Synchronization and Pattern Formation in Coupled Nonlinear Optical Systems

Guido Krüger

gkruger@uni-muenster.de

WWU Münster

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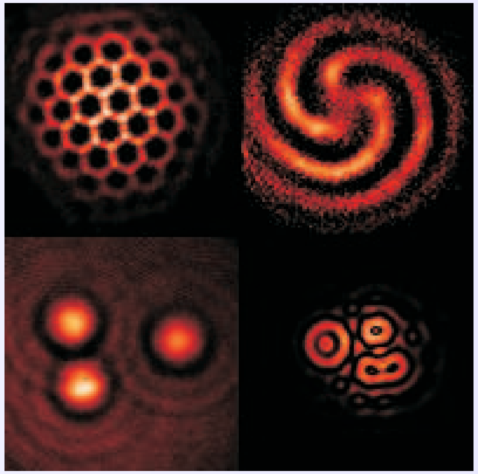
Introduction

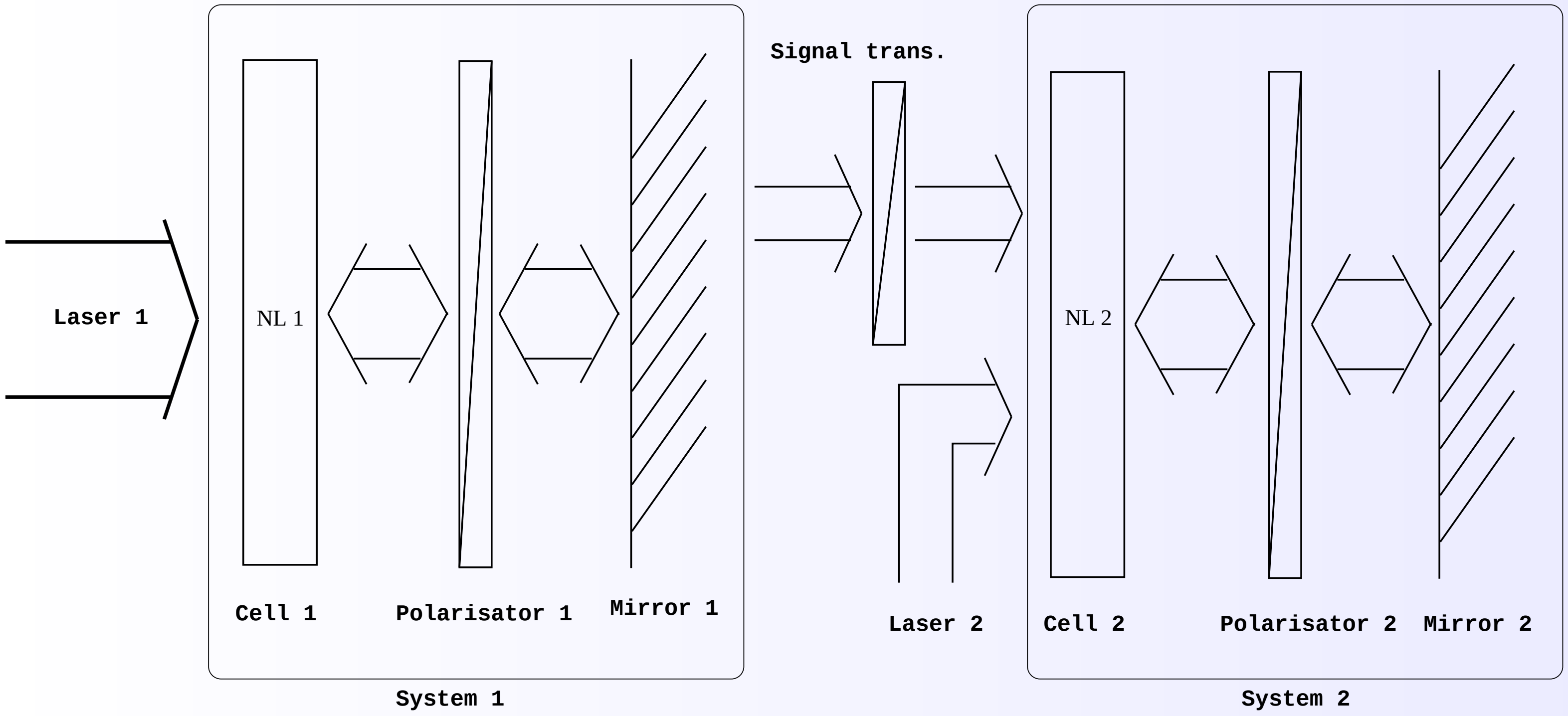
Introduction



- Analyse coupled nonlinear optical systems
- Possible information transfer device
- Interaction of coupled solitary (localized) structures

Coupled Nonlinear Optical Systems with Pattern Formation

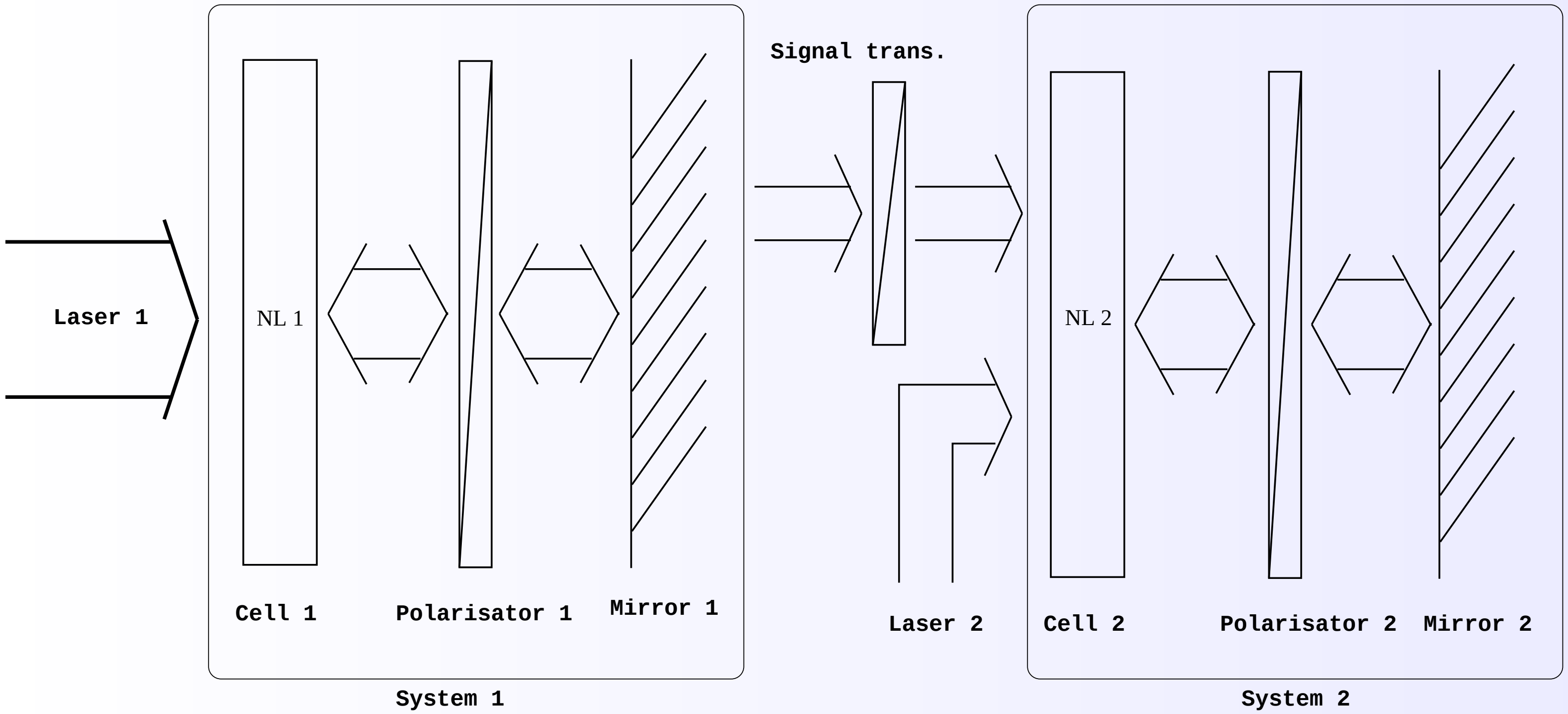
Defining the System



- Two pattern forming single feedback mirror systems
- Unidirectional coupling from system 1 to system 2
- Separation of refraction and diffraction

[Scroggie, Firth, PRA 53,2752 (1996)] [Mitschke et al., PRA 33,3219 (1986)]

Defining the System



- Interesting points
  - Synchronization of patterns (regular and chaotic)
  - Transmission of patterns
  - Solitary structures

[Scroggie, Firth, PRA 53,2752 (1996)] [Mitschke et al., PRA 33,3219 (1986)]

Theoretical Analysis - The Generalized Bloch Type Equation

- Generalized Bloch type equation for $\boldsymbol{m} = (u, v, w)$

(1) $$\dot{\boldsymbol{m}} = \boldsymbol{\Omega} \times \boldsymbol{m} - \gamma_{\text{eff}} \boldsymbol{m} + \boldsymbol{P}$$

  - $\boldsymbol{\Omega} = (\Omega_x, \Omega_y, \Omega_z)$: magnetic field
  - γ_{eff} : effective relaxation rate
  - $\boldsymbol{P}$: pumprate due to the injected electrical field (i.e. the laser)

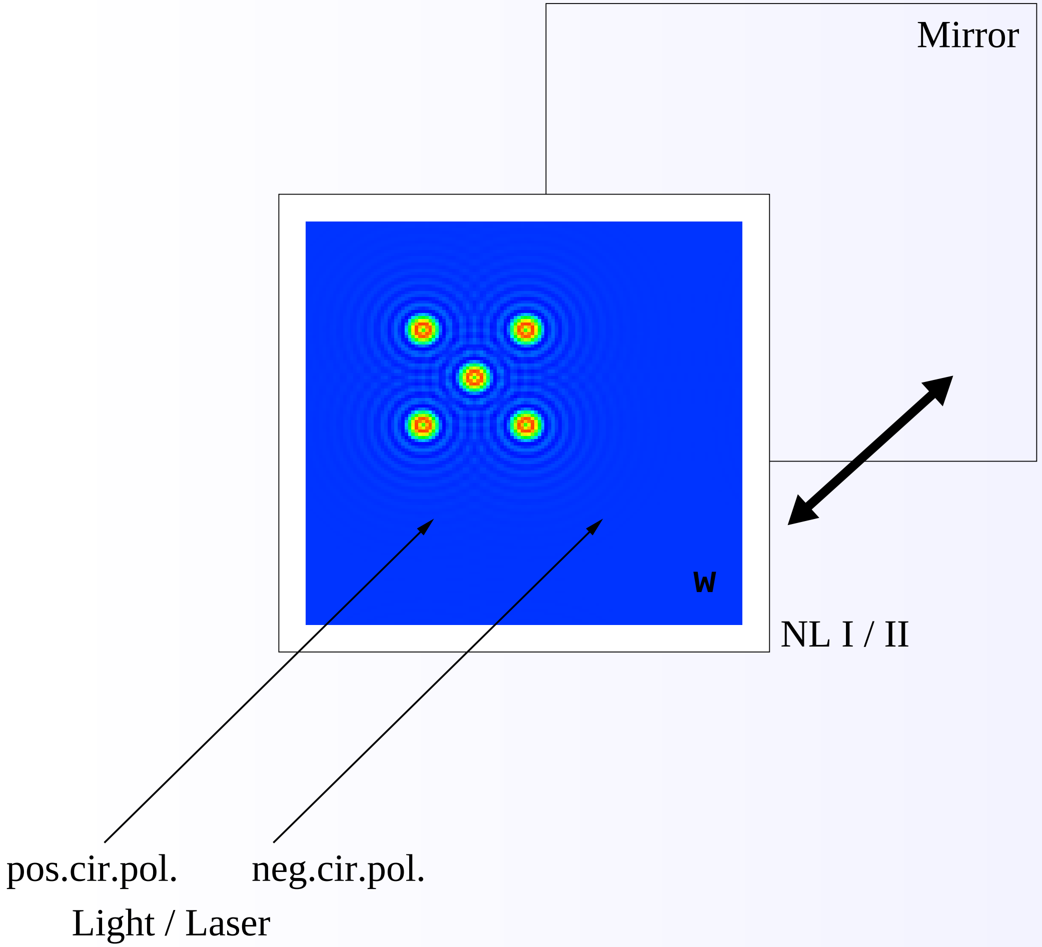
- No magnetic field
- Equations uncouple
- Quasiskalar description

(2) $$\dot{w} = -(\gamma - \text{Diff}\nabla_\perp^2)w + P_+(1-w) - P_-(1+w)$$

  - w : magnetization of the medium
  - $P_\pm$: pumprate of the positive / negative circular polarized light

[Mitschke et al., PRA 33,3219 (1986)] [Scroggie, Firth, PRA 53,2752 (1996)]

Theoretical Analysis - Where Are The Patterns?



- blue = positive magnetization
- red = negatvie magnetization
- w = magnetization of the medium (Na-vapor)
- magnetization $\Leftrightarrow$ Intensity

Synchronization / Correlation

Synchronization

- Transfer fields $\boldsymbol{\phi}_i \in I\!R(n \times n)$ to vectors $\boldsymbol{\phi}_i \in I\!R(n * n)$
- substract mean values, to only calculate the synchronization of the variying fields

(3) $$\boldsymbol{u}_i = \boldsymbol{\phi}_i - < \boldsymbol{\phi}_i > .$$

- The function to derive the synchronization rates is choosen to

(4) $$SyncRate(\boldsymbol{u}, \boldsymbol{v}) = \frac{1}{||\boldsymbol{u}||||\boldsymbol{v}||} \boldsymbol{u}.\boldsymbol{v}$$

Local Correlation

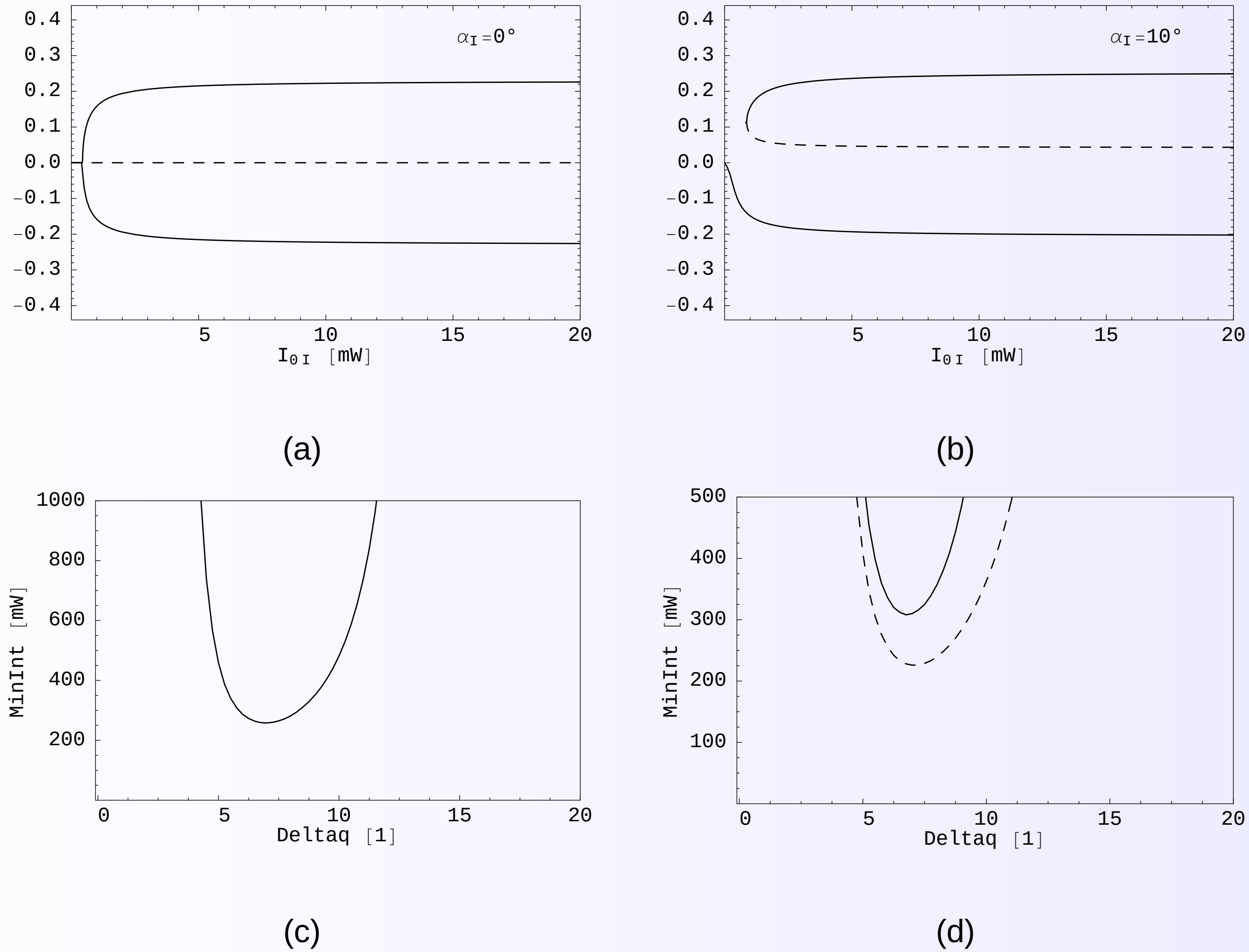
- Time series of one pixel $\phi_i(\boldsymbol{r}, t) \forall \boldsymbol{r}$
- substract mean values (Correlation coefficient from Pearson)
- take last N timesteps $\rightarrow u(\boldsymbol{r}) = (\phi_i(\boldsymbol{r}, t_n))_{n=1..N}$

(5) $$LocCorr(u, v)(\boldsymbol{r}) = \frac{1}{||u||||v||} u.v$$

Numerical Results – One Cell
$\lambda/8$-Case

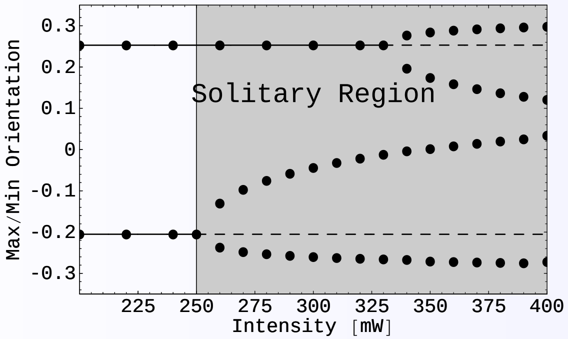
Numerical Results - Solitary Structures Region

- $\lambda/8$-Setting, Bistabel system, Pitchfork-bifurcation

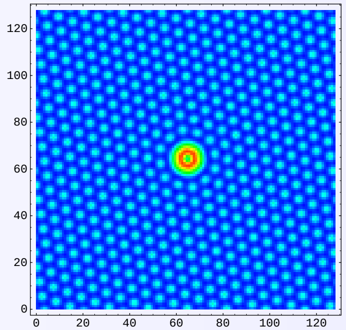


Homogenous state of the system (a) $\alpha = 0^o$, (b) $\alpha = 10^o$. Results of the linear stability Analysis (c) $\alpha = 0^o$, (d) $\alpha = 10^o$ (— positive branch, - - negative branch).

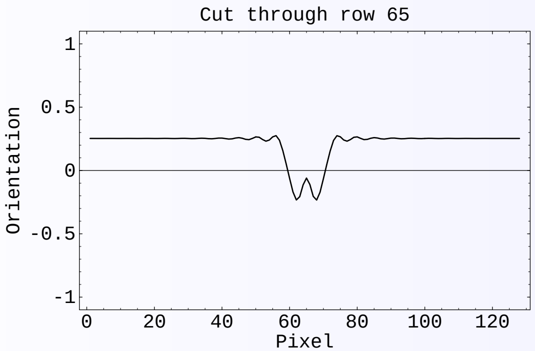
Numerical Results - Solitary Structures Region



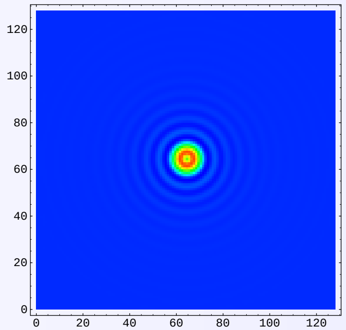
(a)



(b)



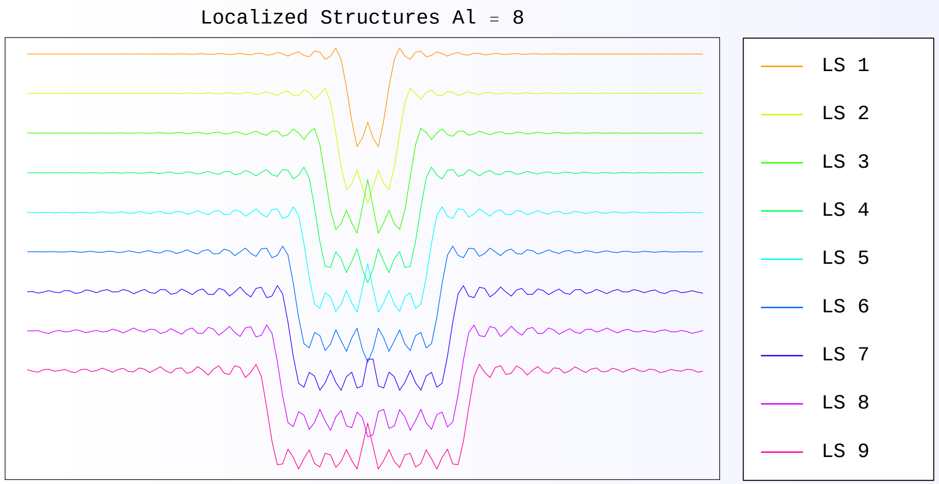
(c)

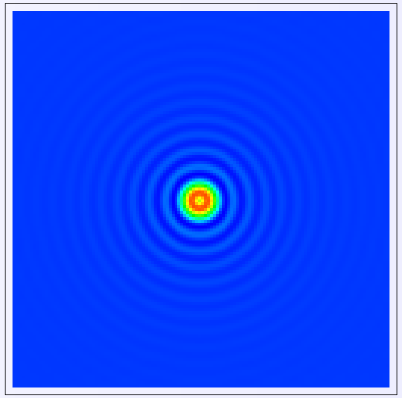


(d)

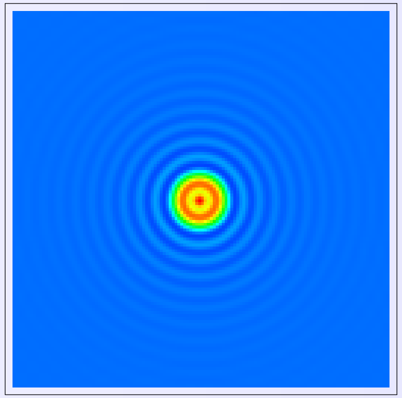
(a) Grey shaded is the region where solitary structures exist. (b) Solitary object in a patterned undergound ($I_{0I} = 400$mW). (c),(d) Cut through, real image of a solitary object on a homogenous underground ($I_{0I} = 300$mW). The parameters are $\gamma = 200$/s, $D = 250\mathrm{mm}^2/\mathrm{s}$, $\alpha_0 = 2.2168$, $1/(2k_0) = 0.468$mm, $L = 15$mm, $d = 0.88$dm, $\alpha_I = 10^o$, $\alpha_{II} = -10^o$, $\varphi_I = \varphi_{II} = \pi/4$, $R_I = R_{II} = 0.99$, $s = 0.0$, $\bar{\Delta} = 6.0$.

Numerical Results - Different Solitary Structures

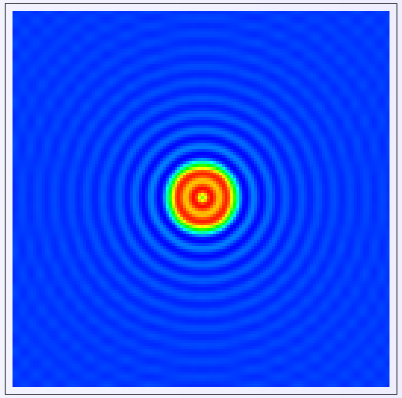




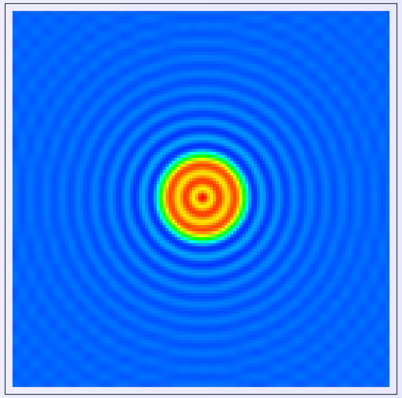
LS 1



LS 2



LS 3

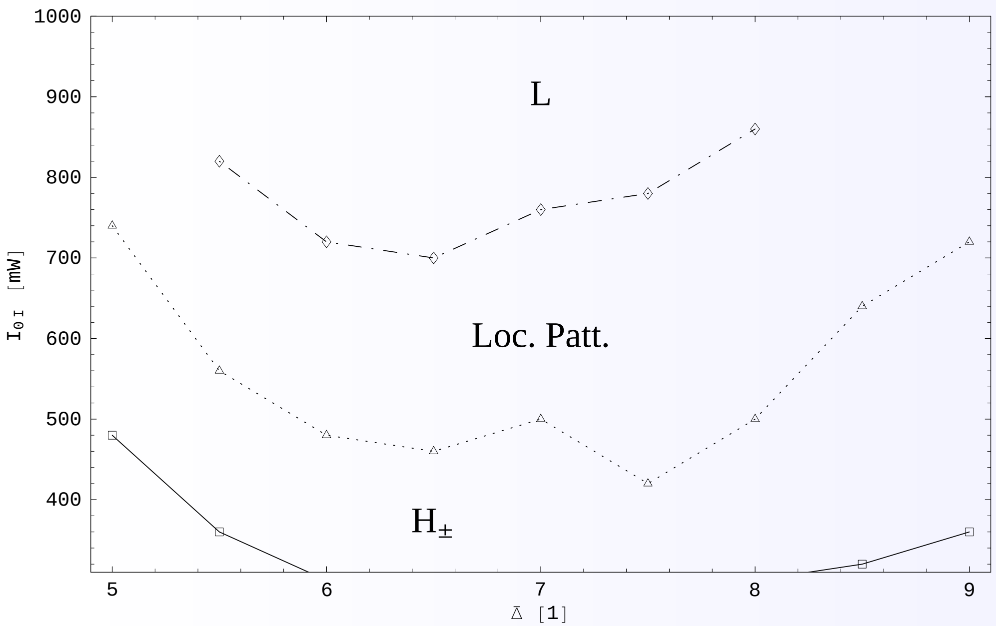


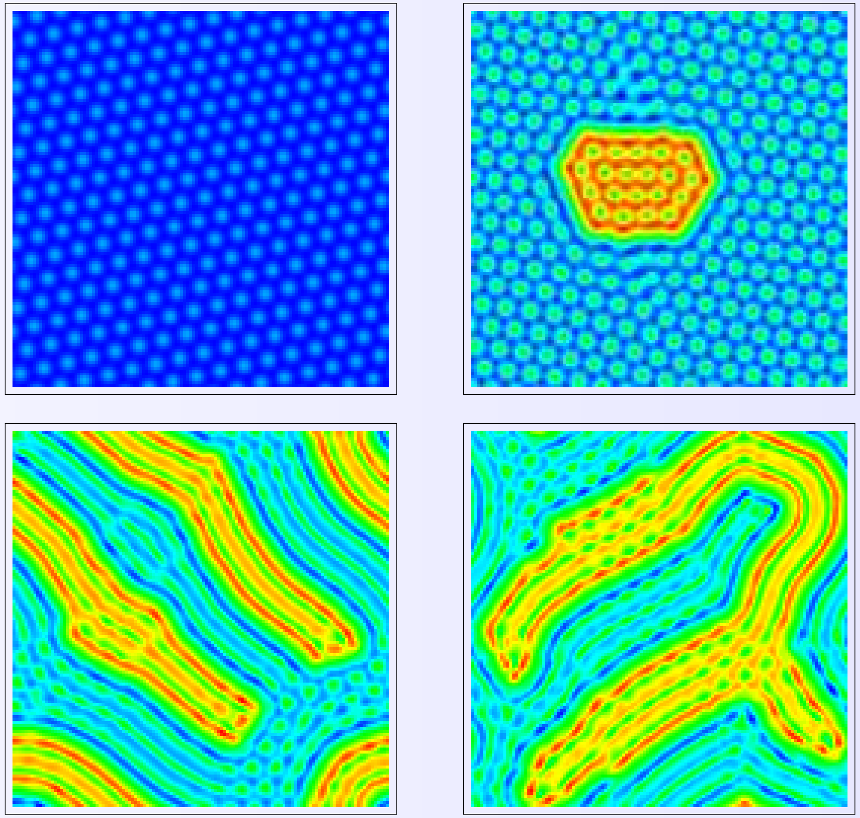
LS 4

- Different solitary structures stable
- Discrete Family of solitary structures

[Pesch et al., PRL 95,143906(2005)]

Numerical Results - Labyrinths





(right) Regions of existance for labyrinths and localized patterns.

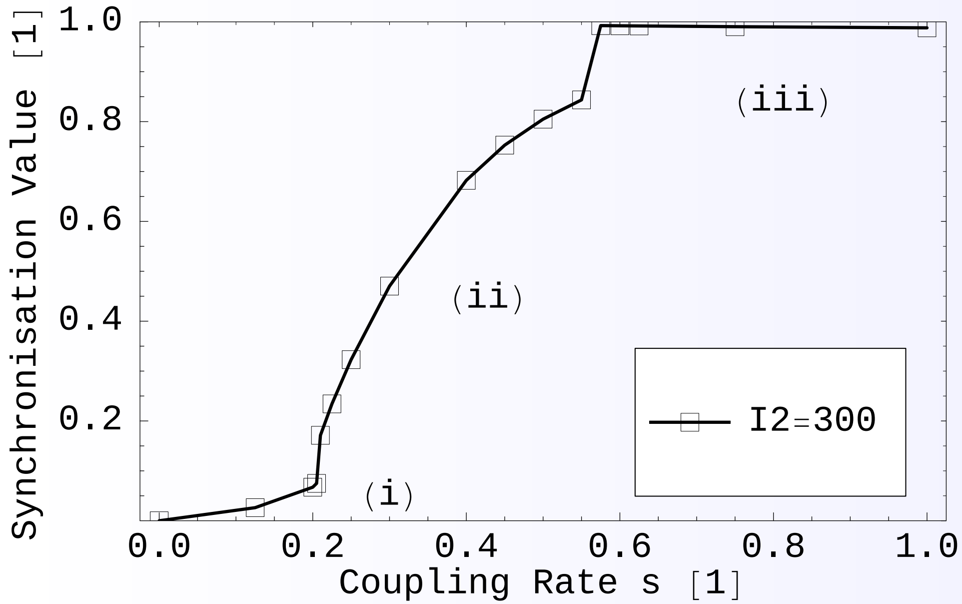
(left) Hexagon, localized pattern (positive and negative Hexagon) and labyrinthine structures.

Parameters : like solitary structures, except $\alpha_I = 0^o$

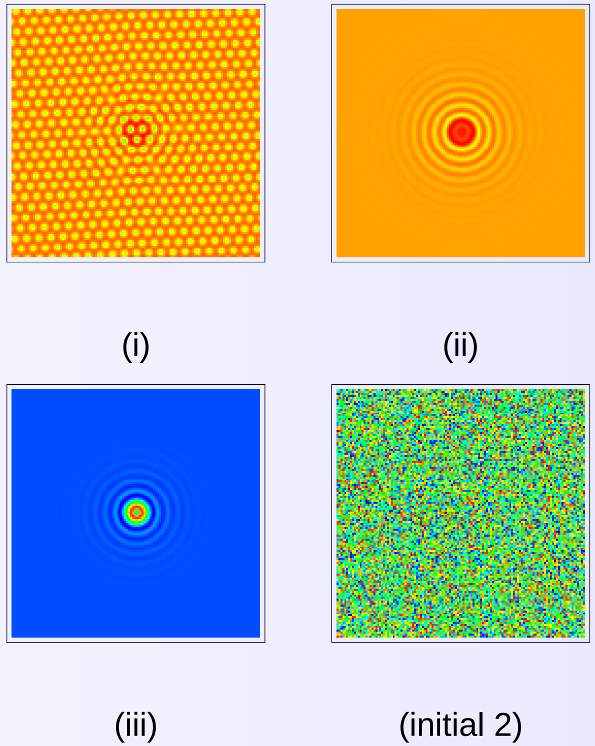
[Diss. Schüttler, 2006]

Numerical Results – Coupling solitons

Numerical Results - Solitary Structures on Random State



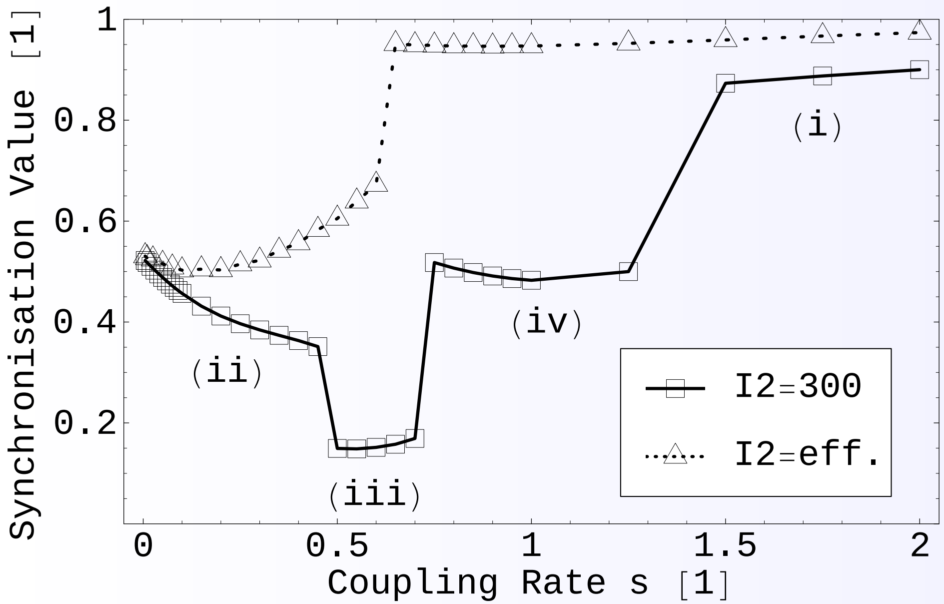
(a)



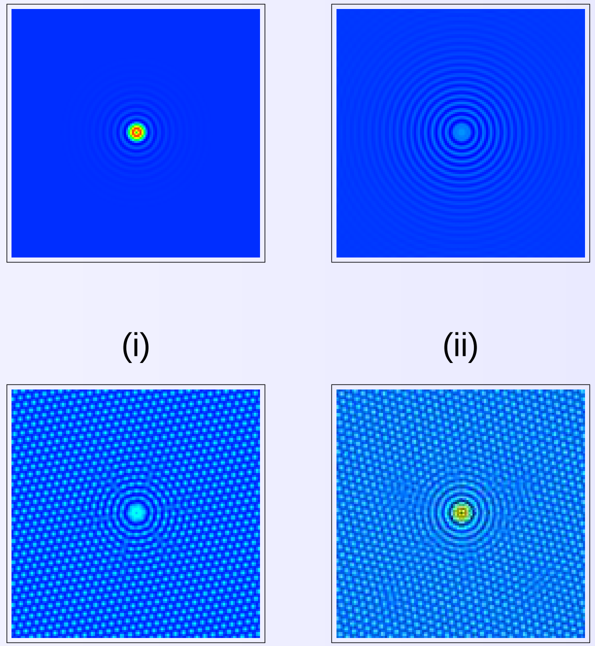
(b)

(a) Synchronisation rate against increasing coupling with $\lambda/8$-plate in both systems, initial condition is a pos. solitary object in system 1 and a random state in system 2. Intensities $I_{0I} = 300$mW, Angles : $\alpha_I = \alpha_{II} = 10^o$. Lines drawn to guide the eye. (b)(i) Weak coupling leads to neg. hexagonal structures with weak imprint of sol. obj. in cell II. (ii) Medium coupling leads to imprint of sol. obj. in cell II on negative background. (iii) Initial positive solitary object of cell I and transmitted sol. obj. for very strong coupling ($I_{0II} = 300$mW).

Numerical Results - Transmission of Solitary Objects



(a)

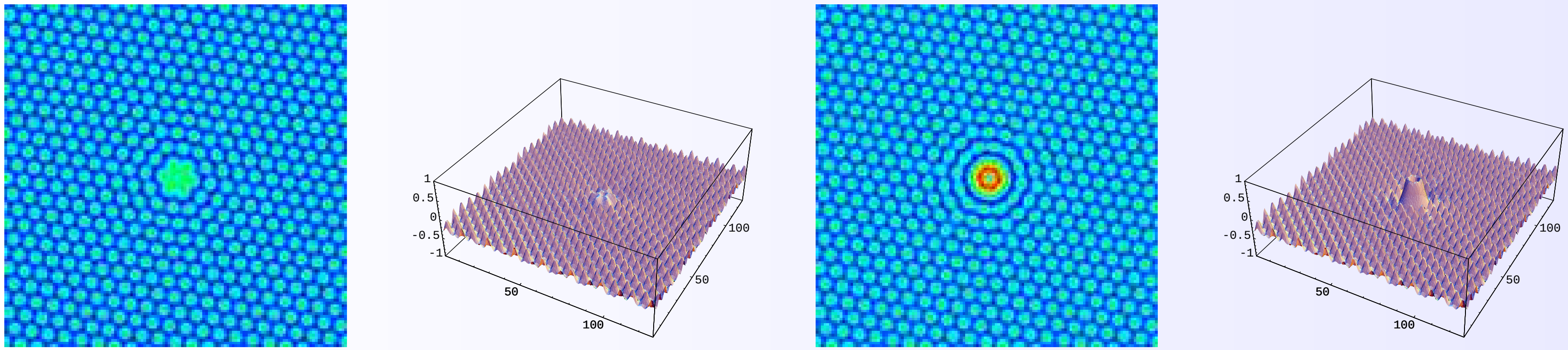


(i) (ii) (iii) (iv)

(b)

(a) Synchronisation rate against increasing coupling with $\lambda/8$-plate in both systems, initial condition is a pos. solitary object in system 1 and a homogenous state in system 2. Intensities $I_{0I} = 300$mW, Angles : $\alpha_I = \alpha_{II} = 10^o$. Lines drawn to guide the eye. (b)(i) Initial positive solitary object of cell I and transmitted sol. obj. for very strong coupling ($I_{0II} = 300$mW) or medium coupling ($I_{0II} = I_{II\text{eff}}$). (ii) Weak coupling leads to weak imprint of sol. obj. in cell II. (iii) Medium coupling leads to formation of hexagonal structures with weak imprint of sol. obj.. (iv) Strong coupling leads to hexagonal structures with transmitted sol. obj..

Numerical Results - Transmission of Solitary Objects

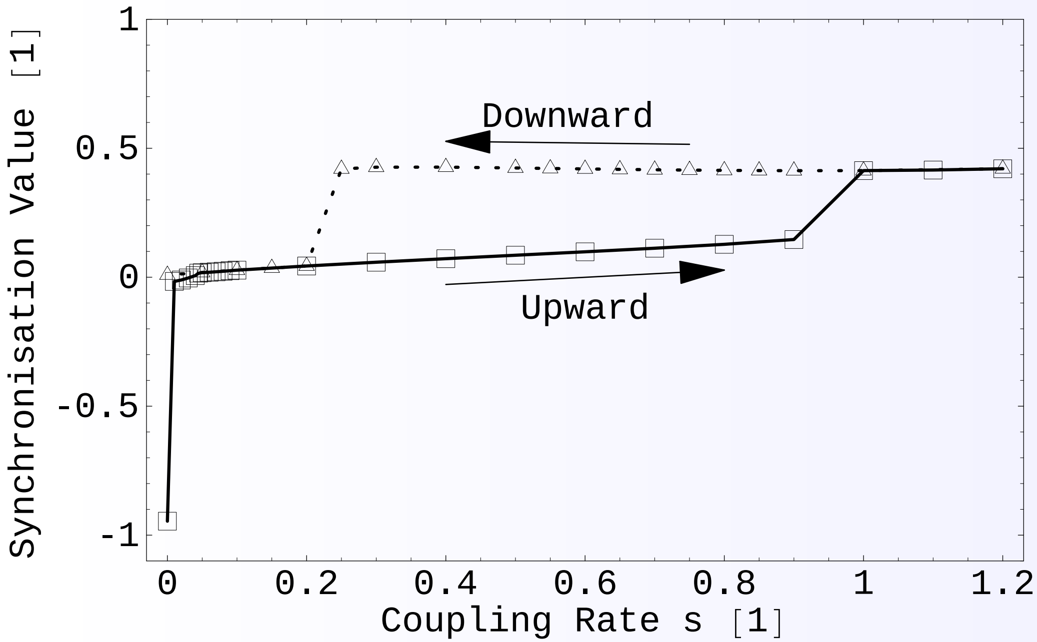


(a) (b) (c) (d)

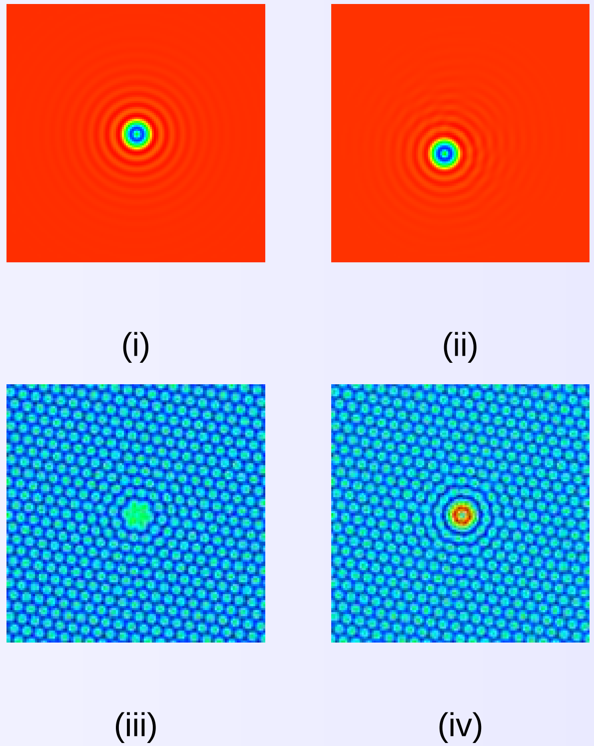
(a) Color coded 2D-plot of cell II with $s = 0.90$, (b) inverted corresponding 3D-plot (for better visability).

(c) Color coded 2D-plot of cell II with $s = 1.00$, (d) inverted corresponding 3D-plot.

Numerical Results - Pos. Sol. Obj. on Neg. Sol. Obj.



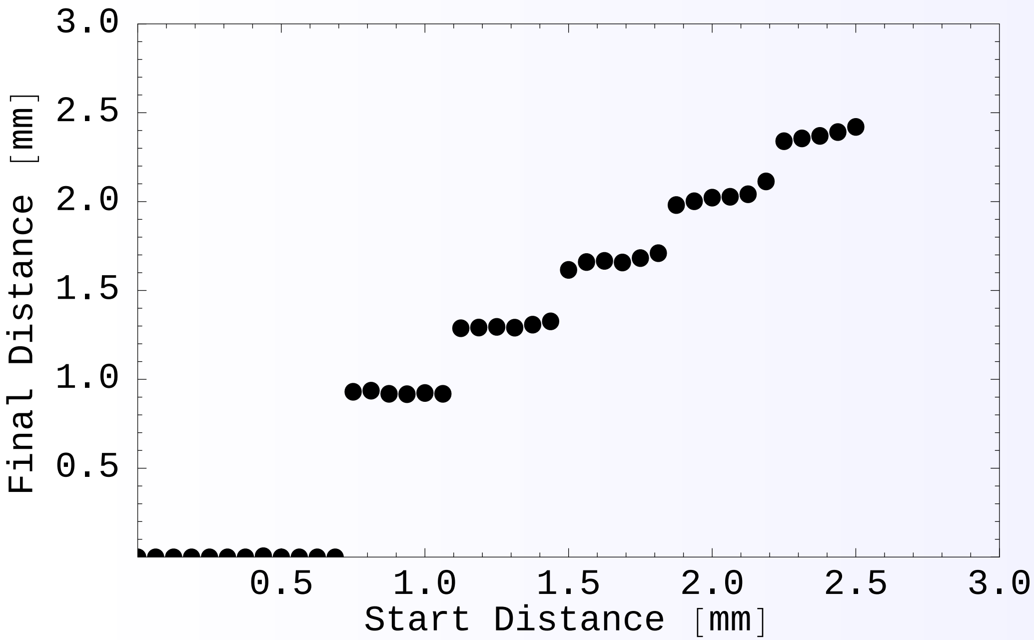
(a)



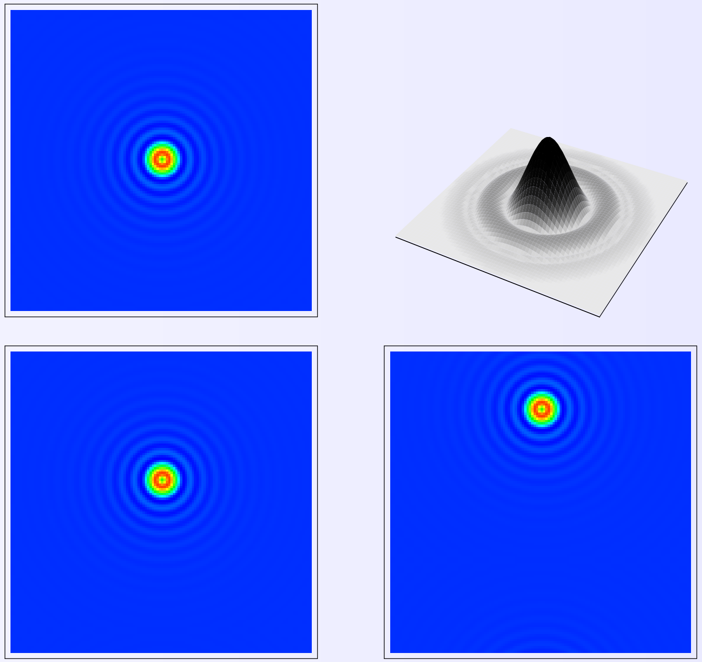
(b)

(a) Synchronisation rate with increasing coupling with $\lambda/8$-plate in both systems, initial condition is a pos. solitary object in system 1 and a negative one in system 2. Intensities $I_{0I} = I_{0II} = 300$mW, Angles : $\alpha_I = -\alpha_{II} = 10^o$. Lines drawn to guide the eye. (b) (i) Initial negative solitary object of cell 2, (ii) “kicked” solitary objects with coupling strength $s = 0.03$, (iii) final field with $s = 0.90$ and (iv) final field with $s = 1.00$.

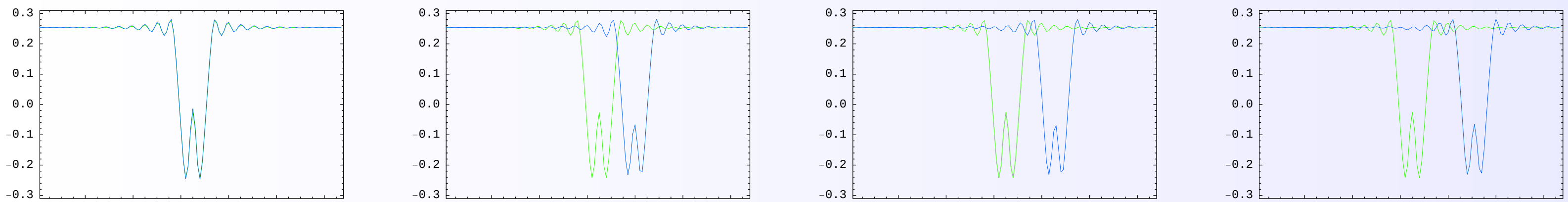
Numerical Results - Coupling identical solitons $s = 0.02$



Overview of stable distances

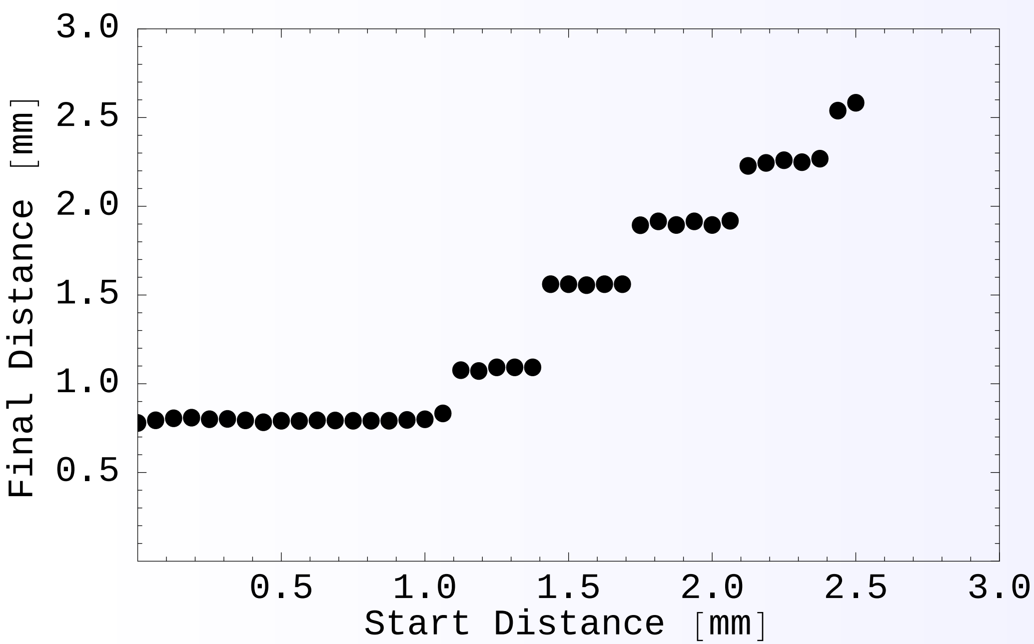


Initial fields : (top) Cell I (bottom) Cell II

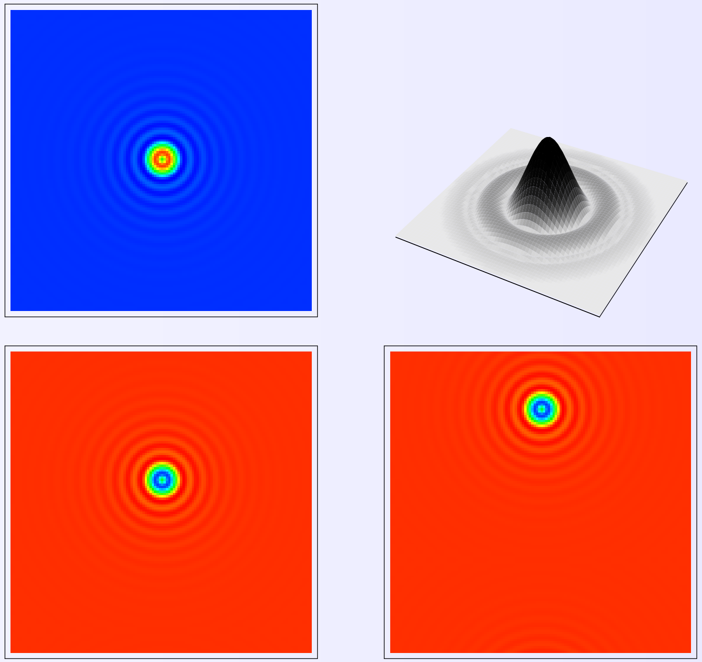


1D-Cuts of stable distances along the connection line (green = Cell I, blue = Cell II)

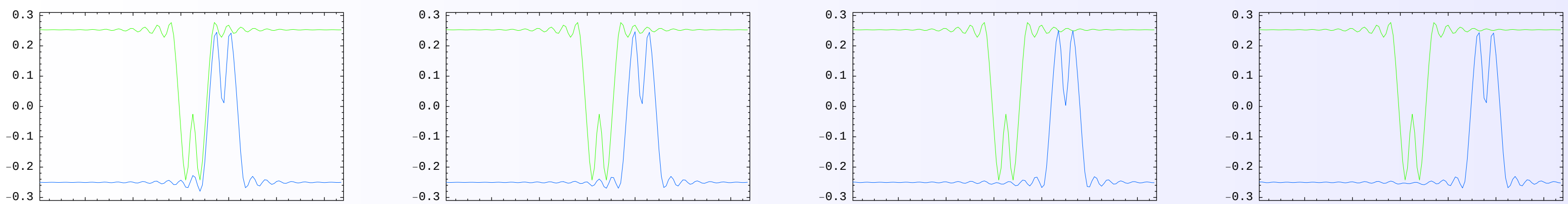
Numerical Results - Coupling inverse solitons $s = 0.03$



Overview of stable distances



Initial fields : (top) Cell I (bottom) Cell II



1D-Cuts of stable distances along the connection line (green = Cell I, blue = Cell II)

Numerical Results - Different Solitons - Mechanisms

- Coupling of solitons yields three basic mechanisms :
  - **Destroying** - Soliton in the second cell is destroyed.

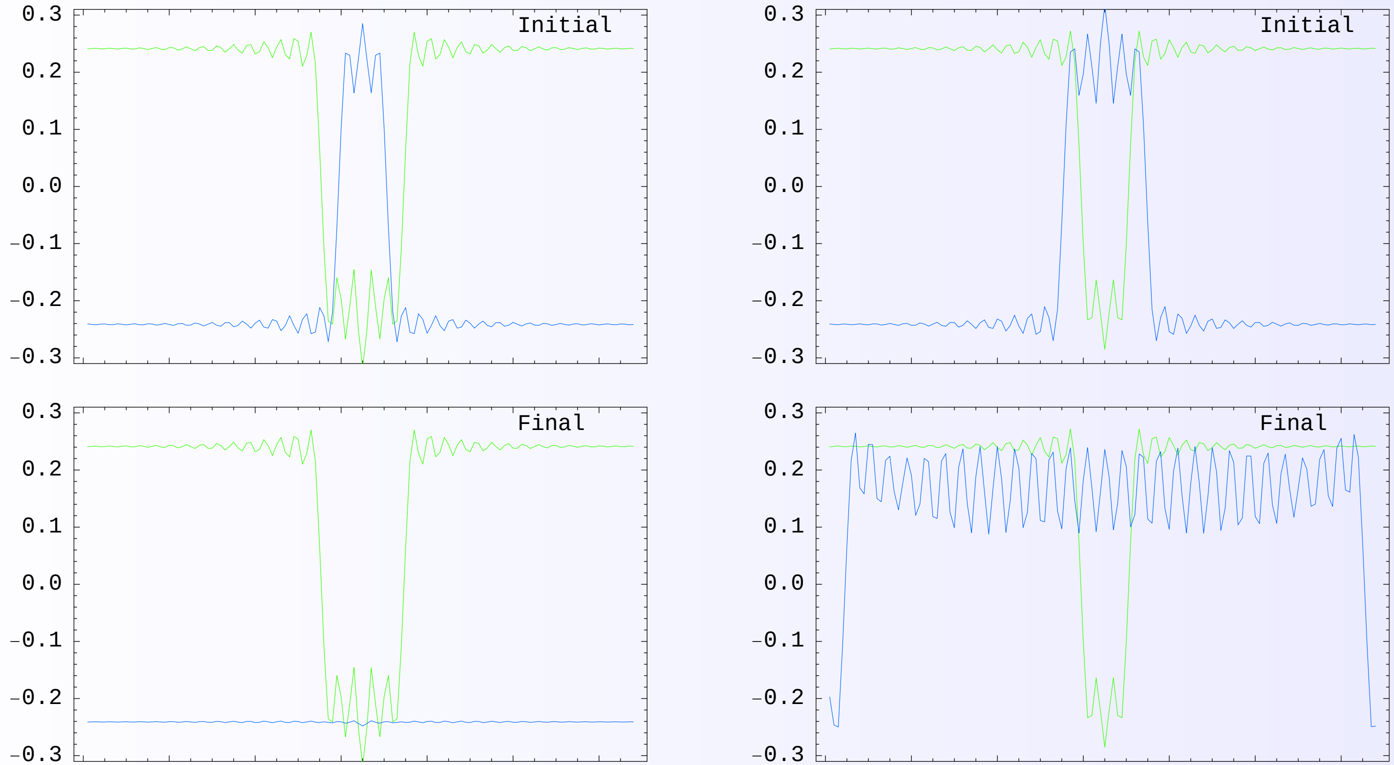
    Resulting pattern :
    - homogenous state
    - generic hexagon
  - **Moving** - Soliton in the second cell is moved.

    Resulting pattern :
    - shape of soliton in the second cell unchanged, position changed
    - shape of soliton in the second cell slightly changed, position changed
  - **Morphing** - Soliton in the second cell is morphed.

    Resulting pattern :
    - soliton in the second cell synchronizes with first soliton

Numerical Results - Different Solitons - Destroying

- solitons of different sizes
- solitons with different direction



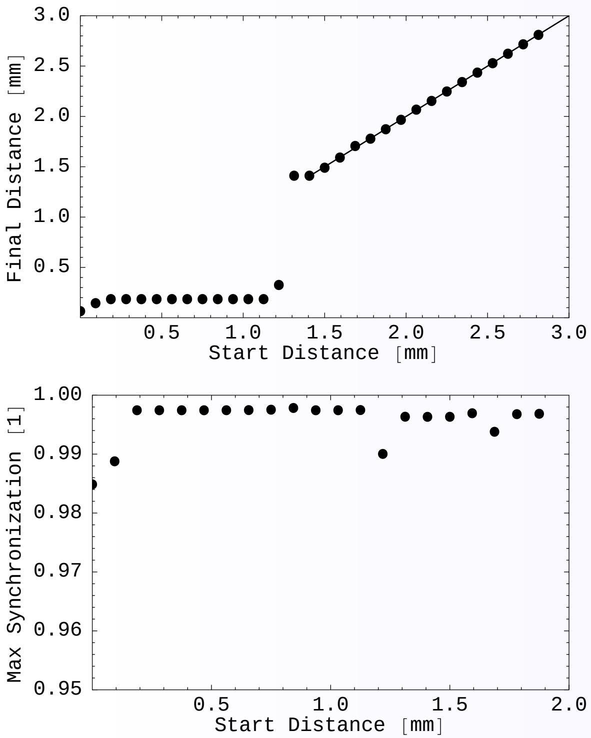
Coupling of different solitons with different direction. (left) LS4 positive - LS2 negative, (right) LS2 positive - LS4 negative.

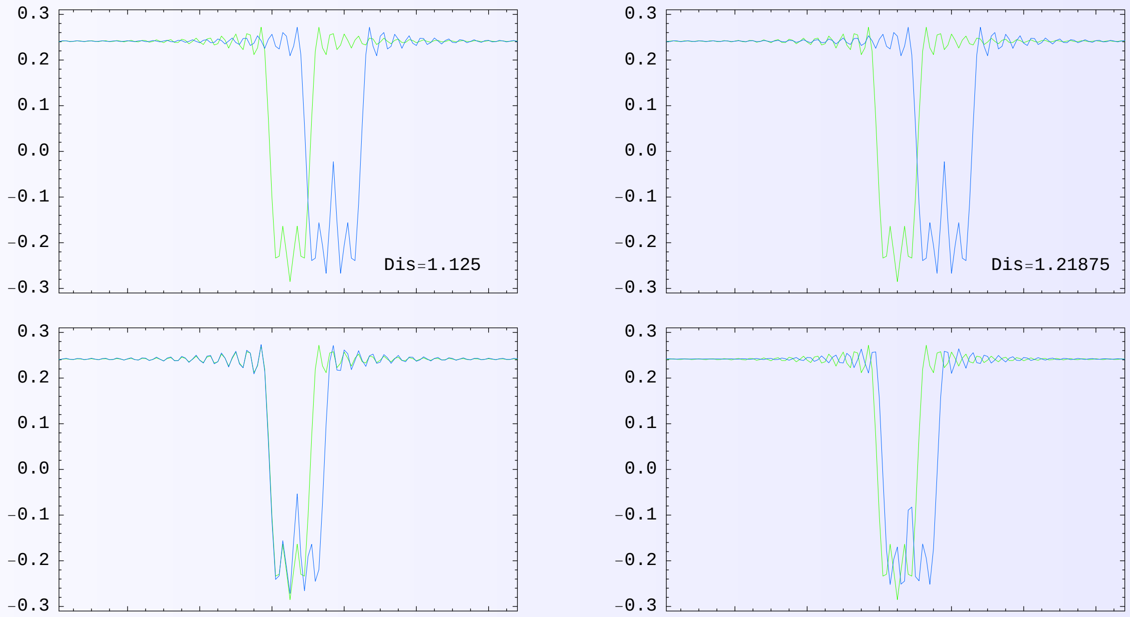
Parameters : $\bar{\Delta} = 6.0$, $\alpha_I = -\alpha_{II} = 5^o$, $I_I = I_{II} = 300$mW. (green = Cell I, blue = Cell II)

Numerical Results - Different Solitons - Moving

- LS1 $\pm$ - LS1 $\pm$
- different solitons, same direction
- small couplings

- LS2 - LS3

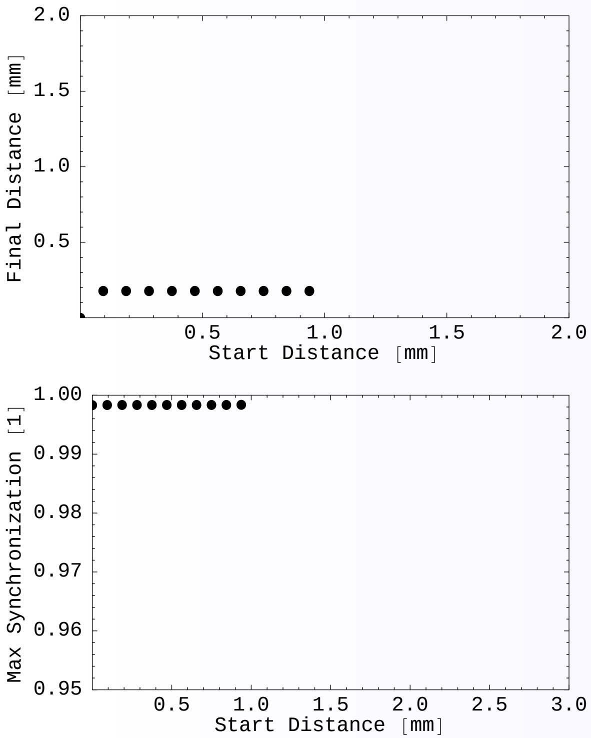


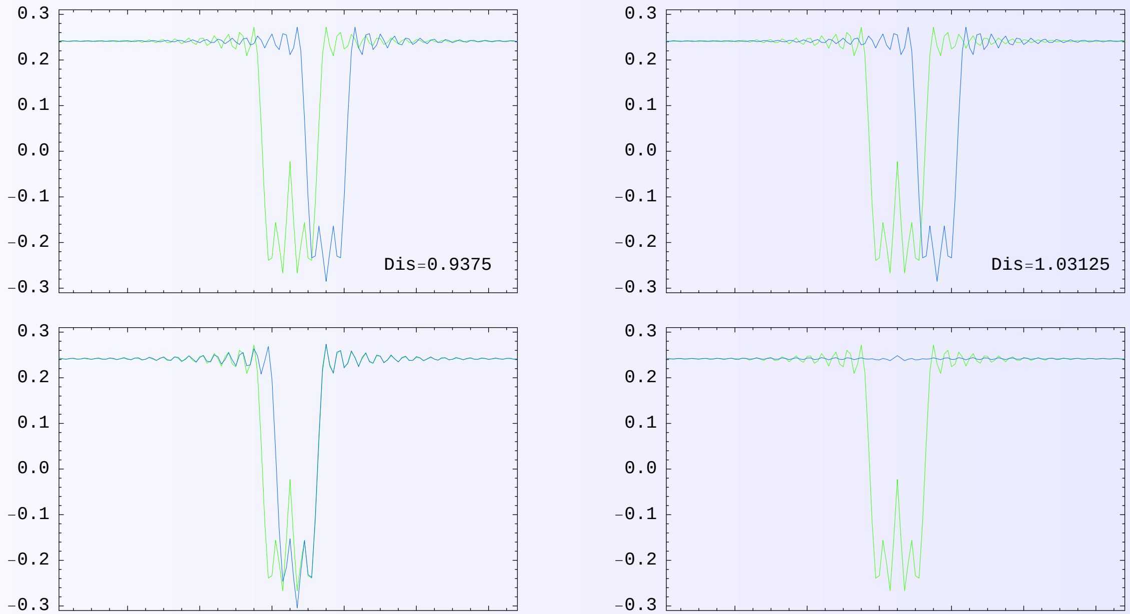


(left) Stable distances and max synchronization with initial solitons in the second cell. (right) 1D-Cuts at the connection line (stable distance, max deformation). Parameters : $\bar{\Delta} = 6.0$, $\alpha_I = -\alpha_{II} = 5^o$, $I_I = I_{II} = 300$mW. (green = Cell I, blue = Cell II)

Numerical Results - Different Solitons - Moving

- LS1 $\pm$ - LS1 $\pm$
- different solitons, same direction
- small couplings
- LS3 - LS2

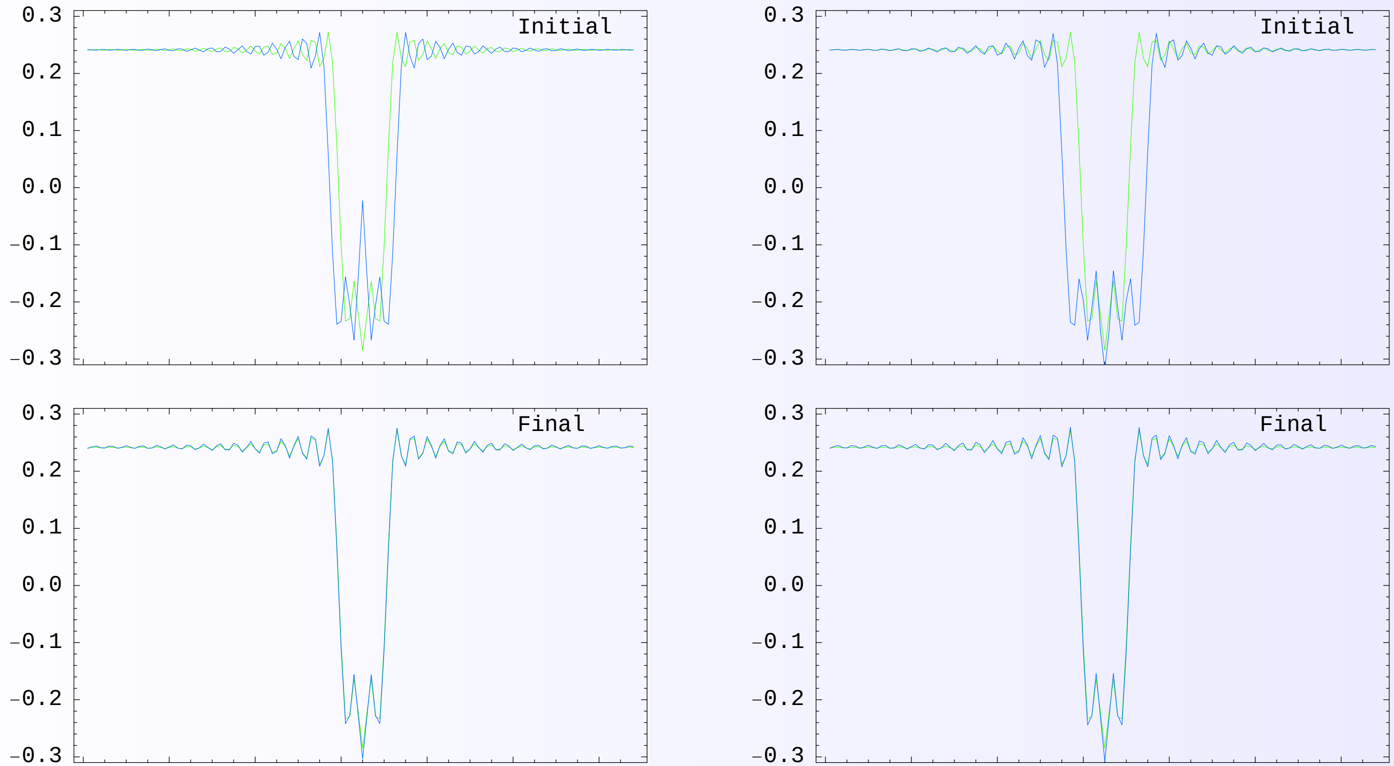




(left) Stable distances and max synchronization with initial solitons in the second cell. (right) 1D-Cuts at the connection line (stable distance, max deformation). Parameters : $\bar{\Delta} = 6.0$, $\alpha_I = -\alpha_{II} = 5^o$, $I_I = I_{II} = 300$mW. (green = Cell I, blue = Cell II)

Numerical Results - Different Solitons - Morphing

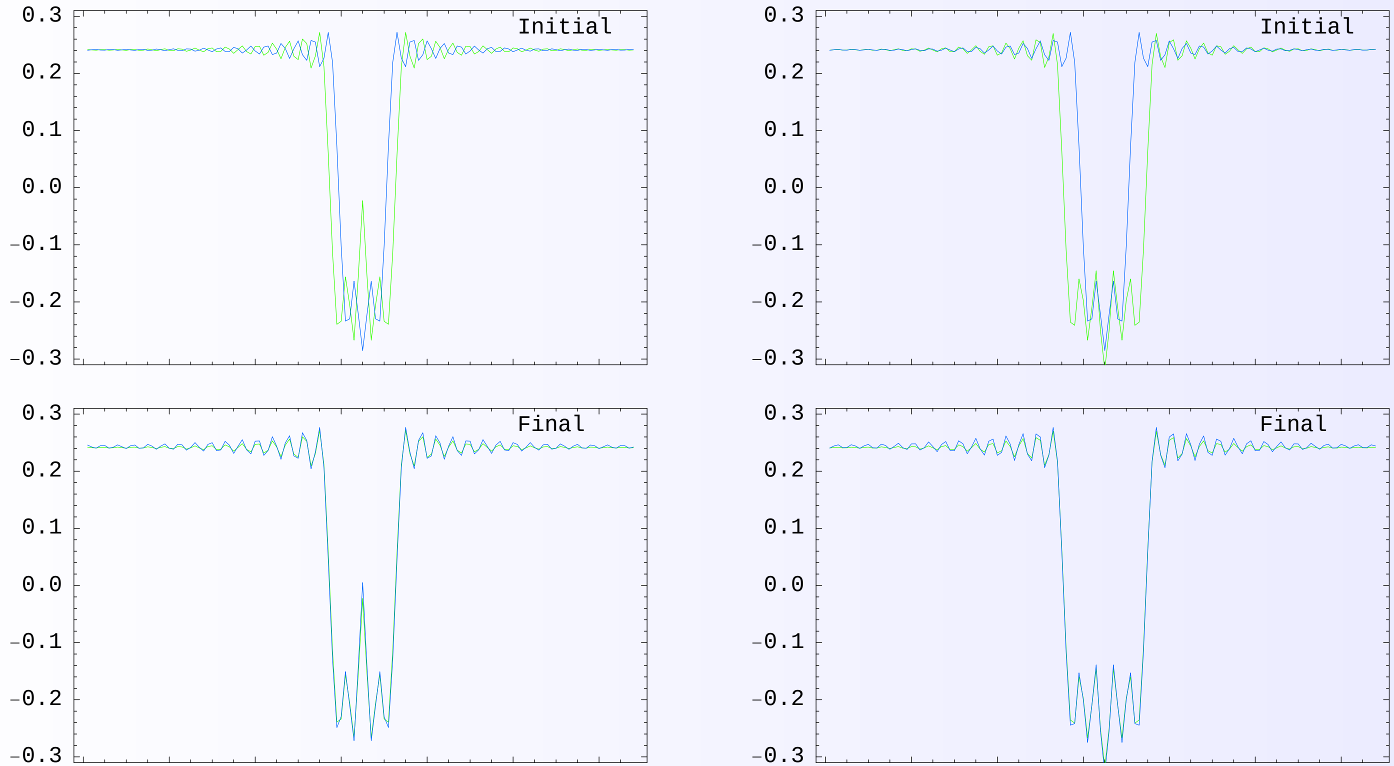
- different solitons
- same direction



Morphing of solitons. (left) LS2 pos. - LS3 pos. $s = 0.010$, (right) LS2 pos. - LS4 pos. $s = 0.030$. Parameters : $\bar{\Delta} = 6.0$, $\alpha_I = -\alpha_{II} = 5^o$, $I_I = I_{II} = 300$mW. (green = Cell I, blue = Cell II)

Numerical Results - Different Solitons - Morphing

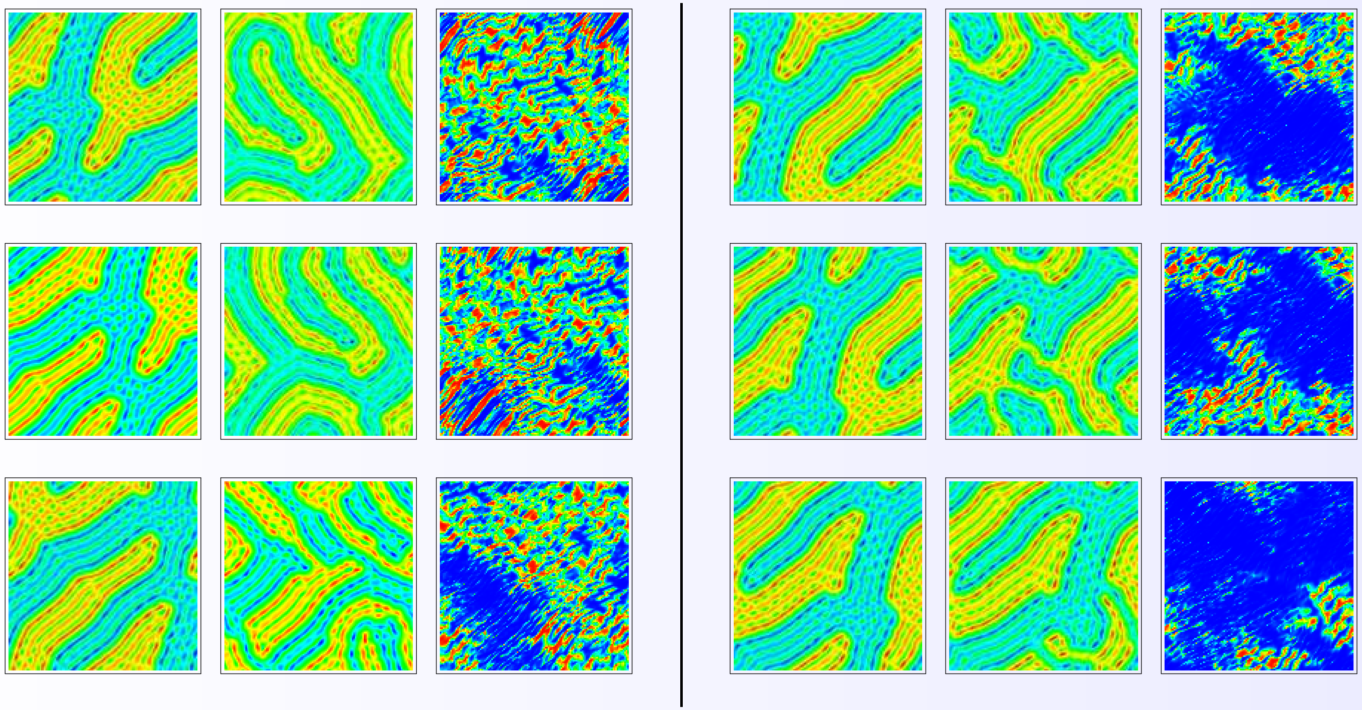
- different solitons
- same direction

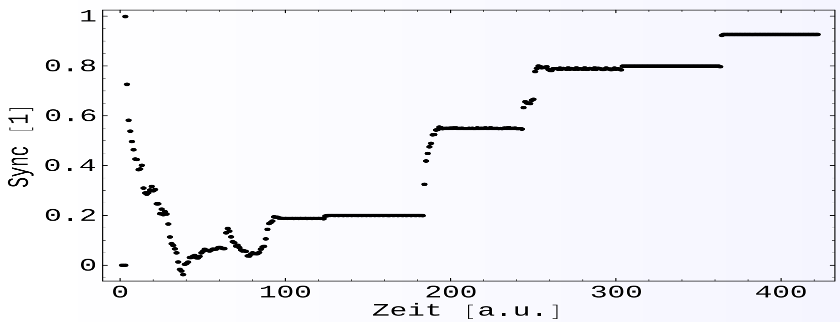


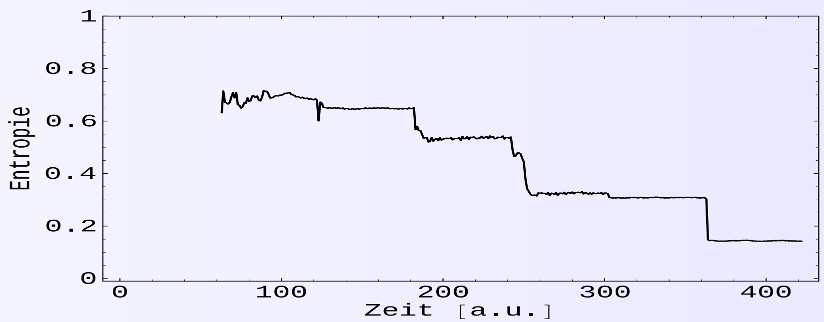
Morphing of solitons. (left) LS3 pos. - LS2 pos. $s = 0.035$ and (right) LS4 pos. - LS3 pos. $s = 0.045$. Parameters : $\bar{\Delta} = 6.0$, $\alpha_I = -\alpha_{II} = 5^o$, $I_I = I_{II} = 300$mW. (green = Cell I, blue = Cell II)

Numerical Results – Labyrinth coupling

Numerical Results - Labyrinth coupling







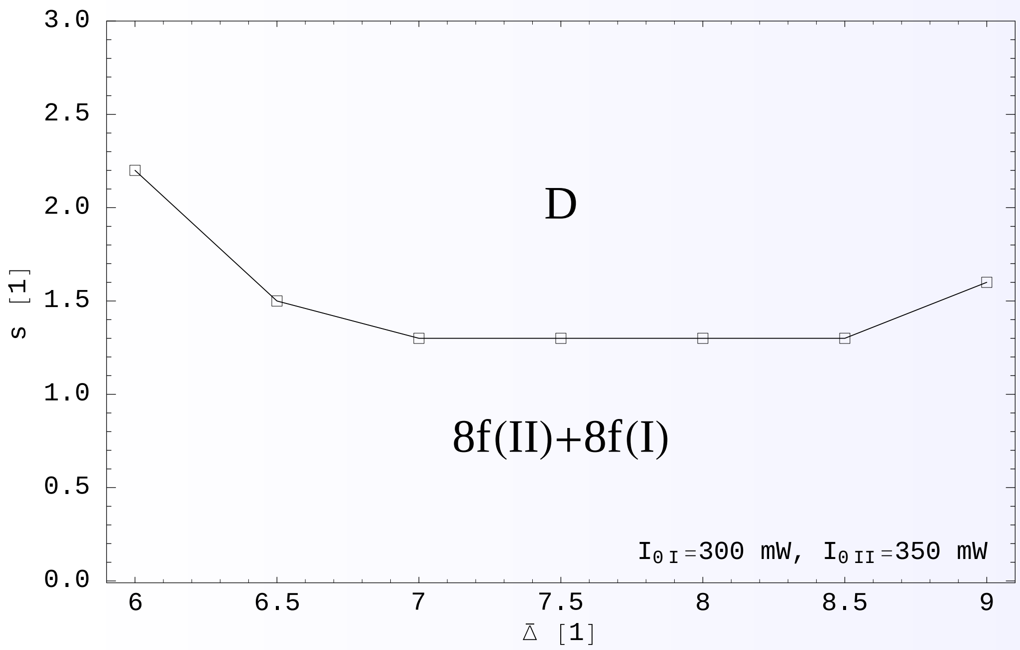
$$Entropy = -\sum_k |c_k^{(2)}| \log\left(\frac{|c_k^{(1)}|}{|c_k^{(2)}|}\right) \qquad \sum_k |c_k^{(i)}| = 1$$

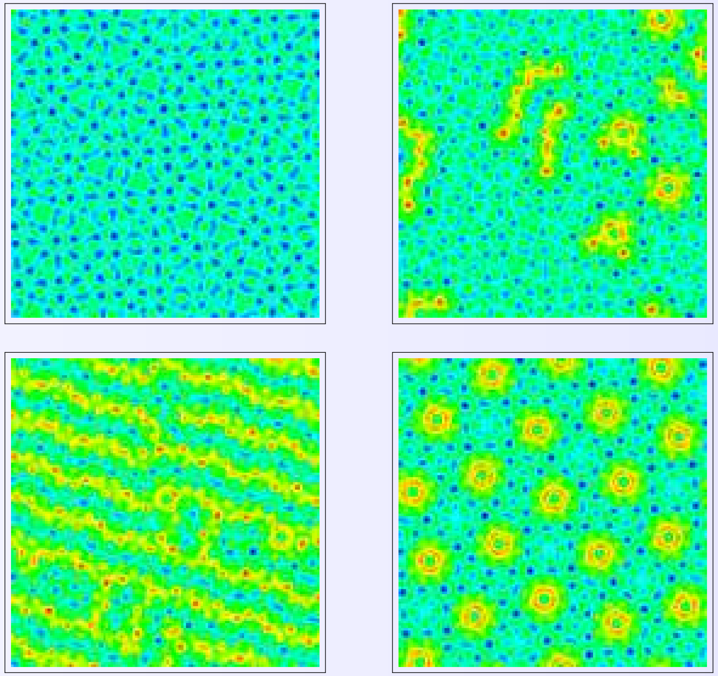
Numerical Results – Domains

$\lambda/4.\lambda/8$

Numerical Results - Domains

- L4.L8 case
- second cell bistable





(left) Region of existance of domain structures (right) Developing of domains with increasing coupling (top left) $8f(II) + 8f(I)$,$s = 2.2$, (top right) domains and fronts D,$s = 2.5$ (bottom left) labyrinthine structure $s = 3.0$. (Parameter : $\bar{\Delta} = 6.0$, $I_{0I} = 300$mW and $I_{0II} = 290$mW). (bottom right) pure circular domains (Parameter : $\bar{\Delta} = 5.5$, $I_{0I} = 355$mW und $I_{0II} = 345$mW).

Conclusions / Outro

Conclusions / Outro

- Coupled two transverse pattern forming nonlinear optical devices
- Transmission of solitary objects
- Interaction of pos. and neg. solitary objects
- Moving of solitary objects
- Destroying of solitary objects
- Morphing of solitary objects
- Coupling of labyrinthine structurse $\longrightarrow$ Synchronisation
- Coupling of two different regluar patterns yields domain structures

Conclusions / Outro

- Coupled two transverse pattern forming nonlinear optical devices
- Transmission of solitary objects
- Interaction of pos. and neg. solitary objects
- Moving of solitary objects
- Destroying of solitary objects
- Morphing of solitary objects
- Coupling of labyrinthine structurse $\longrightarrow$ Synchronisation
- Coupling of two different regluar patterns yields domain structures

Thank you for listening!

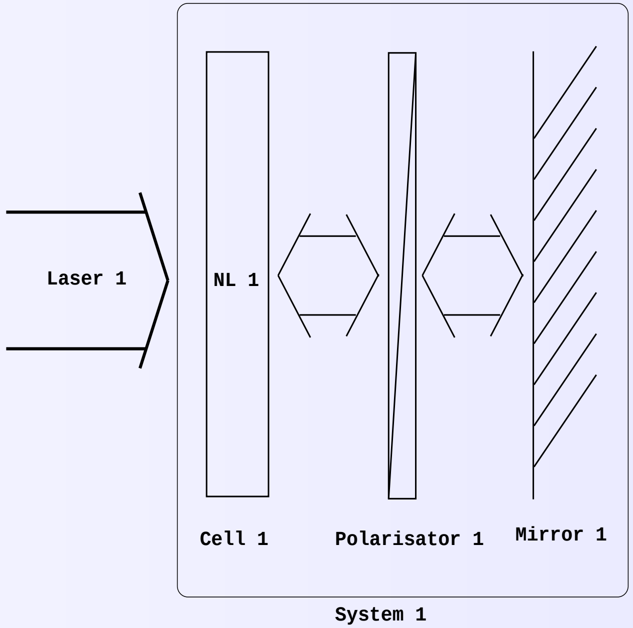
Theoretical Analysis - The Pumprates final

Free propagation	Propagation in the medium	Matrixoperator λ/x-plate
$\mathcal{P}_{FP}(x) = \exp[-ix\nabla_\perp^2/2k_0]$	$\mathcal{P}_{\pm,M}(x,w) = \exp[i\alpha_0\bar{\Delta}x(1\mp w)]$	$M(\phi,\alpha)$

- Matrixoperator for the medium given by

(6) $$P_M(x,w) = \begin{pmatrix} \mathcal{P}_{+,M}(x,w) & 0 \\ 0 & \mathcal{P}_{-,M}(x,w) \end{pmatrix}$$

- Combining the operators easily gives the pumprates at certain points



$$P_\pm \sim |\mathcal{E}_{\pm,f}(0,t)|^2 + |\mathcal{E}_{\pm,b}(0,t)|^2$$

(7) $$\sim |\mathcal{E}_\pm^0|^2 + R|M(\phi,\alpha)\mathcal{P}_{FP}(2d)P_M(L,w)\mathcal{E}_\pm^0|^2$$

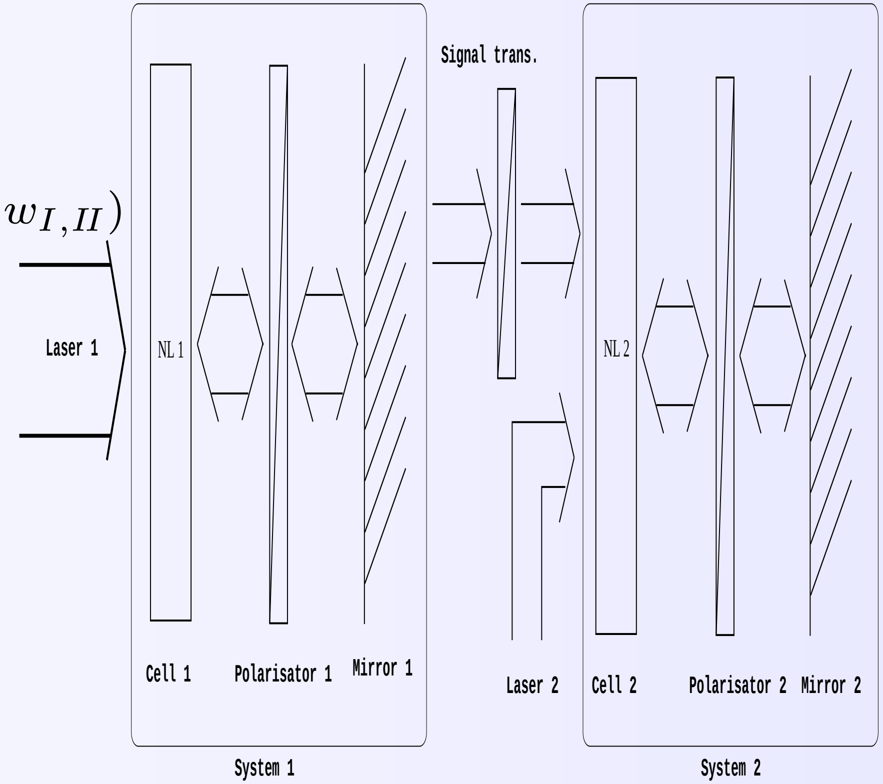
[Grosse-Westhoff et al., J.Opt.B. 2,386 (2000)]

Theoretical Analysis - Inserting the Coupling

- Uncoupled systems gives two differential equations

$$\dot{w}_{I,II} = -(\gamma - \mathrm{Diff}\nabla_\perp^2) w_{I,II} + P_{+I,II}(1 - w_{I,II}) - P_{-I,II}(1 + w_{I,II})$$

$$= \mathrm{NL}_{I,II}(w_{I,II}) \quad (8)$$



- Coupling the systems with **coupling strength** k
- Input in cell two is

$$\mathcal{E}^0_{\pm,II} + \mathcal{E}_{\pm,I,b}(L,t) \quad (9)$$

- Neglegting interference terms

$\mathcal{E}^{0*}_{\pm,II}\mathcal{E}_{\pm,I,b}(L,t) \to 0, \mathcal{E}^0_{\pm,II}\mathcal{E}_{\pm,I,b}(L,t)^* \to 0$

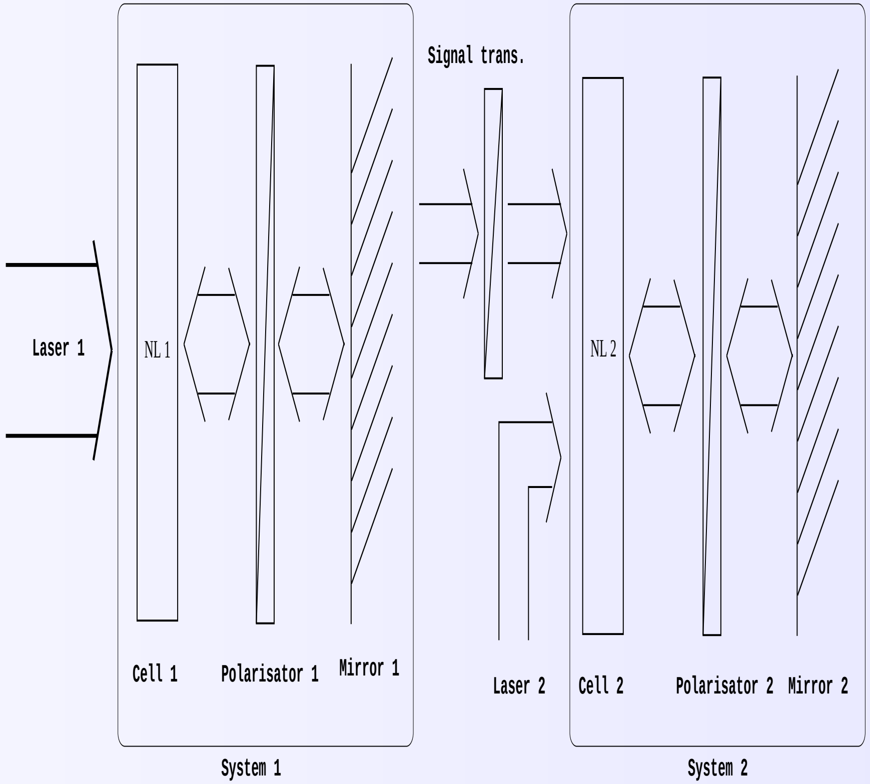
Theoretical Analysis - Inserting the Coupling

- Coupled System gives rise to differential eq. system

$$\dot{w}_I = \mathrm{NL}_I(w_I) \tag{10}$$

$$\dot{w}_{II} = \mathrm{NL}_{II}(w_{II}) + k\mathrm{NL}_{III}(w_I, w_{II}) \tag{11}$$

  - $\mathrm{NL}_{II}(w_{II})$: Terms due to laser 2
  - $\mathrm{NL}_{III}(w_I, w_{II})$: Terms due to the coupling



- "Coupling" nonlinearity is

$$\mathrm{NL}_{III}(w_I, w_{II}) \sim |\mathcal{E}_{\pm,I,b}(L,t)|^2 + R|M(\phi_{II}, \alpha_{II})\mathcal{P}_{FP}(2d)P_M(L, w_{II})\mathcal{E}_{\pm,I,b}(L,t)|^2$$