Excited states of the QCD flux tube

Bastian Brandt

Forschungsseminar Quantenfeldtheorie

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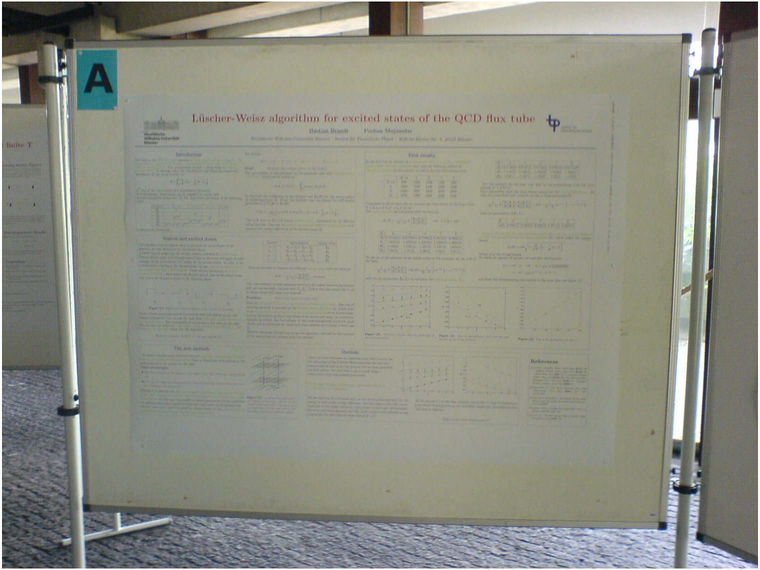
A
Lüscher-Weisz algorithm for excited states of the QCD flux tube
Bastian Brandt
Pushan Majumdar

Contents

1. Why flux tubes?

Introduction

- When QCD came up there was one big question:

Where are the quarks?

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Where are the quarks?

- No isolated color non-singlets are seen in nature.
- Color confinement:

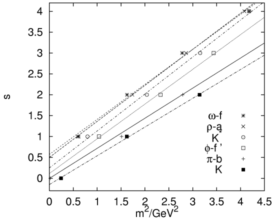
There are no isolated particles in nature, corresponding to a color non-singlet!

- The exact mechanism of color confinement is still unknown!
- Important question:

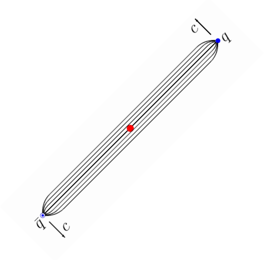
What is the nature of the confining force?

Regge trajectories and flux tubes

Regge trajectories in the hadron spectrum:



[G. Bali: hep-ph/0001312]



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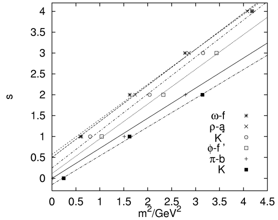

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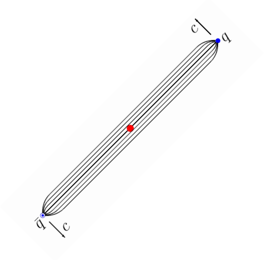

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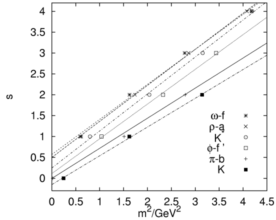

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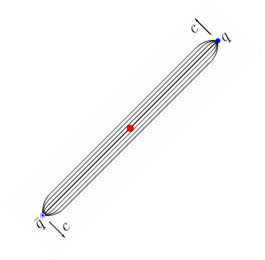

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  Quarks connected with a straight tube-like object, with a constant energy density $\sigma = \frac{1}{2\pi\ \alpha}$.

  $\longrightarrow$ Flux-tube

  System rotates with almost speed of light.

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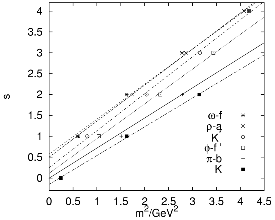

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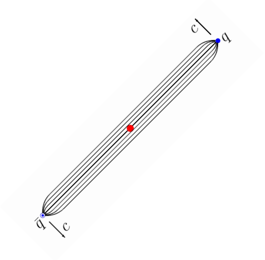

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  Also allow fluctuations of the flux tube.

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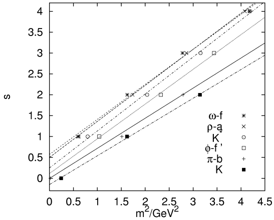

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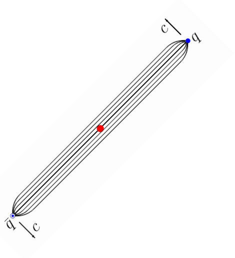

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- More realistic:
  Also allow fluctuations of the flux tube.
- Large distance R between quarks:
  Ratio thickness to length of the flux-tube is going to 0.

  $\longrightarrow$ Effective string models

2. Flux tubes and Wilson loops

Flux tubes in QCD

Can flux tubes appear in QCD?

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  But:

  No derivation from first principles!

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  But:

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$\longrightarrow$ Look at the flux tubes in lattice calculations

In order to do this we need an operator corresponding to the flux tube.

The correlation function for a flux tube in quenched QCD

We now look at quenched QCD!

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Correlation function

$$\left\langle \mathcal{Q}(R,0)\,\mathcal{Q}^{+}(R,T)\right\rangle = \frac{1}{Z_E}\int \mathcal{D}A\,\mathcal{D}q\,\mathcal{D}\bar{q}\;\mathcal{Q}(R,0)\,\mathcal{Q}^{+}(R,T)\,e^{-S^E_{YM}}$$

Operator for a flux tube at time τ:

$$\hat{\mathcal{Q}}(R,\tau) \equiv \bar{q}(\underline{x}_1,\tau)\left(\prod_{\underline{y},j\in\mathcal{V}(\tau)} U_j(\underline{y},\tau)\right) q(\underline{x}_2,\tau)$$

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Spectral representation:

$$\left\langle \mathcal{Q}(R,0)\, \mathcal{Q}^{+}(R,T) \right\rangle = \alpha(R)\, e^{-E_0(R)\, T} \left(1 + \sum_{k=1}^{\infty} \beta_k(R)\, e^{-\Delta E_k(R)\, T} \right)$$

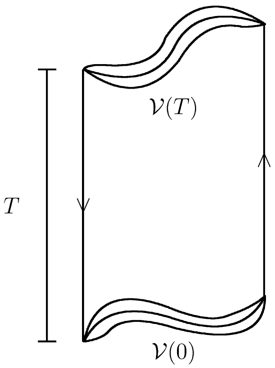
with

$$\alpha(R) = \eta(R)\; |\langle \mathcal{Q}(R,0) \mid 0 \rangle|^2 , \quad \beta_k(R) = \tfrac{1}{\alpha}\, |\langle \mathcal{Q}(R,0) \mid k \rangle|^2$$

and $\quad \Delta E_k(R) = E_k(R) - E_0(R)$

From flux tubes to Wilson loops

Integrate out the static quarks:



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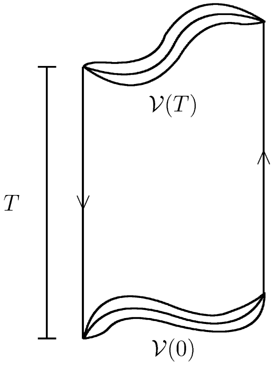
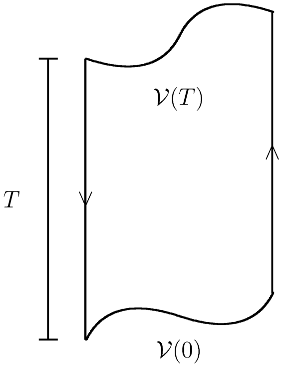
Integrate out the static quarks:



Limit
$R \to \infty$:
$\longrightarrow$



3. Effective string theories

Nambu string theory

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$\longrightarrow$ Nambu-Goto action:

$$S_{NG} = -\frac{1}{2\pi\,\alpha} \int d\tau \int_0^\pi d\kappa \sqrt{\left(\dot{X}^\mu X'_\mu\right)^2 - \left(\dot{X}^\mu\right)^2 \,(X'^\mu)^2}$$

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Search for a solution with the boundary conditions:

$$X^1(\pi, \tau) = R, \quad X^1(0, \tau) = X^i(0, \tau) = X^i(\pi, \tau) = 0 \quad \forall\, \tau \in \mathbb{R}$$

and $\quad X^0(0, \tau) = X^0(\pi, \tau) = p^0\, \tau$

Nambu string theory

Energies in formal canonical quantization:

$$E_n = \sigma \, R \, \sqrt{1 + \frac{2\pi}{\sigma \, R^2} \, \left(n - \frac{1}{24} \, (d-2) \right)}$$

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But:
The quantisation is only consistent with the Lorentz algebra for

$$d = 26 \quad \text{and} \quad \sigma = \frac{1}{2\pi}.$$

Weyl anomaly

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Weyl anomaly

$\longrightarrow$ We need other effective stringtheories for the Wilson loop in 4 dimensions.

Lüscher-Weisz effective string theory

Effective string theory for the partition function of Polyakov loop correlators.

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Open-closed duality

The Polyakov loop can also wind around a compactified spatial dimension.

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$\longrightarrow$ The partition function corresponds to a closed string, winding around the compactified dimension.

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Duality and Nambu string theory

Duality holds excactly for Nambu string theory.
The energies of the closed string are:

$$\tilde{E}_n = \sigma\, T \sqrt{1 + \frac{8\pi}{\sigma\, T^2}\left(n - \frac{1}{24}(d-2)\right)}$$

Lüscher-Weisz effective string theory

The effective action consists of all terms that are consistent with locality and conformal invariance.

$$
\begin{aligned}
S_{LW} &= \frac{1}{2}\int d\tau d\kappa \left[\partial_a X^i\,\partial_a X_i\right] \\
&+ \frac{1}{4}\,b\int d\tau \left[\partial_\kappa X^i\,\partial_\kappa X_i\Big|_{\kappa=0} + \partial_\kappa X^i\,\partial_\kappa X_i\Big|_{\kappa=R}\right] \\
&+ \frac{1}{4}\int d\tau d\kappa \left[c_1\;\partial_a X^i\,\partial_a X_i\,\partial_b X^j\,\partial_b X_j + c_2\,\partial_a X^i\,\partial_b X_i\;\partial_a X^j\,\partial_b X_j\right]
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&+ \frac{1}{4}\int d\tau d\kappa \left[c_1 \; \partial_a X^i \, \partial_a X_i \, \partial_b X^j \, \partial_b X_j + c_2 \, \partial_a X^i \, \partial_b X_i \; \partial_a X^j \, \partial_b X_j\right]
\end{aligned}
$$

Open closed duality for S_{LW}

Demands for the coupling constants:

$$b = 0 \quad \text{and} \quad (d-2)\, c_1 + c_2 = \frac{d-4}{2\,\sigma}$$

Lüscher-Weisz effective string theory

Resulting energies:

$$
\begin{aligned}
E_{n,l} \quad &= E_n^0 + \frac{\pi^2}{R^3}\, c_1 \,\left[n\, \left(\tfrac{1}{12}\,(d-2) - n\right) + \alpha_{n,l}\,(c_2 + 2\,c_1)\right] \\
&+ \left(\tfrac{\pi}{24}\right)^2\, \tfrac{d-2}{2\,R^3}\, \left[2\,c_1 + (d-1)\,c_2\right]
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Especially for $d = 3$:

$$E_n = \sigma\,R + \frac{\pi}{R}\left(n - \frac{1}{24}\right) - \frac{\pi^2}{2\,\sigma\,R^3}\left(n - \frac{1}{24}\right)^2 + \mathcal{O}\left(\frac{1}{R^5}\right) = \sigma\,R\,\sqrt{1 + \frac{2\,\pi}{\sigma\,R^2}\left(n - \frac{1}{24}\,(d-2)\right) + \mathcal{O}(1/R^6)}$$

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Agrees with Nambu up to $\mathcal{O}(1/R^5)$!

Polchinski-Strominger effective string theory

Consider the string theory as a conformal field theory on the world sheet $X^\mu(\kappa^+, \kappa^-)$ of the string.
(We work with light-cone coordinates $\kappa^\pm$ and in radial quantisation)

The action consists of all terms in powers of $1/R$ that avoid the Weyl anomaly.

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$$S_{PS} = \frac{1}{2\pi\,\alpha} \int d\kappa^+ d\kappa^- \left[H + \alpha\,\beta\, \frac{\partial_+^2 X^\mu\, \partial_- X_\mu\, \partial_+ X^\nu\, \partial_-^2 X_\nu}{H^2} \right]$$

$$\text{with} \quad \alpha = \frac{1}{2\pi\,\sigma} \quad \text{and} \quad H \equiv \partial_+ X^\mu\, \partial_- X_\mu$$

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$$\text{with} \quad \alpha = \frac{1}{2\pi\,\sigma} \quad \text{and} \quad H \equiv \partial_+ X^\mu\, \partial_- X_\mu$$

Expand around the classical solution X^μ_{cl} of a closed string, wrapped around the compactified dimension x^1 with length R:

$$Y^\mu \equiv X^\mu - X^\mu_{cl} \quad \text{with} \quad X^\mu_{cl} = e^\mu_+ \frac{R}{2\pi}\,\kappa^+ + e^\mu_- \frac{R}{2\pi}\,\kappa^-$$

Polchinski-Strominger effective string theory

After a long calculation one arrives at the energies:

$$E_n = \sigma\, R \sqrt{1 + \frac{8\,\pi}{\sigma\, R^2}\, \left(n - \frac{1}{24}\,(d-2)\right) + \mathcal{O}(1/R^6)}$$

It agrees with the closed string case of Nambu string theory up to $\mathcal{O}(1/R^5)$!

4. Lüscher-Weisz algorithm

Looking for string effects in QCD

How to compare effective string theories with QCD?

$\longrightarrow$ Use computer simulations of lattice QCD and compare the results with the predictions of the effective string models.

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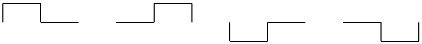
- Use special operators for the spatial line of the Wilson loops to couple to the excited string states.



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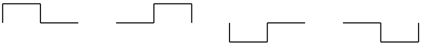

- Looking at the excited string states, we see that the ground states of the four (C, P) channels corresponds to the groundstate and the three lowest excited states.

Looking for string effects in QCD

Calculating the eigenstates of the correlation matrix we find:

(C,P)	superposition	energy
$(+,+)$	$S_1 + S_2 + S_3 + S_4$	E_0
$(+,-)$	$S_1 + S_2 - S_3 - S_4$	E_1
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$(-,+)$	$- S_1 + S_2 - S_3 + S_4$	E_3

Problem: We need to take loops with large time T and space R extends.
$\rightarrow$ Very small signal to noise ratio!

(Area law for wilson loops)

Locality and factorization of path integrals

Consider pure gauge theory on the lattice with the regular plaquette action.

- local
- gauge freedom

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Idea: Use the gauge freedom to keep all spatial links in some time slices fixed.
(We are not allowed to fix two links of one time-like plaquette!)

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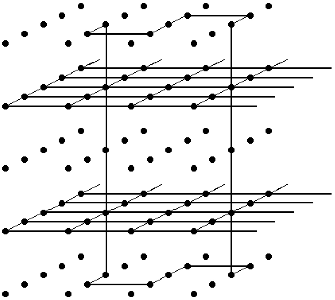
The physics in the sublattices, separated by the fixed time-slices is independent of each other because of the locality!



Locality and factorization of path integrals

Factorization of the action:

$$S^G[U] = \beta \,\mathrm{Tr}\left(\sum_P \left[1 - \frac{1}{2N}\,\left(U_P + U_P^{+}\right)\right]\right) = S^G_{sub,1} + S^G_{sub,2} + \ldots$$

with $$S^G_{sub,i} = \beta \,\mathrm{Tr}\left(\sum_{P_i} \left[1 - \frac{1}{2N}\,\left(U_{P_i} + U_{P_i}^{+}\right)\right]\right)$$

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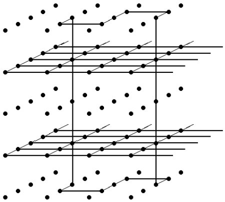
$\longrightarrow$ Factorization of path integrals:

$$\begin{aligned} \langle O \rangle &= \frac{1}{Z} \int \mathcal{D}U \, O[U] \, e^{-S^G[U]} \\ &= \prod_i \left[\frac{1}{Z_{sub,i}} \int \mathcal{D}U_{sub,i} \, O_{sub,i}[U] \, e^{-S^G_{sub,i}[U]} \right] \\ &\equiv \prod_i \left\langle O_{sub,i} \right\rangle_{sub,i} \end{aligned}$$

with $$O[U] = \hat{\mathcal{P}} \left(\prod_i O_{sub,i}[U] \right)$$

Factorization of a Wilson loop

We are interested in Wilson loops:



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Newer version of the algorithm: The spatial lines of the Wilson loops lie in the middle of the sublattice.

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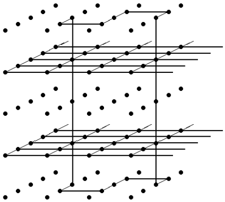

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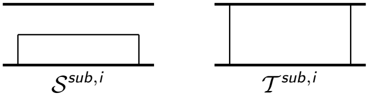

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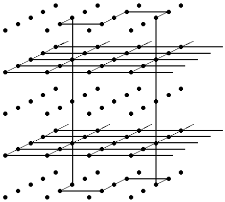

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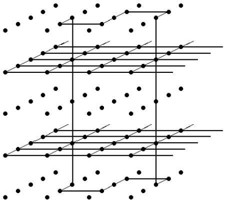

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We have to use the right multiplication to get a path-ordered Wilson loop from these operators!

Factorization of a Wilson loop

Multiplication for two $\mathcal{T}^i$:

$$\left[\mathcal{T}^i \circ \mathcal{T}^{i+1}\right]_{abcd} = \mathcal{T}^i_{a\mu c\nu}\, \mathcal{T}^{i+1}_{\mu b\nu d}$$

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Multiplication for the Wilson loop (on M sublattices):

$$\langle W(R,T)\rangle = \mathcal{S}^{1+}_{ac}\, \left[\mathcal{T}^2 \circ \mathcal{T}^3 \circ \cdots \circ \mathcal{T}^{M-1}\right]_{abcd}\, \mathcal{S}^M_{bd}$$

We have used the abbreviations:

$$\mathcal{T}^i \equiv \left\langle \mathcal{T}^{sub,i} \right\rangle_{sub,i} \quad \text{and} \quad \mathcal{S}^i \equiv \left\langle \mathcal{S}^{sub,i} \right\rangle_{sub,i}$$

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Algorithm leads to an exponential error reduction!

5. Results

Background of the calculations

To get enough statistics we work with $SU(2)$ in 3-dimensions, which is very cheap.

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We use the algorithm with only one level.
Our parameters are:

	T	N_L	subupdates	meas
$\beta = 5$	4	24	12000:1000	2000
$R = 4 - 9$	6	24	12000:1500	
$tsic = 2$	8	24	12000:2000	
	12	24	12000:2500	
$\beta = 5$	4	48	24000:1000	2000
$R = 10 - 12$	6	48	24000:2000	
$tsic = 2$	8	48	24000:6000	
	12	48	24000:12000	
$\beta = 7.5$	6	38	36000:1500	4400
$R = 7 - 20$	10	40	36000:3000	
$tsic = 4$	14	42	36000:9000	
	18	38	36000:18000	

Observables

Naive energies:

$$\bar{E}_n(R) = -\frac{1}{T_2 - T_1} \ln\left[\frac{W_n(R, T_2)}{W_n(R, T_1)}\right]$$

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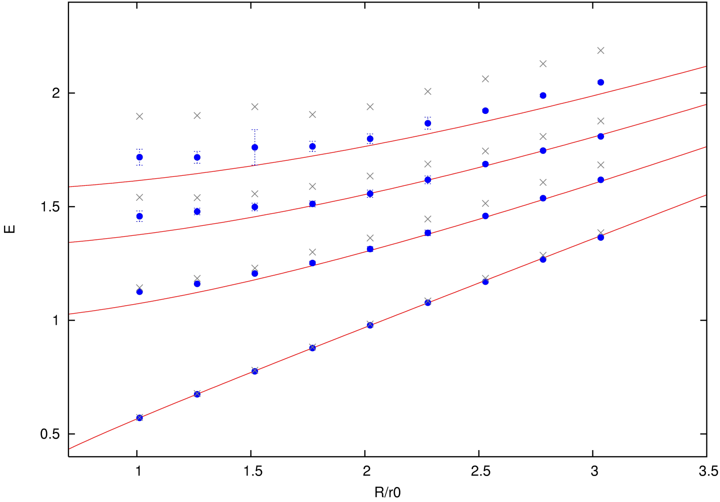
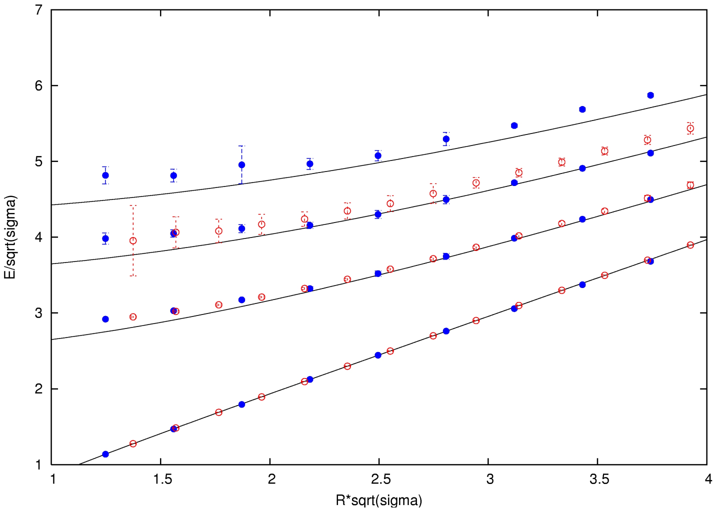
Energy differences:

$$\Delta\bar{E}_{n0} = -\frac{1}{T_2 - T_1} \ln\left[\frac{W_n(R, T_2)\, W_0(R, T_1)}{W_n(R, T_1)\, W_0(R, T_2)}\right] - \frac{1}{T_2 - T_1} b e^{-c\, T_1} \left(1 - e^{-c\,(T_2 - T_1)}\right)$$

Energies and comparison to Nambu string theory $\beta = 5$



Comparison between $\beta = 5$ and $\beta = 7.5$



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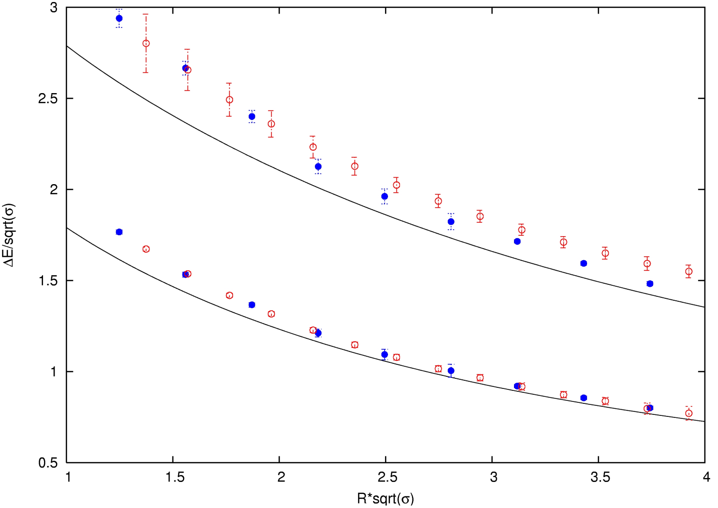

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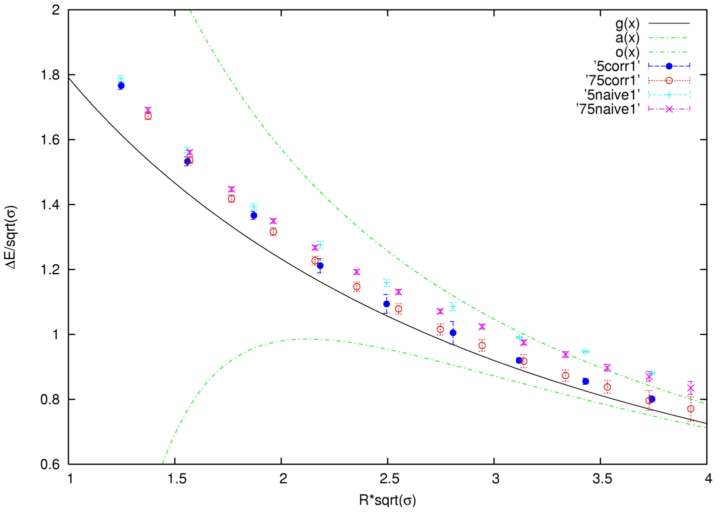

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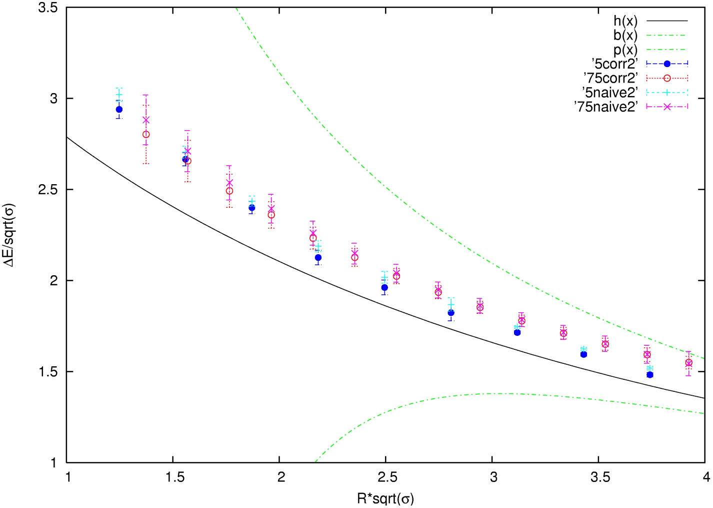

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Fit for $\beta = 5$ to the form:

$$\Delta E_{10}(R) = R \left[\sqrt{1 + \frac{6.021}{R^2}} - \sqrt{1 - \frac{0.2618}{R^2} + \frac{a}{R^6}} \right]$$

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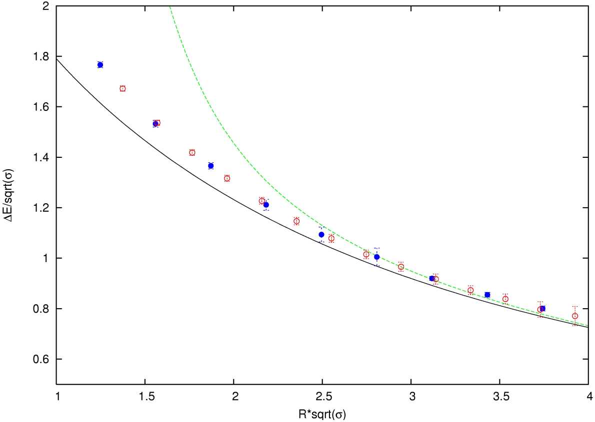

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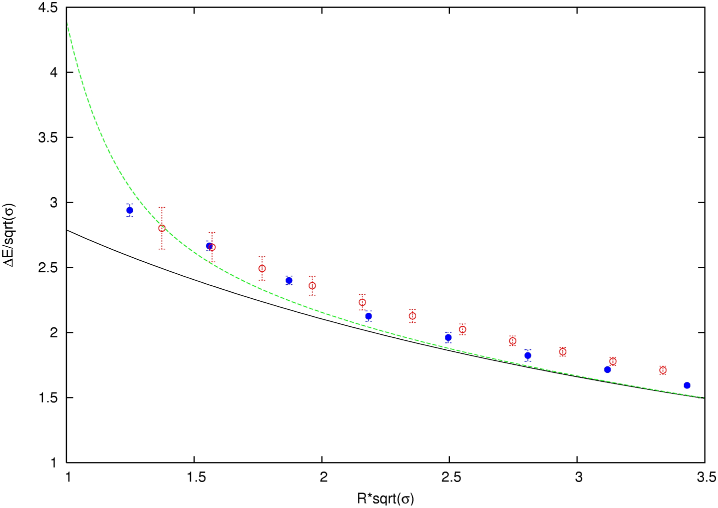

Conclusions

- Found an algorithm which has an error reduction that is sufficient to take a look at the excited states of the QCD flux tube.
- The results show good agreement with Nambu string theory.
- But:
  We work in quenched QCD, where no string breaking appears!
- The runs for the continuum limit are on the run and we hope to have the results in the new year.