Passive tracers and passive scalar in 3d incompressible turbulence

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Contents

Introduction

What is a passive scalar?

- context: Navier-Stokes turbulence $\mathbf{v}(\mathbf{x}, t)$
- additional scalar field $\theta(\mathbf{x}, t)$
- advected by the velocity field but does not contribute to its dynamics
- is subject to diffusion
- examples are:
  - temperature fields in liquids or gases
  - dissolved chemicals of low concentration
- interest for passive scalar lies in engineering and physics
- coupled to understanding mixing properties, combustion and chemical reactions

Passive scalar dynamics

- Navier-Stokes equations

$$\partial_t v_i + v_j \partial_j v_i = -\partial_i p + \nu \partial_{jj} v_i$$
$$\partial_i v_i = 0$$

- the passive scalar advection-diffusion equation

$$\partial_t \theta + v_i \partial_i \theta = \kappa \partial_{jj} \theta$$

- κ: passive scalar diffusity, Schmidt-Number $Sc = \nu/\kappa$
- θ: fluctuation of the passive scalar around a constant mean value Θ
- the full passive scalar field is:

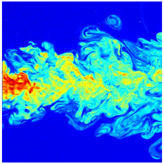
$$T = \Theta + \theta(\mathbf{x}, t)$$
$$\Theta = const.$$

- passive scalar energy

$$E_\theta = \int \theta^2(\mathbf{x}) \mathrm{dV}$$

$\Rightarrow$ conserved in the limit $\kappa = 0$

- passive scalar characteristics
  - equation contains only linear terms
  - dynamics governed by the velocity fields
  - produces rich dynamics
  - highly intermittent
  - *ramp-cliff* or *mesa-canon* structures



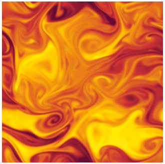


Figure: experiment $\leftrightharpoons$ simulation

Structure functions, spectra & transport

- like kinetic energy spectra, the scalar energy spectrum is believed to show power law scaling in the inertial range.

$$E_\theta(k) \sim k^{-5/3}$$

- scalar dissipation rate is $\chi = \kappa \langle (\nabla\theta)^2 \rangle$
- structure function of order i for a field f is defined as:

$$S_i^f = \langle \mid \delta_{\mathbf{l}}(f) \mid^i \rangle = \langle \mid f(\mathbf{x}) - f(\mathbf{x}+\mathbf{l}) \mid^i \rangle$$

- analogon to Kolmogorovs 4/5-law:

$$S_{12} = \langle \mid \delta_{\mathbf{l}}\mathbf{u} \mid\mid \delta_{\mathbf{l}}\theta \mid^2 \rangle = -\frac{4}{3}\chi l$$

$\Rightarrow$ this last result is exact

The Lagrangian point of view

- instead of using a fixed frame of reference we are now moving along with the velocity fields
- the transformation to Lagrangian coordinates is

$$\mathbf{x} \longrightarrow \mathbf{X}(\mathbf{x}_0, t)$$

$$\frac{d}{dt}\mathbf{X}(\mathbf{x}_0, t) = \mathbf{u}(\mathbf{X}(\mathbf{x}_0, t), t)$$

- $\mathbf{x}_0 = \mathbf{X}(\mathbf{x}_0, 0)$: initial position
- passive scalar equation in Lagrangian coordinates for $\kappa = 0$

$$\partial_t \theta + v_i \partial_i \theta = 0 \qquad \Rightarrow \qquad \frac{d}{dt}\theta = 0$$

Why a Lagrangian description?

- should eliminate the mixing effect of the velocity fields
- no mesa-canon events
- only diffusive effects for finite κ

Software & simulations

The simulation code

- the passive scalar module is an extension to the existing simulation code of H. Homann, which models full 3D turbulence and implements handling of tracer particles
- introduces an extra computational effort of about 30%
- a pseudo-spectral scheme is used for advancing velocity as well as as passive scalar fields
  - derivatives are calculated in Fourier-space using the FFTW library
  - products are calculated in real space
  - spectral method enforces periodic boundary conditions
- timestepping via a Runga-Kutta integrator of 3*rd* order

Forcing

- a substancial amount of experiments use grid generated turbulence
- passive scalar is forced via a temperature gradient
- numerically this works via changing the mean value as follows

$$\partial_i \Theta = g_i, \qquad \mathbf{g} \in \mathbb{R}^3$$
$$\Rightarrow \partial_t \theta + v_i(\partial_i \theta + g_i) = \kappa \partial_{jj} \theta$$

- for comparison a second driver is implemented
- this driver freezes low wave number mode shells in Fourier space

The simulations

- the simulations were carried out on a 64 CPU Opteron cluster
- the runs falls apart into two phases:

  pre-simulation both velocity and passive scalar are decaying, no driving, timestep adjusted to CFL criterion

  simulation driving is applied, tracers, fixed timestep
- grid extension is 2π
- initial condition for θ: assign random values to low wave number modes

Results

The impact of driving

- resolution: 256^3
- $\kappa = \nu$
- goal: test the effect of the forcing scheme if any
- driving:
  - frozen shells
  - gradient
- same initial condition

Passive scalar field

- snapshot of the passive scalar field
- about one Large Eddy Turnover after introducing the scalar
- the scalar has settled → almost no effect of the initial condition left

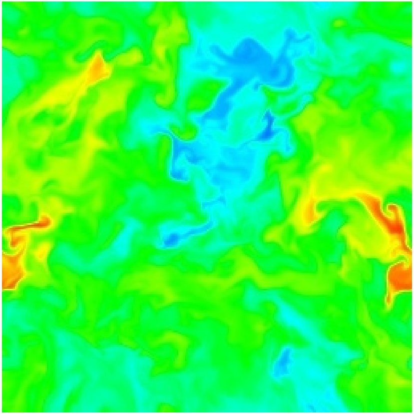


Figure: passive scalar field at $t \simeq T_L$

Ramp-cliff-structure

- the passive scalar field over a line

$\longrightarrow$ ramp-cliff events

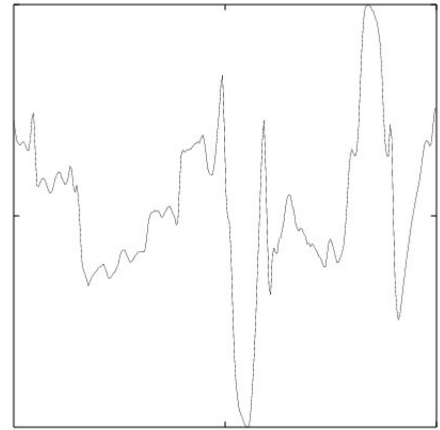


Figure: example of ramp-cliff structure

Movie: Parallel evolution

- the fields for comparison
- timestep resolution is 100 per frame $1/7th$ Largy Eddy Turnover

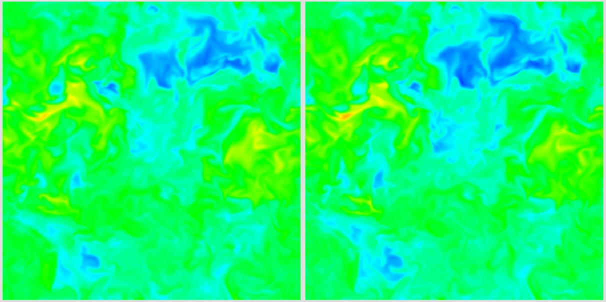
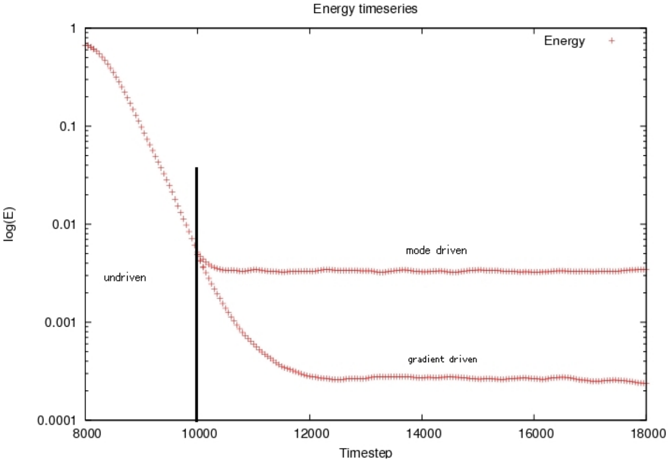


Figure: gradient driven $\leftrightharpoons$ frozen modes

Energy timeseries

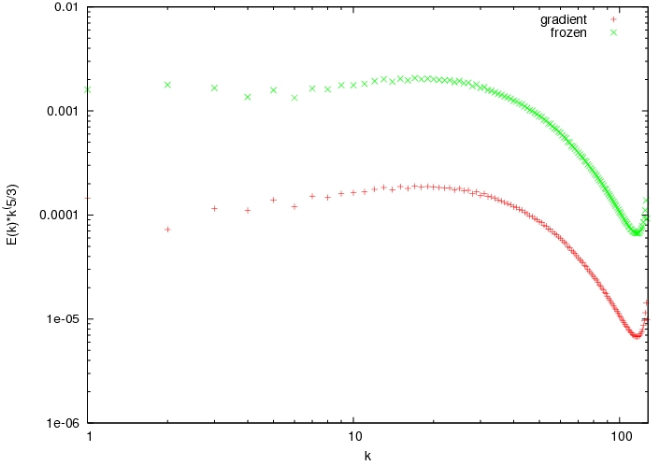
- evolution of the passive scalar energy



Energy spectra

- spectra of the passive scalar
- not normalized

⇒ shift of the spectra



Passive tracers and passive scalar in 3d incompressible turbulence

Intro
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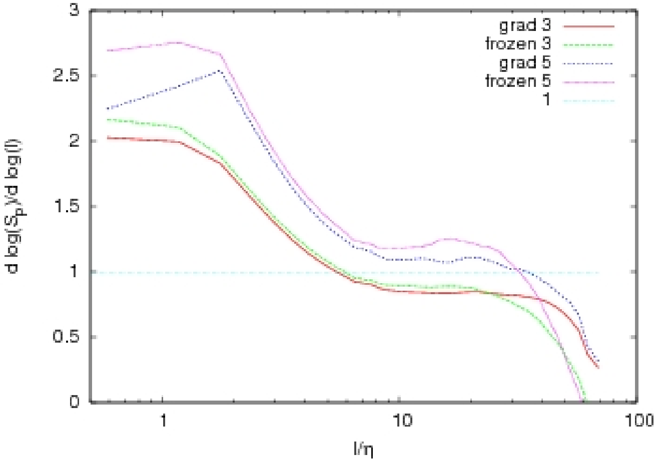
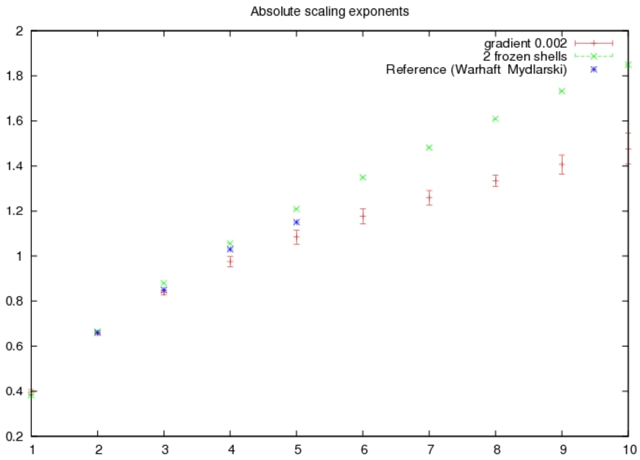
- passive scalar structure functions
- logarithmic derivatives



Scaling exponents

- absolute scaling exponents from 1*st* to 10*th* order
- as reference: experimental data from Wahrhaft and Mydlarski

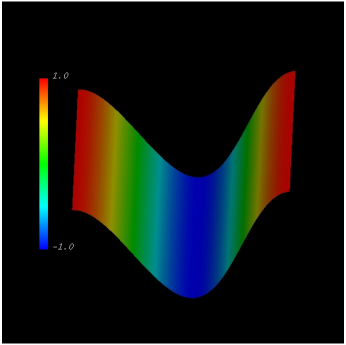


Example of passive scalar evolution

- here initial passive scalar field is a single sine function
- grid resolution is 64^3, $\kappa = \nu$, no driving
- time resolution is 1 timestep per frame

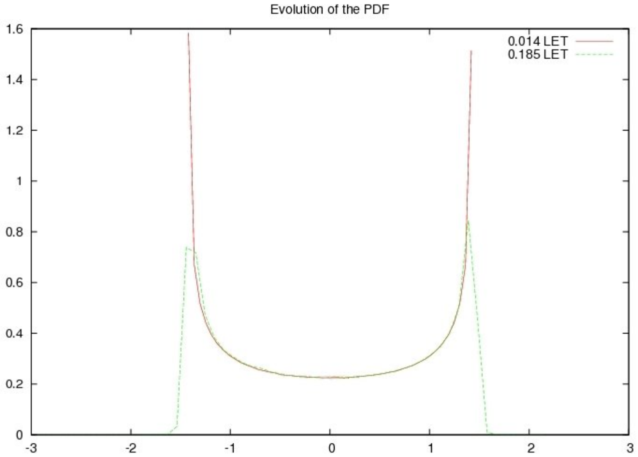
Testing the integrator

- 3rd run: test case with $\kappa = 0$
  - grinds to a halt after about $1/7th$ Large Eddy
  - numerical instable
- 4th run: test case with $\kappa = \nu$
- goal: test the advancer's numerical quality
- start from a single sine field which exibits a distinct pdf
- freely decaying



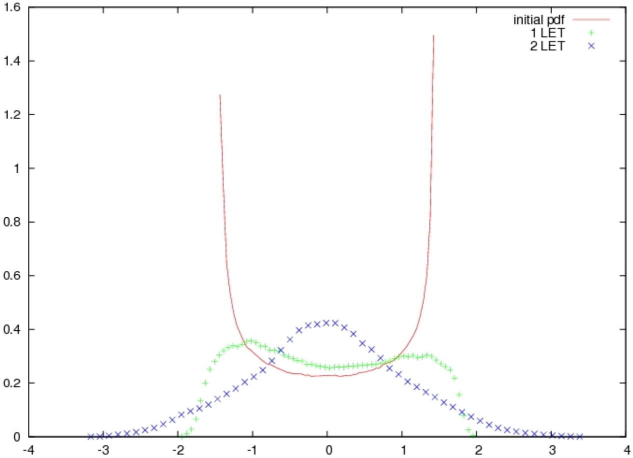
Evolution of the scalar PDF

- $\kappa = 0$
- pdf should stay constant
- numerical instable
- shape stays the same

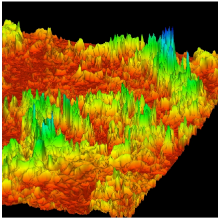
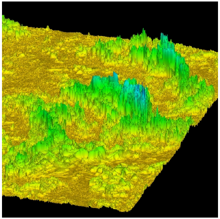
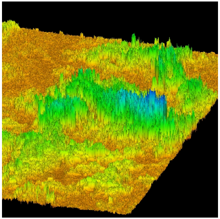


Evolution of the scalar PDF

- $\kappa = \nu$
- dissipation changes the shape
- converges towards gaussian



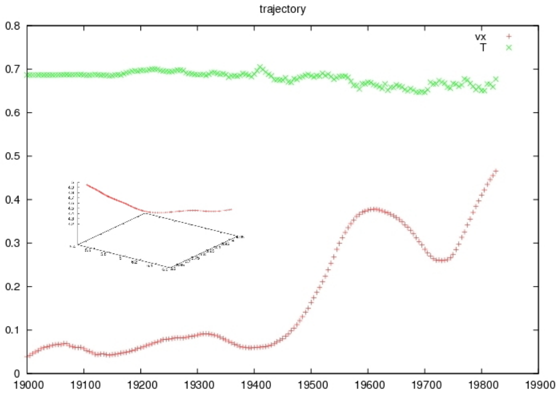
- resolution: 1024^3
- three points in time $1/30th$, $1/7th$ and $1/4th$ Large Eddy Turnover
- grows numerically unstable at $\sim 1/6th$ Large Eddy Turnover
- 10^5 tracer particles

Lagrangian description

Passive scalar values vs velocity values

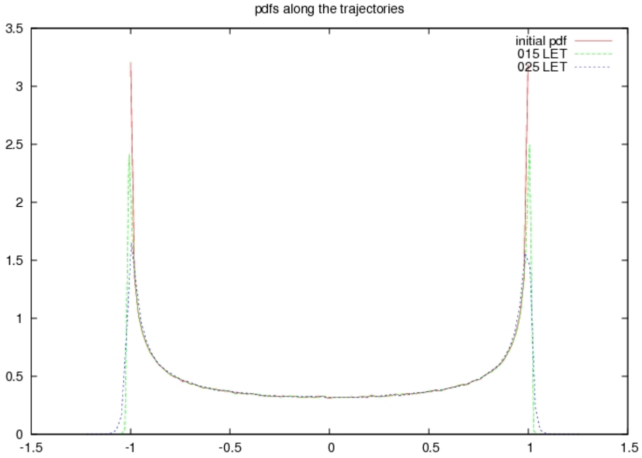
- θ values along the trajectory shown in the inset
- v_x shown for comparison



Lagrangian description
Lagrangian pdfs

- field evaluated at the tracer positions
- no diffusion

⇒ pdfs should be constant



Conclusion

Conclusion

- what we have reached
  - a framework to simulate passive scalar turbulence
  - tested the advancer
  - evaluated the forcing schemes
  - qualitative results show typical characteristics
  - found the expected behaviour for scaling & spectra
- next step: extend Lagrangian description
- long term goal: understand the effect of diffusion

The End

Thank you for your attention