Effective string excitation energies

Peter Weisz

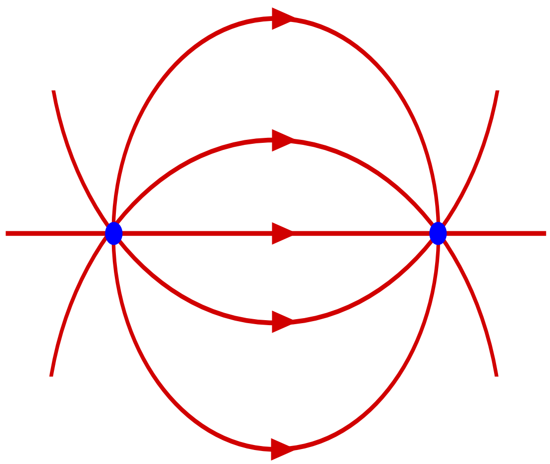
Max-Planck-Institute, Munich

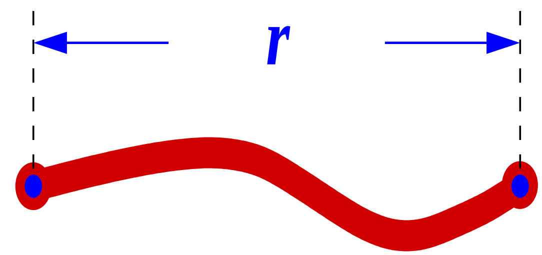
- Effective string model
- Polyakov loop correlation function
- Old results
- "New" results; Martin Lüscher, P.W,JHEP 0407 (2004) 014

Univ. Münster, 25.06.2007

Pure SU(N) gauge theories

Flux distribution in the presence of static color sources





$r < 0.1$ fm | $r >> 1$ fm

perturbative regime | static confinement

In presence of dynamical quarks "the string breaks" due to pair production (not considered in this talk)

static potential $V(r)$: minimum energy of a static $q-\bar{q}$ pair separated by a distance r

$V(r) = -\lim_{t\to\infty}\frac{1}{t}\ln w(\mathcal{C}_{rt})\,,\quad$ $\mathcal{C}_{rt}$: planar $r\times t$ loop

Wilson loop: $\quad w(\mathcal{C}) = \langle \mathrm{tr} P \exp\int_{\mathcal{C}} \mathrm{d}x_\mu A_\mu(x)\rangle$

Pert. Thy: $w(\mathcal{C})$ is multiplicatively renormalizable (Brandt, Ng):

$$w_{\mathrm{ren}}(\mathcal{C}) = \lim_{\Lambda\to\infty} \mathrm{e}^{-P(\mathcal{C})\Lambda F(g)} w(\mathcal{C})$$

$P(\mathcal{C})$: perimeter of (smooth) $\mathcal{C}$, Λ: UV cutoff

force: $F(r) = \frac{\mathrm{d}V}{\mathrm{d}r} = C_{\mathrm{F}}\frac{\alpha_{q\bar{q}}(r)}{r^2}$ is RG invariant

running coupling: $\alpha_{q\bar{q}}(r) \sim \frac{c}{-\ln r}$ for $r\to 0\,,$ (Asympt. Freedom)

For large r need non-perturbative methods:

- $\star$ Lattice gauge theory (LGT)
- $\star$ Loop wave equations

LGT with positive transfer matrix $\Rightarrow$

$V(\mathbf{r})$ along the axis $\mathbf{r} = (r, 0, 0)$ (in infinite volume) is monotonic and concave (Seiler, Bachas 1986)

$\Rightarrow$ Force between opposite charges is always attractive, and at large r, $V(\mathbf{r})$ cannot rise faster than linear.

strong coupling coupling: $V(r) \sim \sigma r$, (Wilson)

$\Rightarrow w(\mathcal{C}) \sim \exp(-\sigma \mathcal{A}(\mathcal{C}))$ for large minimal area $\mathcal{A}(\mathcal{C})$

σ: string tension:

String wave equations (Polyakov, Migdal, Nambu):

$$\frac{\delta^2 w(\mathcal{C})}{\delta x_\mu(\sigma)\delta x_\mu(\sigma)} = \langle \mathrm{tr}\left[(x'_\rho F_{\mu\rho})^2(\sigma) U_{\mathcal{C}}(x(\sigma), x(\sigma))\right]\rangle + \ldots$$

assume finite thickness $\rightarrow$ $(x'_\rho F_{\mu\rho})^2 \sim$ electric flux through the tube independent of posn. along the loop

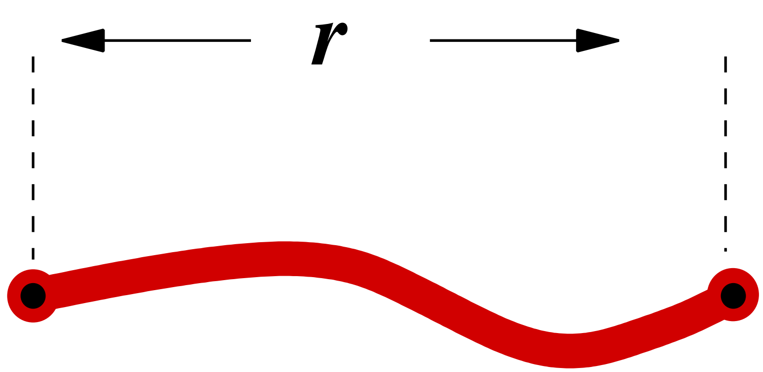
$\Rightarrow$ **free loop equation (for large smooth $\mathcal{C}$)**

$$\left[-\frac{\delta^2}{\delta x(\sigma)^2} + M^4 x'(\sigma)^2\right] w(\mathcal{C}) = 0$$

wave functions: $w(\mathcal{C}) \sim M^p \mathrm{e}^{-M^2 \mathcal{A}(\mathcal{C})} \sum_{\nu=0}^{\infty} M^{-2\nu} w_\nu(\mathcal{C})$

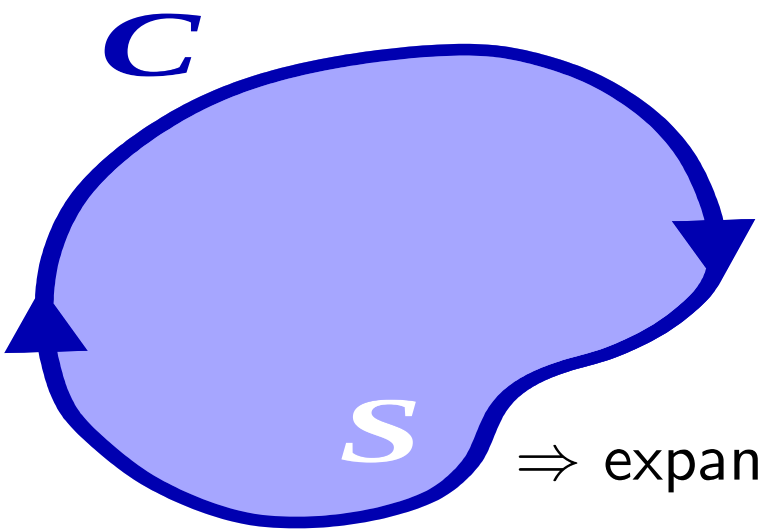
which are *reparametrization invariant* $x'(\sigma)\frac{\delta}{\delta x(\sigma)} w(\mathcal{C}) = 0$, and satisfy a (complicated) local regularized loop equation (Lüscher, Symanzik, P.W. (1980))

$\rightarrow$ hope Nambu-Goto string theory can be quantized without violating “fundamental principles” (Pohlmeyer) tube formed by fluctuating string of thickness $\frac{1}{m_{\mathrm{Glueball}}}$



$r >> 1$ fm

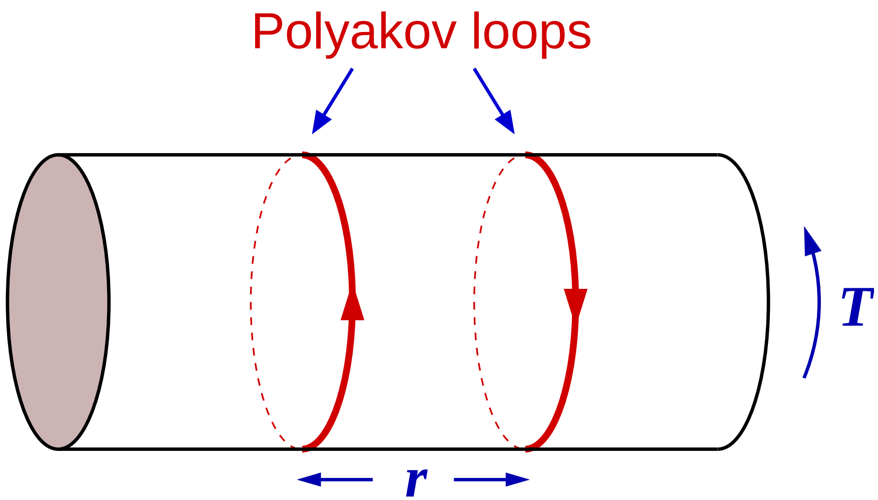
Long distance (low energy) effective string theory



$w(\mathcal{C}) \simeq \int_{\mathrm{surfaces}} \mathrm{e}^{-\sigma A(S)}$ for $\mathcal{C}$ large

$\Rightarrow$ expansion in powers of $\sigma^{-1/2}$ about $S = S_{\mathrm{min}}$

Polyakov loop correlation function



In gauge theories with compact gauge group,

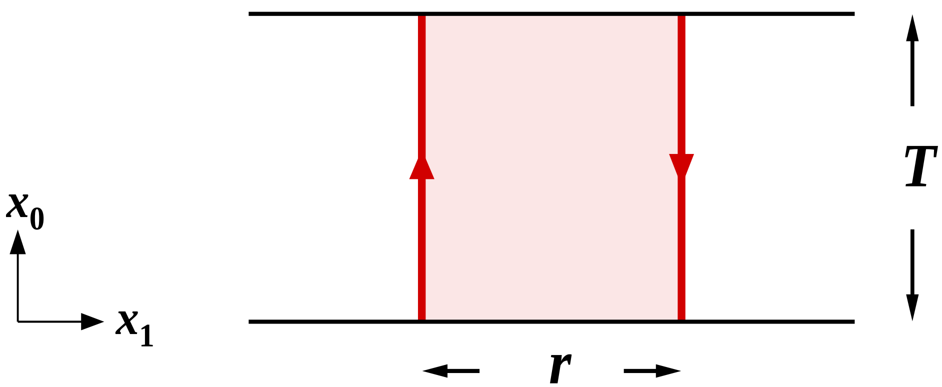
$$\langle P(r, x_\perp)^* P(0, x_\perp) \rangle = \sum_{n=0}^{\infty} w_n \mathrm{e}^{-E_n(r)T}$$

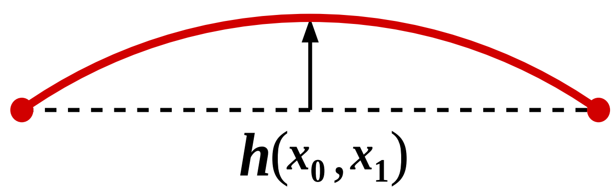
static quark potential: $E_0(r) \equiv V(r)$, $w_0 = 1$

excited states: $E_n(r), n \geq 1$, $w_n \in \mathbb{N}$

in the effective string theory:

$$\langle P(r)^* P(0)\rangle \approx \mathcal{P}(r,T) = \mathrm{e}^{-\sigma r T - \mu T} \times \int_{\text{fluctuations } h} \mathrm{e}^{-S_{\text{eff}}}$$





$h_i\,,\quad i = 1, 2, \ldots, d-2$

d: space-time dimension

effective action has an expansion:

$$S_{\rm eff} = S_0 + S_1 + S_2 + \dots$$

where S_v has couplings of dimension $[\text{length}]^v$

Free string approximation: $S_{\rm eff} = S_0 + S_1$

$$S_0 = \frac{1}{2}\int_0^T \int_0^r \mathrm{d}x_0 \mathrm{d}x_1 \, (\partial_a h)^2$$

$$S_1 = \frac{1}{4} b \int_0^T \mathrm{d}x_0 \left\{ (\partial_1 h \partial_1 h)_{x_1=0} + (\partial_1 h \partial_1 h)_{x_1=r} \right\}$$

$\Rightarrow$ correlation function of required form:

$$\mathcal{P}(r,T)|_{b=0} = \mathrm{e}^{-\sigma r T - \mu T} \left[\det(-\triangle)\right]^{-\frac{1}{2}(d-2)}$$

$$\left[\det(-\triangle)\right]^{\frac{1}{2}} = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1-q^n)\,, \quad q = \mathrm{e}^{-\pi T/r}$$

energy spectrum:

$$E_0 = \sigma r + \mu - \frac{\pi}{24r}(d-2)\left(1 + b/r + \ldots\right)$$

$$\triangle E_n = E_n - E_0 = \frac{n\pi}{r}\left(1 + b/r + \ldots\right)$$

degeneracies: $w_0 = 1,\ w_1 = d-2,\ \ldots$

leading terms are universal! Lüscher '81, Lüscher & P.W. '02

another prediction of the effective theory concerns the **“tube width”**
Lüscher, Münster, P.W. (1981)

$$\lambda^2(r) \equiv \frac{\int d^2x_\perp x_\perp^2 \mathcal{E}(x)}{\int d^2x_\perp \mathcal{E}(x)} \sim \ln(r)\,, \quad \text{for } r \to \infty$$

$\mathcal{E}(x)$: chromo-electric field energy density distribution

$\mathcal{E}(x) \propto \langle q\overline{q}|\mathrm{tr}\mathbf{E}^2(x)|q\overline{q}\rangle - \langle q\overline{q}|q\overline{q}\rangle\langle\Omega|\mathrm{tr}\mathbf{E}^2(x)|\Omega\rangle$

in strong coupling limit the string is rigid ($\lambda(\infty)$ finite),
expect “roughening transition” as g_0 decreases

But Bokko, Gubarev & Morozov (arXiv:0704.1203)
claim $\lambda(r) \to 0$ as $a \to 0$!!

Naive model: doesn't take decay of higher excitations
$\rightarrow$ lower states + glueballs into account

But is it basically correct? If so . . .

- Exactly which string theory?
  - ◇ Alternative string actions ("rigid" string, etc.)
    Polyakov '86, Savvidy & Savvidy '93
  - ◇ String theories with fermionic modes
    Ramond '71, Neveu & Schwartz '71
- At which distances does string behavior set in?

$\Rightarrow$ **lattice gauge theory**

Studying string behavior in LGT

⋆ Ground state energy

$$V'(r) = \sigma + \mathrm{O}(r^{-2})$$

$$V''(r) = \frac{\pi}{12r^3}(d-2) + \mathrm{O}(r^{-4})$$

Lucini & Teper '01 ,Necco & Sommer '02

⋆ Low-lying excited states

$$\Delta E = \frac{\pi}{r} + \mathrm{O}(r^{-2})$$

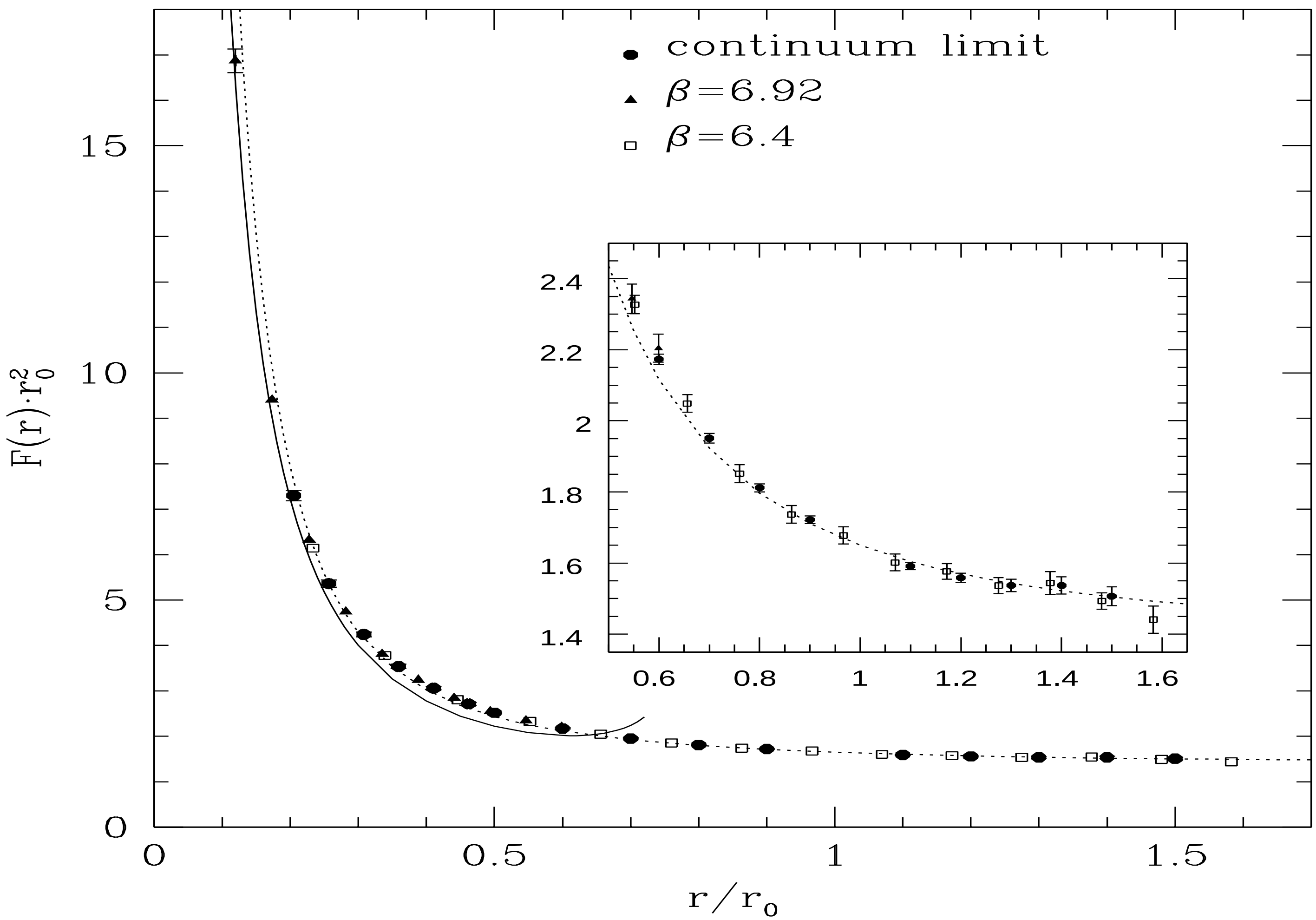
Michael & Perantonis '90 ;Juge, Kuti & Morningstar '98ff, Majumdar '03ff

⋆ Wilson loop expectation values

Caselle et al. '97ff ,Caselle, Hasenbusch & Panero '02ff

Force between static quark sources in pure SU(3) Yang-Mills

Necco, Sommer (2001)



* Sommer's scale: $r_0^2 F(r_0) = 1.65\,, \quad \Rightarrow r_0 \sim 0.5\text{fm}$
* full line: PT with $\Lambda_{\overline{\mathrm{MS}}} r_0 = 0.602$
* dashed line: bosonic string model: $F(r) = \sigma + \frac{\pi}{12 r^2}$

The principal difficulties measuring such correlation fns. accurately:

!The signal $\langle PP \rangle \propto \mathrm{e}^{-\sigma r T}$
decreases exponentially ($\sim 10^{-25}$ at $a = 0.1\,\mathrm{fm}$, $rT = 5\,\mathrm{fm}^2$)

!The significance loss in $-\frac{1}{2} r^3 V''(r) = \frac{\pi}{24}(d-2) + \ldots$
grows proportionally to $\sigma r^4 / a^2$

The multilevel algorithm
achieves exponential reduction of the statistical errors!

(M.Lüscher. & P.W '01, P.Majumdar '03)

Systematic errors

- Excited states contributions

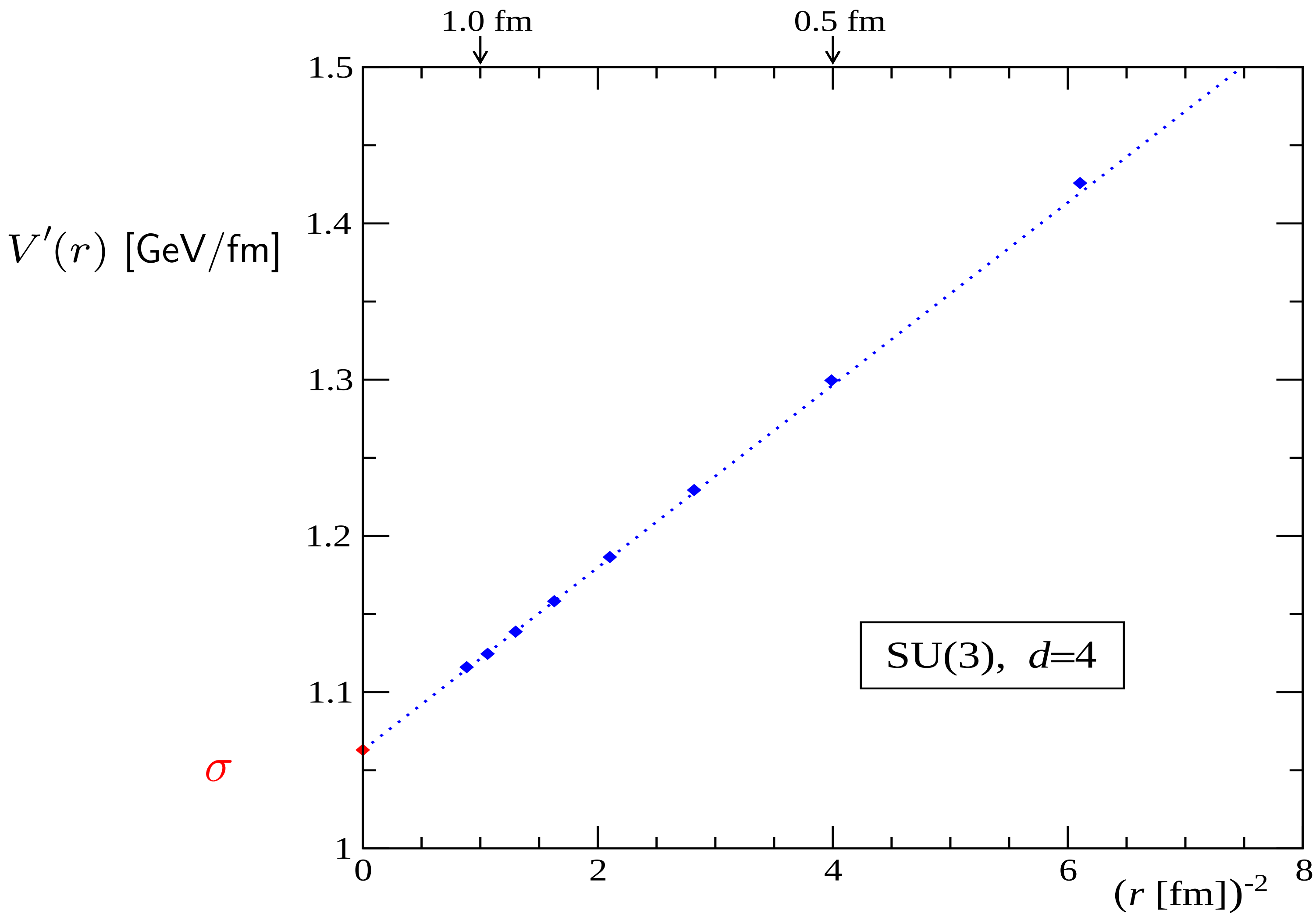
$$V(r) = -\frac{1}{T}\ln\langle P^* P\rangle + \epsilon, \qquad \epsilon \simeq \frac{w_1}{T}\,\mathrm{e}^{-(E_1-E_0)T}$$

should check if negligible

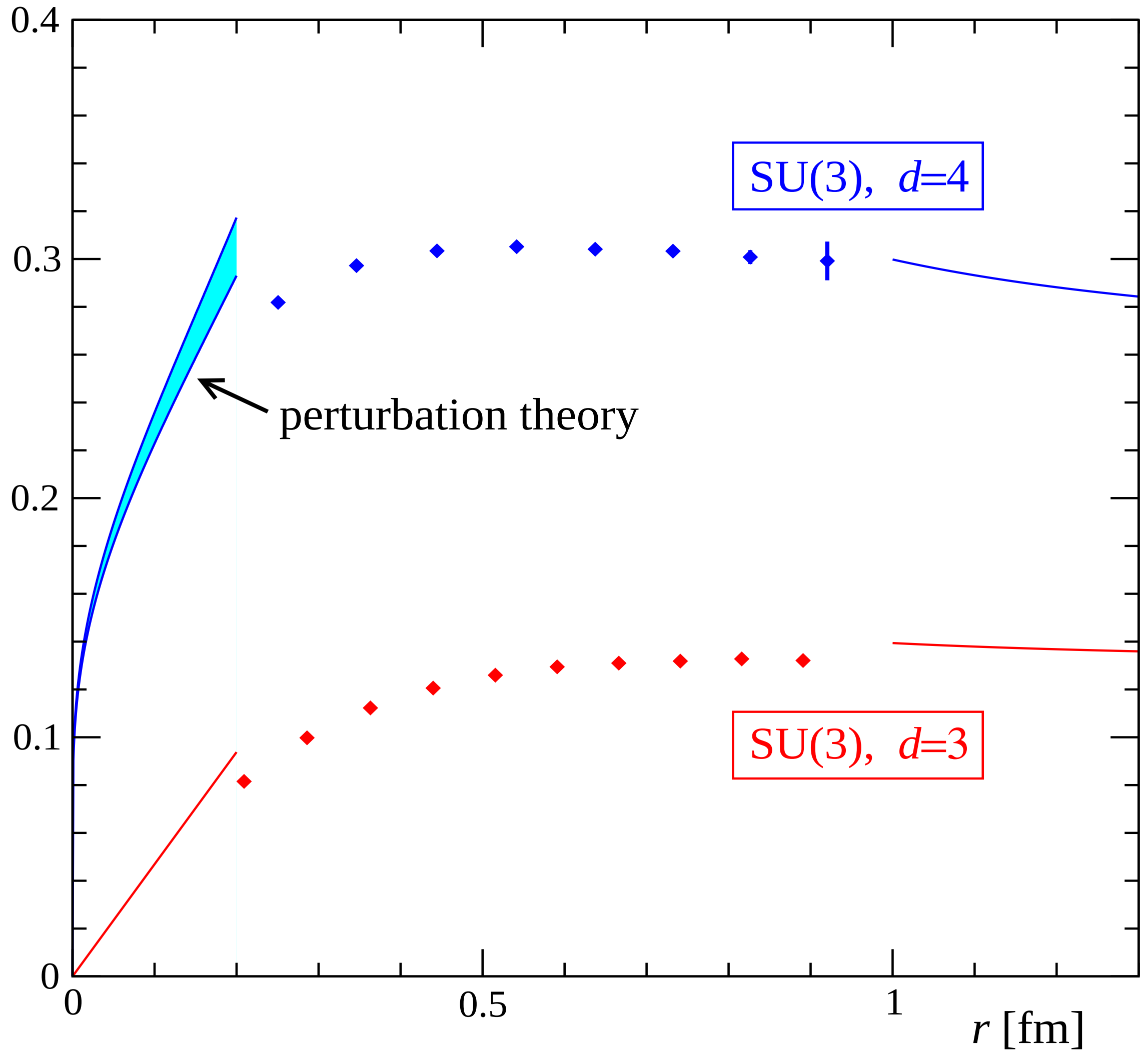
- Lattice spacing effects

Are of order a^2, already small at $a = 0.1$ fm

- Finite-volume effects “around the world”
- Autocorrelations
- Statistical correlations

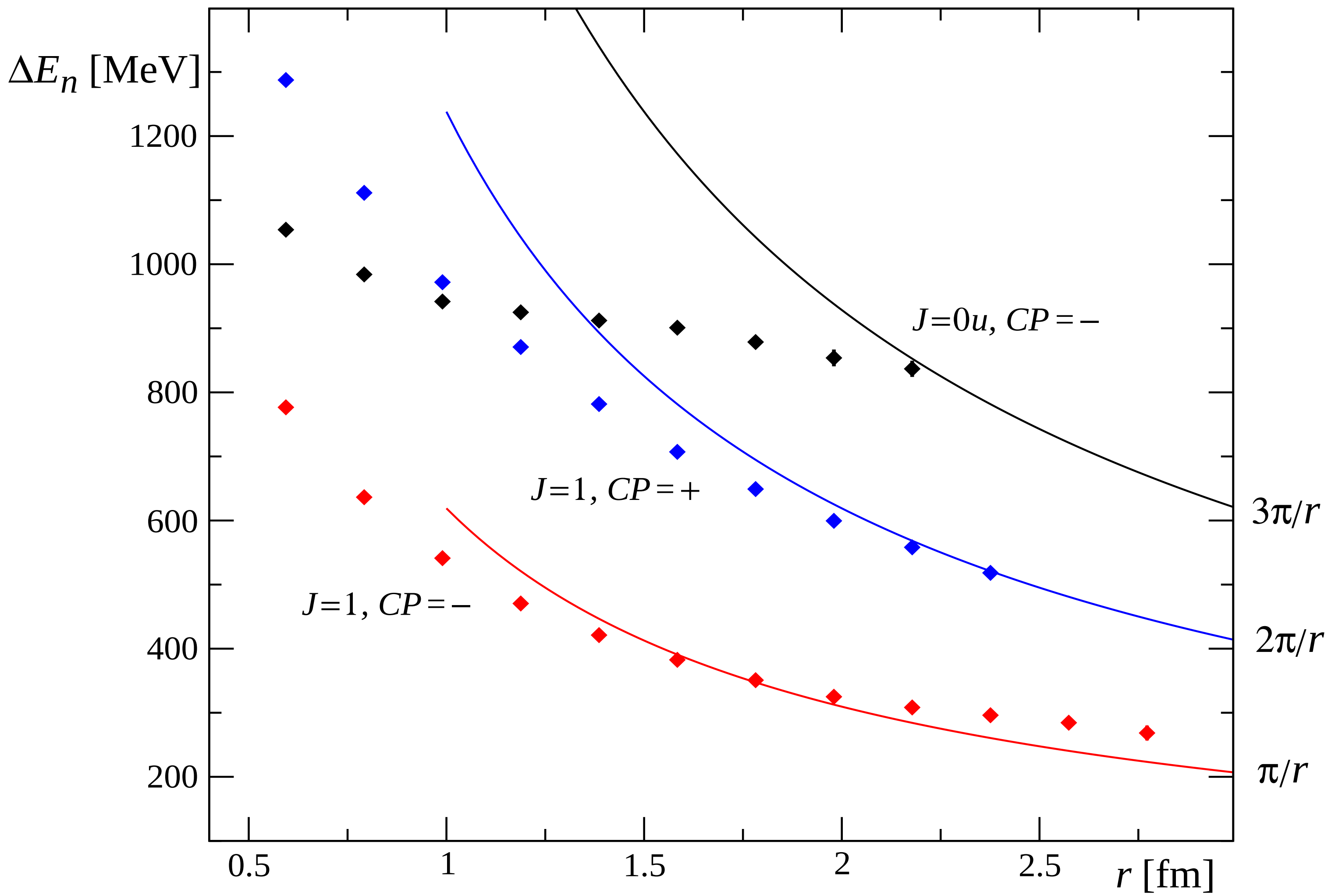

1.0 fm
0.5 fm
1.5
1.4
1.3
1.2
1.1
1
$V'(r)$ [GeV/fm]
σ
SU(3), d=4
0
2
4
6
8
$(r\ [\mathrm{fm}])^{-2}$

$$c(r) \equiv -\tfrac{1}{2}\, r^3 V''(r)$$ (Lüscher. & P.W '02)



$*$ careful continuum limit extrapolation still required!

Excited states [SU(3), $d = 4$] (Juge, Kuti & Morningstar '02)



Precise studies of 3-d gauge theory: SU(2): (Majumdar '03)

$\mathbb{Z}_2$: (Caselle, Fiore, Gliozzi, Hasenbusch, Panero, Pepe, Provero, Rago '97ff)

String self-interaction effects

Classical Nambu–Goto Model:

$$S = \int \mathrm{d}x_0 \mathrm{d}x_1 \sqrt{1 + \partial_a h \cdot \partial_a h + (\partial_0 h \cdot \partial_0 h)(\partial_1 h \cdot \partial_1 h) - (\partial_0 h \cdot \partial_1 h)^2}$$

Arvis '83: Energy levels are given by

$$E_n(r) = \sigma r \sqrt{1 + \frac{2\pi}{\sigma r^2} \left[n - \frac{d-2}{24}\right]}$$

status?? Result for all d but quantum theory claimed consistent only for $d = 26$

see Polchinski & Strominger '91

Effective string interactions with coupling dimension $[\mathrm{length}]^2$:

$$S_2 = \frac{1}{4}\int \mathrm{d}x_0\,\mathrm{d}x_1 \Big\{ c_2\,(\partial_a h \partial_a h)\,(\partial_b h \partial_b h) + c_3\,(\partial_a h \partial_b h)\,(\partial_a h \partial_b h) \Big\}$$

- **contributions to the partition function** (Dietz and Filk '83)

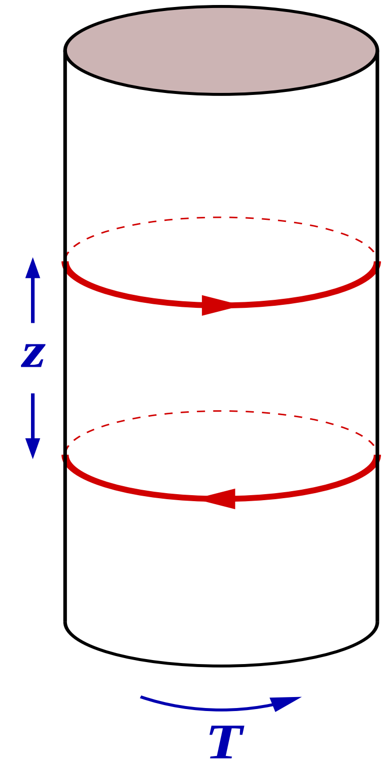
- **effect on energy levels computed for all states $n \leq 3$** (M.Lüscher & P.W '04)

states classified by O$(d-2)$ axial symmetry

e.g. level $n = 3$ has: 1 scalar, 2 vectors, 1 symmetric 2-tensor,
1 antisymmetric 2-tensor, symmetric 3-tensor

- **states for n fixed are degenerate (only) for $c_3 = -2c_2$ for all d** ($\sim$ classical Nambu–Goto case)

“Open-closed string duality”



The Polyakov loop correlation function satisfies

$$\int_{\substack{x_0=0\\x_1=z}} \mathrm{d}^{d-2}x_\perp \, \langle P(x)^* P(0) \rangle = \sum_{n\geq 0} |c_n|^2 \mathrm{e}^{-\widetilde{E}_n z}$$

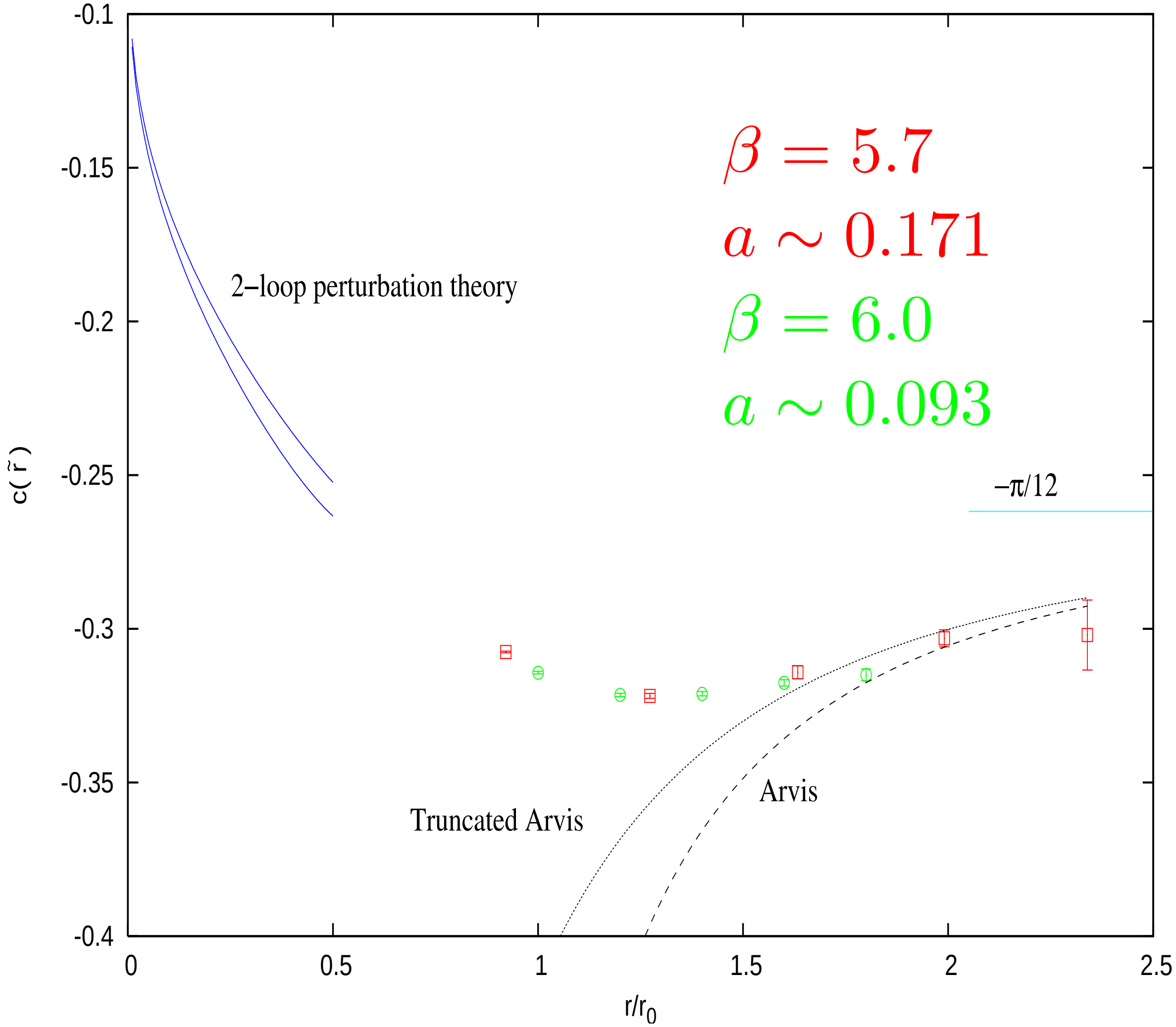
should also be so in the effective theory

$$\Rightarrow \quad b = 0, \qquad (d-2)c_2 + c_3 = (d-4)/(2\sigma)$$

- only one parameter left for $d > 3$; none for $d = 3$ **!**

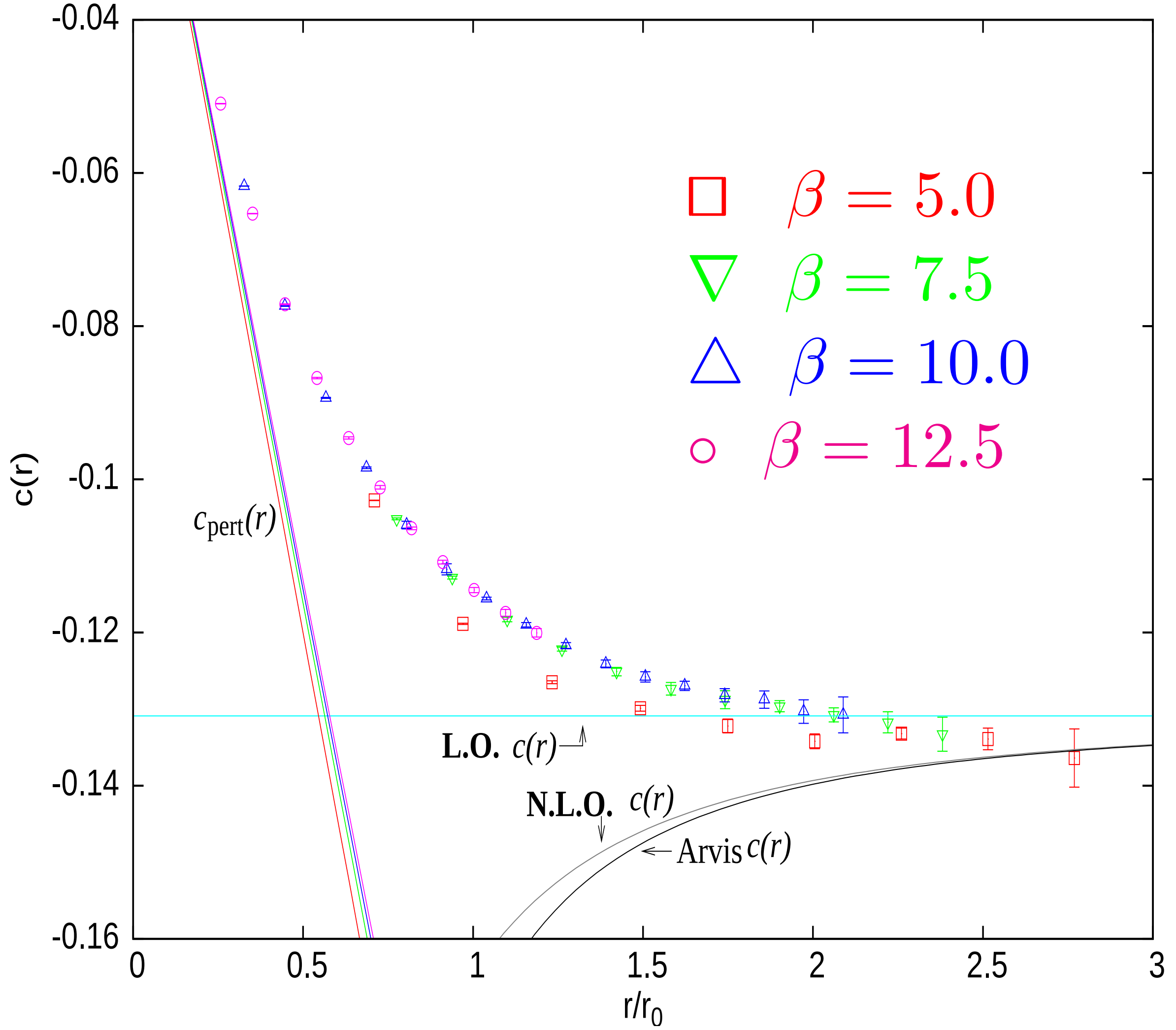
$c(r)$ in 4-d SU(3) Yang-Mills theory

(Majumdar, Hari Dass '06)



$c(r)$ in 3-d SU(2) Yang-Mills theory

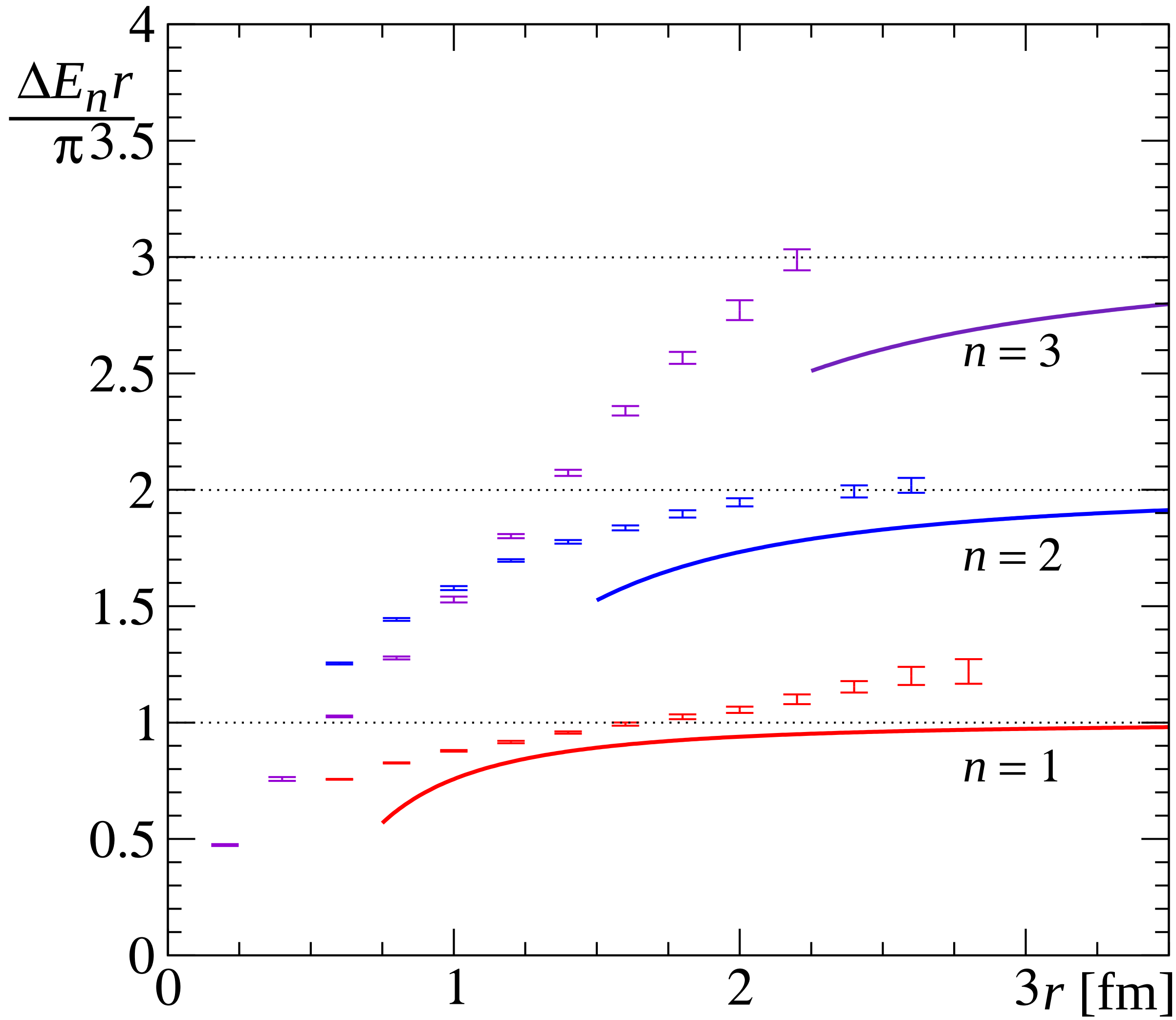
(Majumdar, Hari Dass '07)



case $d = 4$: O-C string duality $\Rightarrow$ $c_3 = -2c_2$

$\Rightarrow$ accidental degeneracies not lifted to this order

curves with $c_2 = (2\sigma)^{-1} \simeq 0.093\text{fm}^2$



Concluding remarks:

small coefficients of $1/r^3$ term in V; e.g. for $d = 4$

$$V = \sigma r + \mu - \frac{\pi}{12r} - \frac{\pi^2 c_2}{144 r^3} + \mathrm{O}(1/r^4)\,,$$

$$\triangle E_1 = \frac{\pi}{r} - \frac{5\pi^2 c_2}{6r^3} + \mathrm{O}(1/r^4)\,,$$

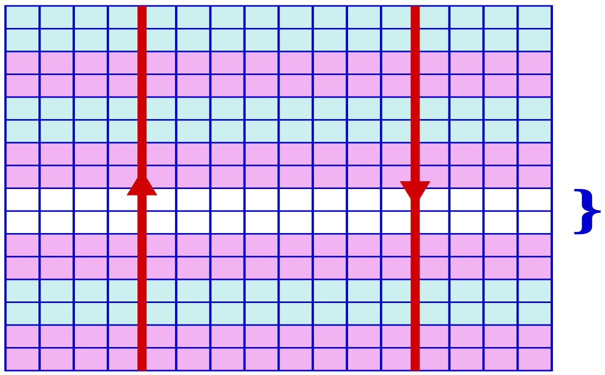
$\sim$ string description for V sets in at relatively short distances where low-lying levels not string-like?

• whether low-lying energy values in gauge theory matched by an effective string theory not yet confirmed.
For detailed comparison need $\mathrm{O}(1/r^3)$ terms and dedicated lattice simulations taking account of systematic error sources

• does the degeneracy for $c_3 = -2c_2$ indicate a (hidden) symmetry?

• Drummond and Haria Dass & Matlock claim that imposing Poincare invariance on the spectrum obtained from the Polchinski-Strominger action fixes the $\mathrm{O}(r^{-3})$ terms (??)

Multilevel algorithm



First average $U^* \otimes U$ here for fixed b.c. and then take product

$$\langle P(r)^* P(0) \rangle = \langle \mathrm{tr}\{[U^* \otimes U]\,[U^* \otimes U] \ldots [U^* \otimes U]\} \rangle$$

$$\uparrow$$

$$\sim \mathrm{e}^{-2\sigma r a}$$

$\Rightarrow$ exponential reduction of the statistical errors!

(M.Lüscher. & P.W '01, P.Majumdar '03)

$$\int_{\substack{x_0=0\\x_1=z}} \mathrm{d}^{d-2}x_\perp \, \langle P(x)^* P(0) \rangle = \sum_{n\geq 0} |c_n|^2 \mathrm{e}^{-\widetilde{E}_n z}$$

the lhs is a **Radon transformation**; inverting yields

$$\langle P(x)^* P(0) \rangle = \sum_{n\geq 0} |c_n|^2 2r \left(\frac{\widetilde{E}_n}{2\pi r}\right)^{-\frac{1}{2}(d-1)} K_{\frac{1}{2}(d-3)}(\widetilde{E}_n r) \, ,$$

$$r = \sqrt{x^2}$$