Heavy quark production in pp, pA and AA collisions

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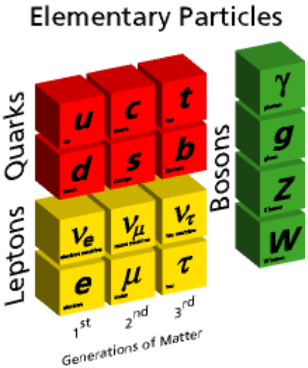
SM particle content



- 3 generations - increasing mass

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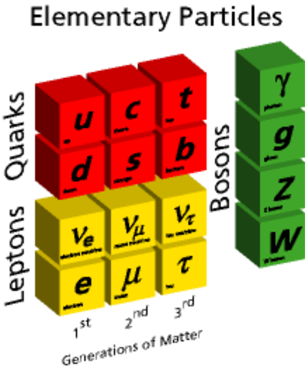

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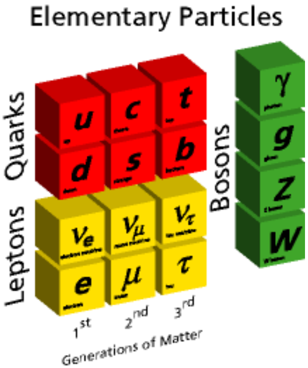

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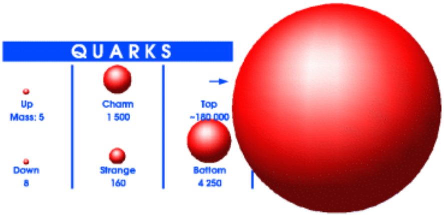

Quark masses / GeV

d	u	s	c	b	t
$3 - 9 \cdot 10^{-3}$	$1 - 5 \cdot 10^{-3}$	$75 - 170 \cdot 10^{-3}$	$1.15 - 1.35$	$4 - 4.4$	174.3 ± 5.1

- $m_{u,d,s} = 0$ in calculations because of small masses
- $m_{c,b,t} \neq 0$

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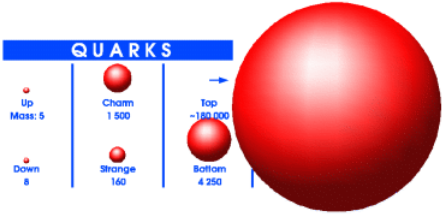

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Heavy mesons

Mesons with $c/\bar{c}$-content

Pseudoscalar mesons	Vector mesons	Quark content
D^0	D^{*0}	$c\bar{u}$
D^+	D^{*+}	$c\bar{d}$
D_s^+	D_s^{*+}	$c\bar{s}$
$\overline{D^0}$	$\overline{D^{*0}}$	$u\bar{c}$
D^-	D^{*-}	$d\bar{c}$
D_s^-	D_s^{*-}	$s\bar{c}$

- Quarkonia: important example J/Ψ with $c\bar{c}$ content

Mesons with $b/\bar{b}$-content

- Same nomenclature with $D \leftrightarrow B$, $c \leftrightarrow b$, $\bar{c} \leftrightarrow \bar{b}$,
- Quarkonia: important example Υ with $b\bar{b}$ content

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Heavy baryons

Baryons with c-content

Baryon	Quark content	
Ω_{ccc}^{++}	ccc	Spin $\frac{3}{2}$
Ω_{cc}^{+}	scc	
Ω_{c}^{0}	ssc	
Ξ_{cc}^{++}	ucc	
Ξ_{cc}^{+}	dcc	
Ξ_{c}^{+}	usc	
Ξ_{c}^{0}	dsc	
Σ_{c}^{++}	uuc	
Σ_{c}^{+}	udc	
Σ_{c}^{0}	ddc	
Λ_{c}^{+}	udc	Spin $\frac{1}{2}$

DIS ...

$e^-(k^\mu) + p(p^\mu) \to e^-(k'^\mu) + X$

- Momentum transfer $Q^2 = -q^2$ with $q^\mu = k^\mu - k'^\mu$
- $M^2 = p^2$ (M: proton mass)
- Define $M\nu = p \cdot q$ and $x = Q^2/(2M\nu)$
- **Bjorken limit:** $Q^2, \nu \to \infty$ with x fixed

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- $\Rightarrow \gamma*$ scatters off pointlike partons

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- $\Rightarrow$ Matrix element calculable ($e^- + u/d \to e^- + u/d$)
  (and we find $x = \xi$ because outgoing u/d is on-shell)

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... and the naive parton model

- One finds $\hat{F}_2 = x\, e_q^2\, \delta(x - \xi) = 2x\, \hat{F}_1$ (partonic)
- $q(\xi)\,\mathrm{d}\xi$ probability that quark q carries momentum fraction $\in [\xi; \xi + \mathrm{d}\xi]$ $0 \leq \xi \leq 1$

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- Proton structure function

$$F_2(x) = \sum_{q,\bar{q}} \int_0^1 \mathrm{d}\xi\, q(\xi)\, x\, e_q^2\, \delta(x - \xi) = \sum_{q,\bar{q}} e_q^2\, x\, q(x)$$

$$F_2 = x \left[\tfrac{4}{9}(u + \bar{u}) + \tfrac{1}{9}(d + \bar{d} + s + \bar{s}) \right]$$

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Experimental results

Proton consists of *uud* and sea of $q\bar{q}$ with $m_q \ll Q$

- At a scale $\mathcal{O}(1\,\mathrm{GeV})$ and assuming sea symmetric in flavours

$$\sum_q \int_0^1 \mathrm{d}x\, x\, [q(x) + \bar{q}(x)] \approx 0.5$$

$\rightarrow$ 50% of proton momentum carried by gluons

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LO processes in $pp \to Q\overline{Q}$

$2 \to 2$ processes at LO

- $q\bar{q} \to Q\overline{Q}$: $\overline{\Sigma}|\mathcal{M}|^2/g_s^4 = \frac{4}{9}\left(\tau_1^2 + \tau_2^2 + \frac{\rho}{2}\right)$
- $gg \to Q\overline{Q}$: $\overline{\Sigma}|\mathcal{M}|^2/g_s^4 = \left(\frac{1}{6\tau_1\tau_2} - \frac{3}{8}\right)\left(\tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2}\right)$

$$\tau_1 = \frac{2p_1\cdot p_3}{s}, \quad \tau_2 = \frac{2p_2\cdot p_3}{s}, \quad \rho = \frac{4m^2}{s}, \quad s = (p_1 + p_2)^2$$

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  $\to$ additional factor of $\frac{1}{36}$
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”Problems” at NLO

UV- and IR-divergencies

- UV due to loops $\sim \int \mathrm{d}^4k \, \frac{1}{k^{2n}}$
  (self-energy diagrams, vertex corrections)
- IR due to participating massless particles
  occur in $2 \rightarrow 3$ processes
  (real gluon emission)

  and in virtual corrections

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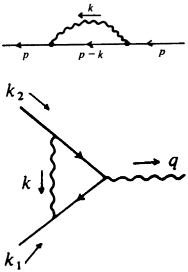

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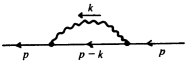

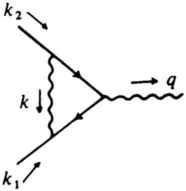

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Regularization

Regularization: method to make divergent integral manageable

Renormalization: getting rid of divergencies
(redefinition of fields, masses and couplings; absorptions)

Regularizations methods

- Dimensional regularization $\int \frac{d^4k}{(2\pi)^4} \to \int \frac{d^Dk}{(2\pi)^D}$ with $D = 4 - (2)\epsilon$
- Mass regularization (gluon mass λ)
- (Cut-off, Pauli-Villars, Analytic, Lattice, ...)

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Identifying types of IR divergencies

Consider vertex correction to $q\bar{q} \to Q\overline{Q}$

$$\Lambda_\mu = g^2 C_F \int \frac{d^4k}{(2\pi)^4 i} \frac{1}{k^2-\lambda^2} \gamma_\rho \frac{1}{m-\not{k}+\not{k}_1} \gamma_\mu \frac{1}{m-\not{k}+\not{k}_2} \gamma^\rho$$

Ignore UV-divergence and $\lambda \to 0$

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$$\Lambda_\mu = \frac{g^2 C_F}{8(2\pi)^3} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{\infty} \frac{d\omega}{\omega} \frac{\dots}{(k_{10}-|\vec{k}_1|\cos\theta)(k_{20}+|\vec{k}_2|\cos\theta)}$$

with $k_{i0} = (\vec{k}_i^2 + m^2)^{\frac{1}{2}}$

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with $k_{i0} = (\vec{k}_i^2 + m^2)^{\frac{1}{2}}$

$\to$ Divergency because of low momentum singularity $1/\omega$

Identifying types of IR divergencies

Consider vertex correction to $q\bar{q} \rightarrow Q\overline{Q}$

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Ignore UV-divergence and $\lambda \rightarrow 0$

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General idea

- Fields, couplings, gauge parameters, masses get Z factors: e.g. $A_\mu^a = \sqrt{Z_3} A_{r\mu}^a$, $g = Z_g g_r$, ...

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and add one heavy flavor $n_f = n_{lf} + 1$

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$\Pi^{ab}_{\mu\nu}(k) = \delta^{ab}(k_\mu k_\nu - q^2 g_{\mu\nu})\Pi(k^2)$
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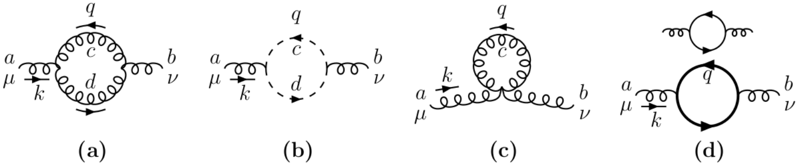

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Calculating the loop $-i\Sigma$ in quark self-energy diagram yields
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Short-distance and long-distance dependences factorize in DIS

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Evolution of parton distributions

Consequence of factorization:

Parton distributions at any scale can be predicted
Evolution from μ to μ' if μ, μ' large ($\rightarrow \alpha_s(\mu), \alpha_s(\mu')$ small)

Similar to RGE but for partons

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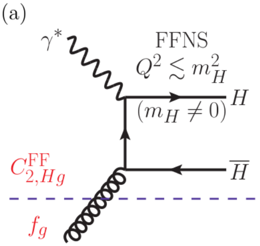

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Factorization theorem (in n_fFS):

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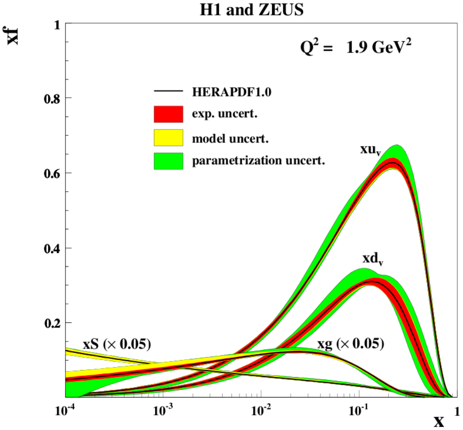
- DIS data taken from HERA, ZEUS and H1 collaborations
- Heavy quarks treated in GM-VFNS, $\overline{\text{MS}}$ factorization scheme



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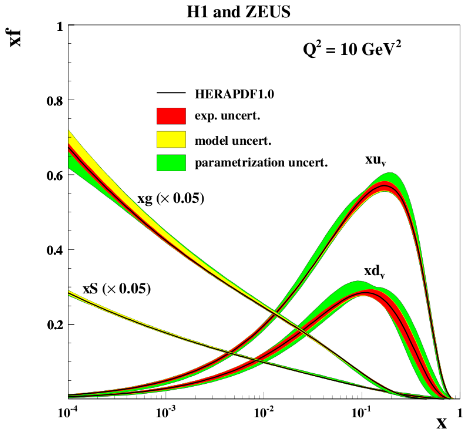

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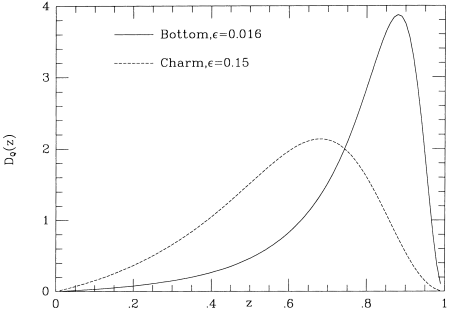

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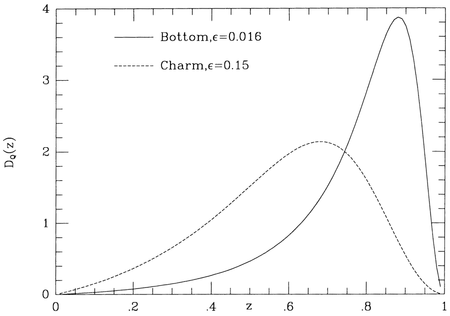

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Partons: free hadrons vs. nucleons

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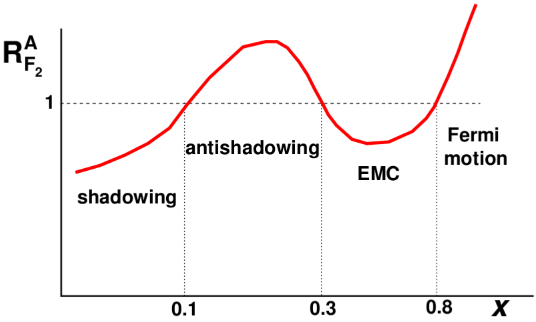

(Armesto; 2006)

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