Introduction to the Standard Model
Exercises 4

Deadline: Monday 9 May 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Self energy in scalar QED, electron self-energy in QED, counterterms.

1. (2 P) Plot, in scalar QED, the Feynman diagrams involved in the determination of the self-energy of the scalar to 1-loop. Then write its expression, using the corresponding Feynman rules. Finally, write down the expression for the effective scalar propagator. (The calculation of the integrals is not required).

2. (3 P) Consider the 1-loop correction to the electron propagator in QED. Plot the Feynman diagram and show, using the Feynman rules, how you determine the corresponding expression:

$$\mathrm{i}\Sigma_2(\not{p}) = (-\mathrm{i}e)^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \frac{\mathrm{i}(\not{k}+m)}{k^2 - m^2 + \mathrm{i}\epsilon} \gamma_\mu \frac{-\mathrm{i}}{(p-k)^2 + \mathrm{i}\epsilon} \; .$$

   Using the Feynman parameters, completing the square and shifting conveniently the momentum k (see Appendix B.1.1 (Regularisation) on the web) derive the following expression:

$$\mathrm{i}\Sigma_2(\not{p}) = 2e^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{x\not{p} - 2m}{[k^2 - \Delta + \mathrm{i}\epsilon]^2} \; ,$$

   where $\Delta = (1-x)(m^2 - p^2 x)$.

3. (2 P) From the last expression of the previous problem, using dimensional regularisation (see Appendix B.3.3 (Regularisation) on the web) determine the following:

$$\Sigma_2(\not{p}) = \frac{\alpha}{2\pi} \int_0^1 dx (x\not{p} - 2m) \left[\frac{2}{\epsilon} + \ln \frac{\tilde{\mu}^2}{(1-x)(m^2 - p^2 x)} \right] \; ,$$

   where $\tilde{\mu}^2 \equiv 4\pi e^{-\gamma_E} \mu^2$ and $\alpha = e^2/(4\pi)$.

4. (2 P) Starting from the expression determined in the previous problem calculate the counterterms δ_2 and δ_m, using $\overline{MS}$ scheme:

$$\delta_2 = -\frac{\alpha}{4\pi} \left(\frac{2}{\epsilon} + \ln\left(4\pi e^{-\gamma_E}\right) \right) \; , \quad \delta_m = -\frac{3\alpha}{4\pi} \left(\frac{2}{\epsilon} + \ln\left(4\pi e^{-\gamma_E}\right) \right) \; .$$