Seminar on Mathematical Methods of Classical Mechanics

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1 Description

The goal of the seminar is to study some of the classical mathematical methods developed for the study of mechanical systems such as our solar system. Our main source will be the book of V. Arnold on "Mathematical Methods of Classical Mechanics."

Prerequisites are the basic analysis and linear algebra courses and basic ODE theory (in particular existence and uniqueness results).

2 Suggested Talks

Newtonian Mechanics

(1) Talk 1: Basic Notions: Space and Time, The Galilean Group and Newton's Equations (Chapter 1-3 of Arnold)

(2) Talk 2: One dimensional mechanical system: Phase portraits (time perimitting: 2dim systems, Lissajous figures) (Chapter 4+5 of Arnold)

(3) Talk 3: Conservative Force Fields and Angular Momentum (Chapters 6+7 of Arnold)

(4) Talk 4: Investigation of motion in a central field, determining potentials for which all bounded orbits are closed (Chapter 8 of Arnold)

Lagrangian Mechanics

(5) Talk 5: Variational Principles, derivation of the Euler-Lagrange Equations (Chapters 12-13 of Arnold)

(6) Talk 6: Legendre Transformations and Hamiltonian Mechanics (Chapters 14+15 of Arnold)

(7) Talk 7: Examples: Brachystochrone, Harmonic Oscillator, (Double)-Pendulum, etc., Kepler problem with Lenz-Runge Vector

(8) Talk 8: Symplectic Vectorspaces and a geometric formulation of Hamilton's equations, Poisson Brackets (Terry Tao's Notes)

(9) Talk 9: Liouville's Theorem and Noether's theorem and applications (Terry Tao's Notes, Chapter 16 of Arnold)

(10) Talk 10: Canonical Transformations (pedestrian viewpoint) + Examples

(11) Talk 11: Canonical Transformations (geometric viewpoint) + Examples

(12) Talk 12: Action Angle variables and integrability

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3 Literature

1. V. I. Arnold, “Mathematical Methods of Classical Mechanics” (Springer)
2. F. Scheck, “Theoretische Physik 1: Mechanik" (Springer)
3. L. D. Landau and E. M. Lifschitz, “Lehrbuch der theoretischen Physik I: Mechanik" (Akademie Verlag Berlin)
4. Terence Tao, “Nonlinear dispersive equations: local and global analysis" (AMS); Chapter 1.4