Statistical Field Theory: Interfaces of Binary Fluid Systems

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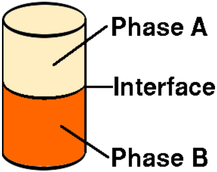
Overview

1. Introduction
   - Interfaces
   - Phenomenology of Binary Fluid Systems
2. Why do *we* care?
   - Phase Transitions and Universality
   - From Ising Model to ϕ^4-Theory
3. Formalism: The Effective Potential
   - Generating Functionals in Field Theory
   - The Effective Potential
4. The Formalism applied to Interfaces

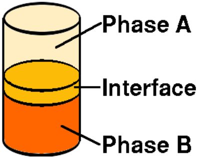
Overview

Interfaces

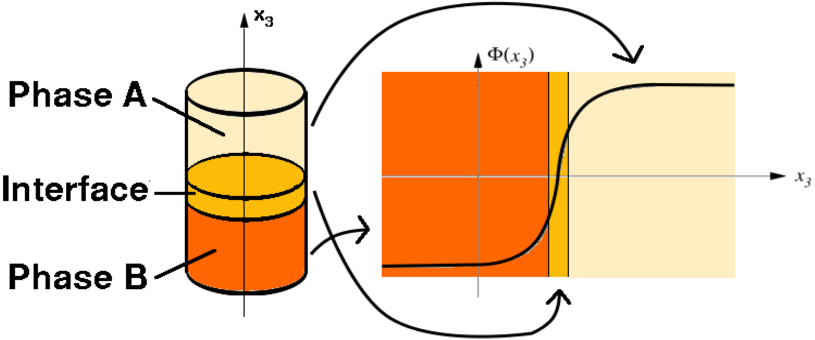


- Consider interfaces between two components/phases of a fluid
- Since Aristoteles' times scientists have been interested in interfaces
- These were assumed to be *sharp* and *discontinuous* like it appears to the human eye

- Poisson (1831): Interface is a diffuse, continuous transition; it can be described by a continuous profile $\phi(x)$
- $\phi(x)$ is e.g. the density difference of the phases



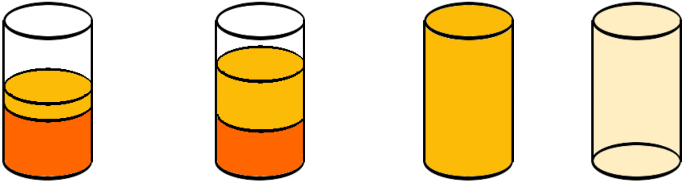
Interfaces



⇒ Definition of interface thickness is arbitrary and therefore a problem on its own

Phenomenology of Binary Fluid Systems

Cyclo-Hexane (C_6H_{12}) and Aniline ($C_6H_5NH_2$)



$T_0 < T_C$ $\quad$ $T_0 < T_1 < T_C$ $\quad$ $T \sim T_C$ $\quad$ $T > T_C$

- Experiment by Atack and Rice (1953)
- Below $T_C = 30.9\ ^{\circ}\mathrm{C}$ both fluids seperate into two pure phases
- Above T_C the fluids mix perfectly
- No latent heat is measured $\Rightarrow$ second order phase transition

Phenomenology of Binary Fluid Systems

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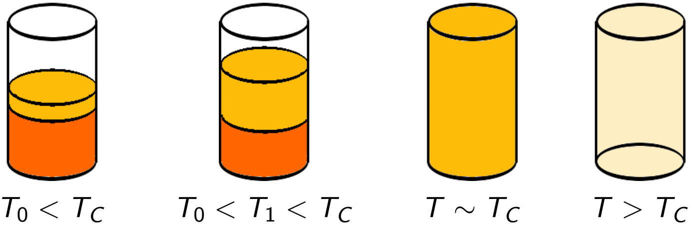


- Near T_C the reduced interface tension $\sigma = \tau / k_B T$ obeys a scaling law

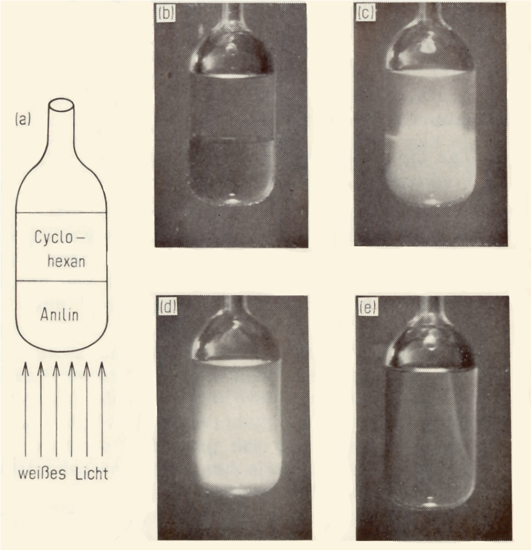
$$\sigma \sim \sigma_0 t^{\mu} \qquad \mu = 1.26 \pm 0.01$$

with reduced temperature $t = \frac{T - T_\mathrm{C}}{T_\mathrm{C}}$

- Correlation length ξ^+ diverges at T_C according to

$$\xi^+ = \xi_0^+ t^{-\nu} \qquad \nu = 0.630 \pm 0.002$$

Phenomenology of Binary Fluid Systems



Phenomenology of Binary Fluid Systems

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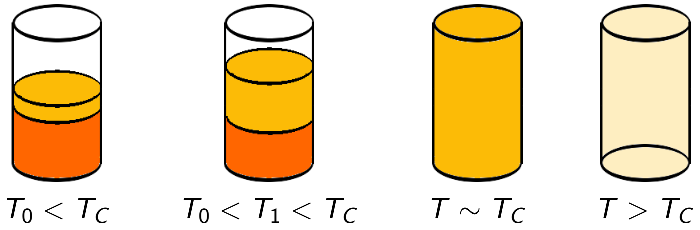
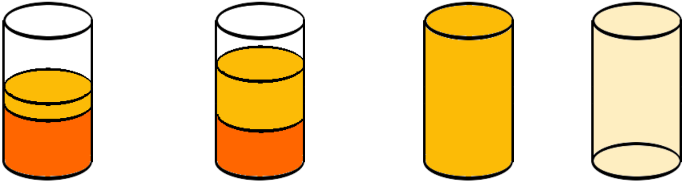


Universality

- Many other BFS show the same behaviour:
  - Isobutyric acid + Water
  - Triethylamine + Water
  - ...
- Even some binary systems of one fluid and one gas behave similar

Phenomenology of Binary Fluid Systems

Cyclo-Hexane (C_6H_{12}) and Aniline ($C_6H_5NH_2$)



$T_0 < T_C$ $T_0 < T_1 < T_C$ $T \sim T_C$ $T > T_C$

Universality

- While $\sigma_0, \xi_0^{\pm}$ vary from system to system...
- ... the critical exponents μ, ν agree almost perfectly

Overview

Remember: Phase Transitions and Universality

Continuous Phase Transitions

- We only consider phase transitions of second order
- Low temperature phase is usually more orderly than high temperature phase because of

$$F = U - TS \qquad F \text{ minimal at equilibrium}$$

- Order parameter $\phi \neq 0$ only for one phase. Examples:
  - spontaneous magnetization $M_S = \frac{1}{V} \sum_i \vec{\mu}_i$ of a magnetical system
  - difference of concentrations $\Delta C = C_A - C_B$ of a binary fluid system

Remember: Phase Transitions and Universality

Critical Exponents

- Near T_{C} many physical quantities behave according to power laws of the reduced temperature t
- Define *critical exponents* λ, λ' to physical quantity F as follows:

$$\lambda = \lim_{t \searrow 0} \frac{\ln |F(t)|}{\ln t} \qquad \lambda' = \lim_{t \nearrow 0} \frac{\ln |F(t)|}{\ln(-t)}$$

$$\Rightarrow F(t) = t^{\lambda} \quad \text{as} \quad t \to 0^{+} \quad \text{and} \quad F(t) = (-t)^{\lambda'} \quad \text{as} \quad t \to 0^{-}$$

Remember: Phase Transitions and Universality

Critical Exponents

specific heat: $C_{\pi=0} \sim \begin{cases} (-t)^{-\alpha'} & \text{for} \quad T < T_\mathrm{C} \\ t^{-\alpha} & \text{for} \quad T > T_\mathrm{C} \end{cases}$

order parameter: $\phi \sim (-t)^{\beta}$ ($\phi = 0$ for $T \geq T_\mathrm{C}$)

susceptibility: $\chi \sim \begin{cases} (-t)^{-\gamma'} & \text{for} \quad T < T_\mathrm{C} \\ t^{-\gamma} & \text{for} \quad T > T_\mathrm{C} \end{cases}$

critical isotherm: $T \sim \phi^{\delta}$ for $T = T_\mathrm{C}$

correlation length: $\xi \sim \begin{cases} (-t)^{-\nu'} & \text{for} \quad T < T_\mathrm{C} \\ t^{-\nu} & \text{for} \quad T > T_\mathrm{C} \end{cases}$

Remember: Phase Transitions and Universality

Hypothesis of Universality (Griffiths, 1970)

- Systems with a phase transition of second order can be sorted in *universality classes*
- Crititical behaviour (i.e. the critical exponents) of systems in the same UC is essentially equal
- Classification depends on
  a) the system's dimension D
  b) the dimension n of the order parameter
  c) the symmetries of the system

From Ising Model to ϕ^4-Theory

How do we describe such Binary Fluid Systems theoretically?

- Values of μ, ν suggest the systems to be in the same universality class as the 3D Ising model

Ising Model

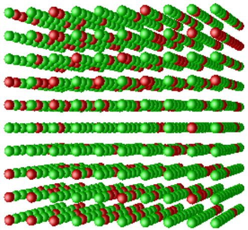
Hamiltonian (ferromagnetic)

$$H = -J \sum_{ij} S_i S_j - \mu B \sum_i S_i$$

$$S_i = \pm 1$$

Order parameter:

$$M = \frac{1}{V} \sum_i \langle S_i \rangle$$



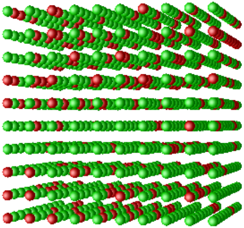
From Ising Model to ϕ^4-Theory

How do we describe such Binary Fluid Systems theoretically?

- Values of μ, ν suggest the systems to be in the same universality class as the 3D Ising model

Ising Model

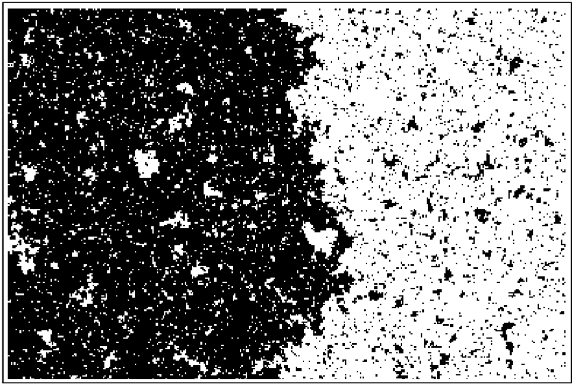
- Fluid systems: Identify
  - $S_i = 1$ ("up") with a volume cell of fluid component A
  - $S_i = -1$ ("down") with a volume cell of component B



From Ising Model to ϕ^4-Theory

Interfaces of Ising Systems

- The Ising Model can be used to treat systems with interfaces numerically



From Ising Model to ϕ^4-Theory

Interfaces of Ising Systems

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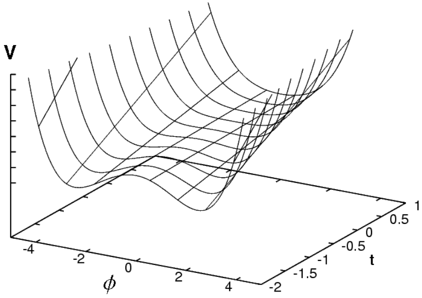
From Ising Model to ϕ^4-Theory

Analytical Treatment: Landau-Ginzburg Model



- System with interface can be described by double-well potential
- Minima correspond to pure phases of the components

From Ising Model to ϕ^4-Theory

Analytical Treatment: Landau-Ginzburg Model

- Order parameter: Continuous variable ϕ $(-\infty < \phi < \infty)$ where $\langle \phi \rangle = M$
- Continuum: $x_i \to x$, $\phi_i \to \phi(x)$, $a \to 0$, $N \to \infty$
- Landau approximation: Neglect fluctuations $\to \phi = \langle \phi \rangle$
- Analyticity assumption: Near T_c free energy $G(T, \phi)$ is expandable in powers of ϕ

$$G(T, \phi) = G_0(T) - \pi\phi + \frac{1}{2} r_0(T)\phi^2 + \frac{1}{4!} u_0 \phi^4 + \dots$$

(symmetry is assumed $\Rightarrow$ no odd powers of ϕ if no external force π is present)

From Ising Model to ϕ^4-Theory

Analytical Treatment: Landau-Ginzburg Model

- The Ginzburg-Landau Model uses the "Hamiltonian" (valid only in vicinity of T_C)

$$\Rightarrow H_{\mathrm{LG}}[\phi] = \int d^D x \left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2} r_0(T)\phi^2 + \frac{1}{4!} u_0 \phi^4\right]$$

- Pertubation expansion terms contain statistical fluctuations

From Ising Model to ϕ^4-Theory

Analytical Treatment: Landau-Ginzburg Model

- Partition function:

$$Z = \sum_{\{S_i\}} e^{-\beta H}$$

- Each configuration $\{S_i\}$ corresponds to a field $\phi(x)$

$$\Rightarrow Z = \int \mathscr{D}\phi(x)\, e^{-\beta \int d^D x \left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2} r_0(T)\phi^2 + \frac{1}{4!} u_0 \phi^4\right]}$$

- Generating functional of correlation functions:

$$Z[B] = \int \mathscr{D}\phi(x)\, e^{-\beta H_{\mathrm{GL}} - \int d^D x\, B(x)\phi(x)}$$

From Ising Model to ϕ^4-Theory

Comparison: Landau-Ginzburg Model vs. ϕ^4-QFT

- Generating functional in QFT:

$$Z[J] = \int \mathscr{D}\phi \, e^{-\frac{1}{\hbar} S_{\mathrm{E}}[\phi] + \int d^D x \, J(x)\phi(x)}$$

- Euclidean action of ϕ^4-QFT

$$S_{\mathrm{E}}[\phi] = \int d^d x \, \underbrace{\left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}g\phi^4\right]}_{\text{Lagrangean } \mathscr{L}}$$

is the formal equivalent of

$$H_{\mathrm{GL}}[\phi] = \int d^D x \, \underbrace{\left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}r_0(T)\phi^2 + \frac{1}{4!}u_0\phi^4\right]}_{\text{Hamiltonian density } \mathscr{H}}$$

From Ising Model to ϕ^4-Theory

Comparison: Ginzburg-Landau-Model vs. ϕ^4-QFT

Order parameter $\phi(x)$	$\leftrightarrow$	Field $\phi(x)$
Temperatur $k_{\mathrm{B}} T = 1/\beta$	$\leftrightarrow$	Planck's $\hbar$
Hamiltonian density $\mathscr{H}(\phi)$	$\leftrightarrow$	Lagrangean $\mathscr{L}(\phi)$
Hamiltonian	$\leftrightarrow$	Euclidean action
$H[\phi] = \int d^D x \, \mathscr{H}(\phi(x))$		$S_{\mathrm{E}}[\phi] = \int d^D x \, \mathscr{L}(\phi(x))$
Landau approximation	$\leftrightarrow$	Classical limit
Statistical fluctuations	$\leftrightarrow$	Quantum fluctuations

So why do *we* care?

- Binary fluid systems happen to be in the same universality class as ϕ^4-theory!
- ϕ^4-theory is part of the Standard Model of high energy particle physics (e.g. electroweak Higgs field)
- Critical behaviour is *universal* $\Rightarrow$ details of binary fluid system can be neglected, methods of QFT can be applied to BFS and vice versa
- Interfaces appear in some systems with spontaneously broken symmetry, consider e.g. domain walls in cosmology

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Generating Functionals in Field Theory

Generating Functional of Correlation Functions

- Addition of a source term to the Lagrangean in the functional integral yields

$$Z[J] = \int \mathscr{D}\phi \, \exp\left\{ - \int d^D x \, [\mathscr{L} + J(x)\phi(x)] \right\}$$

$$\langle 0|\phi(x_1)\ldots\phi(x_n)|0\rangle = \frac{1}{Z_0} \frac{\delta^n Z[J]}{\delta J(x_1)\ldots\delta J(x_n)} Z[J]\Big|_{J=0}$$

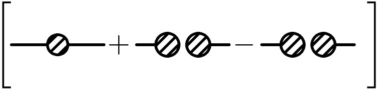
mit $Z_0 = Z[J=0]$

Generating Functionals in Field Theory

Generating Functional of Connected Correlation Functions

- We define another generating functional by $W[J] = \ln Z[J]$

$$\frac{\delta^2 W[J]}{\delta J(x)\delta J(y)} = [<\phi(x)\phi(y)> - <\phi(x)><\phi(y)>]$$

$=$ 

$=$  $=:<\phi(x)\phi(y)>_{\mathrm{conn}}$

- More generally: $\frac{\delta^n W[J]}{\delta J(x_1)\ldots\delta J(x_n)} = \langle\phi(x_1)\ldots\phi(x_n)\rangle_{\mathrm{conn}}$

Generating Functionals in Field Theory

The effective Action Γ

- We define the *classical field* $\phi_{\rm Cl}(x)$ by

$$\frac{\delta W[J]}{\delta J(x)} = \langle\Omega|\phi(x)|\Omega\rangle =: \phi_{\rm Cl}(x)$$

- Analogy to the free energy of statistical mechanics: Legendre transformation of $W[J]$ yields $\Gamma[\phi_{\rm Cl}(x)]$

$$\begin{aligned}
\delta W[J] &= \int d^4x \, \frac{\delta W[J]}{\delta J(x)} \delta J(x) = \int d^4x \, \phi_{\rm Cl}(x) \delta J(x) \\
&= \int d^4x \, \delta(\phi_{\rm Cl}(x) J(x)) - \int d^4x \, J(x) \delta\phi_{\rm Cl}(x)
\end{aligned}$$

$$\Rightarrow \underbrace{\delta(W[J] - \int d^4x \, \phi_{\rm Cl}(x) J(x))}_{\delta\Gamma[\phi_{\rm Cl}]} = -\int d^4x \, J(x) \delta\phi_{\rm Cl}(x)$$

Generating Functionals in Field Theory

The effective Action Γ

- Definition of $\Gamma[\phi_{\mathrm{Cl}}]$ yields

$$\frac{\delta\Gamma[\phi_{\mathrm{Cl}}]}{\delta\phi_{\mathrm{Cl}}(x)} = -J(x)$$

$\Rightarrow$ In abscence of external sources Γ becomes stationary
$\Rightarrow$ equation of motion for VEV $\phi_{\mathrm{Cl}}(x) = \langle\phi(x)\rangle_{J=0}$

- It can be shown that

$$\frac{\delta^2\Gamma[\phi_{\mathrm{Cl}}]}{\delta\phi_{\mathrm{Cl}}(x)\delta\phi_{\mathrm{Cl}}(y)} = -G_C^{-1}(x,y)$$

Generating Functionals in Field Theory

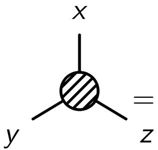
The effective Action Γ

- $\Gamma[\phi_{\mathrm{Cl}}]$ is the generating functional of the proper vertices

$$\frac{\delta^n \Gamma[\phi_{\mathrm{Cl}}]}{\delta\phi_{\mathrm{Cl}}(x_1)\dots\delta\phi_{\mathrm{Cl}}(x_n)} = \langle \phi(x_1)\dots\phi(x_n)\rangle_{\mathrm{1PI}}$$

- This is proved by induction from n-point to $(n+1)$-point correlation functions and explicit calculation for the 3-point correlator

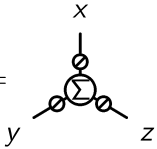
Generating Functionals in Field Theory



$$= \langle \phi(x)\phi(y)\phi(z)\rangle_{\mathrm{conn}} = \frac{\delta^3 W[J]}{\delta J_x \delta J_y \delta J_z}$$

$$\begin{aligned}
&= \int d^4 w\, G_{zw} \frac{\delta}{\delta \phi_w^{\mathrm{Cl}}} \frac{\delta^2 W[J]}{\delta J_x \delta J_y} = -\int d^4 w\, G_{zw} \frac{\delta}{\delta \phi_w^{\mathrm{Cl}}} \left(\frac{\delta^2 \Gamma[\phi_{\mathrm{Cl}}]}{\delta \phi_x^{\mathrm{Cl}} \delta \phi_y^{\mathrm{Cl}}} \right)^{-1} \\
&= \int d^4 w\, d^4 u\, d^4 v\, G_{zw} \left(\frac{\delta^2 \Gamma[\phi_{\mathrm{Cl}}]}{\delta \phi_x^{\mathrm{Cl}} \delta \phi_u^{\mathrm{Cl}}} \right)^{-1} \frac{\delta^3 \Gamma[\phi_{\mathrm{Cl}}]}{\delta \phi_u^{\mathrm{Cl}} \delta \phi_v^{\mathrm{Cl}} \delta \phi_w^{\mathrm{Cl}}} \left(\frac{\delta^2 \Gamma[\phi_{\mathrm{Cl}}]}{\delta \phi_v^{\mathrm{Cl}} \delta \phi_y^{\mathrm{Cl}}} \right)^{-1} \\
&= \int d^4 u\, d^4 v\, d^4 w G_{xu} G_{vy} G_{zw} \frac{\delta^3 \Gamma[\phi_{\mathrm{Cl}}]}{\delta \phi_u^{\mathrm{Cl}} \delta \phi_v^{\mathrm{Cl}} \delta \phi_w^{\mathrm{Cl}}}
\end{aligned}$$

$=$



Generating Functionals in Field Theory

- Induction hypothesis

$$\langle\phi(x_1)...\phi(x_n)\rangle_{\rm conn} = \frac{\delta^n W[J]}{\delta J(x_1)...\delta J(x_n)}$$

$$= \int d^4y_1 d^4y_2...d^4y_n \left\{ G_{x_1y_1} G_{x_2y_2}...G_{x_ny_n} \frac{\delta^n\Gamma[\phi_{\rm Cl}]}{\delta\phi^{\rm Cl}_{y_1}\delta\phi^{\rm Cl}_{y_2}...\phi^{\rm Cl}_{y_n}} + ... \right\}$$

where the ... terms consist of some $G_{x_iy_i}$ and some $\frac{\delta^m\Gamma[\phi_{\rm Cl}]}{\delta\phi^{\rm Cl}_{y_i}\delta\phi^{\rm Cl}_{y_j}...}$, $m < n$ and the derivatives of Γ describe proper vertices

Generating Functionals in Field Theory

- Inductive step: Addition of an external point

$$\langle \phi(x_1)\dots\phi(x_{n+1})\rangle_{\mathrm{conn}} = \frac{\delta}{\delta J_{x_{n+1}}}\left(\frac{\delta^n W[J]}{\delta J_{x_1}\dots\delta J_{x_n}}\right)$$

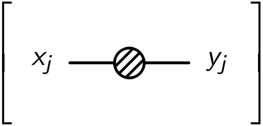
- This yields two types of terms:

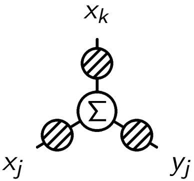
$$\underbrace{\dots\left(\frac{\delta}{\delta J_{x_{n+1}}} G_{x_i y_i}\right)\dots}_{\text{Type I}} \quad \text{and} \quad \underbrace{\dots\frac{\delta}{\delta J_{x_{n+1}}}\frac{\delta^m \Gamma[\phi_{\mathrm{Cl}}]}{\delta\phi^{\mathrm{Cl}}_{y_i}\delta\phi^{\mathrm{Cl}}_{y_j}\dots}}_{\text{Type II}}$$

Generating Functionals in Field Theory

- Terms of type I:

$$
\begin{aligned}
\frac{\delta}{\delta J_{x_k}} G_{x_j y_j} &= \int d^4 u\, G_{x_k u} \frac{\delta}{\delta \phi_u^{\rm Cl}} \frac{\delta^2 W[J]}{\delta J_{x_j} \delta J_{y_j}} \\
&= \int d^4 u\, G_{x_k u} \frac{\delta}{\delta \phi_u^{\rm Cl}} \left(\frac{\delta^2 \Gamma[\phi_{\rm Cl}]}{\delta \phi_{x_j}^{\rm Cl} \delta \phi_{y_j}^{\rm Cl}} \right)^{-1} \\
&= \int d^4 u\, d^4 v\, d^4 w\, G_{x_k u} G_{x_j v} G_{w y_j} \frac{\delta^3 \Gamma}{\delta \phi_u^{\rm Cl} \phi_v^{\rm Cl} \phi_w^{\rm Cl}}
\end{aligned}
$$

$\frac{\delta}{\delta J(x_k)}$ 

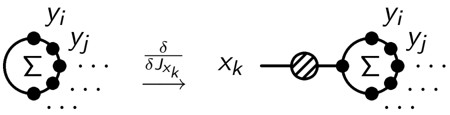
$=$ 

Generating Functionals in Field Theory

- In each of the type I terms one external line is replaced by a three-point function
- Terms of type II:

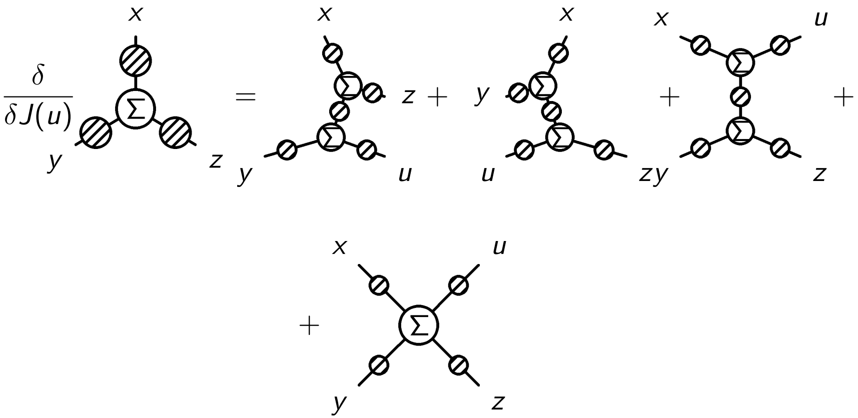
$$\frac{\delta}{\delta J_{x_k}}\left(\frac{\delta^m \Gamma[\phi_{\rm Cl}]}{\delta\phi^{\rm Cl}_{y_i}\phi^{\rm Cl}_{y_j}\ldots}\right) = \int d^4y_k\; G_{x_k y_k}\frac{\delta^{m+1}\Gamma[\phi_{\rm Cl}]}{\delta\phi^{\rm Cl}_{y_i}\phi^{\rm Cl}_{y_j}\ldots\phi^{\rm Cl}_{y_k}}$$

$\Rightarrow$ a new external line is added to what used to be a m-point proper vertex



Generating Functionals in Field Theory

Graphical representation of step $n \longrightarrow (n+1)$:



Generating Functionals in Field Theory

The effective Action Γ

- Γ is the generating functional of the proper vertices!
- Its second functional derivative is the inverse propagator
- Effective action Γ contains all the physics of the corresponding theory!

The Effective Potential

Definition of the Effective Potential

- Definition of Γ yields $\frac{\delta\Gamma[\phi_{\rm Cl}]}{\delta\phi_{\rm Cl}} = 0$ for $J = 0$
- Solutions are the stable VEV: $\phi_{\rm Cl}(x) = \langle\phi(x)\rangle$
- Expansion of Γ in $\phi_{\rm Cl}$ and derivatives:

$$-\Gamma[\phi_{\rm Cl}] = \int d^4x \left[V_{\rm eff}(\phi_{\rm Cl}) + \frac{1}{2}(\partial_\mu\phi_{\rm Cl})^2 Z(\phi_{\rm Cl}(x)) + \dots \right]$$

- We consider systems with Lorentz invariance and conservation of momentum

$$\Rightarrow \phi_{\rm Cl}(x) = \phi_0 = \mathrm{const.}$$

- From above expansion only the first term remains:

$$\Gamma[\phi_0] = -\int d^4x\, V_{\rm eff}(\phi_0) = -V_{\rm eff}(\phi_0)\int d^4x = -V_{\rm eff}(\phi_0)\Omega$$

The Effective Potential

What is the Physical Meaning of V_{eff}?

- Variation of above expansion yields

$$-\delta\Gamma[\phi_{\text{Cl}}] = \int d^4x \left[\frac{\partial V_{\text{eff}}}{\partial \phi_{\text{Cl}}}\delta\phi_{\text{Cl}} + \delta(\text{terms containing derivatives})\right]$$

$$\Rightarrow \frac{\delta\Gamma[\phi_{\text{Cl}}]}{\delta\phi_{\text{Cl}}} = -\left.\frac{\partial V_{\text{eff}}}{\partial\phi_{\text{Cl}}}\right|_{\phi_0}$$

- It can then be shown that

$$\left.\frac{\partial^2 V_{\text{eff}}}{\partial\phi_{\text{Cl}}^2}\right|_{\phi_0} = m_{\text{phys}}^2 \qquad \text{and} \qquad \left.\frac{\partial^4 V_{\text{eff}}}{\partial\phi_{\text{Cl}}^4}\right|_{\phi_0} = g$$

The Effective Potential

Advantages of the Effective Potential Formalism

- If we replace the classical potential by V_{eff} we can apply methods of classical SSB theory to quantum systems
- It can be shown that the divergence structure of a renormalisable theory is not affected by the occurrence of SSB
- Coleman: "Secret symmetry buys us secret renormalizability."

Calculation of the Effective Potential

- For non-trivial models V_{eff} cannot be calculated exactly
- It can be approximated by loop expansion
- The zero-loop term is the classical potential U

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   - From Ising Model to ϕ^4-Theory
3. Formalism: The Effective Potential
   - Generating Functionals in Field Theory
   - The Effective Potential
4. The Formalism applied to Interfaces

The Formalism applied to Interfaces

- Action functional of ϕ^4-Theory

$$S[\phi] = \int d^D x \left\{ \frac{1}{2}(\partial\phi)^2 + U(\phi) \right\} = \int d^D x \left\{ -\frac{1}{2}\phi\partial^2\phi + U(\phi) \right\}$$

$$U(\phi) = \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4$$

- Classical interface profiles obey

$$\frac{\delta S[\phi]}{\delta\phi} = -\partial^2\phi + U'(\phi) = 0$$

$$U'(\phi) = m^2\phi + \frac{g}{3!}\phi^3$$

The Formalism applied to Interfaces

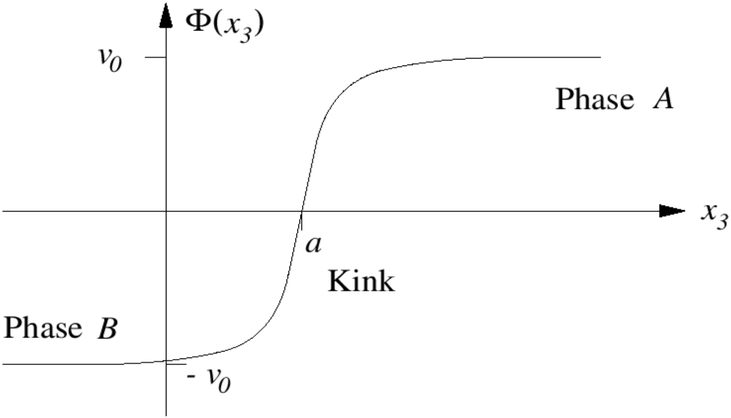
- Assumption: $\phi = \phi(x_3)$
- Multiply equation of motion by $\partial\phi$, integrate over x_3

$$-\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{4}\phi^2 + \frac{g}{4!}\phi^4 = C$$

- Boundary conditions:

$$\phi(x_3) = \begin{cases} v_0, & \text{as} \quad x_3 \to +\infty \\ -v_0, & \text{as} \quad x_3 \to -\infty \end{cases}$$

The Formalism applied to Interfaces



Classical "kink" solution

$$\phi_{\mathrm{Cl}}(x_3) = v_0 \tanh\left(\frac{m}{2}(x_3 - a)\right)$$

The Formalism applied to Interfaces

- Replacing U by V_{eff} yields the equation for interface profiles with loop corrections

$$-\frac{1}{Z_3}\partial^2\phi + V'_{\text{eff}}(\phi) = 0$$

The Formalism applied to Interfaces

- Assumption: $\phi = \phi(x_3)$
- Multiply by $\partial\phi$, integrate over x_3

$$\Rightarrow -\frac{1}{2Z_3}(\partial\phi)^2 + V_{\text{eff}}(\phi) = C$$

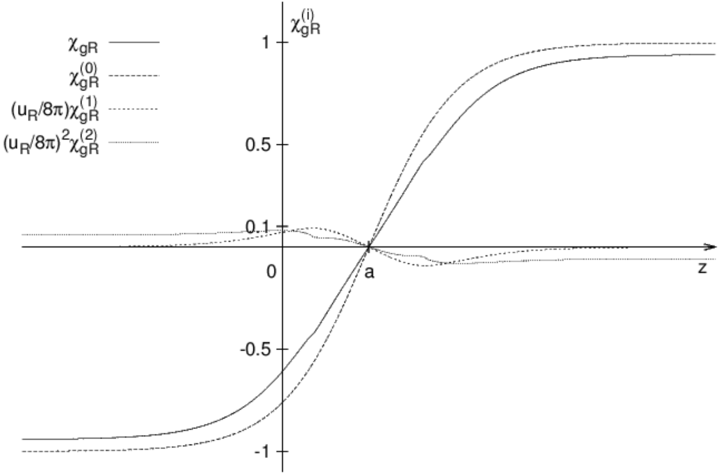
- V_{eff} can be normalized to get $C = 0$

$$(\partial\phi)^2 = 2Z_3 V_{\text{eff}}(\phi)$$
$$\partial\phi = \sqrt{2Z_3 V_{\text{eff}}(\phi)}$$

- We use $\partial\phi = +\sqrt{\ldots}$ in our calculations and retrieve the solution with negative sign simply by substituting x_3 by $-x_3$ in our results

The Formalism applied to Interfaces

Results to second order (Küster, 2001)



Outlook

- The interface profile is to be calculated
- Küster for example approximated the profile by using the effective potential V_{eff}, which is defined for constant ϕ_{Cl} to get a differential equation for a non-constant profile ϕ
- Instead of V_{eff} we are now going to use $\Gamma[\phi_{\mathrm{Cl}}]$ with the classical kink solution ϕ_{Cl}
- The first orders of the loop expansion of Γ will be used to retrieve a new differential equation for the profile ϕ
- The resulting profile will hopefully respect long-wavelength fluctuations - a phenomenon known as interface *roughening* which are ignored by approximations using V_{eff}

Summary

- Interfaces in binary fluid systems can be described by a continuous order parameter field
- BFS belong to the same universality class as the 3D Ising Model and the Ginzburg-Landau Model (ϕ^4-Theory)
- The statistical ϕ^4-Theory is formally equal to the *euclidean* ϕ^4-QFT
- The effective action Γ is the Legendre-Transform of the generating functional of connected correlation functions
- For a constant ϕ_{Cl} an effective potential V_{eff} is defined

Summary

- The transition from classical to quantum field theory can be done by subsituting the classical potential U with the effective potential V_{eff}
- This equals the transition from statistical field theory in Landau approximation to a theory with fluctuations
- V_{eff} can be used to formulate a differential equation for the interface profile
- Instead we will use $\Gamma[\phi_{\mathrm{Cl}}]$ with the classical kink solution ϕ_{Cl} to calculate the profile