F$$

$$\rho = N + W$$

T : stress tensor
F : external forces

The stress tensor

T can be written as:

$$\nabla T = -\nabla p + \mu \nabla^2 v + \lambda \nabla (\nabla v)$$

$p = p^W + p^N$: pressure of the complex fluid
μ : dynamic viscosity
λ : second viscosity takes into acount that the bacterial fluid is compressible.

Pressure terms

Water is assumed to be incompressible, so that p^W is a function of the velocity field v_W of water so that

$$\nabla v_W = 0$$

The pressure of the bacterial phase can be separated in a incompressible and a compressible part proportional to N^2

$$p_N = p_0^N + p_c^N = p_0^N + \gamma(S, W) N^2$$

$\gamma > 0$ rises a force which drives bacteria away from overcrowded areas.

External forces

The external forces can be written as:

$$F = F_s + F_g + F_e$$

- F_s : interaction between the fluid and the agar ($F_s = -\alpha v$)
- F_g : describes changes due to bacterial activity.
- F_e : external forces on the whole system, such as gravity.

The four differential equations

The four basic equations can be written as:

$$\frac{\partial S}{\partial t} = R_S(S,N,W) + D_S\nabla^2 S \qquad (1)$$

$$\frac{\partial N}{\partial t} + \nabla\cdot(Nv) = R_N + \nabla(D^N(S,N,W)\nabla N) \qquad (2)$$

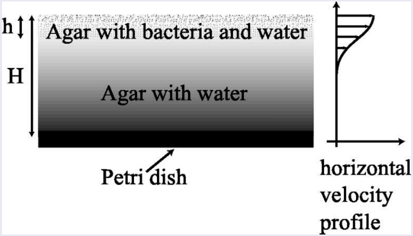
$$\frac{\partial W}{\partial t} + \nabla\cdot(Wv) = R_W + \nabla(D^W(S,N,W)\nabla N) - \nabla(D^N(S,N,W)\nabla N) \qquad (3)$$

$$\frac{\partial v}{\partial t} + (v\cdot\nabla v) = \frac{1}{N+W}(\nabla T + F_s + F_g) \qquad (4)$$

[0] J. Lega, T. Passot (2006)

Reduction on two dimensions

- The fluid motion takes place in a layer of thickness small thickness h.
- The problem can be reduced to a two dimensional one with v(x,y,z) = f(z)u(x,y) and integrating over h



The two dimensional problem

The four equations from above transform to:

$$\frac{\partial S}{\partial t} = R_S(S,N,W) + D_S \nabla_h^2 S \tag{5}$$

$$\frac{\partial N}{\partial t} + \nabla_h \cdot (N\overline{v}) = R_N + \nabla_h (D^N(S,N,W)\nabla_h N) \tag{6}$$

$$\frac{\partial W}{\partial t} + \nabla_h \cdot (W\overline{v}) = R_W + \nabla_h (D^W(S,N,W)\nabla_h N) - \nabla_h (D^N(S,N,W)\nabla_h N) \tag{7}$$

$$\frac{\partial \overline{v}}{\partial t} + \zeta(\overline{v} \cdot \nabla_h \overline{v}) = \frac{1}{N+W}\{-\nabla_h \overline{p} + \mu \nabla_h^2 \overline{v} + \lambda \nabla_h (\nabla_h \cdot \overline{v}) - \eta \overline{v} + \overline{F}_s + \overline{F}_g)\} \tag{8}$$

[0] J. Lega, T. Passot (2006)

Last unknown quanties

- In these equations the following quantities mean:

$$\overline{v}(x,y) = \langle f \rangle u(x,y)$$

$$\zeta = \frac{\langle f^2 \rangle}{\langle f \rangle^2},$$

- $\overline{F} = \langle F \rangle$ means:

$$\langle F \rangle = \frac{1}{h} \int_{-h}^{0} F(z) dz$$

For an arbitrary function F(z)

- The two dimensional approximation is accurate if $\zeta \simeq 1$.

[0] J. Lega, T. Passot (2006)

Reaction terms

Taking into acount nutrient consumption by and growth of bacteria the first reaction terms become:

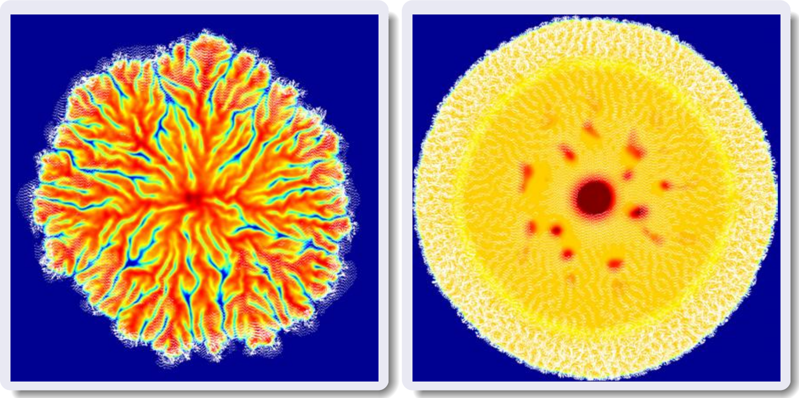
$$R_S(S,N,W) = -R_N(S,N,W) = NS$$

In this model loss of water, e. g. by evaporation at the surface, is neglected:

$$R_W(S,N,W) = 0$$

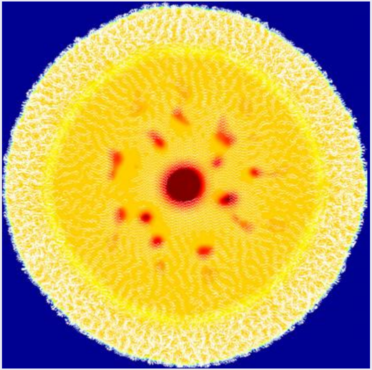
Results

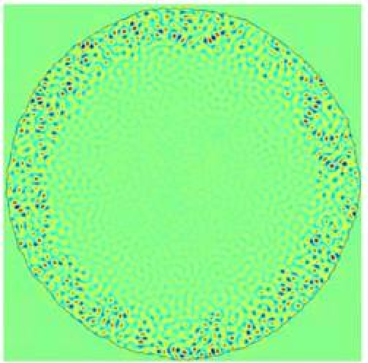
Simulations



J. Lega, T. Passot (2006)

A closer look at the envelope





[0] J. Lega, T. Passot (2006)

Abilities and Linitations of the model

Abilities

- Numerical simulations of the model show the branching in growth of the colony.
- Vortices as seen in the experiment shown above are obtained.

Limitations

- The interaction between bacteria is still almost unknown
- Perhaps the ansatz of a Newtonian fluid for the bacterial phase is not the best.
- Quantitative description of the branching process is still an open question.

Thank you for your attention!

Lyrics

- E. Ben-Jacob et al. : Letters to Nature (1994)*Generic modeling of cooperative growth patterns in bacterial colonies*.
- J. Lega, T. Passot : Journal of Nonlinearity (2006) *Hydrodynamics of bacterial colonies*.
- J. Lega, T. Passot : Physical Review Letters (2003) *Hydrodynamics of bacterial colonies: A model*.
- Y. Kozlovsky et al. : Physical Review Letters (1999) *Lubricating bacteria model for branching growth of bacterial colonies*