Introduction to the Standard Model
Exercises 7

Deadline: Monday 06 June 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Vertex correction, parton model.

1. (3.0 P) The vertex correction to $-\mathrm{i}e_R\gamma^\mu$ is given by $-\mathrm{i}e_R\Gamma_2^\mu$, where:

$$\Gamma_2^\mu = A^\mu \int_0^1 dx \int_0^{1-x} dy \int \frac{d^dk}{(2\pi)^d} \frac{\frac{(d-2)^2}{d}k^2 + Q^2[(2-d)xy + 2x + 2y - 2]}{(k^2 + Q^2xy + \mathrm{i}\epsilon)^3} \ ,$$

with $A^\mu = -2\mathrm{i}\gamma^\mu e_R^2 \mu^{4-d}$. Calculate separately the UV and the IR divergent terms showing that:

$$UV: \ \Gamma_2^\mu/A^\mu = \frac{\mathrm{i}}{16\pi^2}\left(\frac{4\pi}{-Q^2}\right)^{\frac{4-d}{2}} \frac{\Gamma\left(\frac{4-d}{2}\right)\Gamma\left(\frac{d}{2}\right)^2}{\Gamma(d-1)} \ ,$$

$$IR: \ \Gamma_2^\mu/A^\mu = \frac{\mathrm{i}}{16\pi^2}\left(\frac{4\pi}{-Q^2}\right)^{\frac{4-d}{2}} \frac{\Gamma\left(\frac{4-d}{2}\right)\Gamma\left(\frac{d-4}{2}\right)\Gamma\left(\frac{d}{2}\right)}{\Gamma(d-2)} \left(\frac{d^2-8d+24}{4(d-2)}\right) \ .$$

2. (1.0 P) Derive the following expression:

$$\frac{1}{(1-z)^{1+\epsilon}} = -\frac{1}{\epsilon}\delta(1-z) + \frac{1}{[1-z]_+} - \epsilon\left[\frac{\ln(1-z)}{1-z}\right]_+ + \sum_{n=2}^{\infty}\frac{(-\epsilon)^n}{n!}\left[\frac{\ln^n(1-z)}{1-z}\right]_+ \ ,$$

where the plus-distributions have been used.
Hint: one way to do this is to write:

$$\int_0^1 dx\ x^{-1+\epsilon} f(x) = \int_0^1 dx\ x^{-1+\epsilon} f(0) + \int_0^1 dx\ x^{-1+\epsilon}[f(x) - f(0)] \ ,$$

and to evaluate the first term and Taylor expand the second term.

3. (2.0 P) Consider the inelastic electron-proton scattering $ep \to eX$. Let us call k and k' the initial and the final electron momentum respectively, M the mass of the proton, p the momentum of the proton and q the momentum transfer of the virtual photon.

It is common to replace the variables $\nu = (pq)/M$ and q^2, by the dimensionless variables:

$$x = \frac{-q^2}{2pq} = \frac{-q^2}{2M\nu}, \quad y = \frac{pq}{pk} \ .$$

Show that the allowed kinematic region for $ep \to eX$ is $0 \le x \le 1$ and $0 \le y \le 1$.

4. (2.0 P) During the lecture we evaluated the $ep \to eX$ cross section for the electron to be scattered into the $dE'd\Omega$ element in the target proton rest frame (the laboratory frame). Show that:

$$dE'\, d\Omega = \frac{\pi}{EE'}\, dQ^2\, d\nu = \frac{2ME}{E'}\, \pi\, y\, dx\, dy\ ,$$

where E and E' are the initial and final energy respectively, and $Q^2 \equiv -q^2$.

5. (3.0 P) In the center-of-mass frame of the proton process $\gamma^* q_1 \to g q_2$, let us indicate the virtual photon momentum as $q = (q_0, \vec{k})$, the initial parton momentum as $q_1 = (k, -\vec{k})$, the final parton momentum as $q_2 = (k', \vec{k}')$, the gluon momentum as $g = (k', -\vec{k}')$. Show that:

$$\begin{aligned} s &= 2k^2 + 2kq_0 - Q^2 = 4k'^2\ , \\ t &= -Q^2 - 2k'q_0 + 2kk' \cos\theta = -2kk'(1 - \cos\theta)\ , \\ u &= -2kk'(1 + \cos\theta)\ , \end{aligned}$$

where $Q^2 \equiv -q^2$ and θ is the angle between $\vec{k}$ and $\vec{k}'$.
Moreover, prove the following relation:

$$4kk' = -t - u = s + Q^2\ .$$

An interesting quantity is the transverse momentum of the outgoing quark, $p_T = k' \sin\theta$. Show that:

$$p_T^2 = \frac{stu}{(s + Q^2)^2}\ ,$$

and, in the limit of small-angle scattering, $(-t) \ll s$, that

$$p_T^2 = \frac{s(-t)}{(s + Q^2)}\ .$$

Further, show that for small scattering angles ($\cos\theta \simeq 1$),

$$d\Omega = \frac{4\pi}{s} dp_T^2\ .$$