30.04.2007

Lattice Quantum Chromodynamics with a chirally twisted mass term

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1. Introduction

Quantum Chromodynamics

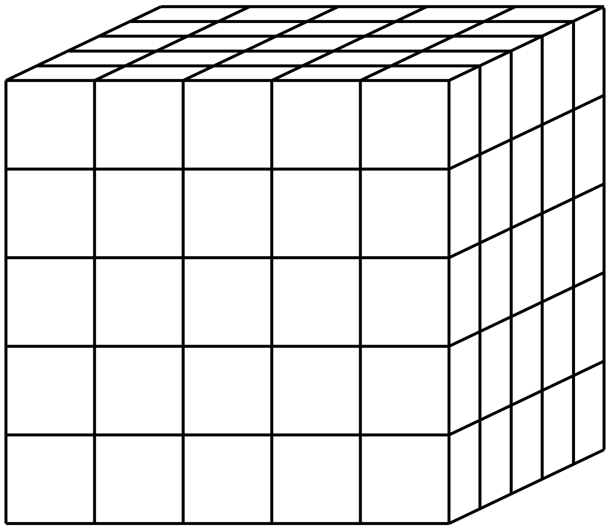
Continuum:

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f \{\gamma_\mu(\partial_\mu - \mathrm{i}\, g A_\mu) + m_f\}\psi_f + \frac{1}{4} F^i_{\mu\nu} F^i_{\mu\nu}, \qquad f = 1, \ldots N_f \quad \text{flavours}$$

Perturbation theory $\sim$ expansion in g $\quad\rightarrow\quad$ high energy regime

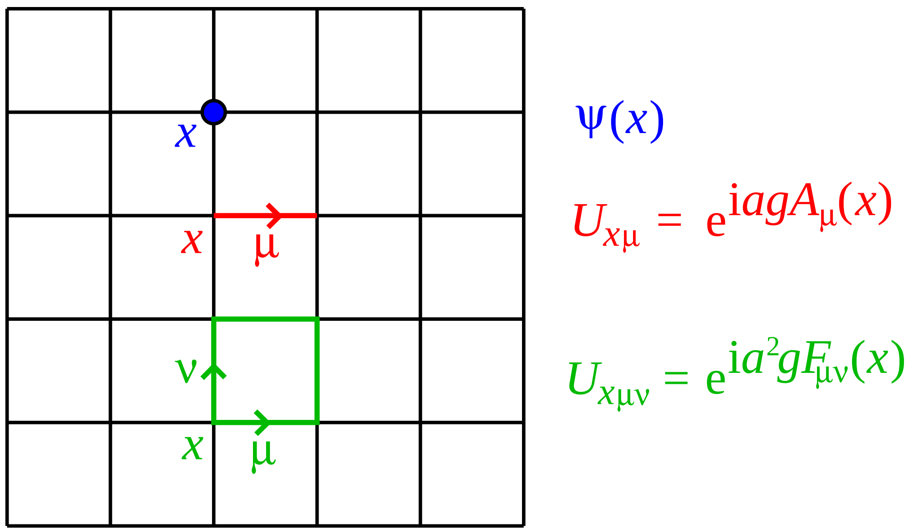
Non-perturbative problems: spectrum, matrix elements, ...

$\rightarrow$ QCD on the lattice



lattice spacing a

Lattice QCD



Wilson action:

$$\mathcal{L} = (m_0 + \frac{4}{a})\bar{\psi}_x\psi_x - \frac{1}{2a}\sum_\mu \{\bar{\psi}_{x+\mu}(1+\gamma_\mu)U_{x\mu}\psi_x + \bar{\psi}_x(1-\gamma_\mu)U^+_{x\mu}\psi_{x+\mu}\}$$

$$-\frac{2}{a^4 g^2}\sum_{\mu<\nu} \mathrm{Re}\,\mathrm{Tr}\,U_{x\mu\nu}$$

$$\simeq \bar{\psi}(\gamma_\mu D_\mu + m_q)\psi - \frac{1}{2a}\bar{\psi}D_\mu D_\mu\psi + \frac{1}{4}F^i_{\mu\nu}F^i_{\mu\nu}$$

quark mass $m_0 = m_c + m_q$

Numerical investigations of Quantum Chromodynamics on the lattice have to fight against problems from

- finite a (lattice spacing) effects,
- finite L (lattice size) effects,
- small quark masses

Quark masses: $m_u = 5\,\text{MeV}, \quad m_d = 9\,\text{MeV}, \quad m_s = 175\,\text{MeV}$

Monte Carlo: slowing down $\propto m_q^p,\ p = 2 - 3$

Extrapolation to small $m_q \quad \rightarrow \quad$ chiral perturbation theory (χPT) (Weinberg; Gasser, Leutwyler, ...): expansion around chiral limit, $m_q = 0$

Lattice spacing a

$$X(a) = X(\text{continuum}) + c_1 a + c_2 a^2 + \ldots$$

Symanzik's improvement program: $\quad c_1 \to 0,\ c_2 \to 0,\ \ldots$

Ansatz:

Twisted mass lattice QCD (Frezzotti, Grassi, Sint, Weisz, Rossi)

$\to \mathcal{O}(a)$ improvement

ETMC: European Twisted Mass Collaboration

O. Bär, D. Bećirević, B. Blossier, Ph. Boucaud, T. Chiarappa, F. De Soto, P. Dimopoulos, F. Farchioni, R. Frezzotti, V. Giménez, G. Herdoiza, K. Jansen, J.P. Leroy, M.P. Lombardo, V. Lubicz, G. Martinelli, C. McNeile, F. Mescia, C. Michael, I. Montvay, G. Münster, K. Nagai, D. Palao, M. Papinutto, O. Pène, J. Pickavance, J. Rodríguez-Quintero, G.C. Rossi, S. Schäfer, L. Scorzato, A. Shindler, S. Simula, T. Sudmann, C. Tarantino, C. Urbach, A. Vladikas, U. Wenger

Berlin, Frascati, Grenoble, Hamburg, Huelva, Liverpool, Milano, München, Münster, Paris XI, Roma III, Roma "La Sapienza", Roma Tor Vergata, València, Villazzano, Zeuthen, Zürich

2. Chiral symmetry of Quantum Chromodynamics

Continuum:

$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f\{\gamma_\mu(\partial_\mu - {\rm i}\, g A_\mu) + m_f\}\psi_f + \frac{1}{4}F^i_{\mu\nu}F^i_{\mu\nu}, \qquad f = 1,\dots N_f \quad \text{flavours}$$

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi\,, \quad \psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi$$

$$\rightarrow \quad \mathcal{L}_q = \bar\psi_L\gamma_\mu D_\mu\psi_L + \bar\psi_R\gamma_\mu D_\mu\psi_R + m_q(\bar\psi_L\psi_R + \bar\psi_R\psi_L)$$

$m_q = 0 \quad \rightarrow \quad$ chiral symmetry $\quad {\rm SU}(N_f)_L \otimes {\rm SU}(N_f)_R \otimes {\rm U}(1)_V \otimes {\rm U}(1)_A$

$$\psi_L \rightarrow L\,\psi_L\,, \quad \psi_R \rightarrow R\,\psi_R \quad \text{with} \quad L \in {\rm SU}(N_f)_L, \quad R \in {\rm SU}(N_f)_R$$

spontaneously broken to ${\rm SU}(N_f)_V$, $\qquad (L \equiv R)$

Goldstone bosons = pseudoscalar mesons: pions, Kaons, η

explicitely broken by quark masses $m_q \neq 0$

Goldstone bosons $\rightarrow$ light Pseudo-Goldstone bosons

For $N_f = 2$: Pseudo-Goldstone bosons = pions π_b , $\quad b = 1,2,3$

Gell-Mann, Oakes, Renner: $\quad m_\pi^2 = B_0(m_u + m_d)$

3. Twisted mass lattice QCD

Frezzotti, Grassi, Sint, Weisz, Rossi

Consider $N_f = 2, \quad m_u = m_d \equiv m_q$

Parameters: quark mass m_q, gauge coupling g

Mass term $\bar{\psi}\, M(\omega)\, \psi$ with

$$M(\omega) = m_q\, \mathrm{e}^{\mathrm{i}\,\omega\gamma_5\tau_3} = m + \mathrm{i}\,\mu\,\gamma_5\tau_3\,, \quad \text{where} \quad m = m_q\cos(\omega), \;\; \mu = m_q\sin(\omega)$$

Continuum: no effect, remove ω by chiral rotation: $\psi = \mathrm{e}^{-\mathrm{i}\omega\gamma_5\tau_3/2}\,\psi'$

Lattice: dependence on ω due to explicit breaking of chiral symmetry through Wilson term

Promising approach for reducing lattice effects in numerical simulations

Full $\mathcal{O}(a)$ improvement for $\omega = \frac{\pi}{2}$ (Frezzotti, Rossi)

4. Low-energy effective Lagrangean for pions

Phase structure, chiral perturbation theory

$\longrightarrow$ Low-energy effective Lagrangean

in terms of $$U(x) = \exp\left(\frac{\mathrm{i}}{F_0}\,\pi_b(x)\tau_b\right)$$

transforms as $$U \to L U R^{-1}, \qquad (L, R) \in \mathrm{SU}(N_f)_L \otimes \mathrm{SU}(N_f)_R$$

Continuum:

Leading order $$\mathcal{L}_2 = \frac{F_0^2}{4}\,\mathrm{Tr}\left(\partial_\mu U^\dagger\,\partial^\mu U\right) - \frac{F_0^2}{4}\,\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)$$

$$\chi = 2B_0 M\,, \qquad M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

Expansion: $$\mathcal{L}_2 = \tfrac{1}{2}(\partial_\mu\pi_a)(\partial_\mu\pi_a) + \tfrac{1}{2}B_0(m_u + m_d)\,\pi_a\pi_a + \ldots$$

with pion masses $$m_\pi^2 = B_0(m_u + m_d) \qquad (\ F_0^2 B_0 = -\langle\bar{u}u\rangle\ )$$

Higher orders: terms with more fields and/or derivatives

Next-to-leading order (NLO)

$$
\begin{aligned}
\mathcal{L}_4 = & \frac{F_0^2}{4}\,\mathrm{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right) - \frac{F_0^2}{4}\,\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right) \\
& - L_1 \left[\mathrm{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right)\right]^2 - L_2\,\mathrm{Tr}\left(\partial_\mu U \partial_\nu U^\dagger\right)\mathrm{Tr}\left(\partial_\mu U \partial_\nu U^\dagger\right) \\
& - L_3\,\mathrm{Tr}\left(\left[\partial_\mu U \partial_\mu U^\dagger\right]^2\right) + L_4\,\mathrm{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right)\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right) \\
& + L_5\,\mathrm{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\left[\chi U^\dagger + U\chi^\dagger\right]\right) - L_6\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 \\
& - L_7\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - L_8\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)
\end{aligned}
$$

Gasser-Leutwyler coefficients L_i

$\mathcal{L}$ non-renormalizable, Weinberg power counting scheme

Lattice:

Symanzik's effective action: $\mathcal{L}_{\text{LQCD}} = \mathcal{L}_{\text{QCD}} + a\,\mathcal{L}_1 + \ldots$ in the continuum

$\rightarrow$ effective chiral Lagrangean $\mathcal{L}$ including lattice artifacts

$\sim$ terms in $\mathcal{L}$ proportional to powers of a

$$\mathcal{L}_2 = \frac{F_0^2}{4}\,\mathrm{Tr}\left(\partial_\mu U^\dagger\,\partial^\mu U\right) - \frac{F_0^2}{4}\,\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right) - \frac{F_0^2}{4}\,\mathrm{Tr}\left(\rho\, U^\dagger + U\rho^\dagger\right)$$

where

$$\chi = 2B_0 m_q'\,, \qquad \rho = 2W_0\,a$$

Expansion: $\mathcal{L}_2 = \frac{1}{2}(\partial_\mu\pi_a)(\partial_\mu\pi_a) + \frac{1}{2}m_\pi^2\,\pi_a\pi_a + \ldots$

with pion masses $m_\pi^2 = 2B_0 m_q' + 2W_0\,a = 2B_0 m_q$

Calculate m_π^2, F_π, etc. in powers of $m_q \sim \chi$ (modified by logarithms)

and $a \sim \rho$

Next-to-leading order: Rupak, Shoresh $\mathcal{O}(a)$; Bär, Rupak, Shoresh $\mathcal{O}(a^2)$

Monte Carlo $\rightarrow$ Gasser-Leutwyler coefficients of χPT

twisted mass lattice QCD:

Twisting of the mass term

$$\chi \longrightarrow \chi(\omega) = 2B_0\, m_q'\, \mathrm{e}^{-\mathrm{i}\omega\tau_3}$$

Leading order

$$\mathcal{L}_2 = \frac{F_0^2}{4}\,\mathrm{Tr}\left(\partial_\mu U^\dagger\,\partial^\mu U\right) - \frac{F_0^2}{4}\,\mathrm{Tr}\left((\chi(\omega)+\rho)\,U^\dagger + U(\chi(\omega)+\rho)^\dagger\right)$$

Next-to-leading order (NLO) to $\mathcal{O}(a^2)$

$$
\begin{aligned}
\mathcal{L}_4 = & \frac{F_0^2}{4} \operatorname{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right) - \frac{F_0^2}{4} \operatorname{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) - \frac{F_0^2}{4} \operatorname{Tr}\left(\rho U^\dagger + U \rho^\dagger\right) \\
& - L_1 \left[\operatorname{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right)\right]^2 - L_2 \operatorname{Tr}\left(\partial_\mu U \partial_\nu U^\dagger\right) \operatorname{Tr}\left(\partial_\mu U \partial_\nu U^\dagger\right) \\
& - L_3 \operatorname{Tr}\left(\left[\partial_\mu U \partial_\mu U^\dagger\right]^2\right) \\
& + L_4 \operatorname{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right) \operatorname{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) \\
& + W_4 \operatorname{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right) \operatorname{Tr}\left(\rho U^\dagger + U \rho^\dagger\right) \\
& + L_5 \operatorname{Tr}\left(\partial_\mu U \partial_\mu U^\dagger \left[\chi U^\dagger + U \chi^\dagger\right]\right) + W_5 \operatorname{Tr}\left(\partial_\mu U \partial_\mu U^\dagger \left[\rho U^\dagger + U \rho^\dagger\right]\right) \\
& - L_6 \left[\operatorname{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)\right]^2 \\
& - W_6 \operatorname{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) \operatorname{Tr}\left(\rho U^\dagger + U \rho^\dagger\right) - W_6' \left[\operatorname{Tr}\left(\rho U^\dagger + U \rho^\dagger\right)\right]^2 \\
& - L_7 \left[\operatorname{Tr}\left(\chi U^\dagger - U \chi^\dagger\right)\right]^2 \\
& - W_7 \operatorname{Tr}\left(\chi U^\dagger - U \chi^\dagger\right) \operatorname{Tr}\left(\rho U^\dagger - U \rho^\dagger\right) - W_7' \left[\operatorname{Tr}\left(\rho U^\dagger - U \rho^\dagger\right)\right]^2 \\
& - L_8 \operatorname{Tr}\left(\chi U^\dagger \chi U^\dagger + U \chi^\dagger U \chi^\dagger\right) \\
& - W_8 \operatorname{Tr}\left(\chi U^\dagger \rho U^\dagger + U \rho^\dagger U \chi^\dagger\right) - W_8' \operatorname{Tr}\left(\rho U^\dagger \rho U^\dagger + U \rho^\dagger U \rho^\dagger\right)
\end{aligned}
$$

Vacuum $\sim$ minimum of $\mathcal{L}$

at non-vanishing pion field (explicit flavour and parity breaking)

Pion mass splitting at $\mathcal{O}(a^2)$

Expansion around vacuum
$\rightarrow$ chiral perturbation theory for twisted mass lattice QCD
(Münster, Schmidt, Scholz, Hofmann, Sudmann,
Scorzato, Sharpe, Wu, Aoki, Bär ...)

5. Phase structure

„The first thing to do is to look for phase transitions“ (G. Parisi, 1988)

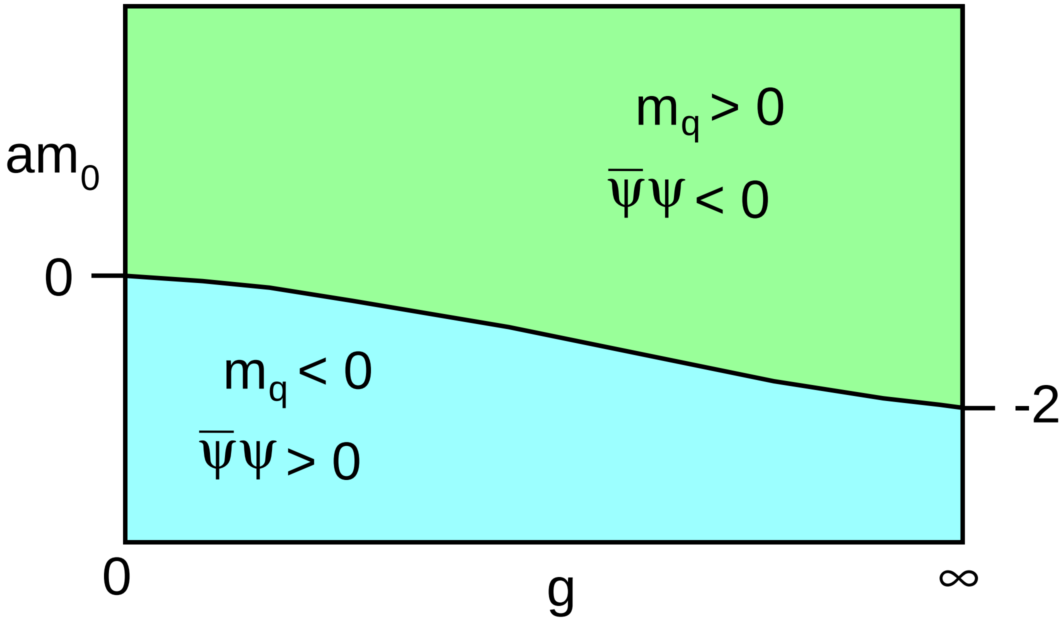
old folklore, see textbooks

$m_0 = m_c + m_q$

$g \to 0$: continuum limit $\sim$ free quarks $\longleftrightarrow$ $m_c = 0$

perturbation theory for quark self-energy: $m_c = -\frac{1}{a}\,(0.4340\, g^2 + \ldots)$

$g \to \infty$: $m_c = -\frac{1}{a} \cdot 2 \longleftrightarrow m_\pi = 0$



physical region: $m_q \geq 0, \quad \bar{\psi}\psi \neq 0,$

at m_c: $m_\pi = 0,$ spontaneously broken chiral symmetry

Analogy:	Ising model	QCD
	magnetic field h	quark mass m_q
	magnetization M	$\bar{\psi}\psi$

This picture is wrong.

More careful look (Aoki 1984, ...)

Discuss symmetry breaking.

For $N_f = 2$: $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \sim \mathrm{SO}(4)$, $\mathrm{SU}(2)_V \sim \mathrm{SO}(3)$

Z_2 (Ising) $\longrightarrow$ SO(4) (Heisenberg model)

No Goldstone bosons at $m_q = 0$?

Phase structure of tmQCD

G.M., JHEP 09 (2004) 035, hep-lat/0407006

Visualize the potential.

SO(4) description: $U = u_0 + \mathrm{i} u_a \tau_a \,, \quad |u|^2 = 1 \,, \qquad 2\chi + 2\rho \equiv h_0 + \mathrm{i} h_3 \tau_3$

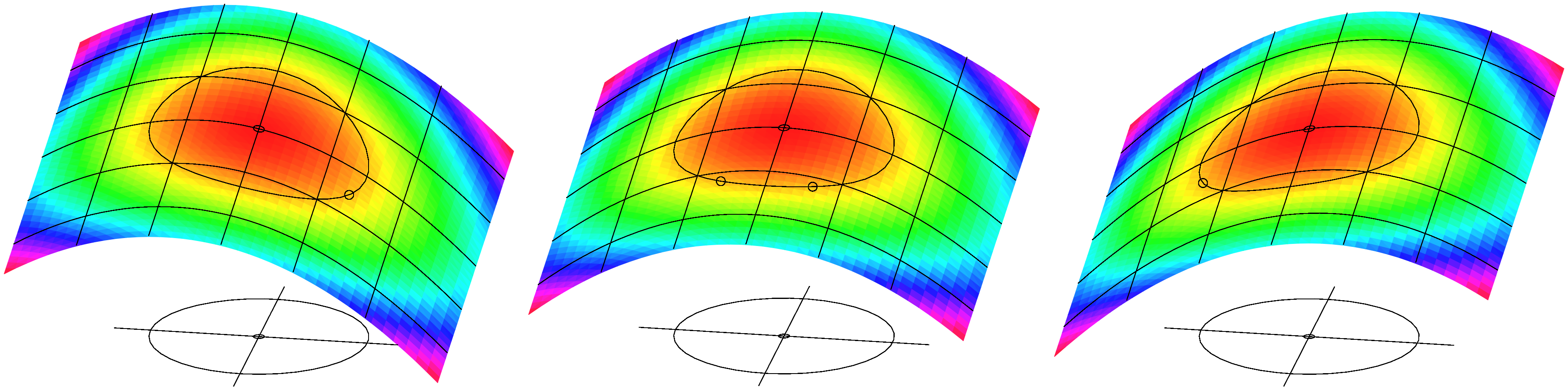
$\mathrm{Tr}\left(U(\chi^\dagger + \rho^\dagger)\right) = u \cdot h \qquad \longleftrightarrow \qquad$ magnetic field in O(4) Heisenberg model

Potential in next-to-leading order:

$$V = -c_1\, u_0 + c_2\, u_0^2 + \; c_3\, u_3 + c_4\, u_3^2 + c_5\, u_0 u_3$$

$c_1 \sim m_q, \qquad c_2 = \mathcal{O}(a^2)$ for small $m_q, \qquad c_3 \sim \mu$

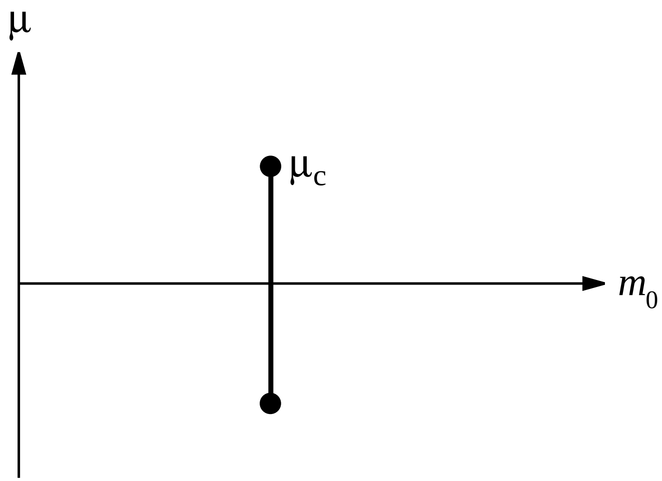
Scenarios at $\mu > 0$: $c_2 < 0$



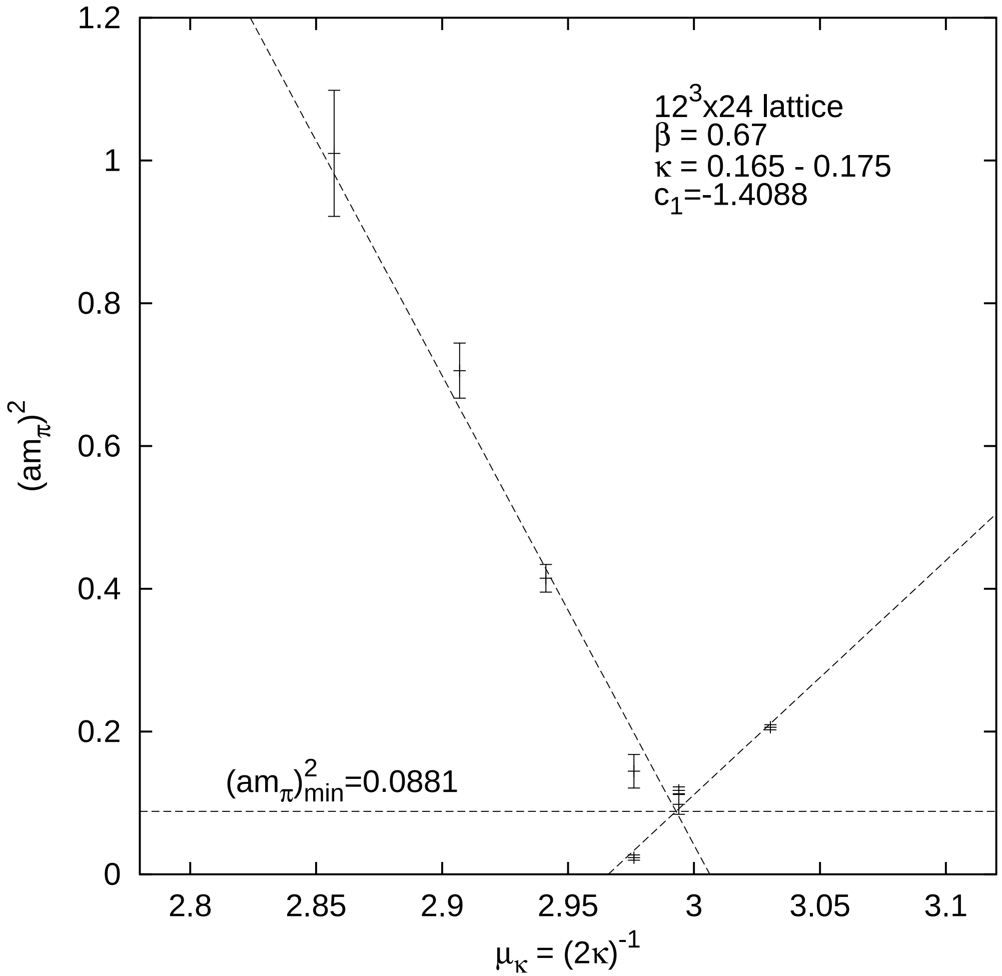
$m_q > 0$ $m_q = 0$ $m_q < 0$

1^{st} order phase transition, 2^{nd} order end point $\mu_c \sim a^2$

normal scenario:



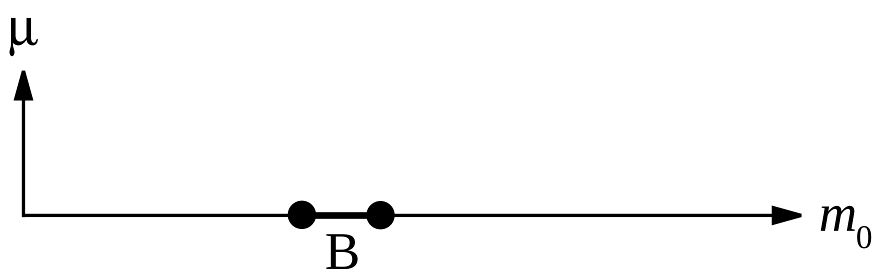
minimal $m_\pi^2 \sim a^2$



from F. Farchioni et al., Eur. Phys. J. C42 (2005) 73-87
(DBW2 action)

Scenarios at $\mu > 0$: $c_2 > 0$

Aoki scenario:

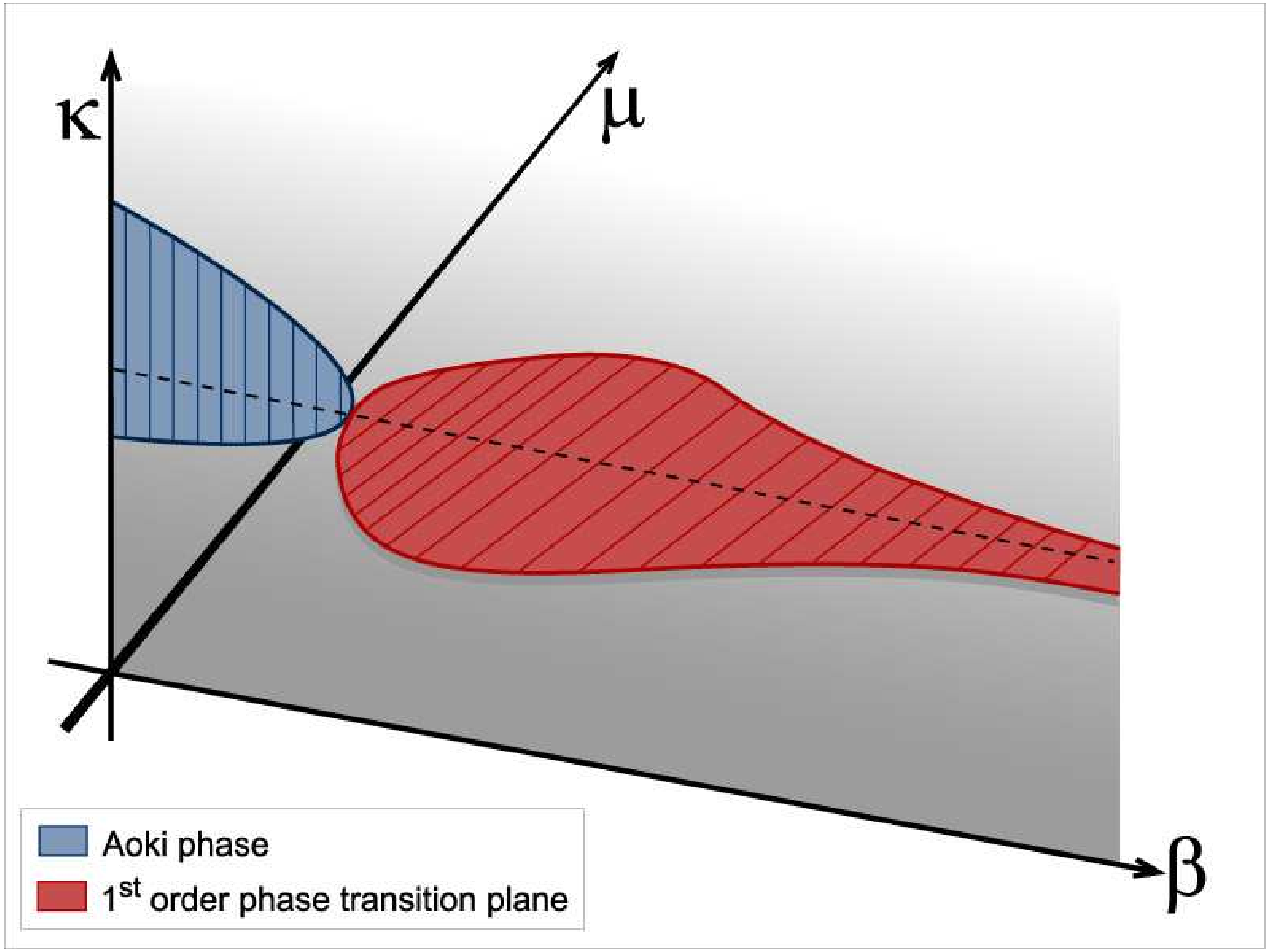


The crucial parameter c_2 depends on g^2, action

Monte Carlo calculations

Müller-Preußker et al.: large g^2 $\leftrightarrow$ Aoki scenario

QCD$^{\mathrm{TM}}$ (Montvay, Farchioni, Jansen et al.): small g^2 $\leftrightarrow$ normal scenario



6. Chiral perturbation theory for twisted mass lattice QCD

Twisting of the mass term

$$\chi \to \chi(\omega) = 2B_0\, m_q\, \mathrm{e}^{-\mathrm{i}\omega\tau_3}$$

shifted to the lattice term by chiral rotation

$$\rho \to \rho(\omega) = \mathrm{e}^{\mathrm{i}\omega\tau_3}\rho = \mathrm{e}^{\mathrm{i}\omega\tau_3}\ 2W_0\, a$$

Vacuum $\sim$ minimum of $\mathcal{L}$

at non-vanishing pion field

$$\tilde{\pi}_3 = F_0 \frac{W_0\, a}{B_0\, m_q}\ \sin\omega \left[1 - \frac{8\chi_0}{F_0^2}(4L_6 + 2L_8 - 2W_6 - W_8)\right] + \mathcal{O}(a^2)$$

Expansion around vacuum $\quad\to\quad$ chiral perturbation theory

powers of $m_q \sim \chi$ (modified by logarithms) and

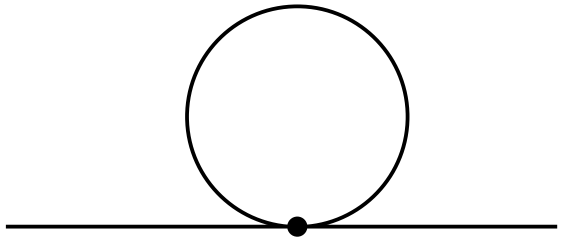
powers of a/m_q, $\quad$ to $\mathcal{O}(a^2)$

6.1 Pion mass

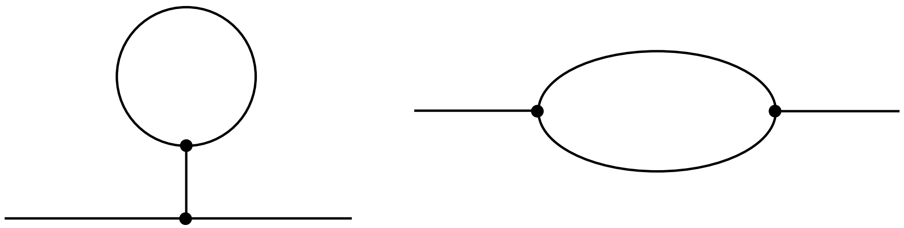
Expansion of $\mathcal{L}$ in pion fields

$$\mathcal{L} = K\frac{1}{2}\{(\partial_\mu \pi_a)(\partial_\mu \pi_a) + m_\pi^2(\text{tree})\,\pi_a \pi_a\} + \ldots$$

Loop contributions to the pion propagator from the LO vertices



New vertices from shifted vacuum yield



$$= \mathcal{O}(a^2)$$

Result:

$$
\begin{aligned}
m_\pi^2 \quad = \quad & \chi_0 + \rho_0 + 8\frac{\chi_0^2}{F_0^2}(4L_6^r + 2L_8^r - 2L_4^r - L_5^r) \\
& +8\frac{\chi_0\rho_0}{F_0^2}(4W_6^r + 2W_8^r - 2W_4^r - W_5^r - 2L_4^r - L_5^r) \\
& +\frac{(\chi_0+\rho_0)^2}{32\pi^2 F_0^2}\ \ln\left(\frac{\chi_0+\rho_0}{\Lambda^2}\right) + \mathcal{O}(a^2)
\end{aligned}
$$

where

$$
\chi_0 = 2B_0\, m_q\,, \qquad \rho_0 = 2W_0\, a \cos\omega
$$

$L_k^r,\ W_k^r$: renormalized chiral parameters, $\quad \Lambda =$ renormalization scale

Dependence on ω: factor $\cos\omega$ (only at $\mathcal{O}(a)$)

Maximal twist, $\omega = \pi/2$ $\quad\rightarrow\quad$ vanishing lattice artifacts, cp. Frezzotti, Rossi

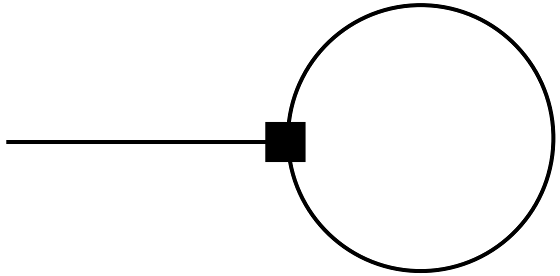
6.2 Pion decay constant

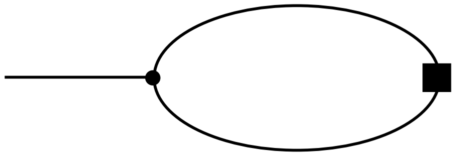
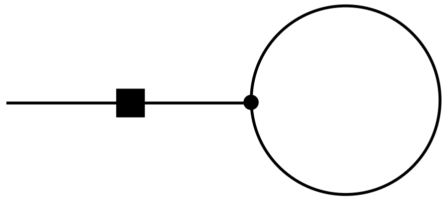
F_π given by $\langle 0 | J_A^{\mu,a} | \pi_b(p) \rangle = \mathrm{i} F_\pi p^\mu \delta_{ab}$

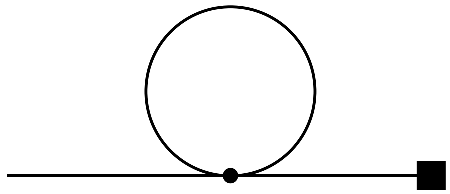
Axial current $J_A^{\mu,a}$: Noether procedure $\longrightarrow$ expression in powers of pion fields

Contributions:

tree level: 

loop correction: 

loop corrections at $\mathcal{O}(a^2)$:  

Wave function renormalization:  

Result:

$$
\begin{aligned}
F_\pi \quad = \quad & F_0 \left(1 + \frac{4}{F_0^2} \left[\chi_0 \left(2L_4^r + L_5^r \right) + \rho_0 \left(2W_4^r + W_5^r \right) \right] \right. \\
& \left. - \frac{1}{16\pi^2 F_0^2} \left[\left(\chi_0 + \rho_0 \right) \ln \frac{\chi_0 + \rho_0}{\Lambda^2} \right] \right) + \mathcal{O}(a^2)
\end{aligned}
$$

$$
\chi_0 = 2B_0\, m_q\,, \quad \rho_0 = 2W_0\, a \cos\omega
$$

6.3 Nondegenerate quark masses

G.M., T. Sudmann, JHEP 08 (2006) 085. hep-lat/0603019

a) $N_f = 2, \qquad m_u \neq m_d$

Mass term $\quad M = (m_d + m_u)\mathbf{1} + (m_d - m_u)\tau_1$

$$M(\omega) = \mathrm{e}^{\mathrm{i}\,\omega\gamma_5\tau_3/2} M\,\mathrm{e}^{\mathrm{i}\,\omega\gamma_5\tau_3/2}$$

b) $N_f = 3, \qquad m_u = m_d \neq m_s$

$m_\pi^2,\ m_K^2,\ m_\eta^2,\ F_\pi,\ F_K,\ F_\eta$ in chiral perturbation theory

7. Monte Carlo calculations

European Twisted Mass Collaboration

Quenched and unquenched calculations

Phase structure, spectrum, scaling, fits to chiral perturbation theory

Algorithms

improved HMC (Urbach, Jansen, Shindler, Wenger)

PHMC with stochastic correction (Montvay, Scholz)

twisted mass removes small eigenvalues from the fermion matrix

algorithmically favourable: safer and faster

Simulations with $N_f = 2$

improved gauge actions

$(1 - 8b_1)$ (plaquette) $+ b_1$ (double plaquette)

DBW2: $b_1 = -1.4088$

tlSym = tree level Symanzik improved: $b_1 = -\frac{1}{12}$

decrease minimal pion mass $\leftrightarrow$ μ_c

DBW2 action

F. Farchioni et al., Eur. Phys. J. C 47 (2006) 453, hep-lat/0512017

$12^3 \cdot 24$, $a \approx 0.19$ fm, $16^3 \cdot 32$, $a \approx 0.12$ fm,

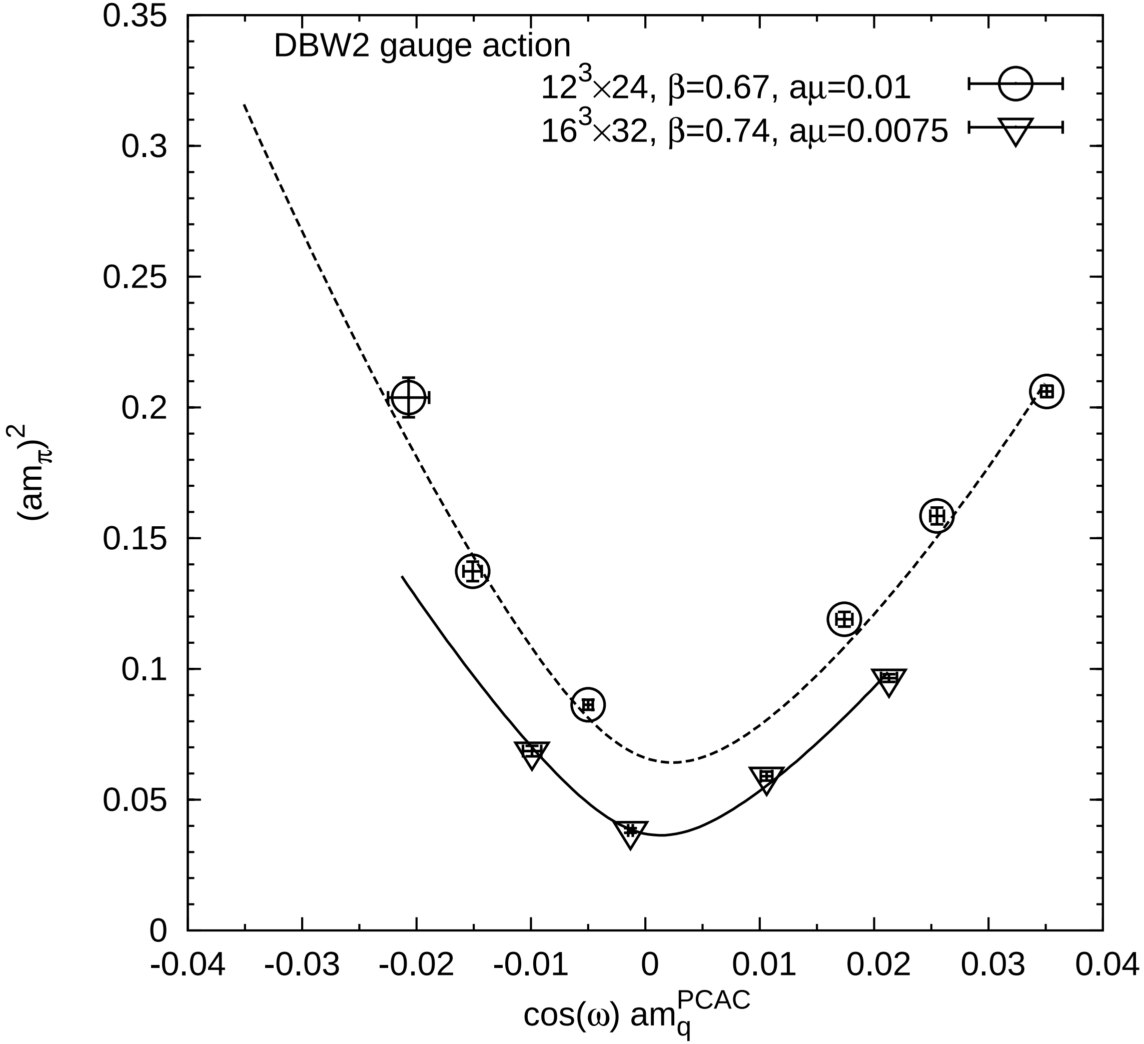
$\mu \approx 11$ MeV

minimal $m_\pi = 320$ MeV

determine m_c: $m_{\mathrm{PCAC}} = 0$

observables: m_π, F_π, G_π, m_{PCAC}, pion mass splitting, ...

Fits to chiral perturbation theory, e.g.:



Estimates of low-energy constants:

$$70\,\mathrm{MeV} \le F_0 \le 85\,\mathrm{MeV}$$

$$2.9\,\mathrm{GeV} \le B \le 3.5\,\mathrm{GeV}$$

$$4.0 \le \Lambda_3/F_0 \le 8.0$$

$$16.0 \le \Lambda_4/F_0 \le 19.0$$

phenomenological values (S. Dürr):

$$F_0 \approx 86\,\mathrm{MeV}$$

$$2.3 \le \Lambda_3/F_0 \le 23.3$$

$$9.3 \le \Lambda_4/F_0 \le 22.1$$

$$\begin{aligned}
\Lambda_3 &= 4\pi F_0 \exp(128\pi^2(2L_4 + L_5 - 4L_6 - 2L_8)) \\
\Lambda_4 &= 4\pi F_0 \exp(32\pi^2(2L_4 + L_5))
\end{aligned}$$

Projects

$N_f = 2$, tlSym gauge action

$24^3 \cdot 48$, $32^3 \cdot 64$

m_π down to 280 MeV

$N_f = 2 + 1 + 1$