Appendix E
Functional Derivative

A functional is a mapping of a function $\psi(\mathbf{x})$ onto a number denoted by $V[\psi(\mathbf{x})]$. This number changes by a variation of the function $\psi(\mathbf{x})$. The functional derivative is defined by the relation

$$V[\psi(\mathbf{x}+\delta\psi(\mathbf{x}]-V[\psi(\mathbf{x}]=\int d\mathbf{x}\,\delta\psi(\mathbf{x})\frac{\delta V}{\delta\psi(\mathbf{x})}\,. \tag{E.1}$$

Let us calculate the functional derivative of the Ljapunov function (**??**)

$$\begin{aligned}
&V[\psi(\mathbf{x},t)+\delta\psi(\mathbf{x},t)]=\int d\mathbf{x}\{\frac{1}{2}\left[(k_c^2+\Delta)\,(\psi(\mathbf{x},t)+\delta\psi(\mathbf{x},t))\right]^2\\
&-\frac{\varepsilon}{2}\,(\psi(\mathbf{x},t)+\delta\psi(\mathbf{x},t))^2+\frac{1}{4}\,(\psi(\mathbf{x},t)+\delta\psi(\mathbf{x},t))^4\\
&-\frac{\delta}{3}\,(\psi(\mathbf{x},t)+\delta\psi(\mathbf{x},t))^3\}\\
&=V[\psi(\mathbf{x},t)]+\int d\mathbf{x}\{\big((k_c^2+\Delta)\psi(\mathbf{x},t)\big)\,\big((k_c^2+\Delta)\delta\psi(\mathbf{x},t)\big)\\
&-\varepsilon\psi(\mathbf{x},t)\delta\psi(\mathbf{x},t)+\psi(\mathbf{x},t)^3\delta\psi(\mathbf{x},t)-\delta\psi(\mathbf{x},t)^2\delta\psi(\mathbf{x},t)\}\,.
\end{aligned} \tag{E.2}$$

In the first integral, we perform a partial integration. This yields

$$\begin{aligned}
&V[\psi(\mathbf{x},t)+\delta\psi(\mathbf{x},t)]-V[\psi(\mathbf{x},t)]=\\
&\int d\mathbf{x}\delta\psi(\mathbf{x},t)\{(k_c^2+\Delta)^2\psi(\mathbf{x},t)-\varepsilon\psi(\mathbf{x},t)+\psi(\mathbf{x},t)^3-\delta\psi(\mathbf{x},t)^2\}\\
&=\int d\mathbf{x}\delta\psi(\mathbf{x},t)\frac{\delta V}{\delta\psi(\mathbf{x},t)}
\end{aligned} \tag{E.3}$$

and we can read off the functional derivative.