

# How *strange* is the nucleon ?

-Hadronic uncertainties  
in direct Dark Matter detection -

Vincent Drach



in collaboration with  
C. Alexandrou, M. Constantinou, K.Hadjiyiannakou  
K. Jansen, G. Koutsou and A. Vaquero.

Institute for Theoretical Physics , Münster, Germany, 13th June 2016



# Outline



- Introduction
  - ★ Why DM ?
  - ★ How to detect DM ?
  - ★ Main assumptions
- Nucleon mass origins :
  - ★ Energy momentum tensor
  - ★ Heavy quark contribution
  - ★ Effective theory and phenomenological results
- Lattice techniques :
  - ★ Setup
  - ★ Indirect approach
  - ★ Challenges of *disconnected* diagrams
- Lattice results :
  - ★ Our setup
  - ★ New Results
  - ★ Comparison with other methods and collaborations
- Summary / Outlook

# Standard Model and beyond...

♦ Standard Model successful « Gauge Yukawa theory »

**BUT**

♦ Theoretical Issues :

★ Naturalness/Fine-tuning

★ Hierarchy

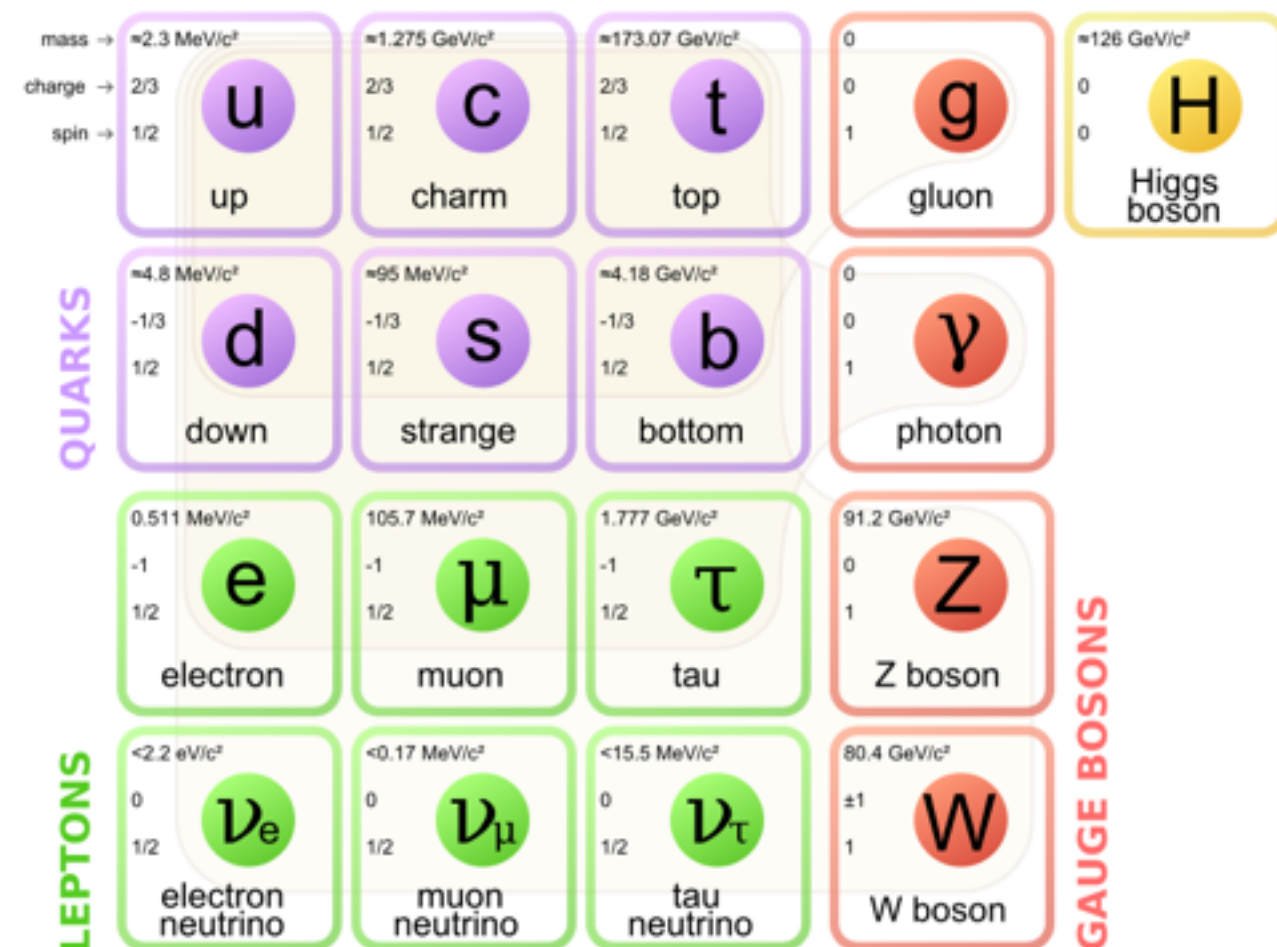
♦ Experimental evidences :

★ **Dark Matter**

★ Neutrino masses

★ Not enough CP violation

★ Hint(s) @ CERN ...



# Dark matter in a nutshell

Planck 2015 results. XIII. Cosmological parameters [1502.01589]

♦ Consistent and accumulating evidences for a large amount of Dark Matter component in the Universe

★ Cosmology

★ Astrophysics ( Rotation of spiral galaxies, velocity dispersion of Galaxies, Galaxy clusters and gravitational lensing)

♦ What do we know :

★ Gravitationally interacts

★ Electrically neutral

♦ Questions :

★ Relation with EW scale ?

★ Cold or Warm ?

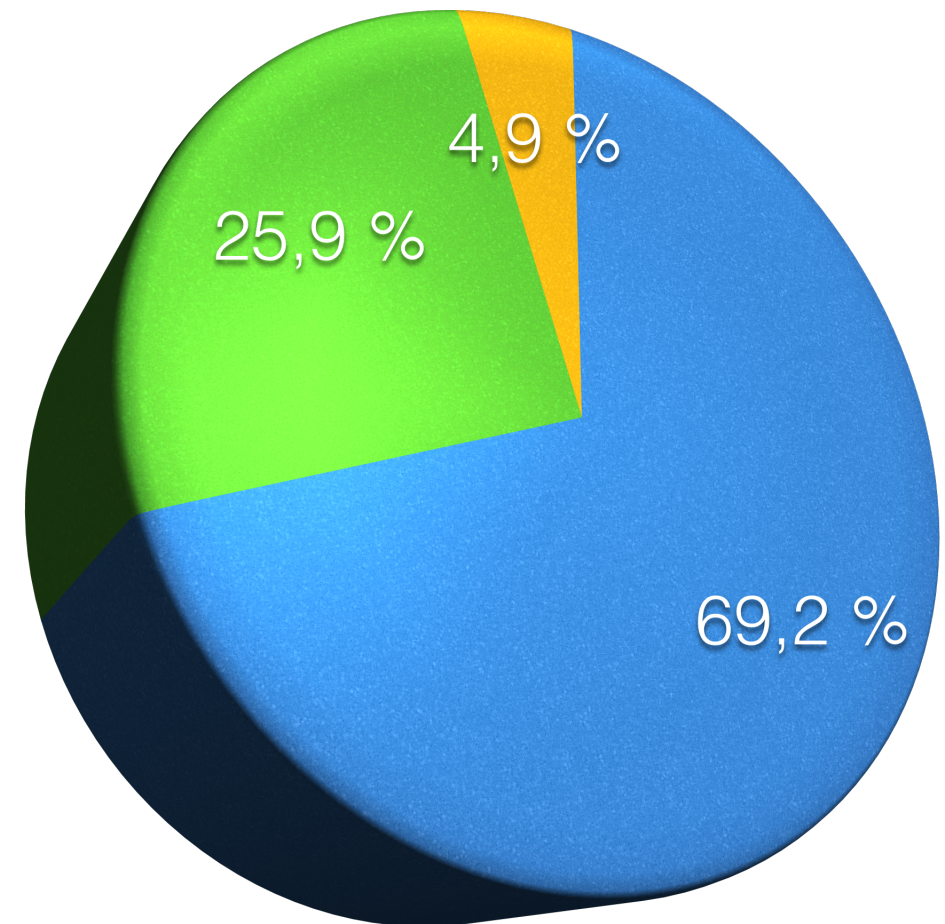
★ size of the self interaction ?

★ Coupled to Higgs boson?

★ Spin ?

★ Is it only one state ?

★ Can it be composite ?



**Energy budget of the universe (Planck)**



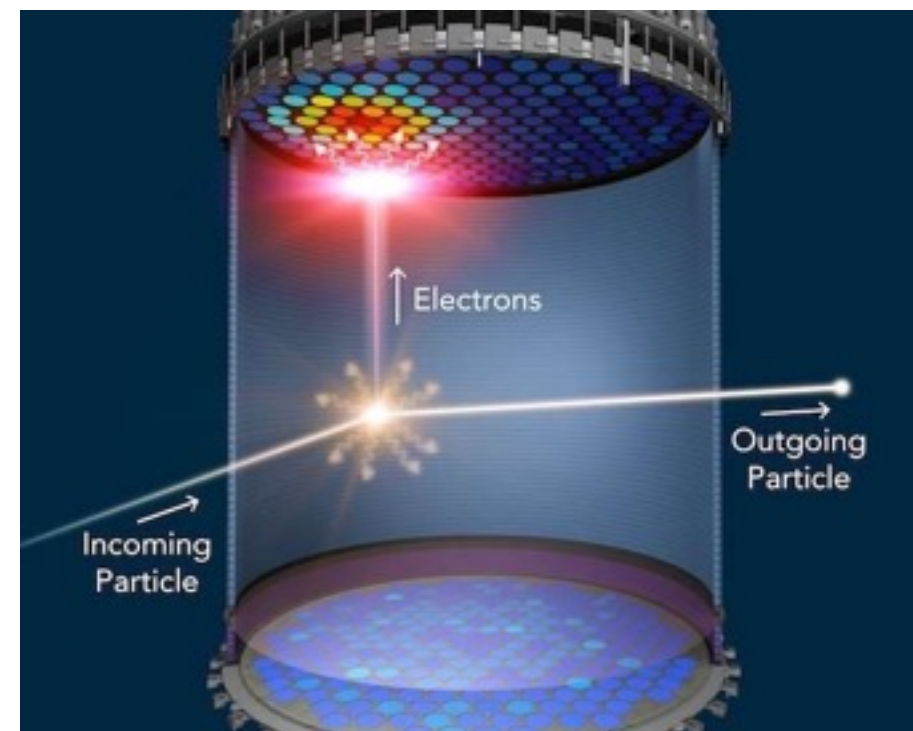
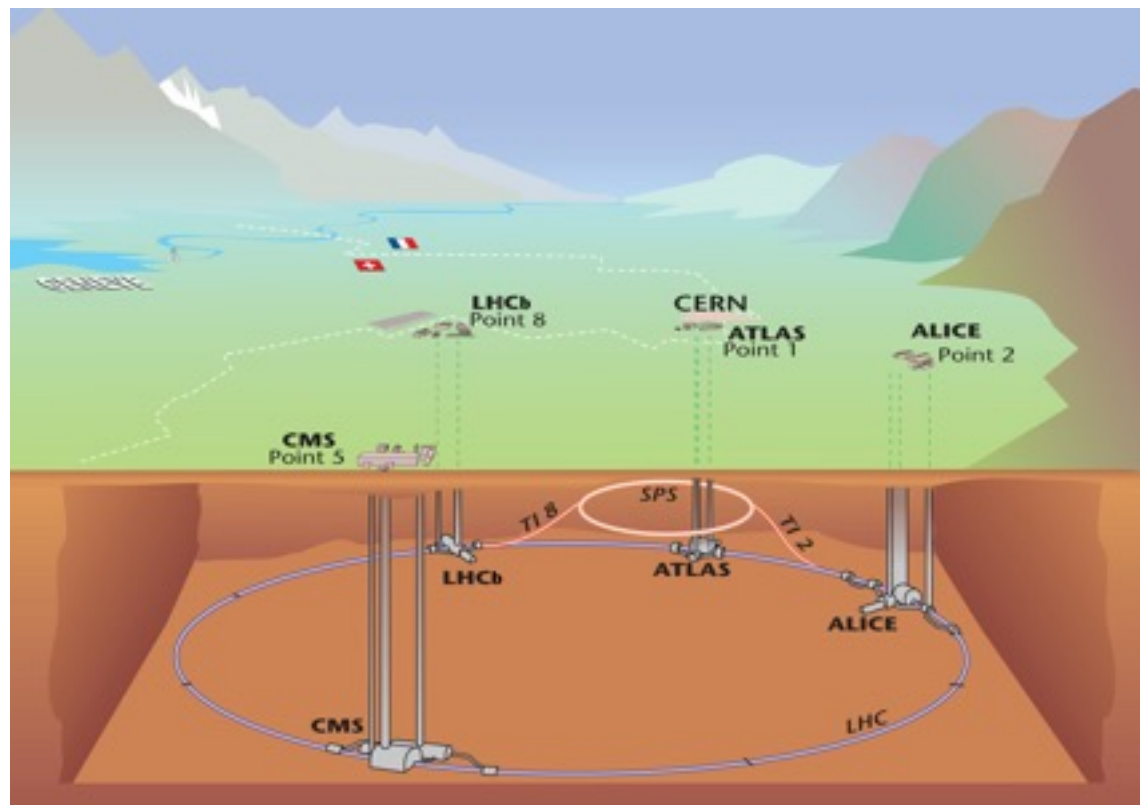
# Dark Matter searches

## Types of searches:

- Indirect detection
- Direct searches
- Colliders



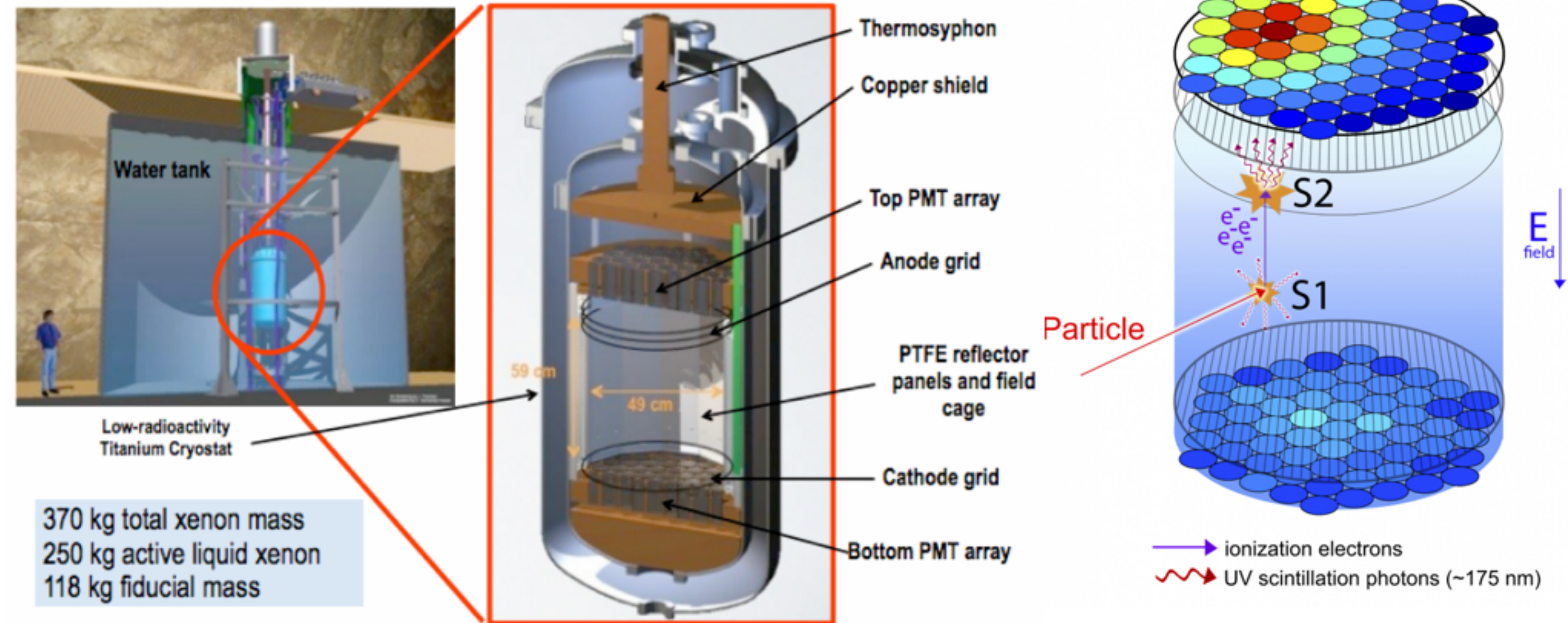
credit NASA



**Complementary searches**

# Direct detection - the LUX experiment

LUX Collaboration, Phys.Rev.Lett. 116 (2016) no.16



**Direct detection experiments constrain the nuclei-DM cross section**

# Direct detection

- ◆ Assumptions:

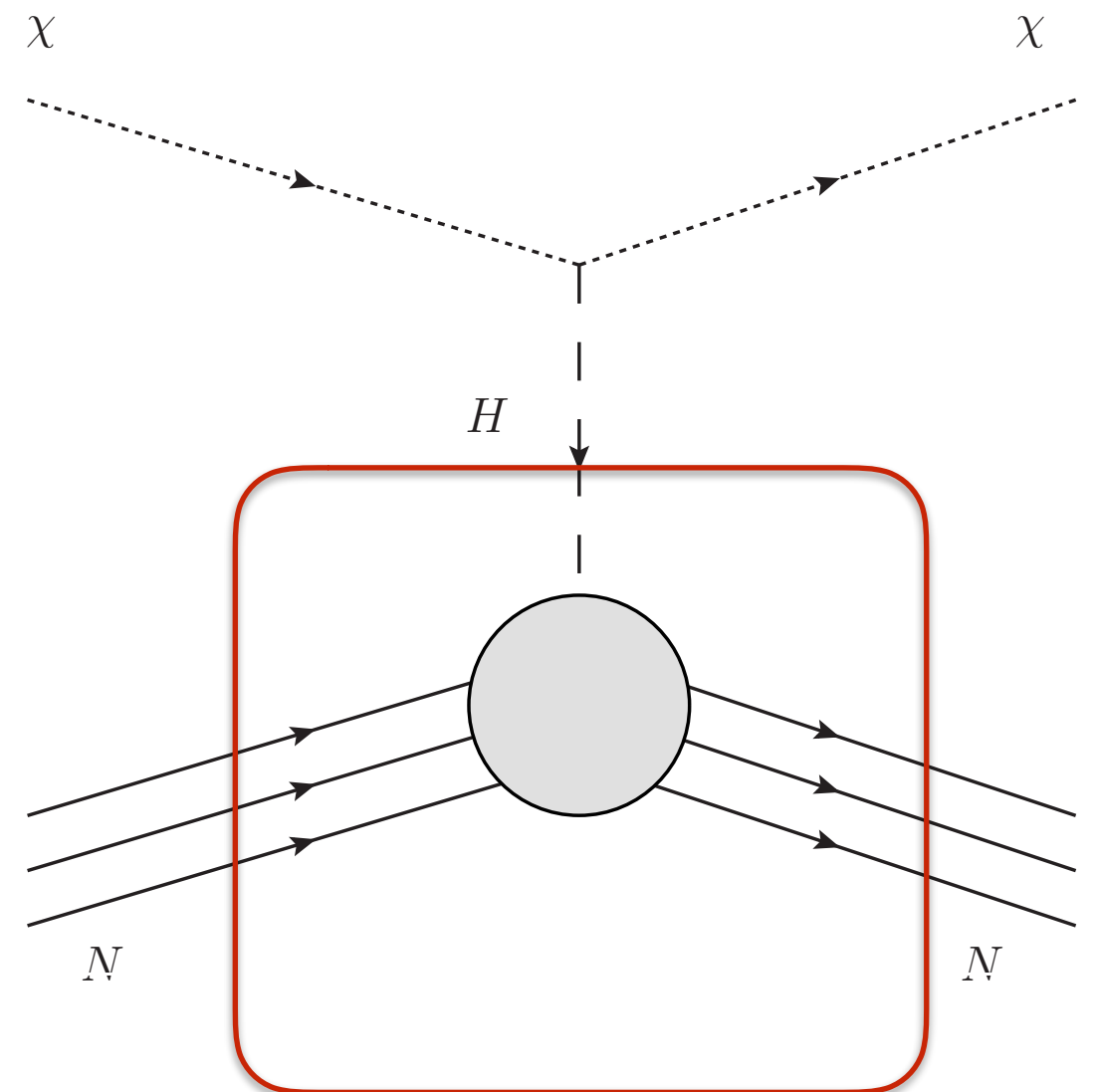
- ◆ Interaction through Higgs exchange
- ◆ zero momentum transfer limit
- ◆ Collective effects in the nuclei are neglected
- ◆ Here : spin-independent

[Detmold \*et al.\* Phys.Rev. D89 \(2014\) 074505](#)

- ◆ Many on-going experiments:

- ◆ LUX
- ◆ Xenon
- ◆ CREST
- ◆ ...

**Hadronic uncertainties**



# From nuclear to nucleon $\sigma$ -terms

Ellis *et al.* Phys.Rev. D77 (2008) 065026

## ◆ Assumptions:

- ◆ Interaction through scalar mediator
- ◆ zero momentum transfer limit
- ◆ Collective effects in the nuclei are neglected
- ◆ Here : spin-independent

◆ Cross section: 
$$\sigma_{\text{SI}} = \frac{4m_r^2}{\pi} (Z f_p + (A - Z) f_n)^2, \quad m_r = \frac{m_{\text{DM}} m_{\text{at.}}}{m_{\text{DM}} + m_{\text{at.}}}$$

- ◆ Characterized by atomic and mass number of the nuclei
- ◆  $f_{Tq}$  : scalar coupling of individual nucleons with flavor  $q$
- ◆  $\alpha_q$  : depends on underlying DM model and on the EW scale

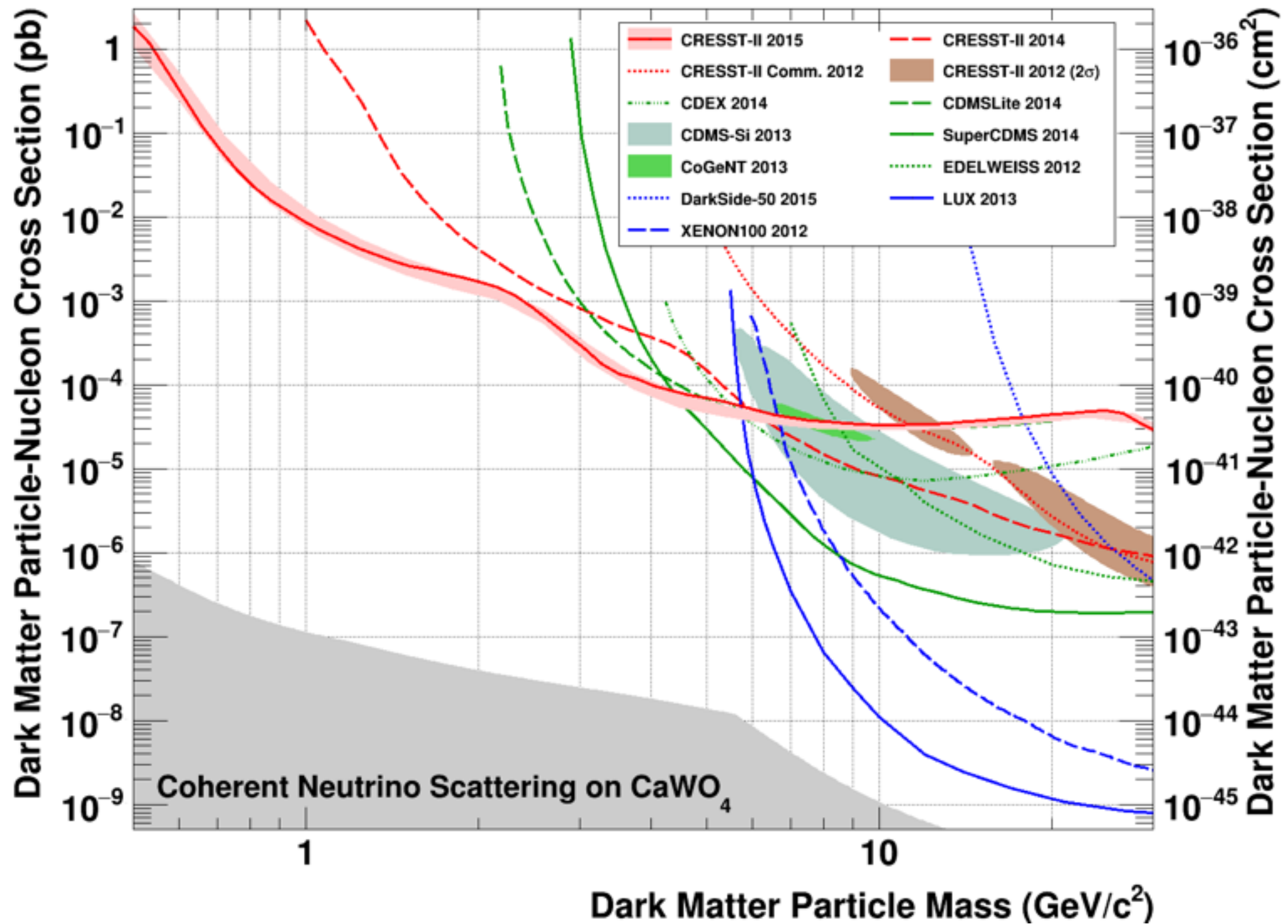
$$\frac{f_{N=n,p}}{m_N} = \sum_{q=u,d,s,c,b,t} f_{Tq} \frac{\alpha_q}{m_q}, \quad f_{Tq} = \frac{m_q \langle N | \bar{q}q | N \rangle}{m_N}$$

**Non perturbative property of the Nucleon**



# Spin independent case

CREST collaboration, Eur.Phys.J. C76 (2016) no.1, 25



**Input to constrain New Physics**



# Origin of the Nucleon mass

**Neglect isospin breaking effect  $m_u=m_d=m_l$**

♦ The energy momentum tensor :

♦ Rigorous decomposition of the Nucleon mass

X.-D. Ji, Phys.Rev.Lett. 74 (1995) 1071-107479

$$m_X = \langle X, \vec{0} | T_0^0 | X, \vec{0} \rangle = \sum_q m_q \underbrace{\langle X, \vec{0} | \bar{q}q | X, \vec{0} \rangle}_{\sigma_q^X} + \text{gauge contribution}$$

♦ Feynman-Hellman theorem :

$$\sigma_q^X \equiv m_q \frac{\partial}{\partial m_q} m_X$$

**Indirect method to compute the  $\sigma$ -terms !**

$$\text{♦ Equivalent to } f_{T_q} = \frac{\sigma_q^X}{m_N}$$

♦ Other quantities of interests :

$$\sigma_{\pi N} = \sigma_l = m_l \langle N | \bar{u}u + \bar{d}d | N \rangle, \quad m_l = \frac{m_u + m_d}{2}$$

$$\sigma_0 = m_l \langle N | \bar{u}u + \bar{d}d - 2\bar{s}s | N \rangle$$

$$y_N = \frac{2\langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle}$$

**the « strangeness » of the nucleon**

# Origin of the Nucleon mass : heavy quarks

Shifman, Vainshtein and Zakharov, Phys.Lett. B78 (1978) 443-446

♦ In the static limit :

♦  $\sigma_h$  in terms of the sum of the  $\sigma$ -terms for which  $m_q < m_h$

$$\sigma_h^X = \frac{2}{27} \left( m_X - \sum_{q=u,d,s} \sigma_q^X \right)$$

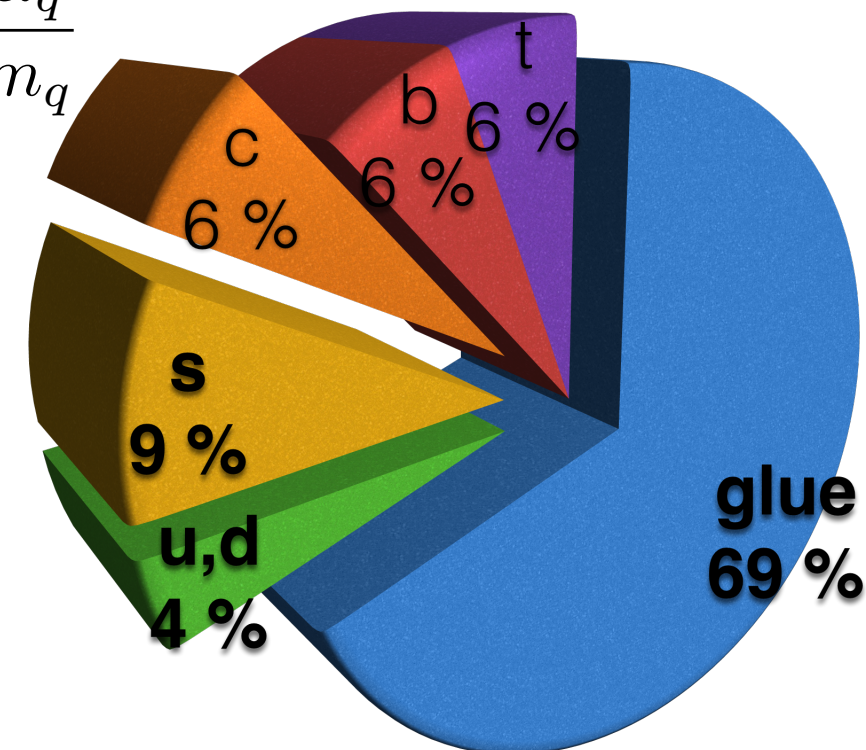
♦ gluon contribution and heavy quark contributions are related !

♦ Radiative corrections [Hill and Solon, Phys.Rev. D91 \(2015\) 043505](#)  
[Vecchi \[1312.5695\]](#)

♦ Explain why  $\frac{f_N}{m_N} \approx \sum_{q=u,d,s} f_{T_q} \frac{\alpha_q}{m_q} + \frac{2}{27} f_{T_G} \sum_{q=c,b,t} \frac{\alpha_q}{m_q}$

♦ Cross section proportional to  $\mathbf{f}_N^2$

♦ Assuming  $\sigma_l \sim 38$  MeV,  $\sigma_s \sim 87$  MeV,  $\sigma_h \sim 60$  MeV



**Heavy quark contribution should be confirmed by a lattice calculation**

# Phenomenological estimates

- ◆  $\sigma_l$  determination :

- ◆  $\pi$ -N scattering data
- ◆ extrapolation at the unphysical Cheng-Dashen point

- ◆  $\sigma_s$  determination :

- ◆ SU(3) breaking in the spectrum :  $\sigma_0$

$$\sigma_s = \frac{1}{2} \frac{m_s}{m_l} (\sigma_l - \sigma_0) \quad y_N = 1 - \frac{\sigma_0}{\sigma_l}$$

- ◆ Examples:

- ◆ GLS :  $\sigma_l = 45(8)$  MeV [J. Gasser, H. Leutwyler, and M. Sainio, Phys. Lett. B 253, 252 \(1991\)](#)
- ◆ GWU :  $\sigma_l = 64(7)$  MeV [M. M. Pavan \*et al\*, PiN Newsl. 16, 110 \(2002\).](#)
- ◆ AMO :  $\sigma_l = 59(7)$  MeV [J. Alarcon, J. Martin Camalich, and J. Oller, Phys. Rev. D85, 051503 \(2012\)](#)
- ◆  $\sigma_0 = 36(7)$  MeV [B. Borasoy and U.-G. Meissner, Ann. Phys. \(Berlin\) 254, 192 \(1997\).](#)
- ◆  $\sigma_0 = 58(8)$  MeV [J. M. Alarcon, \*et al\*, Phys. Lett. B 730, 342 \(2014\)](#)

**First principles answers are needed**

---

# Lattice techniques

---

# Lattice calculations in a nutshell

- **LGT** : Compute non perturbatively euclidean correlation functions:

$$\langle O[\bar{\psi}, \psi, A_\mu] \rangle = \frac{\int D[\bar{\psi}] D[\psi] D[A_\mu] e^{-S[\bar{\psi}, \psi, A_\mu]} O[\bar{\psi}, \psi, A_\mu]}{Z}$$

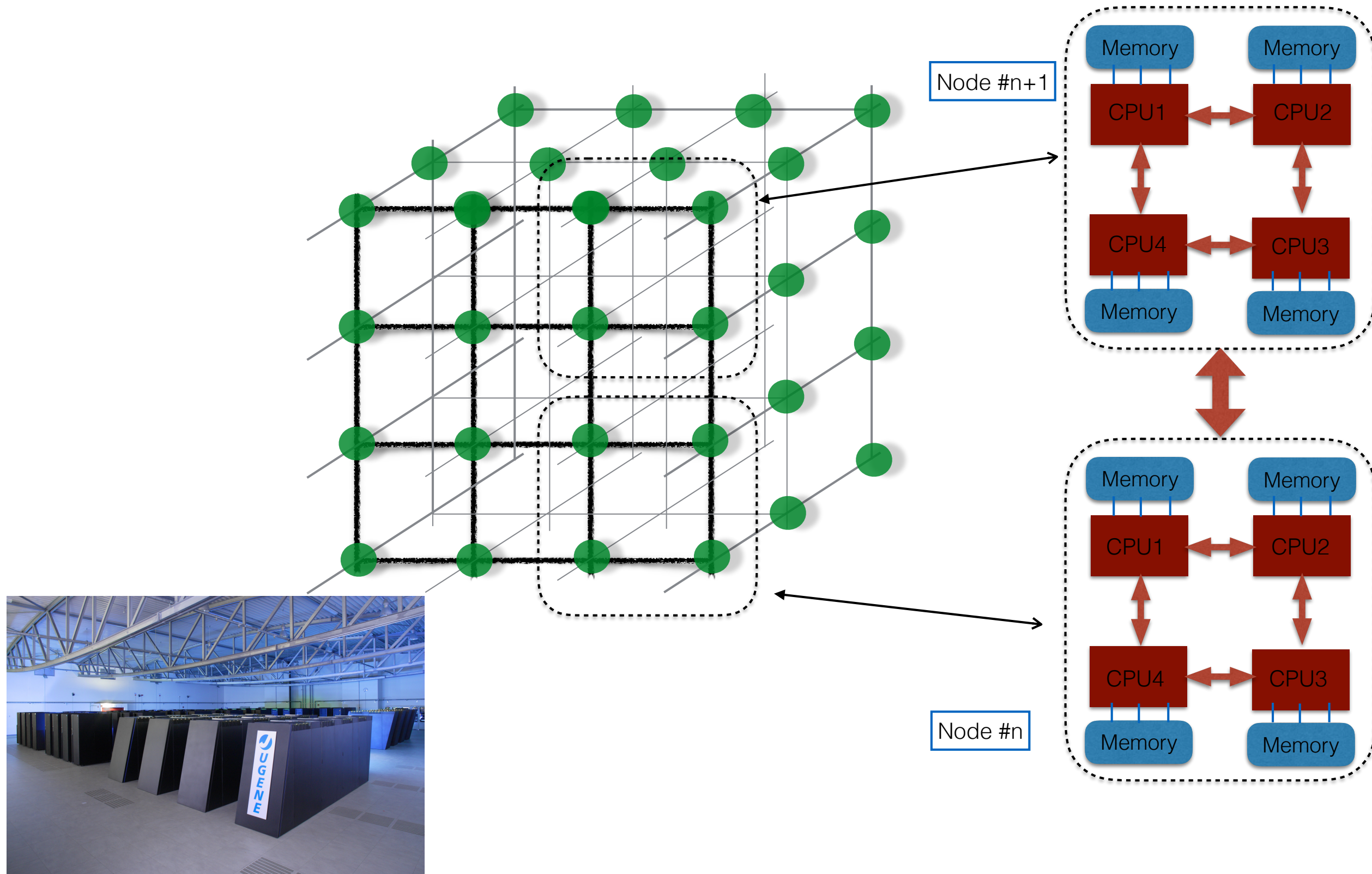
- **Strategy** :

- Discretize : lattice spacing  $a$ , volume  $V$ , mass  $m_f$ ,
- Boltzmann weight: probability distribution
- Sample : HMC algorithm
- Compute correlations functions at finite  $V$ ,  $a$ , and  $m_f$ .
- Renormalize if needed
- Extrapolate to  $V=\infty$ ,  $a=0$  and  $m_f=0$

**Theoretically well defined framework !**  
**Errors can be systematically controlled**



# Lattice calculations in a nutshell



**Gauge configuration generation typically run on ~ 10 000 cores !**

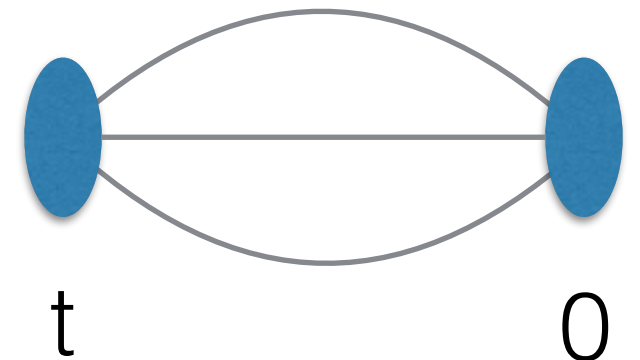
# Extracting masses

- Compute non perturbatively euclidean correlation functions:

$$C_{2\text{pts}}^X(t) = \sum_{\vec{x}} \mathcal{P} \langle J(x) J^\dagger(0) \rangle \propto e^{-M_X t} + \mathcal{O}(e^{-M_{X^*} t}), \quad M_{X^*} > M_X$$

- Sketch of the strategy :

- Choose J : to give the right quantum numbers,
- Study the asymptotic behavior



**No assumptions on the quark and glue content.**

# Extracting (bare) Matrix elements

- Compute non perturbatively euclidean correlation functions:

$$R(t, t_s) = \frac{\sum_{\vec{x}, \vec{y}} \text{Tr} \{ \Lambda \langle J(x) O(y) J^\dagger(0) \rangle \}}{C_{\text{opts}}^X(t_s)} = \langle X | O(0) | X \rangle + \mathcal{O}(e^{-\Delta M_X(t-t_s)}) + \mathcal{O}(e^{-\Delta M_X t_s})$$

- Sketch of the strategy :
  - ◉ Choose  $O, \Lambda$
  - ◉ Extract the asymptotic behavior in a 2D plane  $(t, t_s)$
  - ◉ Obtain the **bare matrix element**

**Asymptotically exact**

# Disconnected contributions

- Jargon :
  - ◉ «**Connected**» correlation functions only involve quark propagator from different space time points.
  - ◉ «**Disconnected**» correlation functions involve quark propagators from the same space time point.

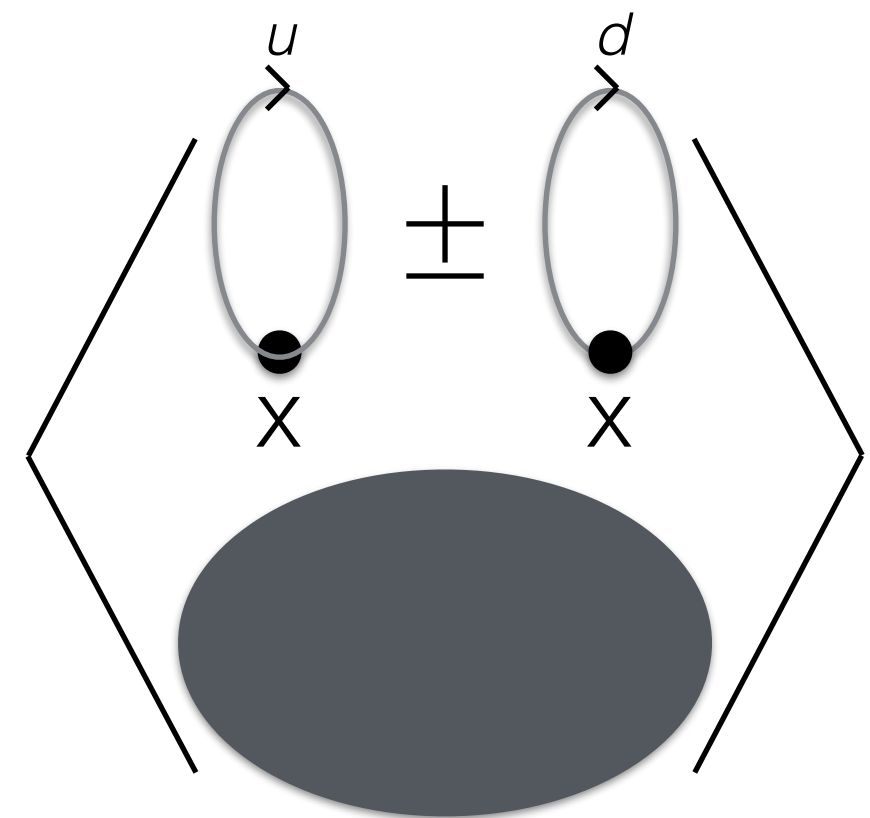
- Examples

- ◉ Connected :

$$\langle \dots [\bar{u}\Gamma u - \bar{d}\Gamma d] (x) \dots \rangle$$

- ◉ Disconnected :

$$\langle \dots [\bar{u}\Gamma u + \bar{d}\Gamma d] (x) \dots \rangle$$



# Why are they fundamental ?

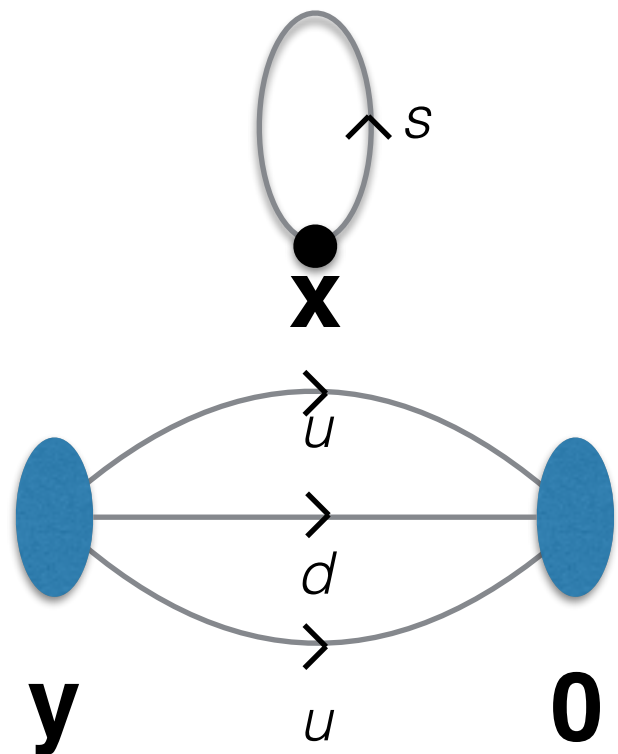
- Some relevant observables:
  - Hadronic contribution to the vacuum polarization
  - $\eta$ ,  $\eta'$ ,  $\sigma$  fermionic operators
  - flavour singlet quantities
  - Isospin breaking quantities (from QED or from mass difference)
  - matrix elements of operator containing only one flavor
- Remark :
  - they are an issue both to compute masses and matrix element



# Why are they difficult to estimate?

- Example:
  - Strange  $\sigma$ -term of the Nucleon

$$\langle \bar{N}(y) \bar{s}s(x) N(0) \rangle, \quad N = \epsilon^{abc} (u^{a,T} C \gamma_5 d^b) u^c$$



$$\sim D_s^{-1}[U](x \rightarrow x)$$

**Large fluctuations !**

$$\sim C_{2\text{pt}}(0 \rightarrow y)$$

$$\sim \langle C_{2\text{pt}}(0 \rightarrow y) D_s^{-1}[U](x \rightarrow x) \rangle$$

**Measures correlations between one object and a UV sensitive quantity !**

# Twisted mass fermions

Frezzotti, Grassi, Sint, Weisz 1999

- Action:

$$S_{(m_0, \mu)}^{\text{tm}} = a^4 \sum_x \bar{\chi}(x) \left[ \gamma_\mu \tilde{\nabla}_\mu + m_0 - r \frac{a}{2} \nabla_\mu^* \nabla_\mu + i\mu \gamma_5 \tau_3 \right] \chi(x)$$

- $m_0$ : bare Wilson mass,  $\mu$ : bare twisted mass
- $\chi$ : doublet of Dirac spinors
- $\tau_3$ : Pauli Matrix
- Wilson fermions :  $\mu=0$

- Properties:

- Break flavor symmetry and parity at finite lattice spacing
- automatic  $O(a)$  improvement if  $m_0$  is properly tuned
- non degenerate doublet can be added

**Theoretically well defined framework !**  
**Errors can be systematically controlled**

# Twisted mass variance reduction: idea

S. Dinter, VD, R. Frezzotti, G. Herdoiza, K. Jansen, G. Rossi JHEP 1208 (2012) 037

- Twisted Mass doublet Dirac operator :

$$D[U] = \begin{pmatrix} D_+[U] & 0 \\ 0 & D_-[U] \end{pmatrix}$$

$$D_{\pm}[U] = D_W[U] + am_0 \pm ia\mu_q\gamma_5$$

- Properties :

$$\frac{1}{D_-} - \frac{1}{D_+} = 2ia\mu_q \frac{1}{D_-} \gamma_5 \frac{1}{D_+} \quad \text{Algebraic property}$$

$$i\bar{\chi}\gamma_5\tau_3\chi \rightarrow \bar{u}u + \bar{d}d$$

**Transformation to the « physical » basis**

- We have shown that

**Bare mass    Bare matrix element**

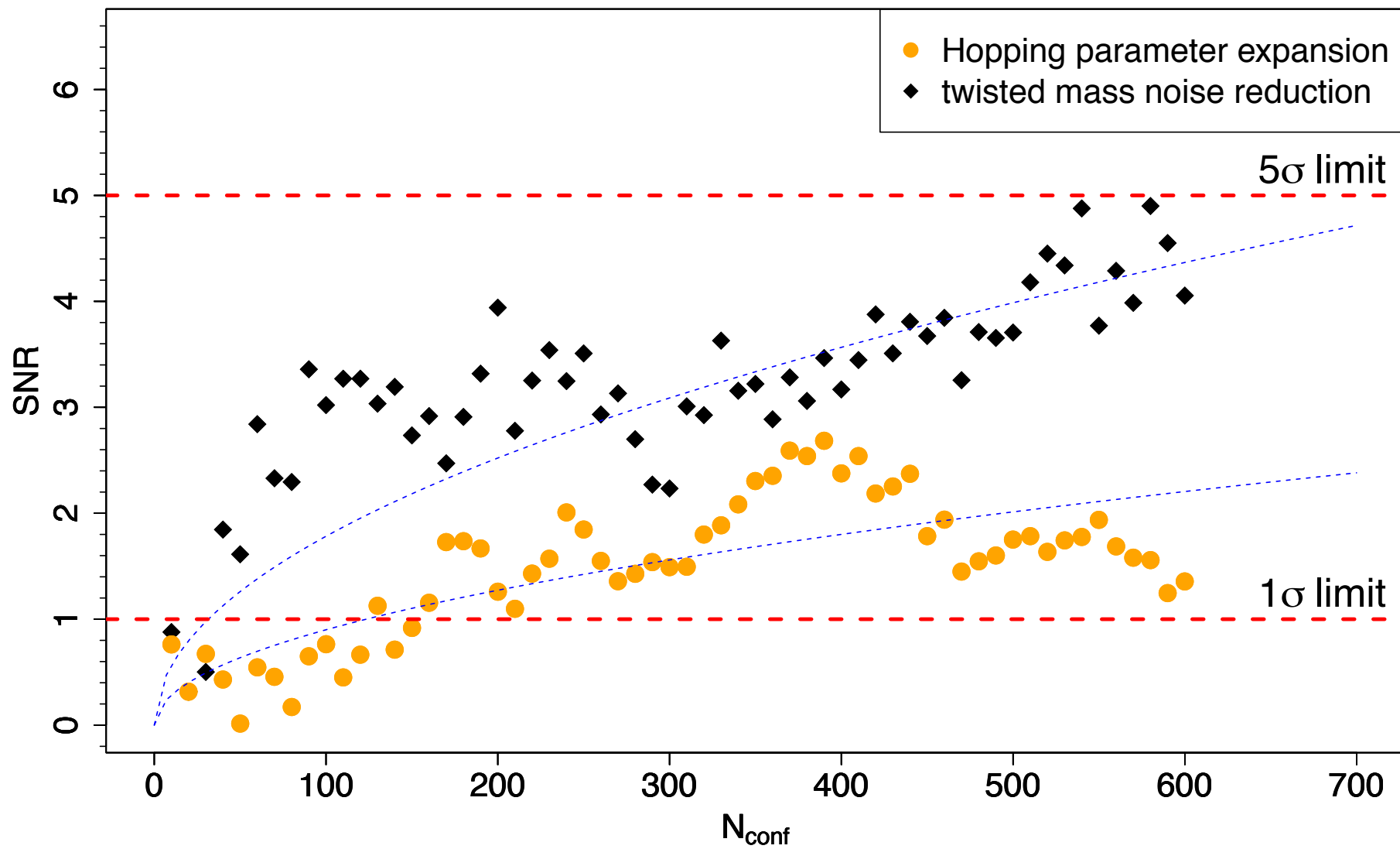
$$\mu_q \langle N, \vec{0} | \bar{\chi}\gamma_5\tau_3\chi | N, \vec{0} \rangle = \sigma_{\pi N}$$

**Renormalization group invariant!**

- Numerically : exploit the fact that the difference is proportional to the product

# twisted mass variance reduction: performances

S. Dinter, VD, R. Frezzotti, G. Herdoiza, K. Jansen, G. Rossi JHEP 1208 (2012) 037




$$\frac{R(t_{\text{op}} = 6a, t_s = 12a)}{dR(t_{\text{op}} = 6a, t_s = 12a)}(N_{\text{conf}})$$

**Huge improvement at fixed numerical cost with twisted mass fermions !**

# Generalisation to the strange sector

- Trick : introduce at the *valence* level a doublet of strange quark !

  
**They differ by O(a) effects**

- The proof goes through

- Recipe :

- \* Tune mass ( $\mu_s$ ) such that  $m_K^{\text{valence}} = m_K^{\text{sea}}$
- \* Write the corresponding Ward-Identities to proof renormalizability
- \* Deduce that

$$\frac{\mu_s}{2} \langle N, \vec{0} | \bar{\chi}_s i \gamma_5 \tau_3 \chi_s | N, \vec{0} \rangle = \sigma_s$$

**Idem for  $\sigma_c$  !**



- **Properties** :
  - **$N_f=2+1+1$  simulations** : degenerate light flavors (u,d), strange (s) and charm(c) bare Wilson mass,  $\mu$  : bare twisted mass
  - **Lightest pion mass** : 230 MeV
  - 3 lattice spacings
  - multiple volumes
- **Many results** :
  - baryon spectrum and structure
  - flavour physics
  - Hadronic contribution to the  $g-2$
  - ....



- Properties :
  - $N_f=2$  simulations : degenerate light flavors (u,d) (with clover term)
  - Physical pion mass :  $\sim 140$  MeV
  - One lattice spacing
  - One volume
- Many results :
  - baryon spectrum and structure
  - flavour physics
  - Hadronic contribution to the  $g-2$
  - ....



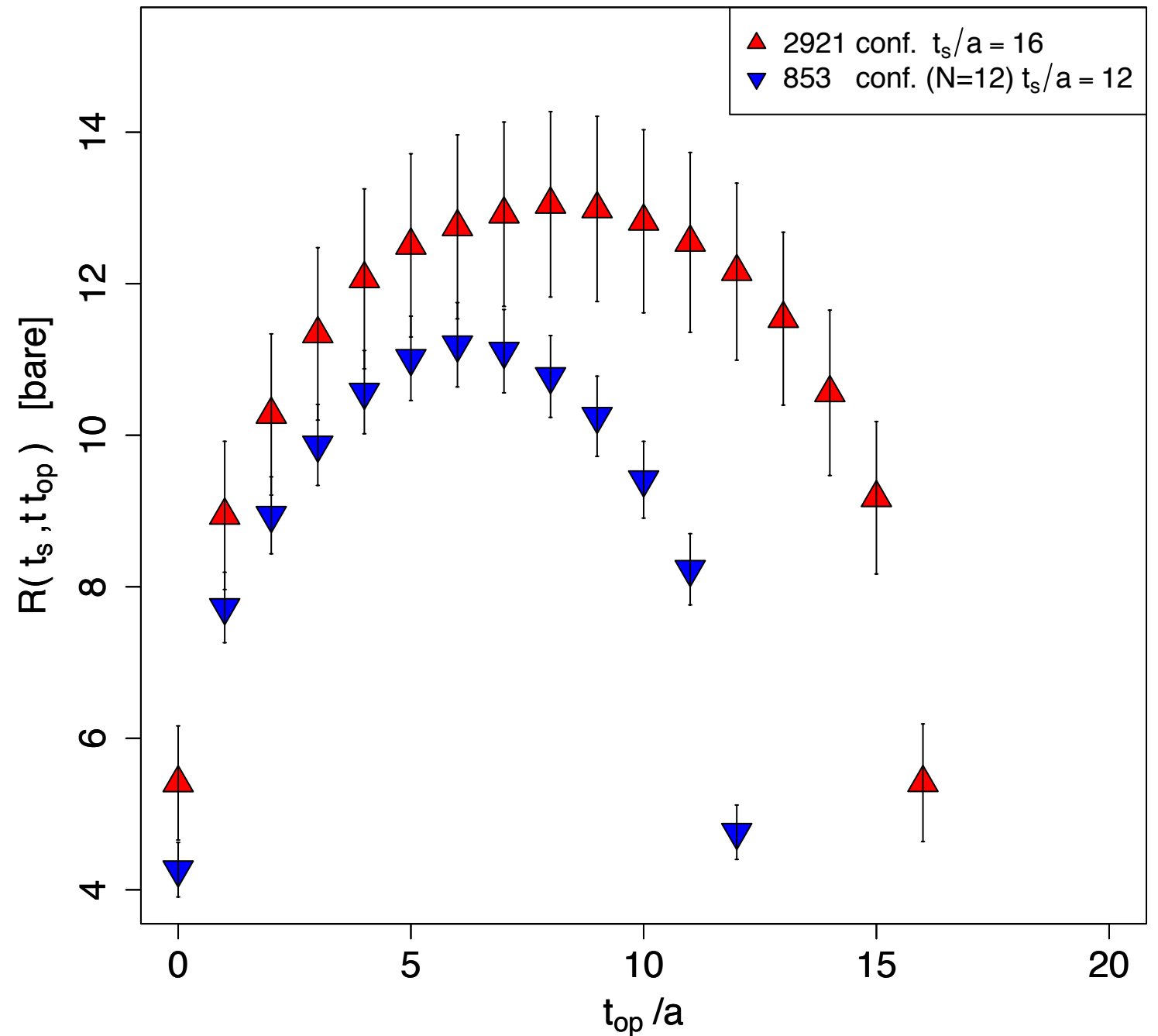
**$N_f=2+1+1$  simulations at the physical pion mass are underway !**

---

# Numerical results & systematics

---

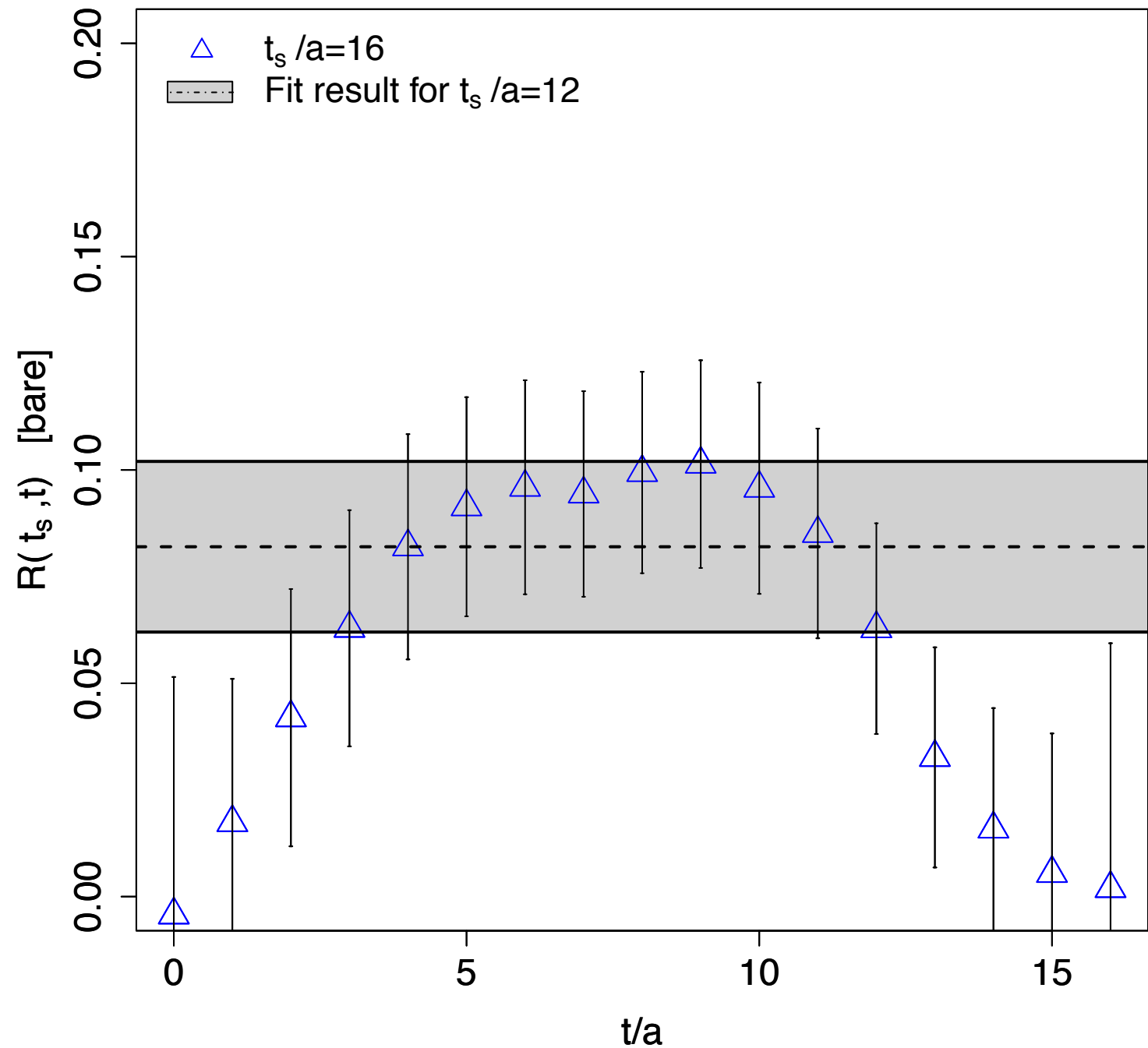
- Example  $\sigma_l$ :
  - $N_f=2+1+1$  simulations
  - pion mass : 380 MeV
  - Large statistics
- Technicalities:
  - Both connected and disconnected contributions
  - large excited states contamination !!!
  - similar in the strange sector



# Alternative : the strangeness

S. Dinter, VD, R. Frezzotti, G. Herdoiza, K. Jansen, G. Rossi JHEP 1208 (2012) 037

- Example  $y_N$ :
  - $N_f=2+1+1$  simulations
  - pion mass : 380 MeV
  - Large statistics
- Technicalities:
  - Both connected and disconnected contributions
  - Excited states contamination cancels out
  - First  $5\sigma$  away from 0 results for  $y_N$



**Excited states under control ! But what about the other systematics ?**

# Heavy Baryon Chiral Perturbation Theory

- ♦ Chiral perturbation theory :

$$m_{\text{PS}}^2 = 2Bm_l + \mathcal{O}(m_l^2)$$

- ♦ Heavy baryon  $\chi$ PT:

- ♦ EFT describing interactions of nucleons and pions
- ♦ Expansion in  $m_{\text{pions}} / m_B$
- ♦ LO in  $m_{\text{pions}} / m_B$

$$m_N(m_{\text{PS}}) = m_N^{(0)} - 4c^{(1)}m_{\text{PS}}^2 - \frac{3g_A^2}{32\pi f_\pi^2}m_{\text{PS}}^3 + \mathcal{O}(m_{\text{PS}}^4)$$

- ♦ FH theorem :

- ♦  $\sigma_q^X \equiv m_q \frac{\partial}{\partial m_q} m_X$

- ♦ Chiral expansion of  $\sigma_l$ :

$$\sigma_l(m_{\text{PS}}) = m_{\text{PS}}^2 \left( -4c^{(1)} - \frac{3}{2} \frac{3g_A^2}{16\pi f_\pi^2} m_{\text{PS}} + \mathcal{O}(m_{\text{PS}}^2) \right)$$

**-4 c<sup>(1)</sup> must be strictly positive**

# What about the strangeness ?

♦ Chiral perturbation theory :  $m_{\text{PS}}^2 = 2Bm_l + \mathcal{O}(m_l^2)$

♦ Heavy baryon  $\chi$ PT:

$$m_N(m_{\text{PS}}) = m_N^{(0)} - 4c^{(1)}m_{\text{PS}}^2 - \frac{3g_A^2}{32\pi f_\pi^2}m_{\text{PS}}^3 + \mathcal{O}(m_{\text{PS}}^4)$$

♦  $\sigma_s$  ansatz :

$$\sigma_s(m_{\text{PS}}) = m_s (d_0 + d_1 m_{\text{PS}}^2 + \mathcal{O}(m_{\text{PS}}^3))$$

♦  $y_N$  expansion : (neglecting the strange quark mass depend of  $m_{\text{PS}}$ )

$$y_N = 2 \frac{\partial m_N}{\partial m_s} \left( \frac{\partial m_{\text{PS}}^2}{\partial m_l} \frac{\partial m_N}{\partial m_{\text{PS}}^2} \right)^{-1}$$

$$y_N = y_N^{(0)} + y_N^{(1)} m_{\text{PS}} + \mathcal{O}(m_{\text{PS}}^2), \quad \text{with } y_N^{(0)} = \frac{d_0}{-4Bc^{(1)}}, \quad y_N^{(1)} = \frac{9d_0 g_A^2}{64\pi B(4c^{(1)})^2 f_{\text{PS}}^2}$$

**$y_N$  should be an increasing function of  $m_{\text{PS}}$ !**

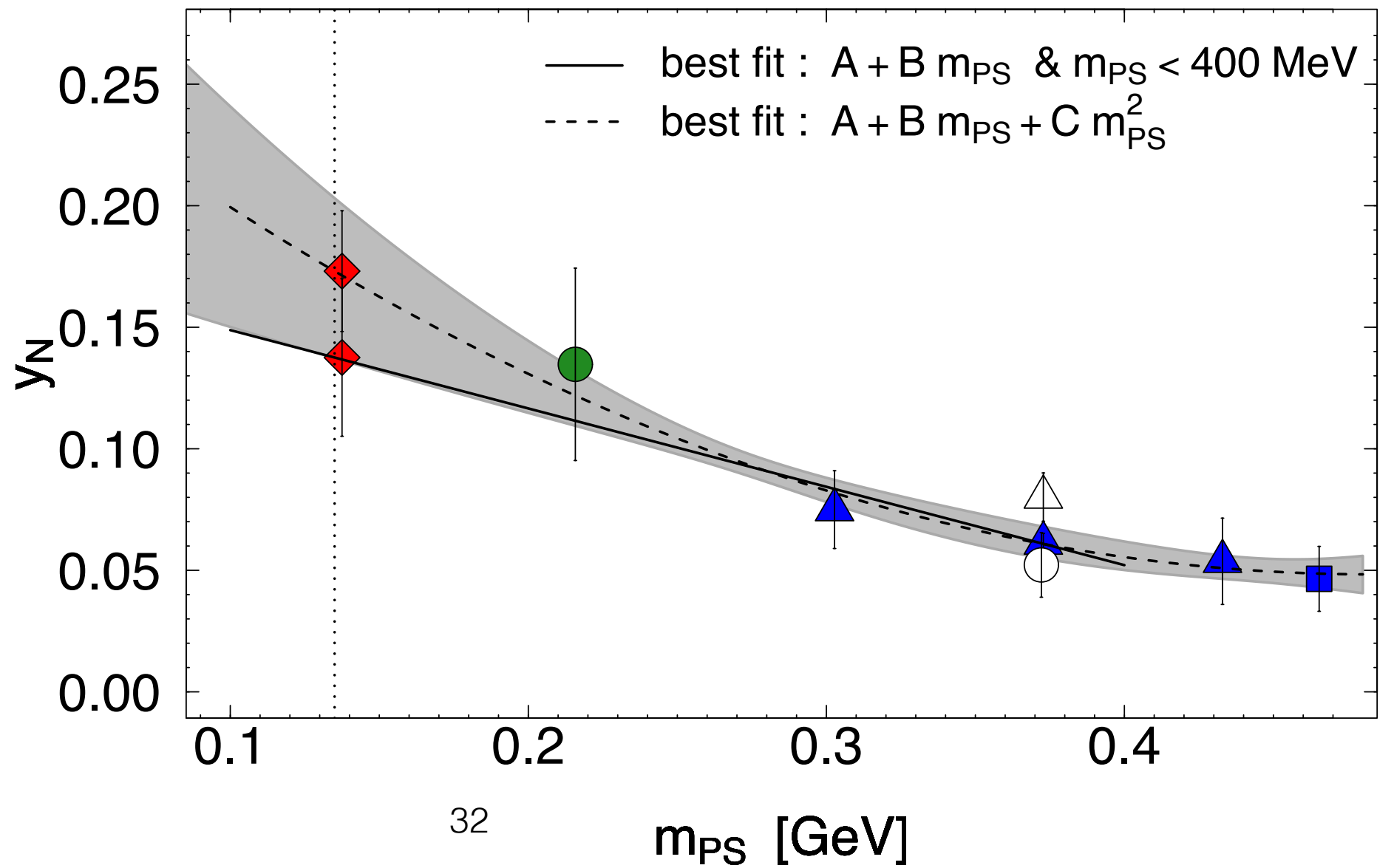
# Chiral behavior $y_N$

- Setup:

- $N_f=2+1+1$  simulations
- Several lattice spacings
- Several volumes
- Chiral extrapolation challenging

ETM Collaboration, Phys.Rev. D91 (2015) no.9, 094503

●  $a \approx 0.064$  fm  $t_s \approx 1.15$  fm  $L \approx 3.07$  fm    △  $a \approx 0.082$  fm  $t_s \approx 1.48$  fm  $L \approx 2.64$  fm  
○  $a \approx 0.064$  fm  $t_s \approx 1.02$  fm  $L \approx 2.05$  fm    ■  $a \approx 0.082$  fm  $t_s \approx 0.98$  fm  $L \approx 1.97$  fm  
▲  $a \approx 0.082$  fm  $t_s \approx 0.98$  fm  $L \approx 2.64$  fm    ◆ extrapolated values



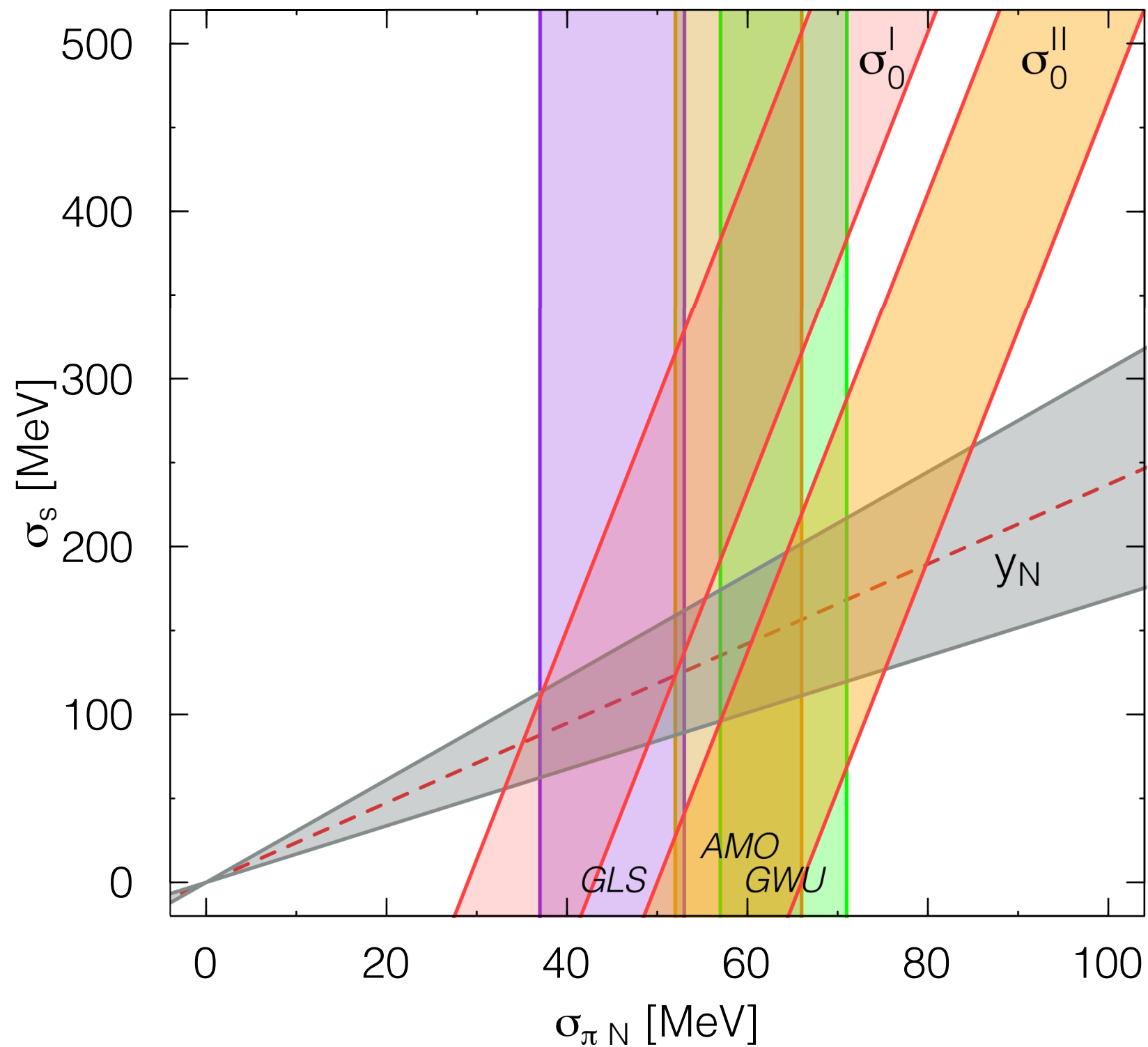


# Chiral behavior $y_N$

- Setup:

- $N_f=2+1+1$  simulations
- Several lattice spacings
- Several volumes
- $y_N=0.17(5)$

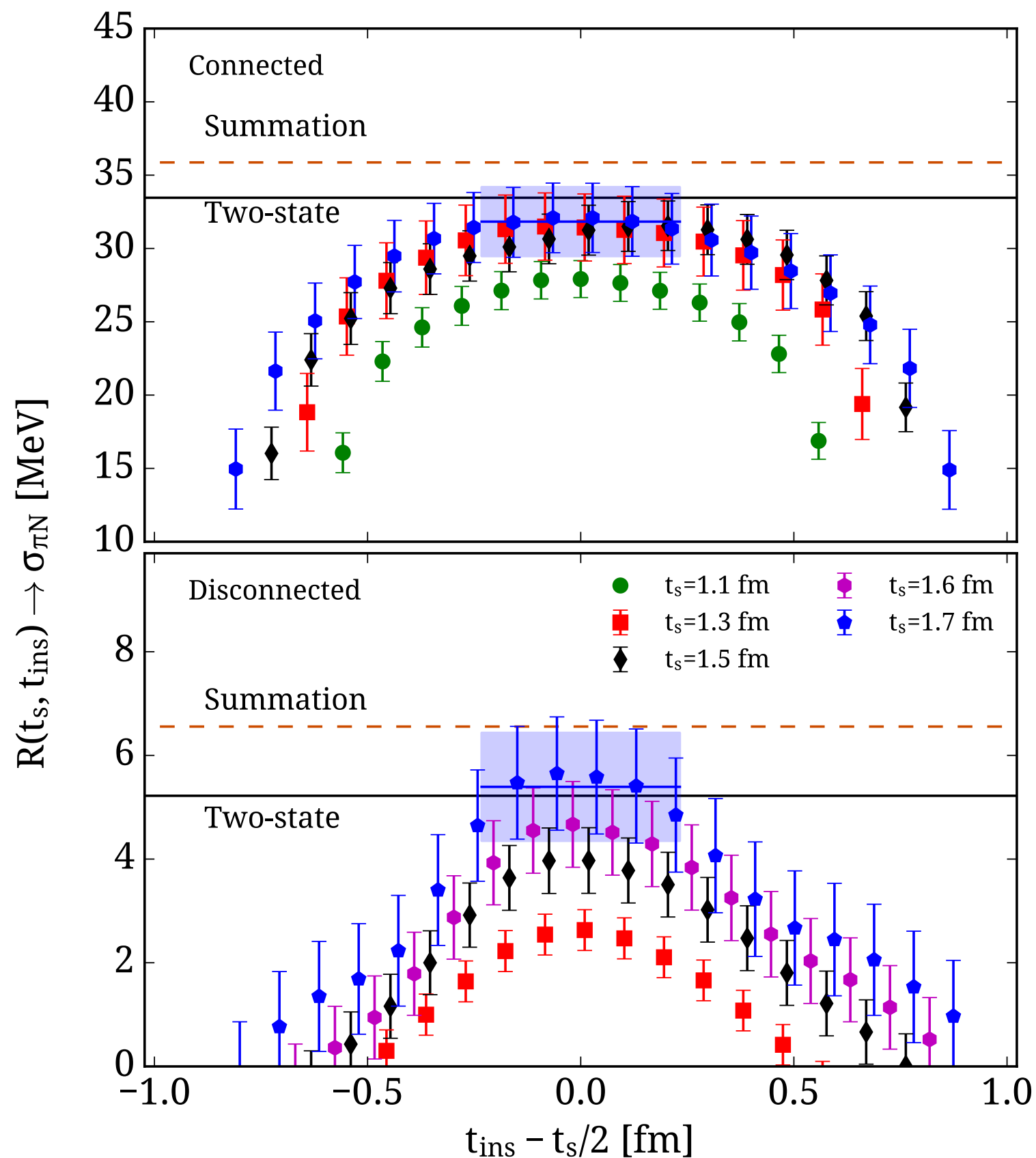
ETM Collaboration, Phys.Rev. D91 (2015) no.9, 094503



# New setup :

- Setup:
  - $N_f=2$  simulations
  - Physical pion mass
  - s and c are «quenched»
  - $\sigma_l=37(3)(10)$  MeV
  - $\sigma_s=41(8)(10)$  MeV
  - $\sigma_c=79(21)(2)$  MeV

ETM Collaboration, [1601.01624]

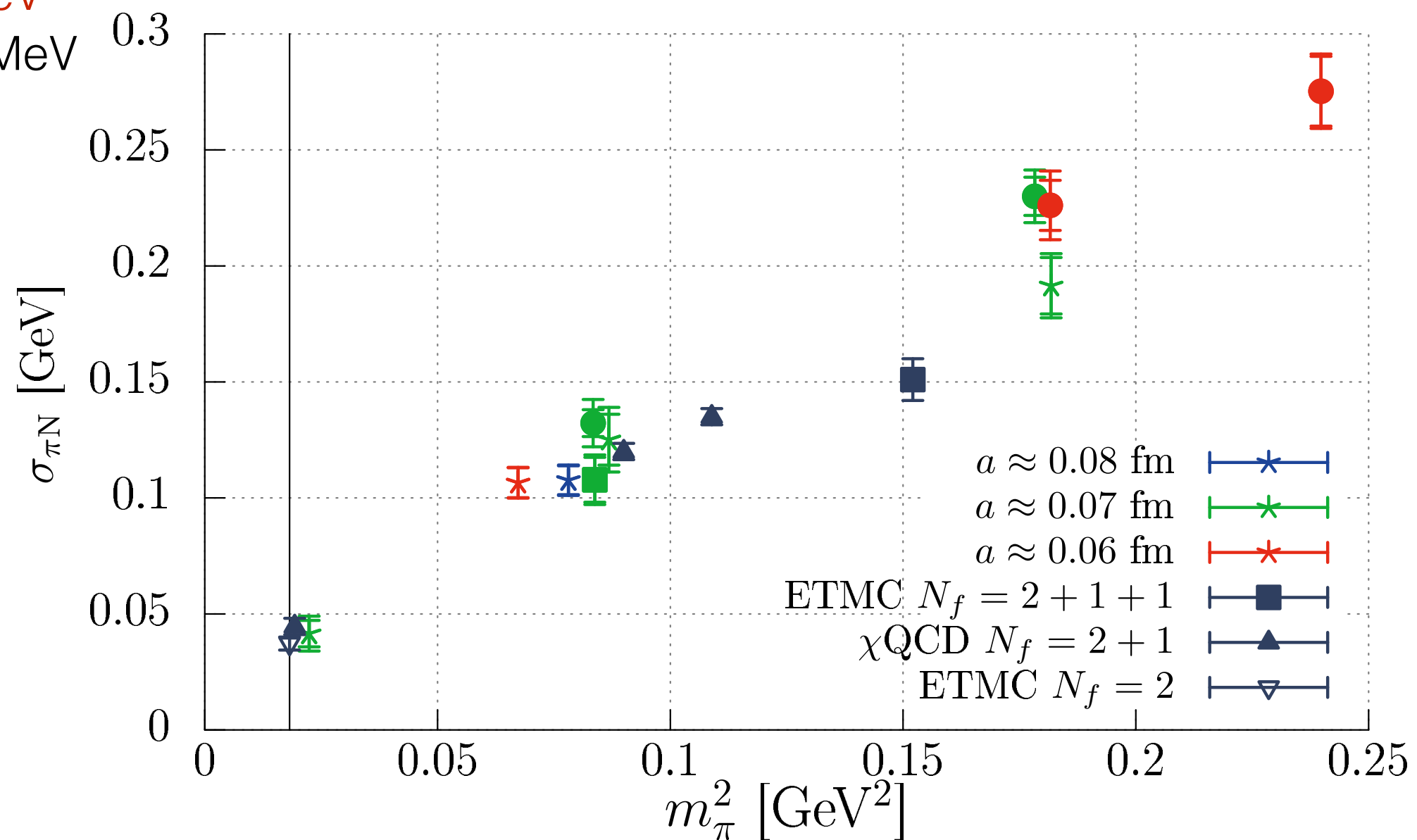


# RQCD collaboration

- Setup:

- $N_f=2$  simulations
- Physical pion mass
- ME approach
- $\sigma_l=35(6)$  MeV
- $\sigma_s=35(12)$  MeV

RQCD Collaboration, Phys.Rev. D93 (2016) no.9, 094504

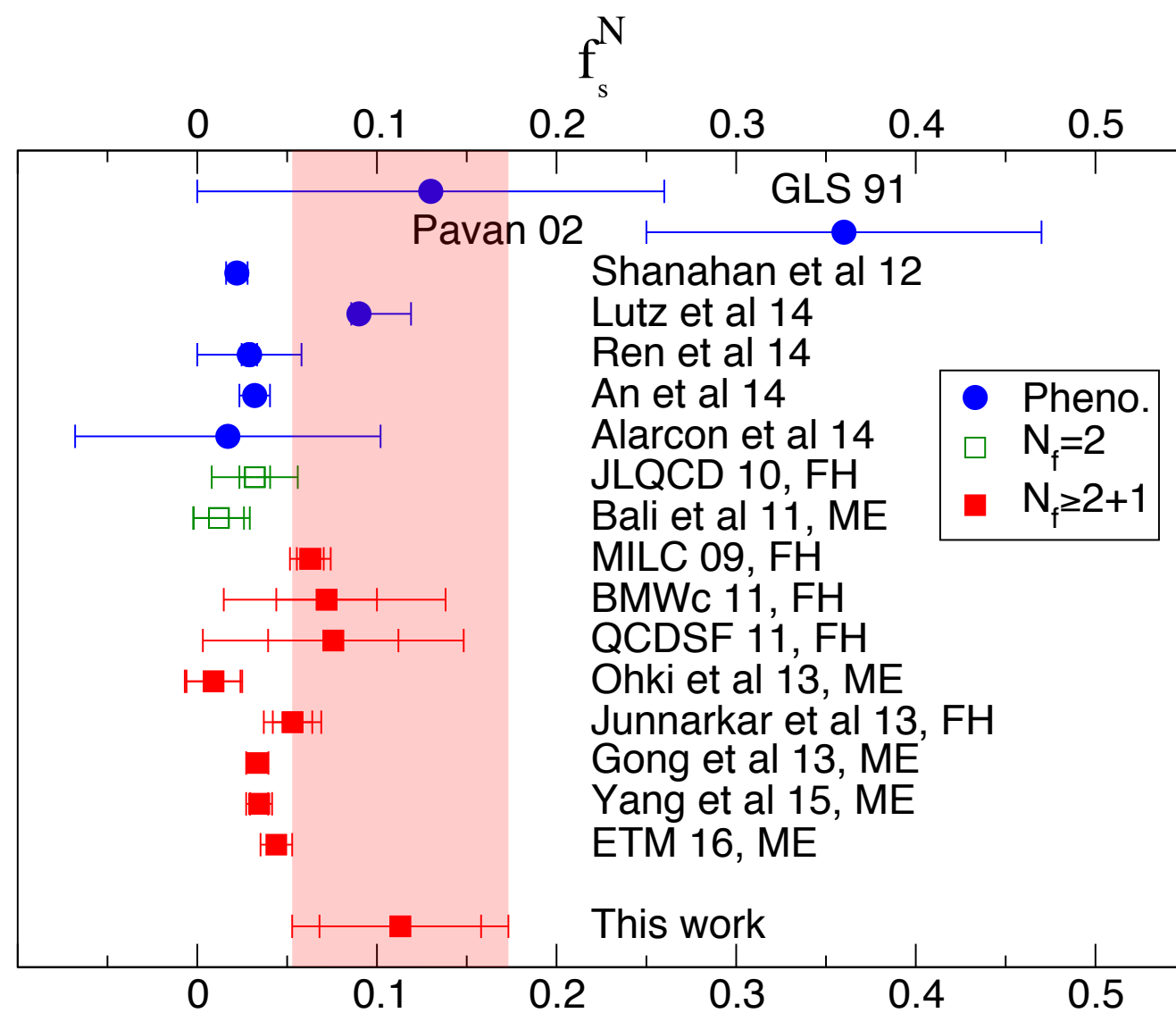
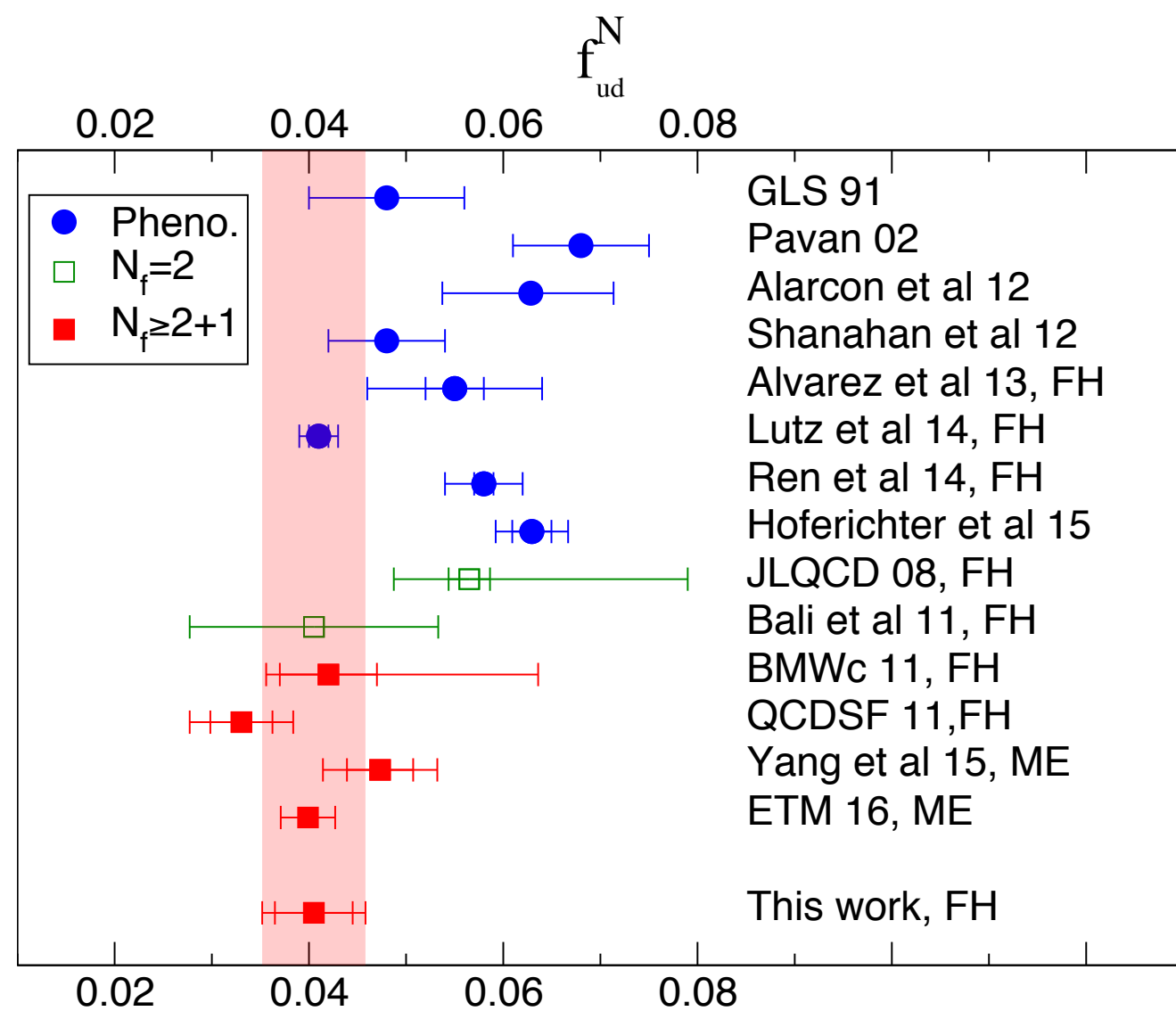


# BMW collaboration

- Setup:

- $N_f=2+1$  simulations
- Physical pion mass
- FH approach
- $\sigma_l=38(3)(3)$  MeV
- $\sigma_s=105(41)(37)$  MeV
- $y_N=0.20(8)(8)$

BMW Collaboration, [PRL 116 (2016) no. 17, 172001]



# *Conclusions & Outlook (I)*

- Hadronic uncertainties to interpret direct detection constraints
- Cross section controlled by the  *$\sigma$ -terms* for each quark flavors
- Theoretical interest : **Dynamical origin of the nucleon's mass**
- Lattice calculations are challenging :
  - ◆ Disconnected diagrams
  - ◆ Systematics are difficult to control
  - ◆ **Heavy quark content should be addressed...**

# Conclusions & Outlook (II)

- ETM Collaboration :
  - ◆ First results of a direct calculation of  $y_N$
  - ◆ Results for  $\sigma_1$  encouraging @ the physical point ( $N_f=2$ )
  - ◆ Looking forward for  $N_f=2+1+1$  simulations
- Other group :
  - ◆ light sector :  $\sigma_1 \sim 38$  MeV
  - ◆ strange sector :  $35 \text{ MeV} < \sigma_s < 150 \text{ MeV}$