Statistics of 3d compressible turbulence

C. Schwarz, C. Beetz,
J. Dreher, R. Grauer

Workshop on Ameland
25.08.2007 - 31.08.2007

Contents

Lagrangian Statistics of Compressible Turbulence

- Interstellar Matter
  - low density

Lagrangian Statistics of Compressible Turbulence

- Interstellar Matter
  - low density
  - high Mach number

Average gas density $\approx 1\text{atom cm}^{-3} \rightsquigarrow$ limit of high Reynoldsnumber $Re \approx 10^8$

Lagrangian Statistics of Compressible Turbulence

- Interstellar Matter
  - low density
  - high Mach number
  - isothermal

rms Mach number of order 20

Lagrangian Statistics of Compressible Turbulence

- Interstellar Matter
  - low density
  - high Mach number
  - isothermal
  - turbulent motion

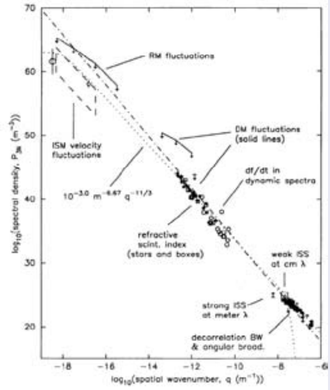
thermal equilibrium time is shorter than hydrodynamical time scale

Lagrangian Statistics of Compressible Turbulence

- Interstellar Matter
    - low density
    - high Mach number
    - isothermal
    - turbulent motion
    - star formation

Amstrong et. al. 1995

scaling behaviour



Lagrangian Statistics of Compressible Turbulence

- Interstellar Matter
  - low density
  - high Mach number
  - isothermal
  - turbulent motion
  - star formation
  - magnetic fields

Lagrangian viewpoint

Lagrangian Statistics of Compressible Turbulence

- Interstellar Matter
  - low density
  - high Mach number
  - isothermal
  - turbulent motion
  - star formation
  - magnetic fields

- important for star formation process
- even weak magnetic fields modify supersonic turbulent flows

- isothermal Euler equations
- isothermal ideal MHD equations

- isothermal Euler equations
- isothermal ideal MHD equations

- Euler equations in primitive variables
  - density ρ
  - flow velocity $\mathbf{v}$

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\mathbf{f}$$

$$p \propto \rho$$

  - $\mathbf{f}$ external force

- Euler equations in primitive variables

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\mathbf{f}$$

- Euler equations in conservative variables
  - density ρ
  - flow momentum $\mathbf{u} = \rho \mathbf{v}$

$$\partial_t \rho + \nabla \cdot \mathbf{u} = 0$$

$$\partial_t \mathbf{u} + \nabla \cdot (\frac{\mathbf{u}\mathbf{u}}{\rho}) = -\nabla p + \mathbf{f}$$

- Euler equations in primitive variables

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$
$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\mathbf{f}$$

- Euler equations in conservative variables

$$\partial_t \rho + \nabla \cdot \mathbf{u} = 0$$
$$\partial_t \mathbf{u} + \nabla \cdot (\frac{\mathbf{u}\mathbf{u}}{\rho}) = -\nabla p + \mathbf{f}$$

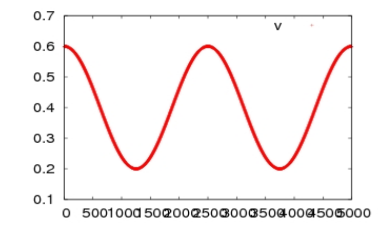
- Euler equations in $\ln(\rho)$

$$\partial_t(\ln \rho) + \mathbf{v} \cdot \nabla(\ln \rho) + \nabla \cdot \mathbf{v} = 0$$
$$\partial_t \mathbf{v} + \nabla \cdot (\mathbf{v}\mathbf{v}) + a\nabla(\ln \rho) = \mathbf{k}$$

Same Type of Differential Equation

$$\partial_t v + \partial_x \frac{1}{2} v^2 = 0$$

Simulation with finite differences



Theorem

Existence and uniqueness of solutions for quasilinear partial differential equations of first order are ensured only in a certain neighbourhood of the initial manifold.

$\rightsquigarrow$ Weak Solutions for 'real world'-problems

Weak Solution

Multiply equation by arbitrary test-function with compact support

$$\Phi(x,t) \in C_0^1(\Omega) \quad ,\Omega = \{(x,t)|x \in \mathcal{R}, t \geq 0\}$$

and integrate by parts over Ω:

$$\partial_t v + \partial_x f(v) = 0$$

Weak Solution

Multiply equation by arbitrary test-function with compact support

$$\Phi(x,t) \in C_0^1(\Omega) \quad ,\Omega = \{(x,t)|x \in \mathcal{R}, t \geq 0\}$$

and integrate by parts over Ω:

$$\int_\Omega \Phi\partial_t v + \Phi\partial_x f(v) d(x,t) = 0$$

Weak Solution

Multiply equation by arbitrary test-function with compact support

$$\Phi(x,t) \in C_0^1(\Omega) \quad ,\Omega = \{(x,t) | x \in \mathcal{R}, t \geq 0\}$$

and integrate by parts over Ω:

$$\int_\Omega v\partial_t \Phi + f(v)\partial_x \Phi d(x,t) + \int_{-\infty}^{+\infty} v_0(x)\Phi(x,0)dx = 0$$

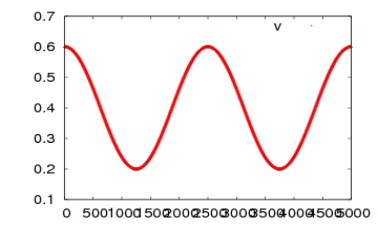
Weak Solution

$$\int_\Omega v\partial_t\Phi + f(v)\partial_x\Phi d(x,t) + \int_{-\infty}^{+\infty} v_0(x)\Phi(x,0)dx = 0$$

every v which fulfill this equation is called a weak solution of $\partial_t v + \partial_x f(v) = 0$ and need not necessarily be smooth.

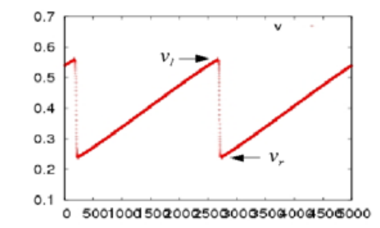
Disadvantage: loss of uniqueness.

Simulation



Loss of Uniqueness (example)

$$\partial_t v + \partial_x f(v) = 0$$



Velocity $\tilde{v}$ of shock can be determined analytically

Rankine-Hugoniot condition:

$$(v_l - v_r)\tilde{v} = f(v_l) - f(v_r)$$

Loss of Uniqueness (example)

Rankine-Hugoniot condition for Burgers shock

$$\partial_t v + \partial_x \frac{1}{2} v^2 = 0$$

Loss of Uniqueness (example)

Rankine-Hugoniot condition for Burgers shock

$$\partial_t v + \partial_x \frac{1}{2} v^2 = 0$$

$$\Rightarrow \tilde{v} = \frac{1}{2}(v_l + v_r)$$

Loss of Uniqueness (example)

Rankine-Hugoniot condition for Burgers shock

$$\partial_t v + \partial_x \frac{1}{2} v^2 = 0$$

$$\Rightarrow \tilde{v} = \frac{1}{2}(v_l + v_r)$$

$$\partial_t v + \partial_x \frac{1}{2} v^2 = 0 \quad | \cdot v^2$$

$$\partial_t \frac{1}{3} v^3 + \frac{1}{4} \partial_x v^4 = 0 \quad | u = v^3$$

$$\partial_t u + \partial_x \frac{3}{4} u^{4/3} = 0$$

Loss of Uniqueness (example)

Rankine-Hugoniot condition for Burgers shock

$$\partial_t v + \partial_x \frac{1}{2} v^2 = 0 \qquad \partial_t u + \partial_x \frac{3}{4} u^{4/3} = 0$$

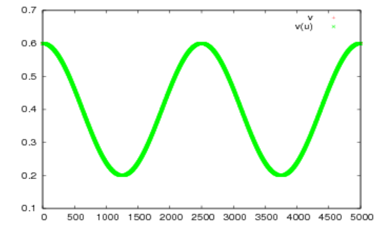
$$\Rightarrow \tilde{v} = \frac{1}{2}(v_l + v_r) \qquad \Rightarrow \tilde{v} = \frac{3}{4} \frac{v_l^4 - v_r^4}{v_l^3 - v_r^3}$$

Loss of Uniqueness (example)

Rankine-Hugoniot condition for Burgers shock

$$\Rightarrow \tilde{v} = \frac{1}{2}(v_l - v_r) \qquad \Rightarrow \tilde{v} = \frac{3}{4}\frac{v_l^4 - v_r^4}{v_l^3 - v_r^3}$$

Simulation



Loss of Uniqueness

Physics must tell right solution

- propper derivation of Fluid-equations from Vlasov-equation (including collision-term)
- additional entropy condition $\partial_t s + \partial_x (vs) \geq 0$
- limit of vanishing dissipation $\mu \partial_{xx} v \longrightarrow 0$

$\rightsquigarrow$ right Euler equations in conservative formulation

$$\partial_t \rho + \nabla \cdot \mathbf{u} = 0$$

$$\partial_t \mathbf{u} + \nabla \cdot \left(\frac{\mathbf{u}\mathbf{u}}{\rho}\right) = -\nabla p + \mathbf{f}$$

$$p \propto \rho$$

Why a central scheme

- no (approximate) Rieman solver needed
- dimension by dimension approach
- high order
- monotone, WENO, TVD reconstruction possible
- easy for complex problems

Kurganov Levy (2001)

First, consider a 1D conservation law

$$\partial_t v(x,t) + \partial_x f(v(x,t)) = 0$$

discretization using cell averages

$$v_j^n = v(j\Delta x, n\Delta t)$$

$$\bar{v}_j^n \equiv \frac{1}{\Delta x}\int_{x_{j-1/2}}^{x_{j+1/2}} v(x,t^n)dx$$

Kurganov Levy (2001)

fully discrete scheme

$$\bar{v}_j^{n+1} = \bar{v}_j^n - \frac{1}{\Delta x} \int_{t^n}^{t^{n+1}} \left[f(v(x_{j+1/2}, \tau)) - f(v(x_{j-1/2})) \right] d\tau$$

Kurganov Levy (2001)

fully discrete scheme

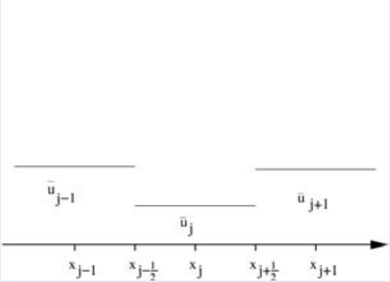
$$\bar{v}_j^{n+1} = \bar{v}_j^n - \frac{1}{\Delta x} \int_{t^n}^{t^{n+1}} \left[f(v(x_{j+1/2}, \tau)) - f(v(x_{j-1/2})) \right] d\tau$$

needed: reconstruction of $v(x)$ from cell averages

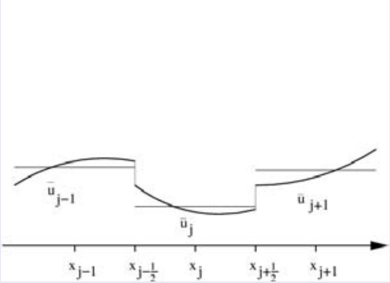
$$v(x, t^n) \approx \tilde{v}(x, t^n) = \sum_j P_j(x) \chi_{[x_{j-1/2}, x_{j+1/2}]}$$

use non-oscillating parabolic reconstruction to achive third order scheme (e.g. CWENO).

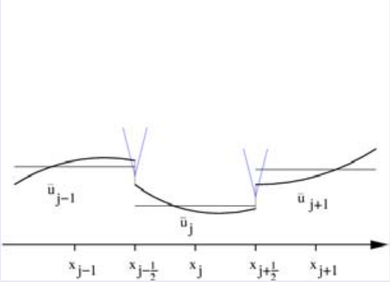
- $\tilde{v}(x, t^n)$ are discontinous at cell boundaries $x_{j+1/2}$.
- upper bound for propagation speed of this discontinuities $a^n_{j+1/2}$
- integrate smooth and non-smooth regions independently
- $\bar{v}_j^{n+1}$ follow from $w^{n+1}_{j(\pm 1/2)}$ by reconstruction or weighted averaging.



- $\tilde{v}(x, t^n)$ are discontinous at cell boundaries $x_{j+1/2}$.
- upper bound for propagation speed of this discontinuities $a^n_{j+1/2}$
- integrate smooth and non-smooth regions independently
- $\bar{v}_j^{n+1}$ follow from $w^{n+1}_{j(\pm 1/2)}$ by reconstruction or weighted averaging.

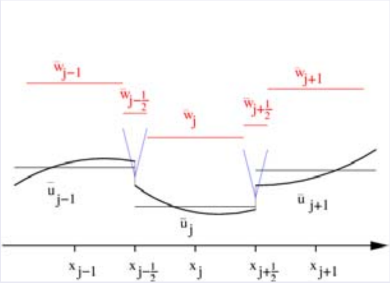


- $\tilde{v}(x, t^n)$ are discontinous at cell boundaries $x_{j+1/2}$.
- upper bound for propagation speed of this discontinuities $a^n_{j+1/2}$
- integrate smooth and non-smooth regions independently
- $\bar{v}_j^{n+1}$ follow from $w^{n+1}_{j(\pm 1/2)}$ by reconstruction or weighted averaging.

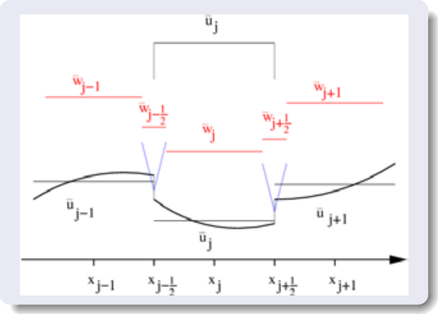


$$= \max_{v \in (v^{n,-}_{j+1/2}, v^{n,+}_{j+1/2})} \text{abs}\left(\frac{\partial f}{\partial v}(v)\right)$$

- $\tilde{v}(x, t^n)$ are discontinous at cell boundaries $x_{j+1/2}$.
- upper bound for propagation speed of this discontinuities $a^n_{j+1/2}$
- integrate smooth and non-smooth regions independently
- $\bar{v}^{n+1}_j$ follow from $w^{n+1}_{j(\pm 1/2)}$ by reconstruction or weighted averaging.



- $\tilde{v}(x, t^n)$ are discontinous at cell boundaries $x_{j+1/2}$.
- upper bound for propagation speed of this discontinuities $a^n_{j+1/2}$
- integrate smooth and non-smooth regions independently
- $\bar{v}_j^{n+1}$ follow from $w^{n+1}_{j(\pm 1/2)}$ by reconstruction or weighted averaging.



consider the limit of $\Delta t \rightarrow 0$ to derive the semi-discrete scheme:

$$\frac{d}{dt}\bar{v}_j(t) = \lim_{\Delta t \rightarrow 0} \frac{v_j^{n+1} - v_j^n}{\Delta t}$$

Result

$$\begin{aligned}\frac{d\bar{v}_j}{dt} = &-\frac{1}{2\Delta x}\Big[f(v^+_{j+1/2}(t)) + f(v^-_{j+1/2}(t)) \\ &\qquad - f(v^+_{j-1/2}(t)) + f(v^-_{j-1/2}(t))\Big] \\ &+\frac{a_{j+1/2}(t)}{2\Delta x}\Big[v^+_{j+1/2}(t) + v^-_{j+1/2}(t) \\ &\qquad - v^+_{j-1/2}(t) + v^-_{j-1/2}(t)\Big]\end{aligned}$$

central Weighted ENO

reconstruct quadratic polynomial approximation to real solution from cell averages

$$P_j(x) = w_L P_L(x) + w_R P_R(x) + w_C P_C(x)$$

with positive weights $w_i > 0$ and $\sum_i w_i = 1$, where $i \in L, R, C$.

central Weighted ENO

reconstruct quadratic polynomial approximation to real solution from cell averages

$$P_j(x) = w_L P_L(x) + w_R P_R(x) + w_C P_C(x)$$

with positive weights $w_i > 0$ and $\sum_i w_i = 1$, where $i \in L, R, C$.

P_L

- left one-sided linear reconstruction
- $$P_L(x) = \bar{v}_j^n + \frac{\bar{v}_j^n - \bar{v}_{j-1}^n}{\Delta x}(x - x_j)$$
- uniquely determined, conserve on-sided cell-averages $\bar{v}_{j-1}^n$, $\bar{v}_j^n$.

central Weighted ENO

reconstruct quadratic polynomial approximation to real solution from cell averages

$$P_j(x) = w_L P_L(x) + w_R P_R(x) + w_C P_C(x)$$

with positive weights $w_i > 0$ and $\sum_i w_i = 1$, where $i \in L, R, C$.

P_R

- right one-sided linear reconstruction
- $$P_R(x) = \bar{v}_j^n + \frac{\bar{v}_{j+1}^n - \bar{v}_j^n}{\Delta x}(x - x_j)$$
- uniquely determined, conserve on-sided cell-averages $\bar{v}_j^n$, $\bar{v}_{j+1}^n$.

central Weighted ENO

reconstruct quadratic polynomial approximation to real solution from cell averages

$$P_j(x) = w_L P_L(x) + w_R P_R(x) + w_C P_C(x)$$

with positive weights $w_i > 0$ and $\sum_i w_i = 1$, where $i \in L, R, C$.

P_C

- centered parabola, chosen so as to satisfy
  $$P_{\mathrm{EXACT}}(x) = c_L P_L(x) + c_R P_R(x) + (1 - c_L - c_R) P_C(x)$$
- P_{EXACT} is the unique parabola that conserves the three-cell-average, $\bar{v}^n_{j-1}, \bar{v}^n_j$ and $\bar{v}^n_{j+1}$.

central Weighted ENO

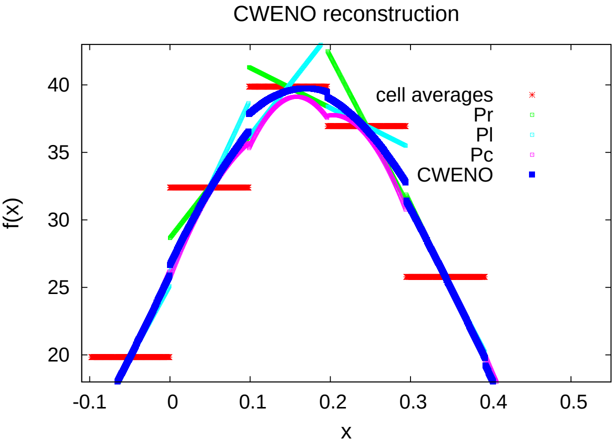
reconstruct quadratic polynomial approximation to real solution from cell averages

$$P_j(x) = w_L P_L(x) + w_R P_R(x) + w_C P_C(x)$$

with positive weights $w_i > 0$ and $\sum_i w_i = 1$, where $i \in L, R, C$.

Weights

- automatically adapt reconstruction to smoothness of solution
- smooth regions: third order for max. precision
- non-smooth regions: second order to provide essentially non-oscillating behaviour


CWENO reconstruction
f(x)
x
40
35
30
25
20
-0.1
0
0.1
0.2
0.3
0.4
0.5
cell averages
Pr
Pl
Pc
CWENO

- isothermal Euler equations
- isothermal ideal MHD equations

ideal MHD-equations (isothermal)

$$\partial_t \rho + \nabla \cdot \rho \mathbf{v} = 0$$

$$\partial_t \mathbf{v} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + (\nabla \times \mathbf{B}) \times \frac{1}{\mu_0} \mathbf{B}$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

$$p \propto \rho$$

additional initial condition : $\operatorname{div} \mathbf{B} = 0$

ideal MHD-equations (isothermal) conservative formulation

$$\partial_t \rho + \nabla \cdot \mathbf{u} = 0$$

$$\partial_t \mathbf{u} + \nabla \cdot \left[\frac{\mathbf{u}\mathbf{u}}{\rho} + (p + \frac{1}{2\mu}\mathbf{B}^2)\mathrm{I} - \frac{1}{\mu}\mathbf{B}\mathbf{B} \right] = 0$$

$$\partial_t \mathbf{B} + \nabla \times (\frac{\mathbf{u}}{\rho} \times \mathbf{B}) = 0$$

$$p \propto \rho$$

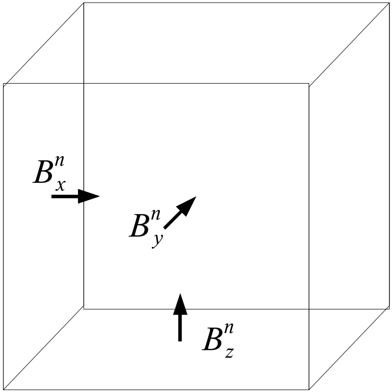
$$\mathrm{div}\, \mathbf{B} = 0$$

staggered collocation of magnetic field components as face averages

$$\bar{B}_{x,i-1/2,j,k} = \frac{1}{\Delta y \Delta z} \int_{y_{j-\frac{1}{2}}, z_{k-\frac{1}{2}}}^{y_{j+\frac{1}{2}}, z_{k+\frac{1}{2}}} B(x, y, z) dydz$$

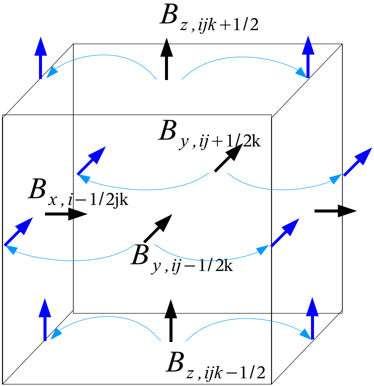
$$\bar{B}_{y,i,j-1/2,k} = \dots$$

$$\bar{B}_{z,i,j,k-1/2} = \dots$$



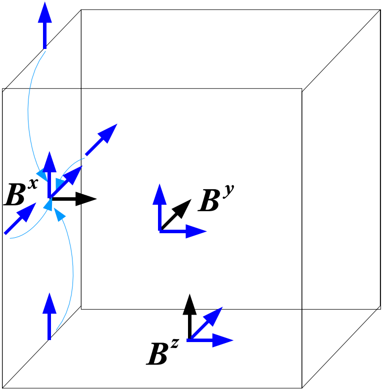
reconstruction of B to edges

$B_{y,i-1/2,j-1/2,k}$ = CWENO



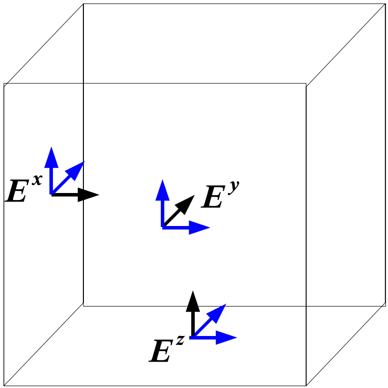
average to face centered

$$B_{y,i-\frac{1}{2},j,k} = \frac{1}{2}(B_{y,i-\frac{1}{2},j-\frac{1}{2},k} + B_{y,i-\frac{1}{2},j+\frac{1}{2},k})$$



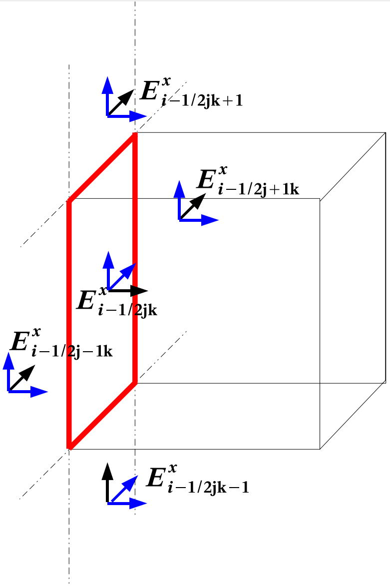
calculation of electric field fluxes with Kurganov central scheme

$$E^x_{i-\frac{1}{2},j,k} = v^x_{i-\frac{1}{2},j,k} \times B^x_{i-\frac{1}{2},j,k}$$



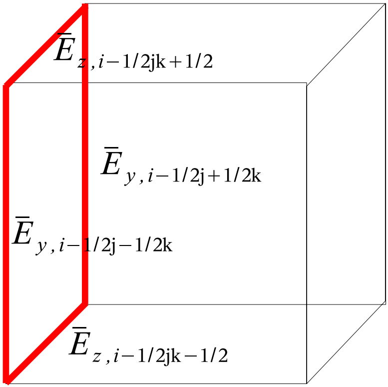
face to edge interpolation of electric field components

$$\bar{E}_{y,i-\frac{1}{2},j,k-\frac{1}{2}} = \frac{1}{4}(E^x_{z,i-\frac{1}{2},j,k} + E^x_{z,i-\frac{1}{2},j,k-1} - E^z_{x,i,j,k-\frac{1}{2}} - E^z_{x,i-1,j,k-\frac{1}{2}})$$



face to edge interpolation of electric field components

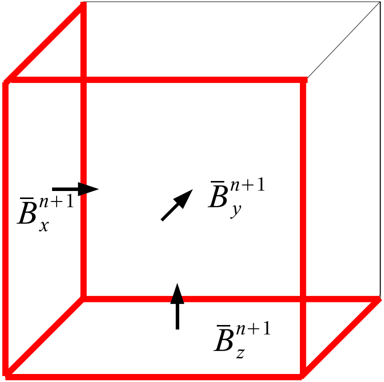
$$\bar{E}_{y,i-\frac{1}{2},j,k-\frac{1}{2}} = \frac{1}{4}(E^{x}_{z,i-\frac{1}{2},j,k} + E^{x}_{z,i-\frac{1}{2},j,k-1} - E^{z}_{x,i,j,k-\frac{1}{2}} - E^{z}_{x,i-1,j,k-\frac{1}{2}})$$



update magnetic field components

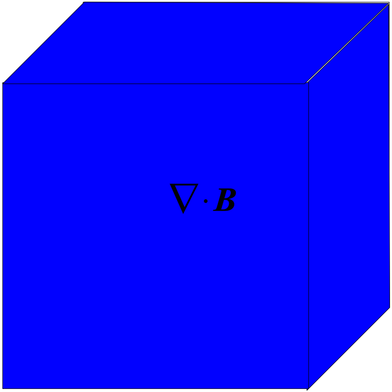
$$\frac{d}{dt}\bar{B}_{x,i-1/2,j,k} = -\int_{y,z} \nabla \times \mathrm{E}^x(t)\,dydz$$

$$= -\frac{\bar{E}_{z,i-1/2,j+1/2,k}(t) - \bar{E}_{z,i-1/2,j-1/2,k}(t)}{\delta y} + \frac{\bar{E}_{y,i-1/2,j,k+1/2}(t) - \bar{E}_{y,i-1/2,j,k-1/2}(t)}{\delta z}$$



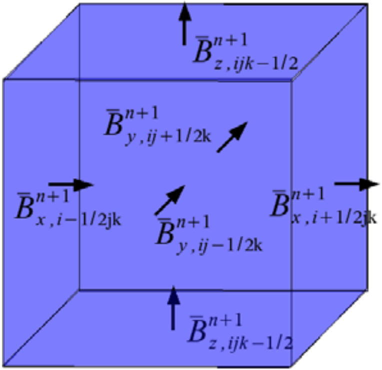
evolution of divergence ?

$$\frac{d}{dt}\,\mathrm{div}\,\bar{\mathrm{B}} = \frac{d}{dt}\int_V \nabla \cdot \mathrm{B}\,dxdydz$$



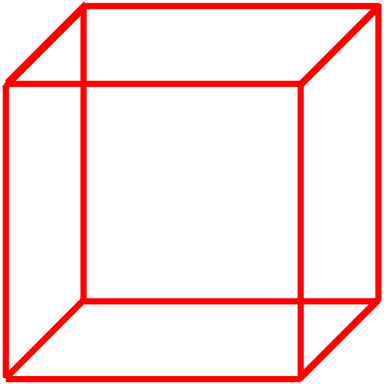
evolution of divergence ?

$$\frac{d}{dt}\,\mathrm{div}\,\bar{\mathrm{B}} = \int_{\partial V} \frac{d}{dt}\mathrm{B}\,dF$$



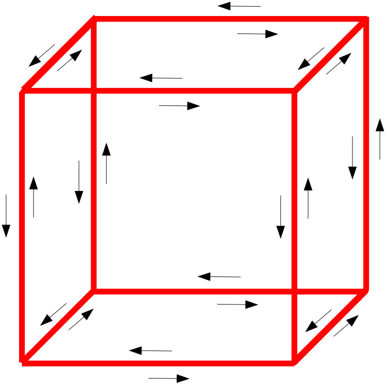
evolution of divergence ?

$$\frac{d}{dt}\,\mathrm{div}\,\bar{\mathsf{B}} = \frac{\frac{d}{dt}\bar{B}_{x,i+\frac{1}{2},j,k} - \frac{d}{dt}\bar{B}_{x,i-\frac{1}{2},j,k}}{\Delta x} + \frac{\frac{d}{dt}\bar{B}_{y,i,j+\frac{1}{2},k} - \frac{d}{dt}\bar{B}_{y,i,j-\frac{1}{2},k}}{\Delta y} + \frac{\frac{d}{dt}\bar{B}_{z,i,j,k+\frac{1}{2}} - \frac{d}{dt}\bar{B}_{x,i,j,k-\frac{1}{2}}}{\Delta z}$$



evolution of divergence ?

$$\frac{d}{dt}\,\mathrm{div}\,\bar{\mathrm{B}} = \frac{\frac{d}{dt}\bar{B}_{x,i+\frac{1}{2},j,k} - \frac{d}{dt}\bar{B}_{x,i-\frac{1}{2},j,k}}{\Delta x} + \frac{\frac{d}{dt}\bar{B}_{y,i,j+\frac{1}{2},k} - \frac{d}{dt}\bar{B}_{y,i,j-\frac{1}{2},k}}{\Delta y} + \frac{\frac{d}{dt}\bar{B}_{z,i,j,k+\frac{1}{2}} - \frac{d}{dt}\bar{B}_{x,i,j,k-\frac{1}{2}}}{\Delta z} = 0$$



Setup

- three runs for isothermal compressible euler equations and mean Mach numbers $\tilde{M}$ of 0.4, 1.4, 4.6

Setup

- three runs for isothermal compressible euler equations and mean Mach numbers $\tilde{M}$ of 0.4, 1.4, 4.6
- periodic boundary conditions and a resolution of 512^3

Setup

- three runs for isothermal compressible euler equations and mean Mach numbers $\tilde{M}$ of 0.4, 1.4, 4.6
- periodic boundary conditions and a resolution of 512^3
- Orszag-Tang like initial condition for $\mathbf{u}$ and $\rho = 1$

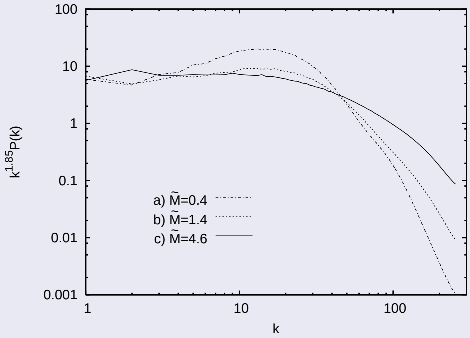
Setup

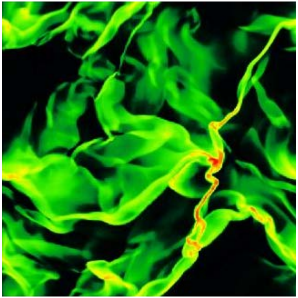
- three runs for isothermal compressible euler equations and mean Mach numbers $\tilde{M}$ of 0.4, 1.4, 4.6
- periodic boundary conditions and a resolution of 512^3
- Orszag-Tang like initial condition for $\mathbf{u}$ and $\rho = 1$
- integrated by a third order Runke-Kutta scheme

Setup

- three runs for isothermal compressible euler equations and mean Mach numbers $\tilde{M}$ of 0.4, 1.4, 4.6
- periodic boundary conditions and a resolution of 512^3
- Orszag-Tang like initial condition for $\mathbf{u}$ and $\rho = 1$
- integrated by a third order Runke-Kutta scheme

Spectrum





$\tilde{M}$	$\mathcal{C}$	Reynolds
0.4	0.07	250
1.4	0.26	700
4.6	0.50	1756

log-normal

- Passot & Vazquez-Semadeni and also Nordlund & Paduan found a log-normal density distribution for isothermal compressible flows

log-normal

- Passot & Vazquez-Semadeni and also Nordlund & Paduan found a log-normal density distribution for isothermal compressible flows
- central idee: isothermal $\Leftrightarrow$ $p = a\rho$

log-normal

- Passot & Vazquez-Semadeni and also Nordlund & Paduan found a log-normal density distribution for isothermal compressible flows
- central idee: isothermal $\Leftrightarrow$ $p = a\rho$
- euler equations with $\ln \rho$:

$$\begin{aligned} \partial_t (\ln \rho) + \mathbf{v} \cdot \nabla (\ln \rho) + \nabla \cdot \mathbf{v} &= 0 \\ \partial_t \mathbf{v} + \nabla \cdot (\mathbf{v}\mathbf{v}) + a \nabla (\ln \rho) &= \mathbf{k} \end{aligned}$$

log-normal

- Passot & Vazquez-Semadeni and also Nordlund & Paduan found a log-normal density distribution for isothermal compressible flows
- central idee: isothermal $\Leftrightarrow$ $p = a\rho$
- euler equations with $\ln\rho$:

$$\begin{aligned}\partial_t(\ln\rho) + \mathbf{v}\cdot\nabla(\ln\rho) + \nabla\cdot\mathbf{v} &= 0\\ \partial_t\mathbf{v} + \nabla\cdot(\mathbf{vv}) + a\nabla(\ln\rho) &= \mathbf{k}\end{aligned}$$

- only derivatives of $\ln\rho$

log-normal

- Passot & Vazquez-Semadeni and also Nordlund & Paduan found a log-normal density distribution for isothermal compressible flows
- central idee: isothermal $\Leftrightarrow$ $p = a\rho$
- euler equations with $\ln \rho$:

$$\begin{aligned} \partial_t (\ln \rho) + \mathbf{v} \cdot \nabla (\ln \rho) + \nabla \cdot \mathbf{v} &= 0 \\ \partial_t \mathbf{v} + \nabla \cdot (\mathbf{v}\mathbf{v}) + a \nabla (\ln \rho) &= \mathbf{k} \end{aligned}$$

- only derivatives of $\ln \rho$

log-normal

- only derivatives of $\ln \rho$
- euler equations invariant under $\ln \rho \rightarrow \ln \rho + \ln K$ or $\rho \rightarrow K\rho$

log-normal

- only derivatives of $\ln \rho$
- euler equations invariant under $\ln \rho \rightarrow \ln \rho + \ln K$ or $\rho \rightarrow K \rho$
- shocks increase or decrease the density in a local area

log-normal

- only derivatives of $\ln \rho$
- euler equations invariant under $\ln \rho \to \ln \rho + \ln K$ or $\rho \to K\rho$
- shocks increase or decrease the density in a local area
- in each area $\ln \rho$ is random variable around a mean value

log-normal

- only derivatives of $\ln \rho$
- euler equations invariant under $\ln \rho \rightarrow \ln \rho + \ln K$ or $\rho \rightarrow K \rho$
- shocks increase or decrease the density in a local area
- in each area $\ln \rho$ is random variable around a mean value
- summation over areas yields normal distribution for $\ln \rho$

log-normal

- only derivatives of $\ln \rho$
- euler equations invariant under $\ln \rho \rightarrow \ln \rho + \ln K$ or $\rho \rightarrow K\rho$
- shocks increase or decrease the density in a local area
- in each area $\ln \rho$ is random variable around a mean value
- summation over areas yields normal distribution for $\ln \rho$

$\Rightarrow$ ρ is log-normal distributed

log-normal

- only derivatives of $\ln \rho$
- euler equations invariant under $\ln \rho \longrightarrow \ln \rho + \ln K$ or $\rho \longrightarrow K\rho$
- shocks increase or decrease the density in a local area
- in each area $\ln \rho$ is random variable around a mean value
- summation over areas yields normal distribution for $\ln \rho$

$\Rightarrow$ ρ is log-normal distributed

log-normal distribution

$$L = \frac{1}{\rho\sqrt{2\pi\sigma^2_{\ln\rho}}} \cdot e^{-\frac{\left(\ln\rho - \mu_{\ln\rho}\right)^2}{2\sigma^2_{\ln\rho}}}$$

with $\mu_{\ln\rho} = \mathbb{E}(\ln\rho)$

and $\sigma^2_{\ln\rho} = \mathbb{V}(\ln\rho)$

log-normal parameters

log-normal distribution

$$L = \frac{1}{\rho\sqrt{2\pi\sigma^2_{\ln\rho}}} \cdot e^{-\frac{\left(\ln\rho - \mu_{\ln\rho}\right)^2}{2\sigma^2_{\ln\rho}}}$$

$$\text{with} \quad \mu_{\ln\rho} = \mathbb{E}\left(\ln\rho\right)$$

$$\text{and} \quad \sigma^2_{\ln\rho} = \mathbb{V}\left(\ln\rho\right)$$

log-normal parameters

$$\mathbb{E}\left(L\right) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$

$$\mathbb{V}\left(L\right) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) = \sigma^2_\rho$$

log-normal distribution

$$L = \frac{1}{\rho\sqrt{2\pi\sigma^2_{\ln\rho}}} \cdot e^{-\frac{\left(\ln\rho - \mu_{\ln\rho}\right)^2}{2\sigma^2_{\ln\rho}}}$$

$$\text{with} \quad \mu_{\ln\rho} = \mathbb{E}\left(\ln\rho\right)$$

$$\text{and} \quad \sigma^2_{\ln\rho} = \mathbb{V}\left(\ln\rho\right)$$

log-normal parameters

$$\mathbb{E}\left(L\right) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$

$$\mathbb{V}\left(L\right) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) = \sigma^2_{\rho}$$

log-normal parameters

$$\mathbb{E}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$

$$\mathbb{V}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) = \sigma^2_\rho$$

- $\mathbb{E}_\rho \equiv 1$

log-normal parameters

$$\mathbb{E}\left(L\right) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$

$$\mathbb{V}\left(L\right) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) \quad = \quad \sigma^2_\rho$$

- $\mathbb{E}_\rho \equiv 1 \quad \Rightarrow \quad \mu_{\ln\rho} = -\sigma^2_{\ln\rho}/2$

log-normal parameters

$$\mathbb{E}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$

$$\mathbb{V}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) = \sigma^2_\rho$$

- $\mathbb{E}_\rho \equiv 1 \quad \Rightarrow \quad \mu_{\ln\rho} = -\sigma^2_{\ln\rho}/2$
- variance σ^2_ρ is related to compressibility and mean Mach number $\tilde{M}$

log-normal parameters

$$\mathbb{E}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$

$$\mathbb{V}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) \quad = \quad \sigma^2_\rho$$

- $\mathbb{E}_\rho \equiv 1 \quad \Rightarrow \quad \mu_{\ln\rho} = -\sigma^2_{\ln\rho}/2$
- variance σ^2_ρ is related to compressibility and mean Mach number $\tilde{M}$
- observation: $\sigma_\rho \sim \tilde{M}, \quad \sigma^2_\rho = \beta^2 \tilde{M}^2$

log-normal parameters

$$\mathbb{E}(L) = e^{\mu_{\ln\rho}+\sigma^2_{\ln\rho}/2}$$
$$\mathbb{V}(L) = e^{\mu_{\ln\rho}+\sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) = \sigma^2_\rho$$

- $\mathbb{E}_\rho \equiv 1 \quad \Rightarrow \quad \mu_{\ln\rho} = -\sigma^2_{\ln\rho}/2$
- variance σ^2_ρ is related to compressibility and mean Mach number $\tilde{M}$
- observation: $\sigma_\rho \sim \tilde{M}, \quad \sigma^2_\rho = \beta^2\tilde{M}^2$

$\Rightarrow \quad \sigma^2_{\ln\rho} = \ln\left(1+\beta^2\tilde{M}^2\right)$

log-normal parameters

$$\mathbb{E}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$

$$\mathbb{V}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) \quad = \quad \sigma^2_\rho$$

- $\mathbb{E}_\rho \equiv 1 \quad \Rightarrow \quad \mu_{\ln\rho} = -\sigma^2_{\ln\rho}/2$
- variance σ^2_ρ is related to compressibility and mean Mach number $\tilde{M}$
- observation: $\sigma_\rho \sim \tilde{M}, \quad \sigma^2_\rho = \beta^2 \tilde{M}^2$
  $\Rightarrow \quad \sigma^2_{\ln\rho} = \ln\left(1 + \beta^2 \tilde{M}^2\right)$
- only one free parameter β determined by fitting to the three runs, obtaining $\beta \simeq 0.37$

log-normal parameters

$$\mathbb{E}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$
$$\mathbb{V}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) \quad = \quad \sigma^2_\rho$$

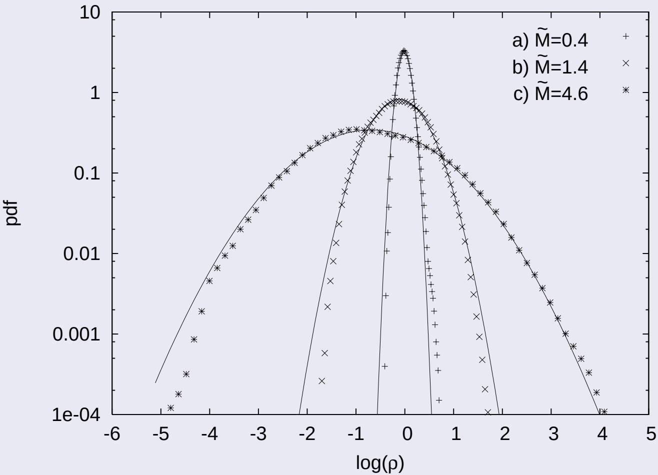
- $\mathbb{E}_\rho \equiv 1 \quad \Rightarrow \quad \mu_{\ln\rho} = -\sigma^2_{\ln\rho}/2$
- variance σ^2_ρ is related to compressibility and mean Mach number $\tilde{M}$
- observation: $\sigma_\rho \sim \tilde{M}, \quad \sigma^2_\rho = \beta^2 \tilde{M}^2$
  $\Rightarrow \quad \sigma^2_{\ln\rho} = \ln\left(1 + \beta^2 \tilde{M}^2\right)$
- only one free parameter β determined by fitting to the three runs, obtaining $\beta \simeq 0.37$
- Kritsuk *et al.* $\beta = 0.26$

log-normal parameters

$$\mathbb{E}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}$$
$$\mathbb{V}(L) = e^{\mu_{\ln\rho} + \sigma^2_{\ln\rho}/2}\left(e^{\sigma^2_{\ln\rho}} - 1\right) = \sigma^2_\rho$$

- $\mathbb{E}_\rho \equiv 1 \quad \Rightarrow \quad \mu_{\ln\rho} = -\sigma^2_{\ln\rho}/2$
- variance σ^2_ρ is related to compressibility and mean Mach number $\tilde{M}$
- observation: $\sigma_\rho \sim \tilde{M}, \quad \sigma^2_\rho = \beta^2 \tilde{M}^2$
  $\Rightarrow \quad \sigma^2_{\ln\rho} = \ln\left(1 + \beta^2 \tilde{M}^2\right)$
- only one free parameter β determined by fitting to the three runs, obtaining $\beta \simeq 0.37$
- Kritsuk *et al.* $\beta = 0.26$

log-normal figure



particle distribution and it's dynamics

- $N = 2\,000\,000$ tracer particles inserted radomly and advected by $\mathbf{v}$

particle distribution and it's dynamics

- $N = 2\,000\,000$ tracer particles inserted radomly and advected by $\mathbf{v}$
- divide the box into $n = 40^3$ subdomains

particle distribution and it's dynamics

- $N = 2\,000\,000$ tracer particles inserted radomly and advected by $\mathbf{v}$
- divide the box into $n = 40^3$ subdomains
- How many particles are in a subdomain?

particle distribution and it's dynamics

- $N = 2\,000\,000$ tracer particles inserted radomly and advected by $\mathbf{v}$
- divide the box into $n = 40^3$ subdomains
- How many particles are in a subdomain?

$\Leftrightarrow$ What is the propability $T(k)$ to find k particles in one of the boxes?

particle distribution and it's dynamics

- $N = 2\,000\,000$ tracer particles inserted radomly and advected by $\mathbf{v}$
- divide the box into $n = 40^3$ subdomains
- How many particles are in a subdomain?

$\Leftrightarrow$ What is the propability $T(k)$ to find k particles in one of the boxes?

- what we know: expected value $\mathbb{E}(T) = \frac{N}{n}$

particle distribution and it's dynamics

- $N = 2\,000\,000$ tracer particles inserted radomly and advected by $\mathbf{v}$
- divide the box into $n = 40^3$ subdomains
- How many particles are in a subdomain?

$\Leftrightarrow$ What is the propability $T(k)$ to find k particles in one of the boxes?

- what we know: expected value $\mathbb{E}(T) = \frac{N}{n}$
- conservated property arbitrary of the distribution

particle distribution and it's dynamics

- $N = 2\,000\,000$ tracer particles inserted radomly and advected by $\mathbf{v}$
- divide the box into $n = 40^3$ subdomains
- How many particles are in a subdomain?

$\Leftrightarrow$ What is the propability $T(k)$ to find k particles in one of the boxes?

- what we know: expected value $\mathbb{E}(T) = \frac{N}{n}$
- conservated property arbitrary of the distribution

initally

- initially particles are randomly distributed over the boxes

initally

- initially particles are randomly distributed over the boxes

⇒ *Binomial* distribution:

initally

- initially particles are randomly distributed over the boxes

$\Rightarrow$ *Binomial* distribution:

$$B_{N,n}(k) = \binom{N}{k} \cdot \left(\frac{1}{n}\right)^k \cdot \left(1 - \frac{1}{n}\right)^{N-k}$$

initally

- initially particles are randomly distributed over the boxes

⇒ *Binomial* distribution:

$$B_{N,n}(k) = \begin{pmatrix} N \\ k \end{pmatrix} \cdot \left(\frac{1}{n}\right)^k \cdot \left(1 - \frac{1}{n}\right)^{N-k}$$

- set $\lambda := \frac{N}{n}$

initally

- initially particles are randomly distributed over the boxes

⇒ *Binomial* distribution:

$$B_{N,n}(k) = \begin{pmatrix} N \\ k \end{pmatrix} \cdot \left(\tfrac{1}{n}\right)^k \cdot \left(1 - \tfrac{1}{n}\right)^{N-k}$$

- set $\lambda := \frac{N}{n}$

$$B_{N,n}(k) \xrightarrow{n \gg 1} \frac{\lambda^k}{k!} e^{-\lambda} = P_\lambda(k) \qquad \textit{Poisson} \text{ distribution}$$

initally

- initially particles are randomly distributed over the boxes

⇒ *Binomial* distribution:

$$B_{N,n}(k) = \begin{pmatrix} N \\ k \end{pmatrix} \cdot \left(\frac{1}{n}\right)^k \cdot \left(1 - \frac{1}{n}\right)^{N-k}$$

- set $\lambda := \frac{N}{n}$

$$B_{N,n}(k) \xrightarrow{n \gg 1} \frac{\lambda^k}{k!} e^{-\lambda} = P_\lambda(k) \qquad \textit{Poisson} \text{ distribution}$$

- variance: $\mathbb{V}(k) = \lambda$

initally

- initially particles are randomly distributed over the boxes

⇒ *Binomial* distribution:

$$B_{N,n}(k) = \begin{pmatrix} N \\ k \end{pmatrix} \cdot \left(\tfrac{1}{n}\right)^k \cdot \left(1 - \tfrac{1}{n}\right)^{N-k}$$

- set $\lambda := \frac{N}{n}$

$$B_{N,n}(k) \xrightarrow{n \gg 1} \frac{\lambda^k}{k!} e^{-\lambda} = P_\lambda(k) \qquad \textit{Poisson} \text{ distribution}$$

- variance: $\mathbb{V}(k) = \lambda$

incompressible turbulence

- in incompressible turbulence with $\nabla \cdot \mathbf{v} = 0$ and $\rho = const$ the distribution is always the same Poissonian

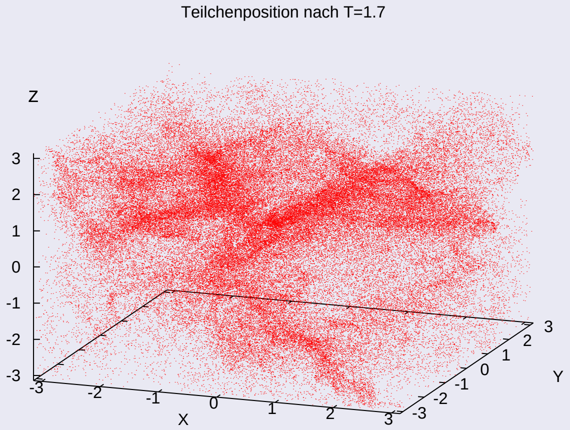
incompressible turbulence

- in incompressible turbulence with $\nabla \cdot \mathbf{v} = 0$ and $\rho = const$ the distribution is always the same Poissonian
- in compressible turbulence a different behaviour is expected

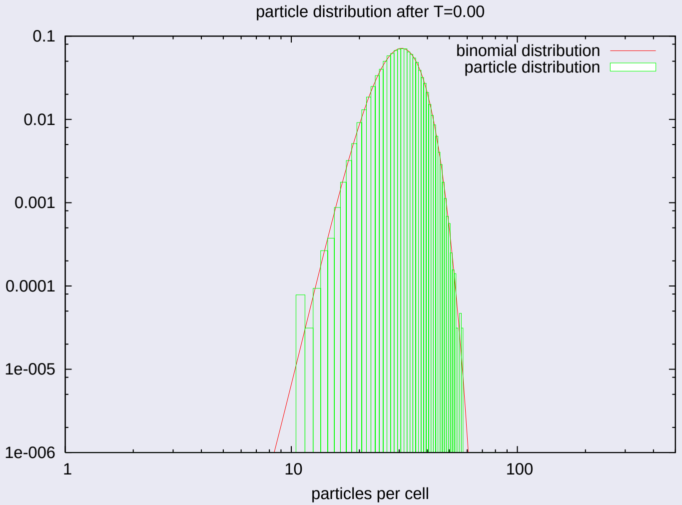
incompressible turbulence

- in incompressible turbulence with $\nabla \cdot \mathbf{v} = 0$ and $\rho = const$ the distribution is always the same Poissonian
- in compressible turbulence a different behaviour is expected

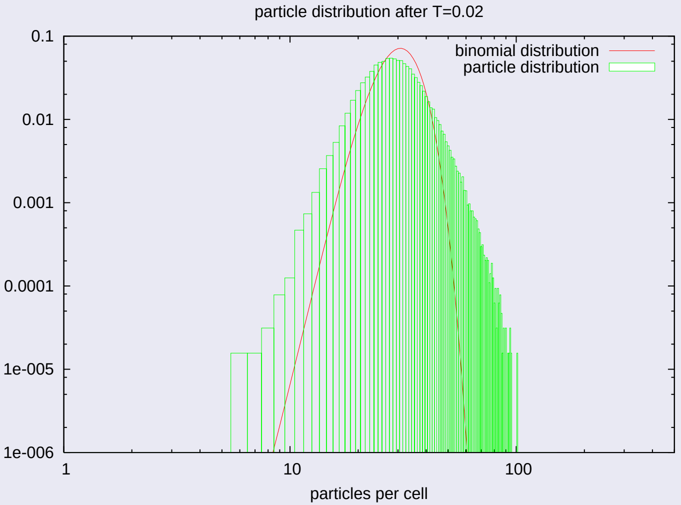
evolution (5% of the particles)



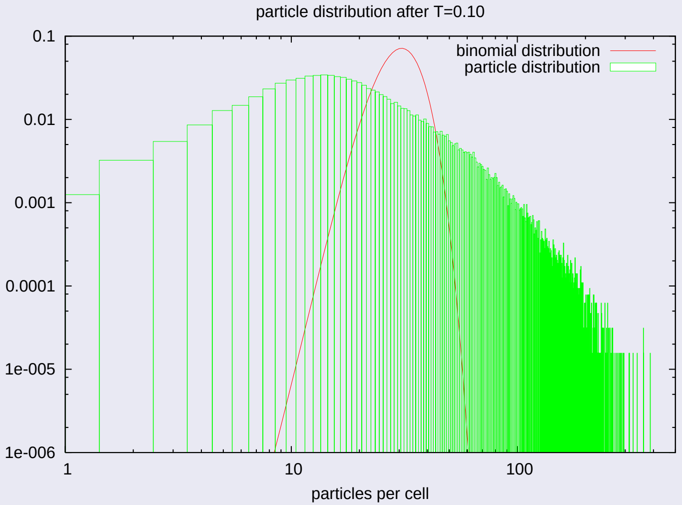
the PDF


particle distribution after T=0.00
binomial distribution
particle distribution
0.1
0.01
0.001
0.0001
1e-005
1e-006
1
10
100
particles per cell

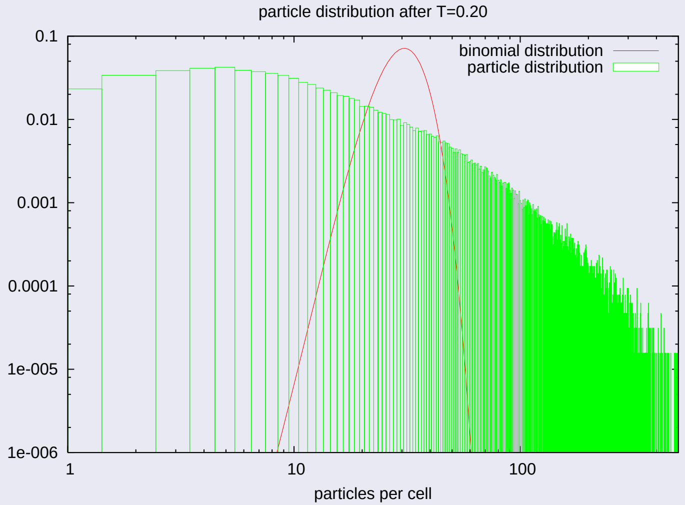
the PDF


particle distribution after T=0.02
binomial distribution
particle distribution
0.1
0.01
0.001
0.0001
1e-005
1e-006
1
10
100
particles per cell

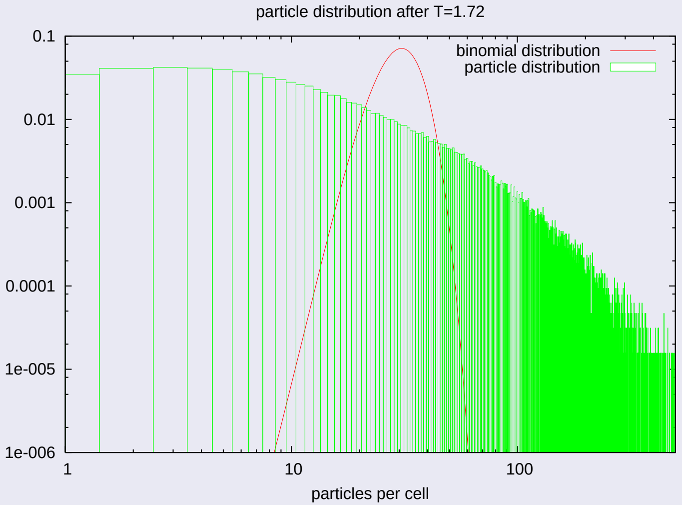
the PDF


particle distribution after T=0.10
binomial distribution
particle distribution
0.1
0.01
0.001
0.0001
1e-005
1e-006
1
10
100
particles per cell

the PDF


particle distribution after T=0.20
binomial distribution
particle distribution
0.1
0.01
0.001
0.0001
1e-005
1e-006
1
10
100
particles per cell

the PDF


particle distribution after T=1.72
binomial distribution
particle distribution
0.1
0.01
0.001
0.0001
1e-005
1e-006
1
10
100
particles per cell

remarks

- density and particle distribution become stationary

remarks

- density and particle distribution become stationary
- these distributions should be correlated

remarks

- density and particle distribution become stationary
- these distributions should be correlated

properties

property	density PDF $R(\rho)$	particle PDF $T(k)$
expected values	$\mathbb{E}(R) = \rho_0$	$\mathbb{E}(T) = \lambda = \frac{N}{n}$
	continues in ρ	discrete in k
incompressible turbulence $\rho = const$	$\delta(\rho - \rho_0)$	$P(k) = \frac{\lambda^k}{k!} e^{-\lambda}$
compressible turbulence high $\tilde{M}$	$\frac{1}{\rho\sqrt{2\pi\sigma^2_{\ln\rho}}} \cdot e^{-\frac{(\ln\rho - \mu_{\ln\rho})^2}{2\sigma^2_{\ln\rho}}}$	? ? ?

idea

- mapping $R \longrightarrow T$ as

idea

- mapping $R \longrightarrow T$ as

$$\hat{T}(k) = \int P_{\lambda=\frac{\mathbb{E}(T)}{\mathbb{E}(R)}\rho}(k)\, R(\rho)\, d\rho$$

idea

- mapping $R \longrightarrow T$ as

$$\hat{T}(k) = \int P_{\lambda = \frac{\mathbb{E}(T)}{\mathbb{E}(R)}\rho}(k)\, R(\rho)\; d\rho := (R \circledast P_\lambda)(k)$$

idea

- mapping $R \longrightarrow T$ as

$$\hat{T}(k) = \int P_{\lambda = \frac{\mathbb{E}(T)}{\mathbb{E}(R)}\rho}(k)\, R(\rho)\; d\rho := (R \circledast P_\lambda)(k)$$

- solves the problem for incompressibility, because for $R(\rho) = \delta(\rho - \rho_0)$ holds

idea

- mapping $R \longrightarrow T$ as

$$\hat{T}(k) = \int P_{\lambda = \frac{\mathbb{E}(T)}{\mathbb{E}(R)}\rho}(k)\, R(\rho)\; d\rho := (R \circledast P_\lambda)(k)$$

- solves the problem for incompressibility, because for $R(\rho) = \delta(\rho - \rho_0)$ holds

$$\int P_{\lambda = \frac{N/n}{\rho_0}\rho}(k)\,\delta(\rho - \rho_0)\,d\rho \;=\; P_{\lambda = N/n}(k)$$

idea

- mapping $R \longrightarrow T$ as

$$\hat{T}(k) = \int P_{\lambda=\frac{\mathbb{E}(T)}{\mathbb{E}(R)}\rho}(k)\, R(\rho)\, d\rho := (R \circledast P_\lambda)(k)$$

- solves the problem for incompressibility, because for $R(\rho) = \delta(\rho - \rho_0)$ holds

$$\begin{aligned} \int P_{\lambda=\frac{N/n}{\rho_0}\rho}(k)\, \delta(\rho - \rho_0)\, d\rho &= P_{\lambda=N/n}(k) \\ &= T(k) \end{aligned}$$

idea

- mapping $R \rightarrow T$ as

$$\hat{T}(k) = \int P_{\lambda = \frac{\mathbb{E}(T)}{\mathbb{E}(R)}\rho}(k)\, R(\rho)\; d\rho := (R \circledast P_\lambda)(k)$$

- solves the problem for incompressibility, because for $R(\rho) = \delta(\rho - \rho_0)$ holds

$$\begin{aligned} \int P_{\lambda = \frac{N/n}{\rho_0}\rho}(k)\, \delta(\rho - \rho_0)\, d\rho &= P_{\lambda = N/n}(k) \\ &= T(k) \end{aligned}$$

idea

- Each distribution can be approximated by weighted sum over delta distributions.

idea

- Each distribution can be approximated by weighted sum over delta distributions.
- $\hat{T}(k) = (R \circledast P_\lambda)(k)$ should hold for every density distribution

idea

- Each distribution can be approximated by weighted sum over delta distributions.
- $\hat{T}(k) = (R \circledast P_\lambda)(k)$ should hold for every density distribution
- test it with log-normal distributions for isothermal flows

$$\hat{T}(k) = \frac{E^k}{\sqrt{2\pi}\sigma_{\ln\rho}} \frac{1}{k!} \int_0^\infty \rho^{k-1} \exp\left(-\frac{(\ln\rho - \mu_{\ln\rho})^2}{2\sigma_{\ln\rho}^2} - E\rho \right) d\rho$$

$$E = \frac{\mathbb{E}(R)}{\mathbb{E}(T)}$$

idea

- Each distribution can be approximated by weighted sum over delta distributions.
- $\hat{T}(k) = (R \circledast P_\lambda)(k)$ should hold for every density distribution
- test it with log-normal distributions for isothermal flows

$$\hat{T}(k) = \frac{E^k}{\sqrt{2\pi}\sigma_{\ln\rho}} \frac{1}{k!} \int_0^\infty \rho^{k-1} \exp\left(-\frac{(\ln\rho - \mu_{\ln\rho})^2}{2\sigma_{\ln\rho}^2} - E\rho \right) d\rho$$

$$E = \frac{\mathbb{E}(R)}{\mathbb{E}(T)}$$

- note: β is the only fit-parameter and we found before $\beta \simeq 0.37$

idea

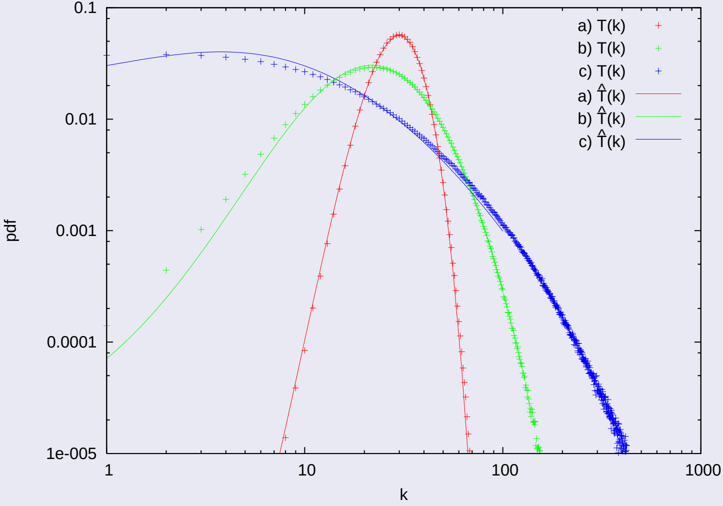
- Each distribution can be approximated by weighted sum over delta distributions.
- $\hat{T}(k) = (R \circledast P_\lambda)(k)$ should hold for every density distribution
- test it with log-normal distributions for isothermal flows

$$\hat{T}(k) = \frac{E^k}{\sqrt{2\pi}\sigma_{\ln\rho}} \frac{1}{k!} \int_0^\infty \rho^{k-1} \exp\left(-\frac{(\ln\rho - \mu_{\ln\rho})^2}{2\sigma_{\ln\rho}^2} - E\rho\right) d\rho$$

$$E = \frac{\mathbb{E}(R)}{\mathbb{E}(T)}$$

- note: β is the only fit-parameter and we found before $\beta \simeq 0.37$

results


0.1
0.01
0.001
0.0001
1e-005
pdf
1
10
100
1000
k
a) T(k)
b) T(k)
c) T(k)
a) $\hat{T}(k)$
b) $\hat{T}(k)$
c) $\hat{T}(k)$

question

What is the characteristic time for $T = P \rightarrow \hat{T}$?

question

What is the characteristic time for $T = P \rightarrow \hat{T}$?

or: How long does the clustering take?

question

What is the characteristic time for $T = P \rightarrow \hat{T}$?

or: How long does the clustering take?

idea

- assume: $T(k,t)$ is a linear combination of $P_\lambda(k)$ and $\hat{T}(k)$ with time depending coefficient $a(t)$

idea

- assume: $T(k,t)$ is a linear combination of $P_\lambda(k)$ and $\hat{T}(k)$ with time depending coefficient $a(t)$

$$T(k,t) = a(t)P_\lambda(k) + [1 - a(t)]\,\hat{T}(k)$$

idea

- assume: $T(k,t)$ is a linear combination of $P_\lambda(k)$ and $\hat{T}(k)$ with time depending coefficient $a(t)$
$$T(k,t) = a(t)P_\lambda(k) + [1 - a(t)]\,\hat{T}(k)$$
- $a(t)$ starts at 1 and drop down close to 0

idea

- assume: $T(k,t)$ is a linear combination of $P_\lambda(k)$ and $\hat{T}(k)$ with time depending coefficient $a(t)$

$$T(k,t) = a(t)P_\lambda(k) + [1 - a(t)]\,\hat{T}(k)$$

- $a(t)$ starts at 1 and drop down close to 0
- time evolution of particle-distribution is reduced to a real function

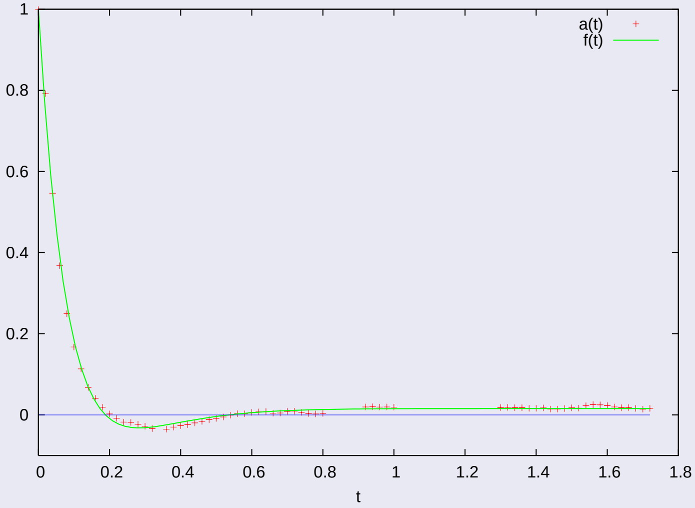
idea

- assume: $T(k,t)$ is a linear combination of $P_\lambda(k)$ and $\hat{T}(k)$ with time depending coefficient $a(t)$

$$T(k,t) = a(t)P_\lambda(k) + [1 - a(t)]\,\hat{T}(k)$$

- $a(t)$ starts at 1 and drop down close to 0
- time evolution of particle-distribution is reduced to a real function

idea



summary

- fitting a damped oscillating function yields a typical time scale t_S

summary

- fitting a damped oscillating function yields a typical time scale t_S
- low Mach numbers (less compressibility) cause strong oscillations

summary

- fitting a damped oscillating function yields a typical time scale t_S
- low Mach numbers (less compressibility) cause strong oscillations
- rough estimate possible

summary

- fitting a damped oscillating function yields a typical time scale t_S
- low Mach numbers (less compressibility) cause strong oscillations
- rough estimate possible

$\tilde{M}$	t_{dyn}	t_S
0.4	7.8	1.0
1.4	2.2	0.33
4.6	0.62	0.11

summary

- fitting a damped oscillating function yields a typical time scale t_S
- low Mach numbers (less compressibility) cause strong oscillations
- rough estimate possible

$\tilde{M}$	t_{dyn}	t_S
0.4	7.8	1.0
1.4	2.2	0.33
4.6	0.62	0.11

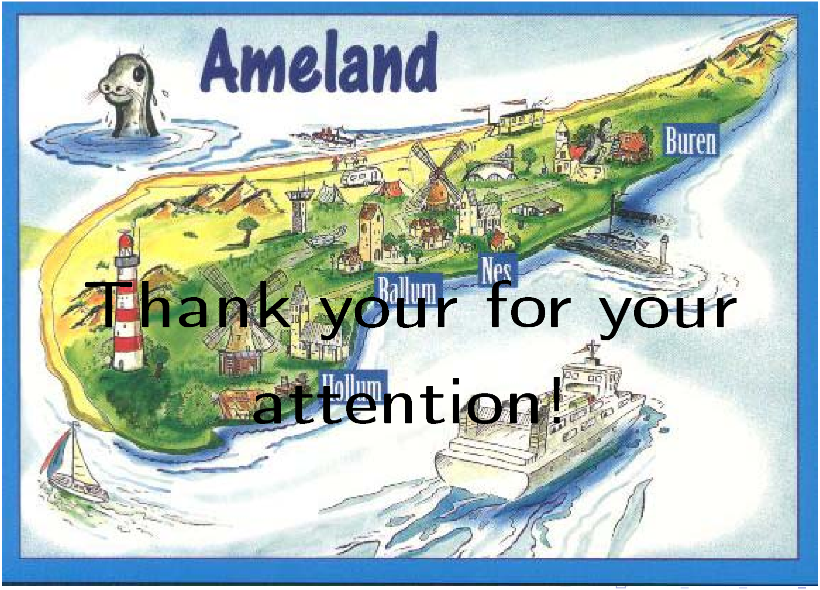
- note: $t_S \ll t_{dyn} = L/2\tilde{M}$

summary

- fitting a damped oscillating function yields a typical time scale t_S
- low Mach numbers (less compressibility) cause strong oscillations
- rough estimate possible

$\tilde{M}$	t_{dyn}	t_S
0.4	7.8	1.0
1.4	2.2	0.33
4.6	0.62	0.11

- note: $t_S \ll t_{dyn} = L/2\tilde{M}$



Thank your for your attention!