issues around $N_f = 1$ QCD

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from $N_f > 1$ QCD to meson masses

$$\mathcal{L}_{QCD} = \sum_{f}^{N_f} \bar{\psi}_f [\gamma_\mu(\partial_\mu - igA_\mu) - m_f]\psi_f + L_g(A_\nu, \partial_\mu A_\nu)$$

chiral symm.: $SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_A \otimes U(1)_V$

- $U(1)_A$ broken by quantum effects (anomaly $\partial_\mu A^\mu \neq 0$)
- spontaneous SB: $SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$
  $\longrightarrow$ massless Goldstone bosons
- $m_q > 0 \leftrightarrow$ explicit SB: $SU(N_f)_L \otimes SU(N_f)_R$
  Goldstone bosons $\longrightarrow$ pseudo Goldstone bosons (pions)

effective theory $(N_f = 3,\ m_u = m_d = m_q)$

- GMOR $m_\pi^2 \propto m_q$ (Gell-Mann, Oakes, Renner)
  imposes $m_q \geq 0$!

$N_f = 1$ case

$$\mathcal{L}_{QCD} = \bar{\Psi}[\gamma_\mu(\partial_\mu - igA_\mu) - m_q]\Psi + L_g(A_\nu, \partial_\mu A_\nu)$$

no Chiral symmetry: $U(1)_A \otimes U(1)_V$

- $U_A(1)$ broken by quantum effects (anomaly)
- no explicit SB for $m_q \neq 0$
- let M_q be the physical quark mass,
  $M_q > 0$ vs $M_q = 0$ ill defined (RGT on m_q ambiguous)

some puzzles in QCD

- **mechanisms spontaneous symmetry breaking?**
- **mechanism generating quark masses?**
  (responsible for the explicit symmetry breaking...)
- **strong CP problem (why is $\theta \simeq 0$?), $m_u \to 0$ way out?**
- **role of instantons effects for the chiral symmetry breaking?**

Answers, indications from the lattice?

a closer look to the anomalous symmetry $U(1)_A$

consider fermion gauge invariant theory:
$\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi + (\frac{1}{2g^2})\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$, $\mathcal{D}_\mu = \partial_\mu + A_\mu$

anomalous axial symmetry $U(1)_A : \psi \rightarrow e^{i\frac{\theta}{2}\gamma_5}\psi$

- functional integral measure: $d\mu \equiv \prod_x [\mathcal{D}A_\mu]\, \mathcal{D}\bar{\psi}\mathcal{D}\psi$
  $U(1)_A : d\mu \rightarrow d\mu\, e^{-i\theta Q[A]}$
  $Q[A] = \int d^4q(x)$, $q(x) = -\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\lambda\sigma}\mathrm{Tr}(F_{\mu\nu}F_{\lambda\sigma})$
- $U(1)_A : m\bar{\Psi}\Psi \rightarrow m\cos(\theta)\bar{\Psi}\Psi + im\sin(\theta)\bar{\psi}\gamma_5\psi$
  $U(1)_A : m \rightarrow me^{i\gamma_5\theta}$ (rotated into the complex plane)

notice:

1. for $\theta = \pi$, $U(1)_A : m \rightarrow -m$!
2. $\bar{\Psi}\gamma_5\Psi$ and $\{q(x), Q\}$ break CP!

strong CP problem

we noticed for $U(1)_A$: $\psi \to e^{i\frac{\theta}{2}\gamma_5}\psi$

- $U(1)_A$: $\omega\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\lambda\sigma}\mathrm{Tr}(F_{\mu\nu}F_{\lambda\sigma}) \to (\omega - \theta)\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\lambda\sigma}\mathrm{Tr}(F_{\mu\nu}F_{\lambda\sigma})$
- $U(1)_A$: $me^{i\gamma_5\phi} \to me^{i(\phi+\theta)\gamma_5}$

$\Rightarrow \bar{\theta} = \theta + \phi$ is unique invariant CP-violating parameter!

experimentally: $\bar{\theta} < 10^{-11}$ (neutron dipole moment)

Strong CP problem: why is $\bar{\theta}$ set to zero?
($\bar{\theta} = 0 \Leftrightarrow$ CP conservation in strong interactions)

- a possible way out? $m_u \to 0$

CP breaking in effective $N_f = 3$ QCD [Creutz 2003]

eff. theory: $L_{eff} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma) - \nu\mathrm{Re}\mathrm{Tr}(\Sigma M)$,

$\Sigma = \exp(i\pi_\alpha\lambda_\alpha/f_\pi) \in SU(3)$

where $M = \mathrm{diag}(m, m_d, m_s) = \frac{m_u+m_d+m_s}{3}1 - \frac{m_d-m_u}{2}\lambda_3 - 2\frac{2m_s-m_d-m_u}{2\sqrt{3}}\lambda_8$,

$$\Sigma_{min} = \{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i(\phi_1+\phi_2)} \end{pmatrix}\},$$

with $m_u\sin(\phi_1) = m_d\sin(\phi_2) = -m_s\sin(\phi_1+\phi_2)$,

degeneracy: $\{\phi_1, \phi_2\} \leftrightarrow \{-\phi_1, -\phi_2\}$ is a symmetry

now recall the following mass relations:

$$m_{\pi_0}^2 \propto \frac{2}{3}\left(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

$$m_\eta^2 \propto \frac{2}{3}\left(m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

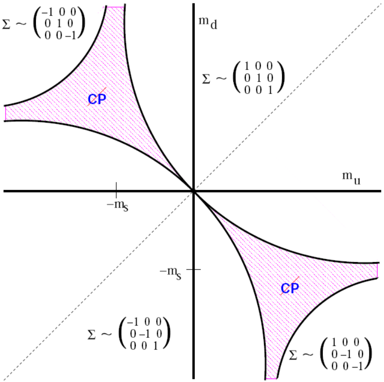
$$\Rightarrow m_{\pi_0}^2 m_\eta^2 \propto m_u m_d + m_u m_s + m_d m_s = 0$$

$$m_{\pi_0} = 0 \Leftrightarrow m_u = -\frac{m_s m_d}{m_s+m_d} \equiv m_c$$

CP breaking in effective $N_f = 3$ QCD [Creutz 2003]

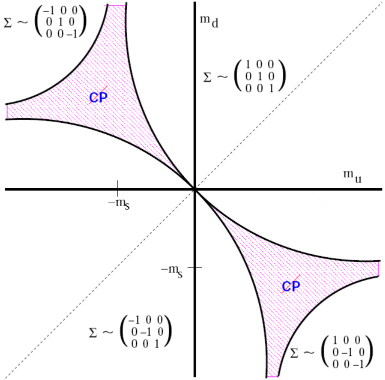
$$m_c = -\frac{m_s m_d}{m_s + m_d}$$

- $m_u \geqslant m_c$: $\Sigma_{min} = 1$ (unique vacuum)
- $m_u < m_c$: $\Sigma_{min} = \Sigma_{min}$ (degenerate vacuum)
  pseudoscalar $\langle \pi_0 \rangle \neq 0$: $\Rightarrow$ CP broken



CP breaking in effective $N_f = 3$ QCD [Creutz 2003]

In $N_f = 3$ QCD, after chiral symmetry breaking ($m_u \neq m_d \neq m_s$, no GMOR relation), CP should be spontaneously broken for $m_u < 0$!



toy model toward $N_f = 1$ QCD [Creutz]

consider effective $N_f = 3$ QCD with quark masses $\{m, M, M\}$

$\Sigma = \exp(i\pi_\alpha\lambda_\alpha/f_\pi) \in SU(3)$,

$V_{eff}(\Sigma) \propto -\mathrm{Re}\,\mathrm{Tr}\,(\mathcal{M}\Sigma) = -\frac{M+m}{2}\,\mathrm{Re}\,\mathrm{Tr}(\Sigma) + \frac{M-m}{2}\,\mathrm{Re}\,\mathrm{Tr}\,(\Sigma h)$,

where $\mathcal{M} = \begin{pmatrix} m & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{pmatrix}$, $h = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

1. *1* is minimum of $-\frac{M+m}{2}\mathrm{Re}\,\mathrm{Tr}(\Sigma)$
2. $e^{\pm 2\pi i/3}h$ degenerated minima of $\frac{M-m}{2}\mathrm{Re}\,\mathrm{Tr}(\Sigma h)$

- $m \geqslant M$: one unique vacuum
- $m = -M$: two degenerated vacua
- $\Rightarrow$ $\exists m_c$, $s.t.$ $-M \leqslant m_c \leqslant M$ where the symmetry breaks spontaneously

CP violating phase diagram in $N_f = 1$ QCD

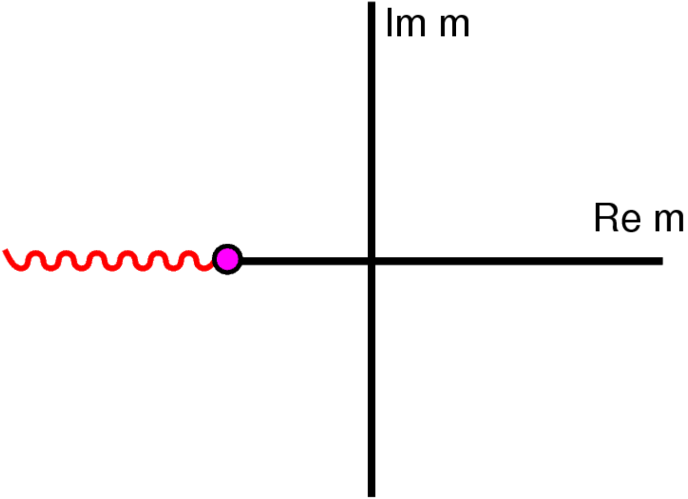
rough argument for CP breaking [Creutz]

$$V_{eff}(\eta', m_1, m_5) = \frac{m_1 + c}{2}\eta'^2 + \lambda\eta'^4 + im_5\eta'$$

- $\eta' \equiv \eta_q = \bar{\Psi}_s\gamma_5\Psi_s$ (pseudoscalar meson state, CP odd)
- $m^2_{\eta'} = m1 + c$ effective-theory like
- $U(1)_A: \ \Psi_s \to e^{i\gamma_5\theta/2}\Psi_s, \ m\bar{\Psi}_s\Psi_s \to m_1\bar{\Psi}_s\Psi_s + im_5\bar{\Psi}_s\gamma_5\Psi_s$

spontaneous SB of CP expected $\langle\eta'\rangle \neq 0$ for $m_1 < -c$

CP violating phase diagram in $N_f = 1$ QCD



our $N_f = 1$ QCD on the lattice

Partial Quenching: add valence, ghost quarks (graded Lie algebra)

$$\mathcal{L} = \bar{\vec{\Psi}} D_{PQ} \vec{\Psi} + L_g, \;\; D_{PQ} \equiv \left(\begin{array}{cc|c} \gamma_\mu D_\mu + m_V & 0 & 0 \\ 0 & \gamma_\mu D_\mu + m_S & 0 \\ \hline 0 & 0 & \gamma_\mu D_\mu + \tilde{m} \end{array} \right)$$

Chiral Symm.: $SU(3|2)_L \otimes SU(3|2)_R \otimes U(1)_A \otimes U(1)_V$

- spontaneous SB $SU(3|2)_L \otimes SU(3|2)_R \to SU(3|2)$

$m_s = m_V = \tilde{m} \to$ **same fermionic determinant as $N_f = 1$ QCD!**

Quantities and analytic evaluations

particle spectrum

- "physical states" $\eta_s = \bar{s}\gamma_5 s$, $\sigma = \bar{s}s$
- "unphysical states" degenerate pions octet π^a $(s, \bar{s}, v, \bar{v})$

PCAC hypothesis

- PCAC quark mass: $am_{PCAC} \equiv \frac{\langle \partial^* A^+_{x\mu} P^-_y \rangle}{2\langle P^+_x P^-_y \rangle}$,

  $m^R_{PCAC} = \frac{Z_A}{Z_p} m_{PCAC}$, $\chi_{PCAC} = 2B_0 m^R_{PCAC}$

PQχPT

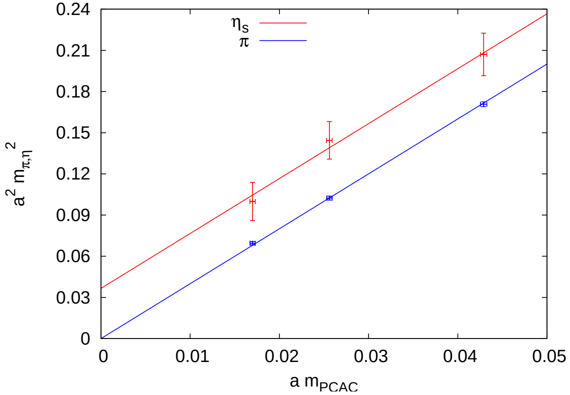
- pion masses $m^2_\pi = \chi_{PCAC} + \frac{\chi^2_{PCAC}}{16\pi^2 F_0^2} \ln \frac{\chi_{PCAC}}{\Lambda^2} + \ldots$ ($\sim$ GMOR)
- decay constant $F_\pi = F_0 \cdot \{1 - \frac{\chi_{PCAC}}{32\pi^2 F_0^2} \ln \frac{\chi_{PCAC}}{\Lambda^2} + \ldots\}$

PQ effective theory

- η_s mass $m^2_\eta = \frac{m^2_\Phi + \chi_{PCAC}}{1+\alpha}$

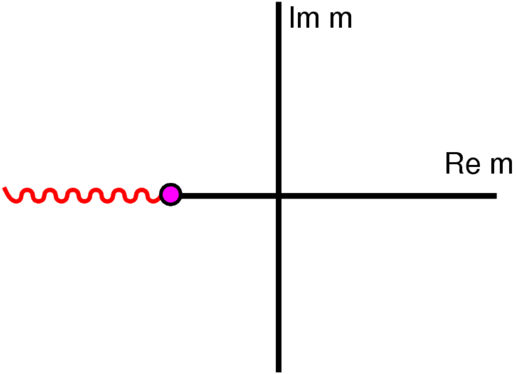
simulation and results

simulation:
tl-improved Symanzik action, PHMC algorithm, stout smearing
main results: PQχPT fits:



work in progress

- refine the statistics on large lattice
- detect CP violation with $\langle \bar{\psi}\gamma_5\psi \rangle \neq 0$,
  decrease m_q controlled by $\kappa = \frac{1}{2am_q+8}$ for simulations



the fermionic determinant sign is an issue

Vafa-Witten theorem:

- No CP violation if fermionic measure is positive

As our goal is precisely to construct a CP-breaking model
$\Rightarrow$ the sign problem has to be an issue!

statistics with negative determinant

$$Z = \int dA\, d\bar{\psi}\, d\psi\, e^{S_g(A)+\bar{\psi}\mathcal{D}(A)\psi} = \int dA\, e^{S_g(A)} \det \mathcal{D}(A),$$

Dirac operator $\mathcal{D}(A) = \gamma_\mu(\partial_\mu + igA_\mu) + m$

- $\sigma \equiv sign(\det \mathcal{D}(A))$,
  $$\langle \mathcal{O} \rangle_{e^{S_g+S_f}} = \frac{\int dA\, \{\mathcal{O}\sigma\}\, |\det \mathcal{D}(A)\, e^{S_g(A)}|}{\int dA\, \{\sigma\}\, |\det \mathcal{D}(A) e^{S_g(A)}|} = \frac{\langle \mathcal{O}\sigma \rangle_{|e^{S_g+S_f}|}}{\langle \sigma \rangle_{|e^{S_g+S_f}|}}$$
- statistical sample: $\langle \mathcal{O} \rangle_{st.} = \frac{\sum_{i=1}^{N} \mathcal{O}_i}{N}$
  $\Rightarrow \langle \mathcal{O} \rangle_{st.} = \langle \sigma\mathcal{O} \rangle_{st.} / \langle \sigma \rangle_{st.}$
  knowledge of σ's $\Rightarrow$ statistical corrections (reweighting)
- σ depends on the eigenvalues of $D(A)$

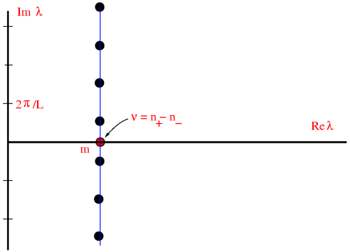
the sign problem...

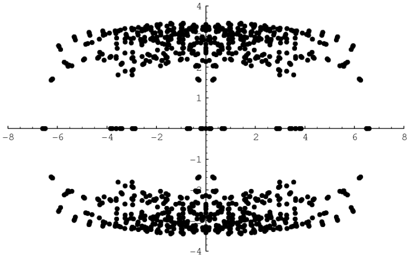
let us rewrite $\langle \mathcal{O} \rangle_{st.} = \frac{\sum_i^{N+} \mathcal{O}_i - \sum_j^{N-} \mathcal{O}_j}{N+ - N-}$

(recall $\langle \mathcal{O} \rangle_{st.} = \langle \sigma \mathcal{O} \rangle_{st.} / \langle \sigma \rangle_{st.}$, with $\langle \mathcal{O} \rangle_{st.} = \frac{\sum_{i=1}^{N} \mathcal{O}_i}{N}$)

- !the sign problem could "destroy" your statistics!

Eigenvalues spectrum: continuum and lattice





properties, symmetries and boundaries

- $D_W = 1 - \kappa Q$ Dirac-Wilson operator
- Q Hopping matrix ($Q = \sum_{\mu=1}\left[1 - \gamma_\mu U_\mu \delta(x + a_\mu - y) + (1 + \gamma_\mu) U_\mu^+(y)\delta(x - a_\mu - y)\right]$)

1. pairing $\{\lambda, \lambda^*\}$/ $\{\lambda, \lambda^*\}$
   (γ_5 hermicity) $Q^\dagger = \gamma_5 Q \gamma_5$
2. pairing $\{\lambda, -\lambda\}$ / $\{1 + \kappa\lambda, 1 - \kappa\lambda\}$
   $\Xi Q \Xi = -Q$, $\Xi(x, y) = (-1)^{x_1 + \dots + x_D}\delta(x - y)$
3. bounded $|\lambda| < 2D$, $D = 4$ / also bounded

$Q \equiv \gamma_5 D$ is the hermitian Dirac operator
$Q^\dagger = Q$ (from $\gamma_5 D \gamma_5 = D^\dagger$)

- real eigenvalues
  - no known relations between λ_Q's and λ_D's in general
  - $D\,|\psi\rangle = 0 \leftrightarrow Q\,|\psi\rangle = 0$ (λ_Q small $\Leftrightarrow$ λ_D small)

sign computation:

available Krilov space methods

$\mathcal{K}(D_W, v, j) = span\{v, D_W v, D_W{}^2 v, ..., D_W{}^j v\}$

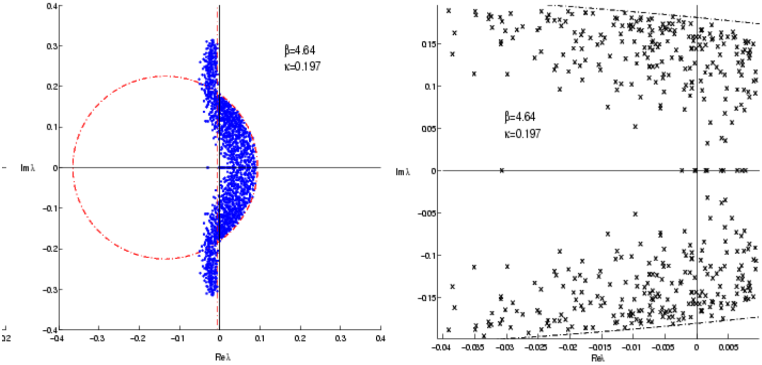
- conjugate gradient method on $\gamma_5 D_W$ (expensive, not exact)
- Arnoldi algorithm on D_W
  algo. efficiency $\propto$
  1. matrix size
  2. computed eigenvalues number
  3. eigenvalues separation/density

Improvement for 2., 3. (acceleration):
Preconditioning $D \rightarrow P_n(D)$, $P_n(D)$ polynomial:

- which polynomial?

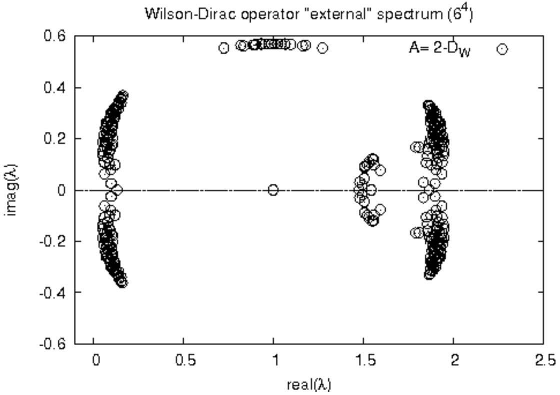
Find a way to get real eigenvalues here...(a year ago)

- the outer spectrum for $N_f = 1$ was observed to be similar to the ones for $N_f > 1$
- only the real negative eigenvalues matters! $\lambda \in \mathcal{C}$, $\lambda\lambda^* = \mid \lambda \mid^2$



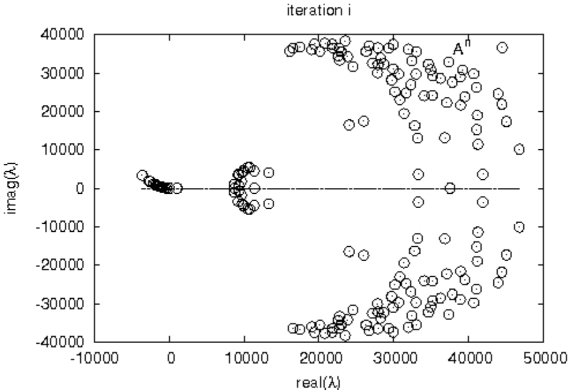
preconditioning techniques

- basic preconditioning $D_W \to P_n(D_W) = (\sigma \mathbf{1} - D_W)^n$:
  rotates away complex eigenvalues
  $(\lambda = \rho e^{i\theta} \to \lambda^n = \rho^n e^{in\theta})$



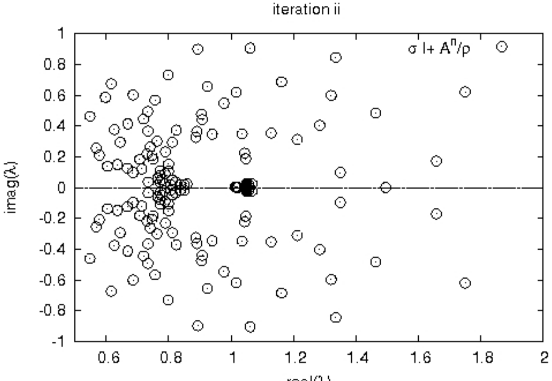
preconditioning techniques

- basic preconditioning $D_W \rightarrow P_n(D_W) = (\sigma \mathbf{1} - D_W)^n$: rotates away complex eigenvalues ($\lambda = \rho e^{i\theta} \rightarrow \lambda^n = \rho^n e^{in\theta}$)

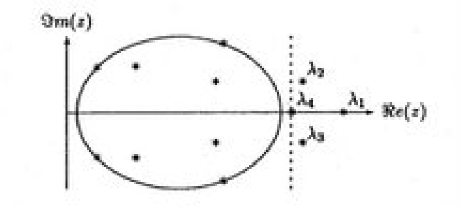


preconditioning techniques

- basic preconditioning $D_W \rightarrow P_n(D_W) = (\sigma \mathbf{1} - D_W)^n$: rotates away complex eigenvalues $(\lambda = \rho e^{i\theta} \rightarrow \lambda^n = \rho^n e^{in\theta})$
- iterated version $P(D_W) \rightarrow (\rho \mathbf{1} - P(D_W)/C_{Ren.})^n$: "peels" complex eigenvalues away (partially)



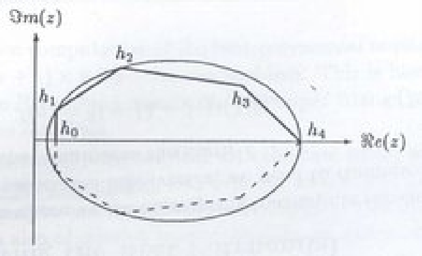
sytematic approach



1. define an ellipsoid hull (wanted/unwanted eigenvalues)
2. formulate a min max problem for a given polynomial order:
   $\{min_{p \in \mathbf{P}_n} max_{\lambda \in E} |p(\lambda)|, \, | \, p(\lambda_1) = 1\}$
3. solve it with Chebyshev polynomials
4. go to 1.

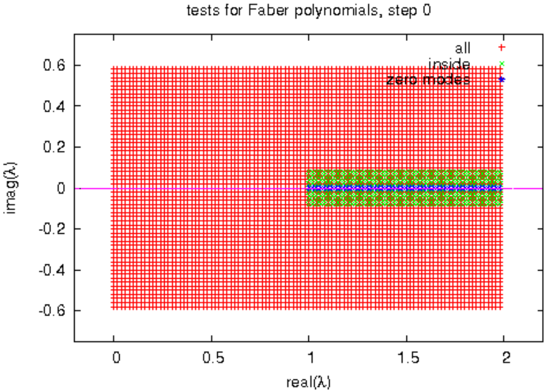
[Y.Saad]

methods with Faber polynomials

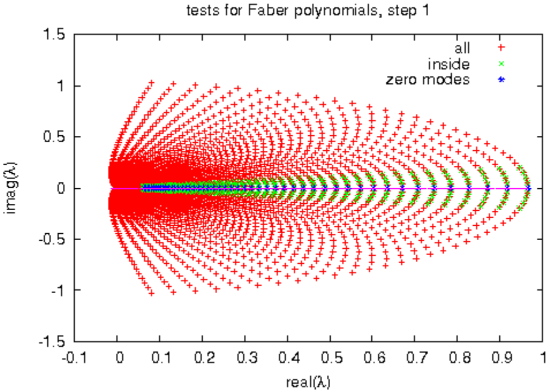
- polygonal hull 

- min max problem: $\{min_{p \in \mathbf{P}_n} \; max_{\lambda \in P} |p(\lambda)|, \; s.t. \; |p(\lambda_1)| = 1\}$
- solve it with Faber polynomials

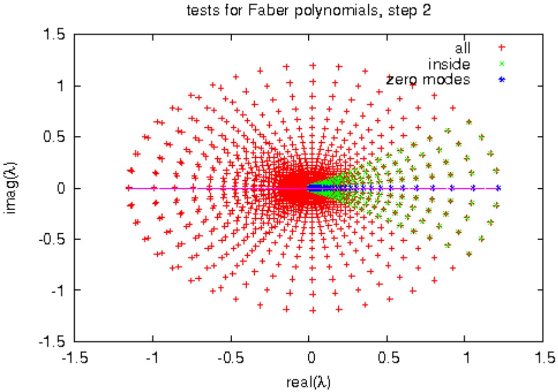
example


tests for Faber polynomials, step 0
all
inside
zero modes
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
imag(λ)
0
0.5
1
1.5
2
real(λ)

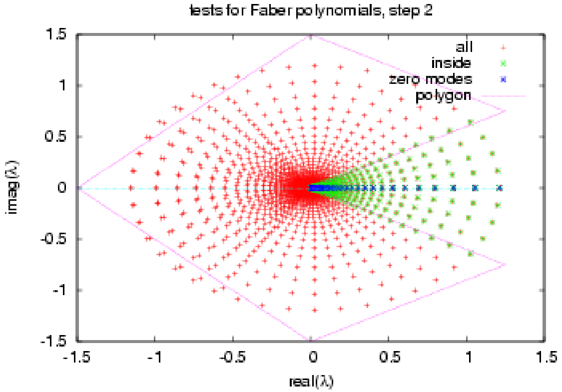
example


tests for Faber polynomials, step 1
all
inside
zero modes
imag(λ)
real(λ)
1.5
1
0.5
0
-0.5
-1
-1.5
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

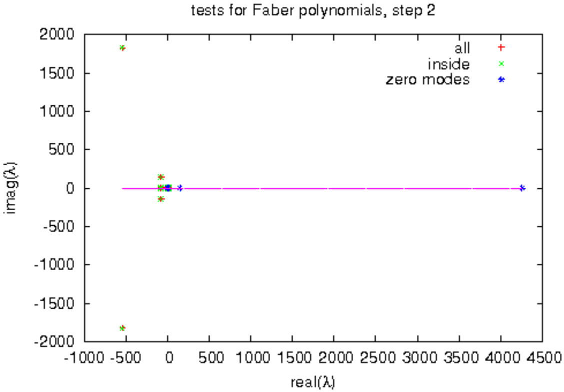
example


tests for Faber polynomials, step 2
all
inside
zero modes
imag(λ)
real(λ)
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5

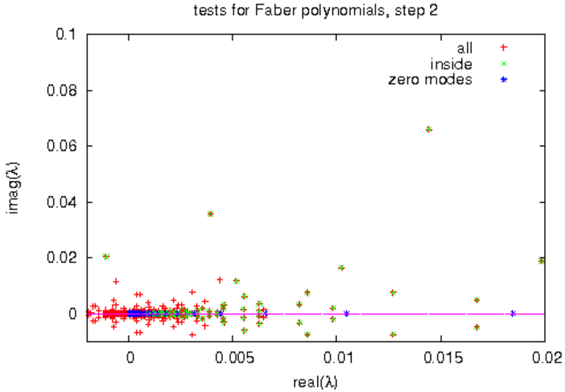
example


tests for Faber polynomials, step 2
all
inside
zero modes
polygon
imag(λ)
real(λ)
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5

example


tests for Faber polynomials, step 2
all
inside
zero modes
imag(λ)
real(λ)
2000
1500
1000
500
0
-500
-1000
-1500
-2000
-1000 -500 0 500 1000 1500 2000 2500 3000 3500 4000 4500

example


tests for Faber polynomials, step 2
all
inside
zero modes
imag(λ)
real(λ)
0.1
0.08
0.06
0.04
0.02
0
0
0.005
0.01
0.015
0.02

limitations, issues for optimisation?

- optimal shape for the polygon?
- mix the methods?
- polynomial order for the transformation?
- number of eigenvalues to look for?
- cpu, memory limitations?
- inner spectrum eigenvalues density?

1. is there an optimal fairly systematic approach? ($\neq$ cooking)?
2. challenge: get all the real eigenvalues!

eigenvalues spectrum, what is interesting there?

configurations generated through a Monte-Carlo algorithm:
"Path integral realisation on the lattice"
eigenvalues spectrum as "fingerprint" of the "lattice" situation
which information is available?

- det. sign

ok

- ...?

some understanding of how the Dirac-Wilson spectrum in continuum is "translated" onto the lattice is needed

zero modes in continuum

we have $\{\mathcal{D}, \gamma_5\} = 0$
the eigenvalues/eigenvectors are paired $\{\psi, (+iE)\}$, $\{\gamma_5\psi, (-iE)\}$
($\mathcal{D}\psi = iE\psi \Leftrightarrow \mathcal{D}(\gamma_5\psi) = -iE(\gamma_5\psi)$)
chirality of a state $\chi = (\psi, \gamma_5\psi)$,
$\chi = (\psi, \gamma_5\psi) \neq 0 \Leftrightarrow E = 0$

- only zero modes can have non vanishing chirality.
- Atiyah-Singer/index Theorem $\nu[A] = n_+ - n_-$
  (connects topological charge to the zero modes chirality)

situation on the lattice with the non-hermitian Dirac-Wilson operator

Here we have $D^\dagger = \Gamma_5 D \Gamma_5$,
$Dv_i = \lambda_i v_i \Leftrightarrow (v_i^+ \Gamma_5) D = (v_i^+ \Gamma_5) \lambda_i^*$
Therefore $v_i^+ \Gamma_5 D v_j = \lambda_i^* (v_i^+ \Gamma_5 v_j) = \lambda_j (v_i^+ \Gamma_5 v_j)$
we find $v_i^+ \Gamma_5 v_j \neq 0 \Leftrightarrow \lambda_i = \lambda_i^*$

- only the real eigenvalues have non-vanishing chiralities

the topological charge on the lattice

the condensate density on the lattice $q(x) \simeq \frac{1}{2} tr(\gamma_5 D(x,x))$
(from the Casher Banks relation)
now $\sum_x q(x) = \frac{1}{2} tr(\gamma_5 D) = -\frac{1}{2} tr(2 - \gamma_5 D) = -\frac{1}{2} \sum_{\lambda_Q} (2 - \lambda_Q)(\phi_{\lambda_Q}, \gamma_5 \phi_{\lambda_Q})$

This leads to

- Index theorem on the lattice: $\nu[A] = n_+ - n_-$

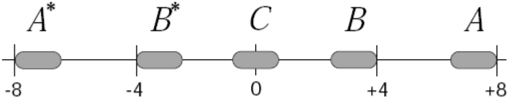
the Dirac-Wilson spectrum real eigenvalues

- real eigenvalues can reasonably be interpreted as remnants of continuum zero modes!

Which informations can be extracted from the zero-modes!?
(dominance of the low eigenmodes(low vs high energy physics)?, spectral decomposition method, topological issues with the fields configurations, consistency checks with random matrix theory,...)

localisation of real eigenvalues: theory vs lattice

16 zeros doublers in the brillouin zone, at $(0,0,0,0)$, ..., (π,π,π,π)
their global contributions to the topological charge cancel
[Bell, Jackiw]

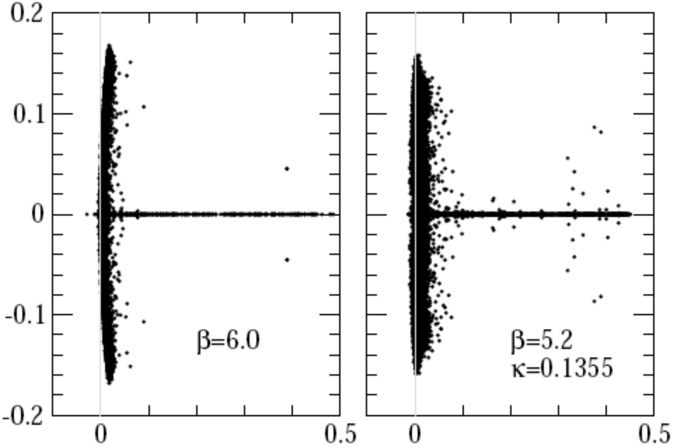


branch	A^*	B^*	C	B	A
$R_- - R_+$	ν	-4ν	6ν	-4ν	ν
corners of B.Z.	(π,π,π,π)	$(\pi,\pi,\pi,0)$ + 3 perm.	$(\pi,\pi,0,0)$ + 5 perm.	$(\pi,0,0,0)$ + 3 perm.	$(0,0,0,0)$

localisation of real eigenvalues: theory vs lattice

!real daily life on the lattice!

[Schierholz 2001]

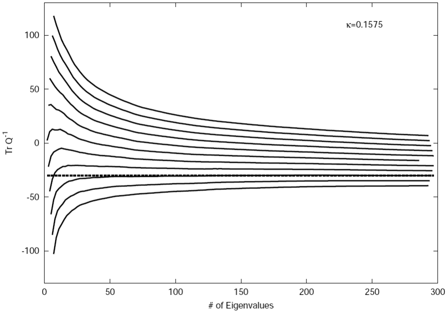


the spectral decomposition approach

idea: reconstruct the Dirac operator from the eigenvectors

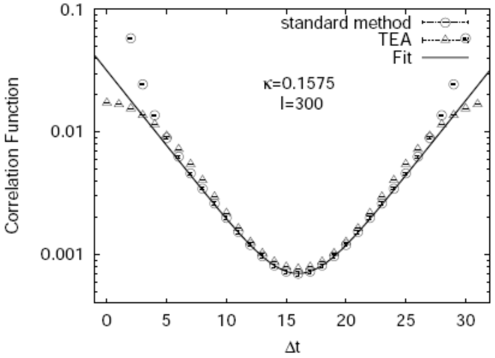
- $D^{-1} = \sum_i \frac{1}{\lambda_i^D} \frac{|\psi_i\rangle\langle\psi_i|\gamma_5}{\langle\psi_i|\gamma_5|\psi_i\rangle}$
- $Q^{-1} = \gamma_5 \sum_i \frac{1}{\lambda_i^Q} \frac{|\psi_i\rangle\langle\psi_i|\gamma_5}{\langle\psi_i|\gamma_5|\psi_i\rangle}$

in continuum limit $\nu[A] = trQ^{-1}$
Does this connection still hold on the lattice(Q, no level crossing)?



correlators from spectral decomposition

- $C_\pi(\Delta t) = \langle \sum_x Q_x^{-1}(t) Q_x^{-1}(t+\Delta t)\rangle$
  $C_\pi^l(\Delta t) = \sum_{i,j,t}^l \frac{1}{\lambda_i \lambda_j} \frac{\langle \psi_i(t)|\psi_j(t)\rangle \langle \psi_j(\tau+\Delta t)|\psi_i(t+\Delta t)\rangle}{\langle \psi_i(t)|\psi_i(t+\Delta t)\rangle \langle \psi_j(t)|\psi_j(t+\Delta t)\rangle}$
- recall for the correlation function:
  $C_2(j) = \sum_{n\neq 0} \mid \langle 0 \mid \mathcal{O} \mid n\rangle \mid^2 \exp\left(-\frac{(E_n - E_0)}{\hbar} ja\right)$
  for $j \sim 5$: $\sim$ only contributions from lower decay (E_1)



Conclusion

After a theoretical introduction, I discussed/introduced:

- **the possibility of CP breaking for negative quark masses in $N_f = 1$ QCD!**
- **a strategy to test this model with a partially quenched extension of the $N_f = 1$ theory**
- **the sign problem**
- **practical techniques to evaluate it with the computation of Dirac-Wilson operator real eigenvalues**
- **the physical meaning of the real eigenvalues as remnants of the zero modes and the fact that low energy physics is contained in the small eigenvalues**