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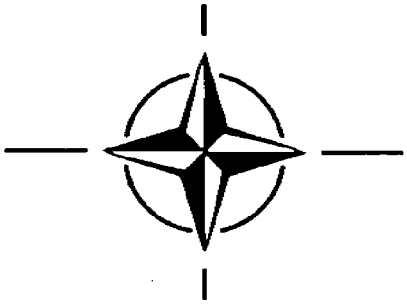

NONCOMMUTATIVE STRUCTURES IN MATHEMATICS AND PHYSICS

Kiev, Ukraine

24 – 27 September, 2000

ABSTRACTS

The North Atlantic Treaty Organization
Max Planck Institute for Physics, Munich, Germany
National Academy of Sciences of Ukraine
Ministry of Ukraine for Education and Science
Kharkov National University, Kharkov, Ukraine
National Scientific Centre for Medical and Biotechnical Research, Ukraine

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1. Quantum groups
2. Supersymmetry

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A B S T R A C T S

(A — Z)

Compiling, design and making-up by Steven Duplij

SEMICLASSICAL DYNAMICS OF SU(2) MODELS

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The spin coherent state path integral visualizes the quantum dynamics as pseudoclassical spin rotations on the two-sphere. Here, the dynamics of SU(2) models is considered. Employing a regularization with a spherical Wiener measure, continuous dominant paths emerge with jumps at the endpoints. Although SU(2) models are not supersymmetrically exact in general, we show that the resulting semiclassical propagator without fluctuations coincides with the exact quantum mechanical propagator. A non-linear transformation of the angle variables is introduced allowing the determination of the dominant path and the jumps without solving a boundary-value problem.

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NEW SUPERSYMMETRIC GAUGE THEORIES WITH 2-FORM GAUGE POTENTIALS

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During the past few years new 4d supersymmetric gauge theories have been discovered, some of which are expected to be relevant to the description of superstring vacua. Common to all these theories are unusual (`exotic') interactions of 2-form gauge potentials. This contribution is to present a connecting perspective on these theories, summarize the state-of-the-art and outline possible future projects.

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VECTOR FIELDS AND DIFFERENTIAL OPERATORS FORNONCOMMUTATIVE GEOMETRY

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In contrast with differential geometry, where vector fields over a manifold are one of the main objects, noncommutative differential geometry is constructed using the language of differential forms. We define an algebraic notion of Cartan pairs that replaces the notion of vector filed for the non-commutative case. Let k be a field. A (right) Cartan pair over an associative k-algebra $\boldsymbol{A}$ is defined to be an $\boldsymbol{A}$-bimodule $\boldsymbol{M}$ together with a right action

$$\P: \boldsymbol{M} \to \mathbf{End}_k \boldsymbol{A}$$

such that

$$(f.X)^{\P}(g) = f.X^{\P}(g) \quad \text{and} \quad X^{\P}(fg) = X^{\P}(f)g + (X.f)^{\P}(g).$$

The image of M under ¶ is called a module of *algebraic vector fields*. This generalises the standard definition of vector fields as derivations of an algebra of smooth functions over a manifold. If a first order differential calculus (of one forms) is given, i.e.

$$\mathbf{d}: A \to M$$

is a derivation then the right dual

$$M^* = \mathbf{Hom}_{(\text{-},A)}(M,A)$$

is canonically a right Cartan pair over A. The action is given by the Cartan-like formulae

$$X^{¶}(f) = \langle X, df \rangle.$$

A converse statement is also true: to each Cartan pair one can canonically associate a first order differential calculus. This correspondence restores the classical correspondence between vector fields and one forms. This duality concept can be, in particular, applied to the universal differential calculus over an algebra A. As a result the notion of universal vector fields is introduced. Another interesting situation appears when A is a Hopf algebra with bijective antipode (quantum group) and the differential $\mathbf{d}$ is bicovariant in the sense of Woronowicz. In this case the dual bimodule of algebraic vector fields is bicovariant over a co-opposite Hopf algebra. Like in the classical - Lie algebra case, the quantum Lie algebra of Woronowicz vector fields consists of left or right invariant algebraic vector fields. We also propose a definition for an algebra of (linear) differential operators associated with a given Cartan pair and discuss its natural Fock type representation. Algebraic vector fields are just first order differential operators. In particular, the algebra $\mathbf{End}_k(A)$ can be seen as an algebra of universal differential operators over A.

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ROTATING SUPER BLACK HOLE AS SPINNING PARTICLE

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It is known that Kerr geometry of rotating black hole displays some remarkable features suggesting certain relationships with the spinning elementary particles. In particular, the gyromagnetic ratio of the Kerr-Newman solution is the same as that of the Dirac electron (Carter 1968). Some stringlike structures were obtained in Kerr geometry which are linked with a singular ring of the Kerr solution and with a complex representation of the Kerr geometry, in which the Kerr-Newman solution is considered as a retarded-time field generated by a mysterious complex source propagating along a complex world line (Lind &Newman 1974). It was mentioned (Burinskii 1994) that the complex world line is really a world sheet or a special type of string. A very important period was started by Witten (1992)who pointed out the role of black holes in string theory and also by Sen (1992) who gave a supergeneralization of the Kerr solution to low energy string theory leading to a treatment of black holes as fundamental string states, and to a conclusion that some black holes should be treated as elementary particles (Sen 1995). However, a description of spinning particle based only on the bosonic fields cannot be complete, and involving fermionic degrees of freedom is required. A nontrivial supergeneralization of the Kerr-Newnan solution has been given recently by author (Class. Quant. Grav. 16(1999)3497). It represents a natural combination of the Kerr solution and a superparticle model. We

show that a nontrivial rotating super black hole can be constructed in full analogue with complex structure of Kerr geometry by the replacement of the mysterious complex source of Kerr geometry by a complex supersource which can be obtained by an extra supershift. An extra nonlinear operation «body slice», which is a generalization of the real slice procedure of the complex Kerr geometry, is necessary. We consider the resulting super-structures of the super-Kerr geometry and show that there appears a nonlinear realization of broken supersymmetry (Volkov-Akulov 1972) generalized to broken supergravity by Deser and Zumino (1976). The resulting super black hole solution represents a Kerr-Newman background dressed by fermionic hairs in the form of a nilpotent radiation along principal null congruence of the Kerr geometry. Next we analyze a «hard core» region of the Kerr geometry which is naked for the case of large angular momentum corresponding to spinning particles. By using a method of smooth deformations of the Kerr-Newman solution we show that the «hard core» can be represented as a (highly oblated) rotating bag. In a singular limit this bag tends to the known model of the Kerr source in the form of rotating elliptic shell (Lopez 1984). Finally, analyzing the models of bags, we give some arguments in favor of the superconducting bag models constructed on the base of the supersymmetric U(I) x U(I) Witten field model which was used for description of superconducting strings (Witten1985). These bags possess the properties of the standard bag models and contain in addition an extra sector with long range gauge field. It allows one to obtain a solution when the bag is a source of the external Kerr-Newman e.m. field.

We would like to thank organizers of this NATO Research Workshop for kind invitation and financial support.

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ADELIC QUANTUM MECHANICS: NONARCHIMEDEAN AND NONCOMMUTATIVE ASPECTS

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Recall that all experimental data belong to the field of rational numbers Q, and completion of Q with respect to the absolute value $|\ |_?$ and p-adic norms $||_p$ (p= a prime number) give R and the fields of p-adic numbers Q_p, respectively. R has archimedean and Q_p, $p = 2,3,...$ have nonarchimedean geometry. A restricted topological product $R \times \prod_p Q_p$ makes the space of adeles A, which is a topological ring. Since 1987, there have been significant investigations in construction of physical models with p-adic numbers and adeles (for a review, see e.g. [1] and [2]). Adelic quantum mechanics [3] is a natural generalization of ordinary QM with Feynman's path integral quantization to QM on the space of adeles. Adelic eigenfunctions have the form $\Psi_S(x) = \Psi_\infty(x_\infty) \prod_{p \in S} \Psi_p(x_p) \prod_{p \notin S} \Omega(|x_p|_p)$, where $x \in A$, $\Psi(x) \in C$, S is a finite set of primes, and $\Omega(|x_p|_p) = 1$ if $|x_p|_p \le 1$ and $\Omega(|x_p|_p) = 0$ otherwise. Adelic QM takes into account also p-adic quantum effects and may be regarded as a starting point for

construction of more complete superstring and M-theory. In the low-energy limit adelic QM becomes the ordinary one. In this contribution we will give motivation for *p*-adic numbers and nonarchimedean geometry in physics, first of all at the Planck scale. A short introduction to *p*-adic numbers and adeles, and their analysis will be recalled. *p*-Adic QM based on Feynman's path integral will be presented. Adelic QM and remarkable space-time discreteness will be emphasized. Some simple models of (non)relativistic quantum mechanics and quantum cosmology will be used as illustrations [4,5]. Perspectives in the theory will be briefly considered. Some interesting and promising connections between *p*-adic and *q*-analysis, as well as *p*-adic analogues of *q*-deformation and noncommutativity will be discussed and suggested for further investigations.

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[4] G.S.Djordjevic, B.Dragovich, Lj. Nesic, Mod. Phys. Lett. **A14** (1999) 317.
[5] B. Dragovich , Lj. Nesic, Grav. Cosm. 3 (1999) 222.

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p-ADIC STRINGS AND NONCOMMUTATIVITY

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A notion of p-adic string was introduced in 1987 [1] and refers to strings whose some properties may be described by p-adic numbers. Many approaches have been proposed depending on the way how one makes p-adic analogue (for a review, see e.g. [2] and [3]). Such investigations are mainly motivated by: 1) possible existence of nonarchimedean geometry at the Planck scale; 2) the fact that rational numbers are dense not only in the field of real but also in the field of p-adic numbers (Qp), and 3) a conjecture that fundamental physical laws should be invariant under a change of the number fields. A very attractive and promising is an adelic proposal (as a new version, see [4]). At the beginning of this contribution we will give a review of p-adic numbers and p-adic string theory emphasizing new results and perspective investigations. We will point out some connections between p-adic analysis and q-analysis, and discuss possible p-adic generalizations of archimedean noncommutative geometries and their relevance for string theory.

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[2] L. Brekke and P. G. O. Freund, Phys. Rep. **233** (1993) 1.
[3] V. S. Vladimirov, I. V. Volovich and E. I. Zelenov, *p-Adic Analysis and Mathematica Physics*, World Scientific, Singapore, 1994.
[4] B. Dragovich, *On Adelic Strings*, hep-th/0005200.

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SUPERSYMMETRY, NONINVERTIBILITY AND CATEGORIES

Steven Duplij
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In supermathematics noninvertible objects are naturally arise initially, but it is common to deal with invertible ones only "factoring" former out in some extent. We first propose to reconsider this ansatz and try to redefine such fundamental notions as supermanifolds, fiber bundles and homotopies using some weakening invertibility conditions. The prefix ``semi-" reflects the fact that underlying morphisms form corresponding semigroups consisting of a known group part and a new ideal (disjoint) noninvertible part. That can lead to construction of noninvertible analogs of Cech cocycles and spectral sequences. Then it is proposed to extend "invertibility" to "regularity" also for categories in general abstract algebraic manner. Higher regularity conditions and "semicommutative" diagrams for general morphisms are introduced. Distinction between commutative and "semicommutative" cases is measured by non-zero obstruction proportional to the difference of some self-mappings (obstructors) from the identity. This allows us to generalize the notion of functor and to "regularize" braidings and related structures in monoidal categories. A "noninvertible" analog of the Yang-Baxter equation is proposed.

S. Duplij, Semigroup methods in supersymmetric theories of elementary particles. Habilitation thesis. Kharkov State University, Kharkov, 483 pp. `math-ph/9910045`.1999

S. Duplij and W. Marcinek, Higher regularity properties of mappings and morphisms, University of Wroclaw preprint IFT UWr 931/00, `math-ph/0005033`, April 2000

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APPROXIMATE MODEL OF THE SPACETIME FOAM

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An approximate model of the foam is offered in which each quantum wormhole (QWH) is a 5D wormhole-like solution. A spinor field is introduced for an effective description of the foam. It is shown that can be two cases: (a) the mouths of QWH are separated (see Fig.1), (b) each QWH is like to dipole (see Fig.2).

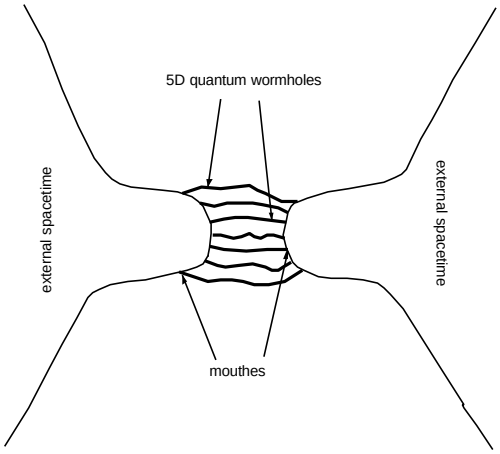

Fig.1

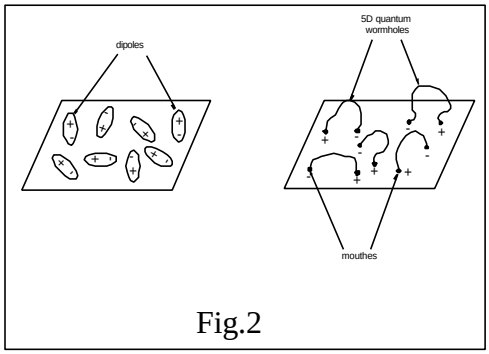

Fig.2

In the first vase the consequences of such model are: (a) preventing a “naked" singularity in the Reissner-Nordström solution with |e|/m > 1, (b) QWH’s can entrap a part of the electric flux lines by such a way that near to the event horizon of the Reissner-Nordstöm black hole will satisfy the condition |e|/m < 1 although at the infinity |e|/m > 1. In the second case it is shown that: (a) the spacetime with the foam is similar to a dielectric with dipoles, (b) supergravity theories with a nonminimal interaction between spinor and electromagnetic fields can be considered as an effective model describing approximately the spacetime foam.

I would like to acknowledge the generosity of NATO in its support for this workshop.

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SUPERSYMMETRIC ODD MECHANICAL SYSTEMS AND HILBERT Q-MODULE QUANTIZATION

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The odd mechanical systems are known in literature for over two decades. However, usually they are not discussed as separate models but only a part of the “odd” formalism is applied or developed in different theories without a reference to them. Specially frequent is an application of the formalism of the odd bracket or the anti-bracket which is used in the BRST quantization. We want to discuss the odd mechanics as the theory with interesting content by itself, with own dynamics, relevant quantization scheme etc. Using the formalism of the Hilbert Q-modules we will show how the odd Grassmannian Hermite polynomials (as well as the odd Laguerre functions) can be derived.

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D-BRANES AND VACUUM PERIODICITY

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We show that the non-Abelian Born-Infeld action defines the finite height potential barrier between the neighboring topologically distinct Yang-Mills vacua. Sphalerons are constructed for the symmetrized trace definition of the effective action. We also find the hybrid monopole-sphaleron solutions to the NBI-Higgs action. Similar solutions are discussed in the case of a concommutative geometry defined through the effective description of the theory in the constant b-field.

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ON FUSION RINGS, CATEGORIES AND SYMMETRIES IN 2D PHYSICS

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The definition of a fusion ring (algebra) is an abstraction of the properties of the Grothendieck ring of a rigid braided semisimple monoidal category C. Fusion rings/algebras appear in many occasions (we consider only finite dimensional ones) : the category C could be Rep(finite (quantum) group); Rep(U_q(g))/Z with q a root of 1, g a simple Lie algebra, and Z the ideal of zero quantum dimensional modules; or C could be the Moore-Siberg category of 2-dimensional rational conformal field theory (2D-RCFT) or the Doplicher-Roberts category of localizable automorphisms of the algebra of observables of a 2D-QFT (quantum field theory). The last three are typically non Tannakian categories and in particular the statistical dimensions of the sectors are in general only algebraic integers thus one cannot reconstruct a Hopf algebra but rather the most general case is C being the rep category of a quasitriangular weak Hopf algebra (or quantum groupoid). On many occasions (2D-RCFT, 2D-QFT) one has more structure with C being ribbon and in fact a Turaev modular category with the irreps comprising a representation of the modular group SL(2,Z) with modular S and T matrices. The S plays the role of characters and diagonalizes the fusion rules (Verlinde's famous formula) while T is diagonal with the balancing phases on the diagonal. Next I will briefly mention three in my view important problems: classification, categorification (i.e., "reversing" the Grothendieck functor or given a fusion ring find the categories having this fusion ring), and explicit formulas (for fusion rules of say 2D-RCFT models).

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CAN THE CABIBBO MIXING ORIGINATE FROM NONCOMMUTATIVE EXTRA DIMENSIONS?

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The approach to hadronic flavor symmetries based on the quantum (q-deformed) algebras $U_q(su(n))$ (i.e., q-analogs of the Lie algebras of SU(n) groups) leads to a number of interesting consequences. Among these are: mass sum rules for vector mesons as well as for octet/decuplet baryons of improved accuracy; possibility to label (heavy quarkonia of) different flavors by topological means using torus winding number; in case of baryons the relevant values of deformation parameter q are linked in a simplest manner to the Cabibbo angle ?, which supposes for this angle the exact value ? = p /14. In the present contribution we discuss the possibility that the Cabibbo angle (and the Cabibbo mixing as a whole) can indeed take its origin in the noncommutativity properties of an additional, with respect to ordinary 3+1, spacetime dimensions.

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POSSIBLE CONSTRAINTS ON STRING THEORY IN CLOSED SPACE WITH SYMMETRIES

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It is well known that certain quadratic constraints have to be imposed on linearized gravity in closed space with symmetries. For example, if the background space is a flat torus, then the total momentum of the system is required to vanish. Thus, the system is

required to be translationally invariant. There are two ways to understand these constraints. One way is to note the analogy between gravity and electromagnetism. In the latter theory, the total charge can be obtained by integrating the electric flux over a surface which encloses the system. If the space is closed, electric flux originating from a positive charge must end at a negative charge because it cannot escape to infinity. Therefore the total charge must vanish. In linearized gravity there are fluxes obeying Gauss' law which couple to momentum if the space is translationally invariant. One obtains the ADM momentum by integrating these fluxes at infinity. Since they cannot escape to infinity, the total momentum must be zero. Another way to understand these constraints is to note that there cannot be any notion of "absolute position" in general relativity in closed space because of diffeomorphism invariance. In other words, the centre of mass cannot be defined, and as a result the total momentum, which is canonically conjugate to the centre of mass, must vanish. These constraints should certainly be present in string theory in closed background space with translation invariance. However, it is not clear how they arise because diffeomorphsm is not manifest in the current formulation of string theory and because it is not clear how to define "gravitational fluxes". We review some work concerning these constraints in

general relativity and provide some basis for further discussion as to how they may arise in string theory.

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NONCLASSICAL REPRESENTATIONS OF THE NONSTANDARD DEFORMATIONS $U'_q(so(n))$, $U_q(iso(n))$ AND $U'_q(so(n,1))$

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The aim of this report is to announce the results on irreducible nonclassical type representations of the nonstandard q-deformations $U'_q(so(n))$, $U_q(iso(n))$ and $U'_q(so(n,1))$ of the universal enveloping algebras of the Lie algebras so(n,C), iso(n) and so(n,1) when q is a real number. These representations are characterized by the property that they are singular at the point q=1. For this reason these representations have no classical analogue. It is shown that q-analogue of Gel'fand–Tsetlin formalism is also appropriate to give the explicit action formulas for representation operators in the case of nonclassical type representations.

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NONSTANDARD (TWISTED) q-DEFORMED ALGEBRA $U_q'(so(n))$ AND ITS APPLICATIONS

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The aim of this report is to review the results on the nonstandard q-deformation $U_q'(so(n))$ of the universal enveloping algebra U(so(n)) of the Lie algebra so(n) which does not coincide with the standard Drinfeld--Jimbo quantum algebra $U_q(so(n))$. Properties of this algebra are given. In particular, it is shown how it is related to Drinfeld--Jimbo quantum algebras. The algebra $U_q'(so(n))$ can be embedded into the quantum algebra $U_q(sl(n))$ as a subalgebra and a Hopf coideal. The PBW theorem for $U_q'(so(n))$ is proved. Formulas for central elements of it is given both for q a root of unity and for q not a root of unity. Irreducible finite dimensional representations of this algebras are described. For q a root of unity ($q^p=1$) these representations act on p^N -dimensional linear space (where N is a number of positive roots of the Lie algebra so(n)) and are given by r=dim so(n) complex parameters. It is related to the algebra of observables in 2+1 quantum gravity on the Riemmanian surfaces. A quantum analogue of the Riemannian symmetric space SU(n)/SO(n) is constructed by means of the algebra $U_q'(so(n))$. A q-analogue of the theory of harmonic polynomials (q-harmonic polynomials on quantum vector space) is constructed by using the algebra $U_q'(so(n))$. In particular, a q-analogue of different separations of variables is given by means of this algebra and its subalgebras.

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EUCLIDEAN GIBBS STATES OF QUANTUM LATTICE SYSTEMS

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A lattice model of N-dimensional quantum anharmonic oscillators with a polynomial anharmonicity and a ferroelectric pair interaction is considered. The local Gibbs states of such model are constructed by means of temporal Green functions defined on commutative subalgebras of the algebras of local observables. For these functions, a representation, which employs probability (Euclidean Gibbs) measures on infinite-dimensional spaces, is used. This allows to study a wide variety of the model properties, in particular, to describe quantum effects caused by noncommutativity.

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QUASIPARTICLES IN HOT NON-COMMUTATIVE FIELD- AND STRING THEORY

Karl Landsteiner
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I study the dispersion relation for scalar quasiparticles in supersymmetric, non-commutative field theories at finite temperature. In N=4 Yang-Mills theory the low momentum modes give rise to a superluminous front velocity. In the massless Wess-Zumino model the minimum of the dispersion relation is at non zero momentum for temperatures above a limiting temperature given by the inverse square-root of the product of the non-commutativity scale and the coupling constant. Finally I investigate how these effects arise in string theory on the world volume of a D3-brane in a B-field background.

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REPRESENTATIONS OF SIMPLE VECTORIAL LIE SUPERALGEBRAS AND SEMI-INFINITE COHOMOLOGY OF SUPERMANIFOLDS

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In this talk I will report on the joint results with P. Grozman and I.Shchepochkina on classification of irreducible representations of simple Lie superalgebras of vector fields with polynomial coefficients continuous with respect to a natural topology. Our result is a superization of a similar result obtained for Lie algebras by Rudakov in 1974. Super case is much more interesting and leads to new phenomena, e.g., to semi-infinite cohomology of the underlying supermanifold. We mainly consider the cases of exceptional Lie

superalgebras which have no analogs for Lie algebras and might be connected with the Grand Unification Theories.

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SPACE-TIME SUPERSYMMETRY AND QUANTUM DEFORMATIONS

Jerzy Lukierski
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We discuss the supersymmetric extensions of the deformations of of space-time described by constant and linear Poisson brackets. The example of quantum deformation of superspace with kappa-Minkowski space-time as even sector will be elaborated. The deformed superfield formalism with graded star-product multiplication will be presented. Supersymmetric extension of kappa-deformed Fourier transform is given.

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HOPF CATEGORIES VIA DERIVED CATEGORIES

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We attempt to get a Hopf category analogue of some quantum groups via equivariant derived categories of l-adic sheaves. They are equipped with the functors of multiplication and comultiplication and coherence isomorphisms. We discuss one of the ingredients of a monoidal 2-category of categories of perverse sheaves - the braiding. For a vector bundle $h:E\to B$ of dimension d over the algebraic closure of a finite field we prove that the braiding functor $Rh_!\circ h^*:D^b(B,Q_l)\to D^b(B,Q_l)$ between derived categories of l-adic sheaves is isomorphic to the (Tate twisted) shift functor $[-2d](-d)$.
The author would like to acknowledge the generosity of NATO in its support for this workshop.

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EXTERNAL FIELDS AS INTRINSIC GEOMETRY

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It is known that some bosonic field theories on the commutative geometry of space-time can be re-expressed as abelian-gauge theory in an appropriate noncommutative geometry. This fact is quite the analogue of the dichotomy in general relativity between the components of a metric considered as external fields in a flat background and the same components considered as defining the metric and therefore a non-flat geometry. We mention very briefly a certain number of examples which have been considered in the past and which exhibit the property of an external field which can be incorporated into a

redefinition of the basic geometry. The noncommutative structure can be considered as containing extra modes all of whose dynamics are given by the one abelian action. An example is afforded by the Yang-Mills-Higgs-Kibble action of the standard model. Somewhat analogous results are also known, for example, for non-relativistic hamiltonians and classical spin. Some of the most illuminating examples are taken from the field of simple hamiltonian mechanics. Complicated non-local non-polynomial hamiltonians can be considered as the free-particle hamiltonian in appropriately chosen geometries. An important dynamical variable which can also be considered as part of the space-time geometry is classical spin; a relativistic spinning particle can be described as an ordinary particle in a noncommutative geometry. We shall be concerned with analogous examples of this sort from noncommutative geometry, involving external fields which can be absorbed into a redefinition of the commutation relations rather than the metric.

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CATEGORIES OF GELFAND-ZETLIN MODULES

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Kiev Taras Shevchenko University, Ukraine

The Gelfand-Zetlin formal method of description of finite-dimensional simple modules over the Lie algebra gl(n,C) is a classical part in the theory of representations of Lie algebras and Lie groups. It was used by several authors for the description of (unitary) Harish-Chandra modules associated with symmetric pairs and for construction of the largest known family of simple modules over gl(n,C), known as generic Gelfan-Zetlin modules. In this talk we are going to discuss the following question:

- Construction of Gelfand-Zetlin modules.
- Structure of modules, induced from Gelfand-Zetlin modules.
- Structure of categories generated by Gelfand-Zetlin modules and their relation to the category of complete modules in the sense of Enright.
- Construction of some classes of Harish-Chandra modules over Lie algebras.
- Analogue for $U_q(gl_n)$.

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C*-ALGEBRAIC DESCRIPTION OF A QUANTUM PARTICLE IN SPACETIME

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(Doplicher et al, 1994, 1995) proposed a simple model of a quantum particle in spacetime. They postulate that the internal degrees of freedom of the particle are described by two vectors e and m in R3 of equal length, satisfying e.m=±1. The Hilbert space representation is such that the position-time operators Qμ satisfy noncanonical commutation relations. Uncertainty relations between time and position become important

on the scale of Planck's length. Our approach to study this model is a constructive one, based on a generalization to covariance systems of the well-known Gelfand-Naimark-Segal construction. In this approach the role of generalized projective representations of the Poincaré group is emphasised. The phase factor f , which appears when composing two unitary operators U(x)U(y)= exp(if) U(xy), is a function of e and m, and hence is an operator after quantization. In particular, the commutator between position and momentum operators Qμ resp. P? is proportional to the inverse of such a phase operator, denoted ?. The natural interpretation of ? is that of a metric tensor. It is used to define the square of the mass operator as M2= ?μ?PμP?. The two submanifolds of internal state, defined by the two possible values of e.m=±1, are the basis of a particle/anti-particle structure. Its analysis requires the introduction of a Dirac-like operator linearizing the equation M2? =m2? . This linearization is complicated by the noncommutativity of the momentum operators.

The authors would like to acknowledge the generosity of NATO in its support for this workshop.

S. Doplicher, K. Fredenhagen, J.E. Roberts, Spacetime quantization induced by classical gravity, Phys. Lett. B331, 39-44 (1994); The Quantum Structure of Spacetime at the Planck Scale and Quantum Fields, Commun. Math. Phys. 172, 187-220 (1995)

J. Naudts, C*-Multipliers, crossed product algebras, and canonical commutation relations, math-ph/9907008.

J. Naudts, M. Kuna, Covariance systems, preprint July 2000.

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QUANTUM CORRECTIONS TO SOLITONS

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We fix the long-standing ambiguity in the 1-loop contribution to the mass of a 1+1-dimensional supersymmetric soliton by adopting a set of boundary conditions which follow from the symmetries of the action and which depend only on the topology of the sector considered, and by invoking a physical principle that ought to hold generally in quantum field theories with a topological sector: for vanishing mass and other dimensionful constants, the vacuum energies in the trivial and topological sectors have to become equal. In the two-dimensional N=1 supersymmetric case we find a result which for the supersymmetric sine-Gordon model agrees with the known exact solution of the S-matrix but seems to violate the BPS bound. We analyze the nontrivial relation between the quantum soliton mass and the quantum BPS bound and find a resolution. For N=2 supersymmetric theories, there are no one-loop corrections to the soliton mass and to the central charge (and also no ambiguities) so that the BPS bound is always saturated. Beyond 1-loop there are no ambiguities in any theory, which we explicitly check by a 2-loop calculation in the sine-Gordon model.

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HOWE DUALITY AND LIE SUPERALGEBRAS

Irina Shchepochkina (Paramonova)
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I will report on an initiated by P. Deligne in 1989 joint work with D. Leites on an interpretation of the Howe duality in terms of Lie superalgebras. An important example of this duality is that between spinor and oscillator representations. We give several new examples of such duality some of which are based on a classification of maximal subalgebras of simple Lie superalgebras (hep-th/9702122). Some known phenomena become manifest in the light of this duality, e.g., the "square root" of the Hodge decomposition due to J. Bernstein. We similarly consider "square roots" of the sp(4)-action on the space of differential forms on the Kählerian manifolds.

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PARA, PSEUDO, AND ORTHO-SUPERSYMMETRIC QUANTUM MECHANICS AND THEIR BOSONIZATION

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Supersymmetry has established an elegant symmetry between bosons and fermions and is one of the cornerstones of modern theoretical physics. Its application to quantum mechanics has provided a powerful method of generating solvable quantum mechanical models. On the other hand, exotic quantum statistics has received considerable attention due to their possible relevance to the fractional quantum Hall effect and anyon superconductivity. By combining both concepts within the framework of quantum mechanics, one gets variants of supersymmetric quantum mechanics (SSQM). Here we review some of them: para, pseudo, and orthoSSQM. We then consider the problem of bosonizing SSQM and its variants, i.e., of describing them in terms of solely boson-like particles, instead of a combination of bosons and fermions or exotic particles. In the SSQM case, this is realized in terms of the generators of the Calogero-Vasiliev algebra (also termed deformed Heisenberg algebra with reflection). In that of the SSQM variants, this is done by considering a generalization of the latter algebra, namely the C_λ-extended oscillator algebra, where C_λ is the cyclic order of order $\sim\lambda$.

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FOCK REPRESENTATION OF A *- ALGEBRAS ALLOWING WICK ORDERING

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We consider a *-algebras defined by generators a_i^* , a_i , $i = 1,\ldots,d$, satisfying the basic relations $a_i^* a_j = d_{ij}\ 1\ + ?\ _{k,l=1}^{d} T_{ij}^{kl} a_l a_k^*$, $i,j,k,l=1,\ldots,d$. These relations allow everyone to represent each element of algebra as a linear combination of Wick ordered monomials. P.E.T. Jorgensen , L.M. Schmitt and R.F. Werner called such algebras a Wick algebras. This family is a generalisation of several types of deformed canonical commutation relations, such as twisted canonical commutation an anticommutation relations defined by V. Pusz and S. Woronowicz, higherdimensional q - CCR defined by M. Bozejko and R. Speicher etc. In terms of a non-commutative differentional calculus the similar relations was considered by B. Zumino and J. Wess. We have investigated the conditions for the positivity of Fock representation of Wick algebras with braided operator T. We have improved the result of Bozejko and Speicher about strict positivity of Fock inner product and , in the case when operator T is contraction, we proved that the kernel of Fock representation is generated by the largest quadratic Wick ideal. As a corollary of these results we have the C*-representability of several deformations of CCR and a description of generators of the largest cubic Wick ideal. Finally we consider the universal bounded representations of Wick algebras corresponding to several deformations of canonical commutation relations and show that Fock representation of these C*-algebras is faithful.

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SUPERANALOGS OF JACK POLYNOMIALS

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Superanalogs of Jack polynomials are obtained as eigenfunctions of a superanalog of Sutherland operator enters an infinite family of differential operators with our Jack polynomials as their common eigenfunctions. For certain particular values of the parameter on which Jack polynomials depend, the polynomials are interpreted (a partial answer to G. Olshannsky's question) as zonal spherical functions on homogeneous super spaces corresponding to pair(GL,OSP) and (Gl,Pe).

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ON STRUCTURE OF NONCOMMUTATIVE ALGEBRAS OF TIME-DEPENDENT SYMMETRIES FOR INTEGRABLE EVOLUTION EQUATIONS

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We derive a general representation for the time-dependent formal symmetries of (1+1)-dimensional scalar evolution equations. Using this representation, we obtain the explicit formula for the leading term of commutator of two high order nonlocal time-dependent symmetries of such equations, thus exhibiting a Virasoro-type structure of the Lie algebra of these symmetries for generic (not necessarily completely integrable) evolution equations and providing a complete solution of the "evaluation from the top" problem, posed by A.M. Vinogradov et al., for this algebra. On the basis of this formula we suggest a modification of the well-known master symmetry approach to the construction of infinite sets of time-independent symmetries for integrable (1+1)-dimensional evolution systems. Namely, we show that under some easily verifiable conditions the presence of one linear in time (and possibly nonlocal) symmetry of sufficiently high order implies the existence of the infinite set of time-independent symmetries for the evolution equation under study. The key advantage of this result with respect to the standard master symmetry approach is that the cumbersome check of commutativity of the set of constructed time-independent symmetries is entirely avoided. We also give some other applications of the above-mentioned representation for time-dependent formal symmetries, including the complete description of structure of differential part for a large class of recursion operators. The generalization of obtained results for a class of systems of evolution equations is presented.

I am sincerely grateful to Profs. M. Blaszak, B. Fuchssteiner, B.A. Kupershmidt and V.V. Sokolov for stimulating discussions on the subject of this work.

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WORLD VOLUME REALIZATION OF AUTOMORPHISMS

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It is already known the relation among spacetime supersymmetry algebras and the world volume approaches to string theory. In particular, the field theory on the brane provides us with a field theory realization of the corresponding supersymmetry algebra of the background. BPS states have their field counterpart in terms of BPS field configurations solving the kappa symmetry eigenvalue problem $\Gamma_\kappa\varepsilon=\varepsilon$. The latter gives us the same

supersymmetry projection conditions appearing in the purely algebraic analysis plus some BPS equations which, when used in the hamiltonian analysis of brane theory give rise to the corresponding BPS mass/energy of the configuration in terms of the corresponding central charges. It has been lately stressed that the full group of automorphisms of the N=1 D=11 SuperPoincaré algebra is GL(32,?). In this work, we will mainly be concerned with the way BPS states are transformed under GL(32,?) transformations, in particular, under the SO(32,?) subgroup preserving their masses. This will be analysed, first of all, from a pure algebraic point of view and we will later move on its world volume realization. A natural question raised in this analysis is whether such automorphisms are good symmetries of brane actions, and if so, whether they are local or non-local symmetries. We will provide evidence in favour of the symmetry character of certain of these transformations on certain world volume theories. Spacetime rotations and boosts are among the automorphisms being linearly realized as local symmetries on the brane. Besides that, we will emphasize the non-local character of some of these transformations when acting on the world volume dynamical fields, such as the S-duality invariance of the D3-brane effective action and/or their T-duals and M-theory liftings. Some examples of world volume solitons and their transformations under such automorphisms will be explicitly examined. We would like to conclude with some remarks concerning the possible relation among the automorphism group and the U-duality group when compactifying the spacetime manifold.

The author would like to acknowledge the generosity of NATO in its support for this workshop.

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TURTLES AND TENSIONS IN HW/RS GEOMETRIES

K.S. Stelle
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The talk will consider the Randall-Sundrum spacetime as an analogue of domain wall spacetimes that have arisen in supergravity realisations of the Horava-Witten orbifold construction. Particular attention will be paid to the nature of the source terms needed to support such solutions.

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ON THE CLASSIFICATION OF N-EXTENDED SUPERSYMMETRIC QUANTUMMECHANICAL SYSTEMS

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Some properties of the irreducible multiplets of representation for the (p,q) - extended supersymmetry in one dimension are discussed. It is shown i) that any finite multiplet with M spin states is equivalent to a multiplet containing 2 spin states; ii) that all such multiplets are in one-to-one correspondence with real-valued Clifford Gamma-matrices of Weyl (block antidiagonal) type. The implication of these results to the theory of the spinning particle are analyzed.

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LOCALLY ANISOTROPIC (SUPER)SPACES AND STRINGS

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The work proposes a geometric background of the theory of field interactions and strings in commutative and noncommutative spaces with higher order ansiotropy. Our approach proceeds by developing the concept of locally anisotropic, in general, noncommutative space which unifies the logical and mathematical aspects of modern noncommutative and Kaluza-Klein theories and generalized Lagrange and Finsler geometry and leads to modelling of quantum and classical physical processes on spaces provided with nonlinear and distinguished connections and metric structures. The presentation is divided into four parts. The first one covers the geometry of higher order anisotropic superspaces. We focus on the geometry of distinguished by nonlinear connection vector superbundles, consider different supersymmetic extensions of Finsler and Lagrange spaces. The second part is devoted to the theory of higher order anisotropic superstrings. In the framework of supersymmetric nonlinear sigma models in Finsler extended backgrounds we prove that the low-energy dinamics of such strings contains also motion equations for locally anisotropic field interactions. In the third part we present new classes of locally anisotropic soliton-dilaton, black hole and cosmological solutions which have been constructed in the framework of general relativity theory and discuss their supersymmetric and superstring generalizations. The last (forth) part is connected with gauge theories on both locally anisotropic and noncommutative spaces.

S. Vacaru: JHEP 9809 (1998) 011; Nucl. Phys. B494 (1997) 590-665; Ann.Phys. 256 (1997) 39-61; J. Math. Phys. 37 (1996) 508-523.

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HIDDEN SYMMETRY OF SOME ALGEBRAS OF Q-DIFFERENTIAL OPERATORS

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In the past few years q-analogues of irreducible Cartan domains and associated prehomogeneous vector spaces have excited a great interest of specialists in quantum group theory. The present work deals with q-analogues of the algebras of differential operators with polynomial coefficients on the prehomogeneous vector spaces. Our main

result is in discovering and an explicit description of a hidden symmetry of these algebras. We present the result in the simplest case of matrix spaces (the corresponding Cartan domains are matrix balls). In conclusion we describe an application of our theory to the problem of deformation quantization of q-Cartan domains. For this purpose we discuss q-analogues of some notions of Hermitian geometry in Cartan domains (such as Kaehler forms, Hermitian metrics on tangent and cotangent bundles).

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QUANTUM CORRECTIONS TO SUPERSPACE CONSTRAINTS

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Low-energy supergravity theories to M-theory or superstring theories and D-brane effective actions receive an infinite set of higher derivative corrections. They are induced string corrections proportional to powers of α' (the inverse of the string tension) and receive contributions from massless as well as massive states. They have various origins from superstring loop corrections to anomaly induced contributions and their presence is highly constrained by supersymmetry. Invariance of the theory under supersymmetry including these corrections obliges for modifications of the supersymmetry algebra. In particular the field dependent structure coefficients of the algebra will receive corrections proportional to powers of alpha' and will modify the superspace constraints. These modifications to the constraints are linked to the recently computed higher-derivative corrections to D-brane actions. For the M-twobrane whether or not these modifications to the torsion constraints need new couplings is still unanswered but is of definitive importance. At a specific order in alpha' there can be various modifications to the field transformations that may lead to different superinvariants. Higher order α' corrections or some non-linear symmetry in the theory can impose extra constraints and relations between invariants. Knowing the number of independent invariants for a specific (superstring) theory would necessitate to take care of all the alpha' modifications of the superalgebra. This difficulty can be overcome by using duality relations. For example only one R^4 superinvariant is compatible with the Sl(2,Z) symmetry of the type IIB theory. We construct supersymmetry invariants for the R^4 corrections to the supergravity effective actions in d=10 and d=11. This construction requires a deformation of the supersymmetry transformation rules for the field by terms of order $(\alpha')^3$. These contributions modify the field dependent structure coefficients of the algebra, and consequently the superspace constraints. We find explicit expressions including fermion bilinears for these invariants, which will have to be re-derived from any microscopic definition of M-theory.

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SUPERSYMMETRY IN SINGULAR SPACES

Antoine Van Proeyen
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We develop the concept of supersymmetry in singular spaces, apply it in an example for 3-branes in D=5 and comment on 8-branes in D=10. The new construction has an interpretation that the brane is a sink for the flux and requires adding to the standard supergravity a (D-1)-form field and a supersymmetry singlet field. This allows a consistent definition of supersymmetry on a $\frac{S_1}{Z_2}$ orbifold, the bulk and the brane actions being separately supersymmetric. Randall-Sundrum brane-worlds can be reproduced in this framework without fine tuning. For fixed scalars, the doubling of unbroken supersymmetries takes place and the negative tension brane can be pushed to infinity. In more general BPS domain walls with 1/2 of unbroken supersymmetries, the distance between branes in some cases may be restricted by the collapsing cycles of the Calabi-Yau manifold. The energy of any static x^5-dependent bosonic configuration vanishes, E=0, in analogy with the vanishing of the Hamiltonian in a closed universe.

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A METRIC-AFFINE FIELD MODEL FOR THE NEUTRINO

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The first part of the talk is an exposition of [2]. We consider the Dirac equation in flat Minkowski 3-space and rewrite it as the polarised Maxwell equation in Minkowski 4-space with torsion. The torsion tensor T is defined as the dual of the electromagnetic vector potential A:

(1) $$T=*A.$$

Our model clearly distinguishes the electron and the positron without resorting to ``negative frequencies'': we produce a real scalar invariant (charge) which indicates whether we are looking at an electron or a positron. Another interesting feature of our model is that the free electron and positron are identified with gradient type solutions of the standard (torsion free) Maxwell equation; such solutions have traditionally been disregarded on the grounds of gauge invariance. In the second part of the talk we attempt to develop, starting from (1), an affine field theory. This means that we view the connection coefficients $\Gamma_{\lambda\mu\nu}$ as the unknowns. The main difference with Einstein-Schrödinger type affine field models is that we assume the connection to be metric compatible. Thus, the actual unknown quantity in our theory is the torsion tensor T which has 24 independent components. We base our analysis on the decomposition of the spaces of torsions and curvatures into invariant subspaces (joint work with A.D.King). For torsions we have a decomposition into 3 subspaces of dimension 4, 4, and 16, respectively. One of the 4-dimensional subspaces is the subspace of torsions which are

dual to vectors. It appears that this subspace has been neglected in works on affine field theory; in our view it is the subspace which is ``responsible" for the interaction between the electron/positron and the electromagnetic field. For curvatures we follow Einstein [1] in writing the basic decomposition in terms of the eigenspaces of the operator R→*R*. (Simultaneous action of the Hodge star on the first and second pairs of indices; note that this operation is independent of orientation.) We pose the following problem: find a torsion from an invariant subspace which generates a curvature from an invariant subspace. Here in the first instance we restrict our interest to geometries with the additional property Ric=0. We identify 3 torsions with such remarkable properties. We conjecture that these 3 torsions can be used as building blocks in the construction of a soliton model for the electron/positron. In the process of decomposing curvatures we also correct a mistake from [1].

[1] A.Einstein, Math. Ann. **97** (1927) 99-103.
[2] D.Vassiliev, preprint http://xxx.lanl.gov/abs/math-ph/0006019.

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GENERALIZED TAUB-NUT METRICS AND KILLING-YANO TENSORS

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The necessary condition that a Staeckel-Killing tensor of valence 2 be the contracted product of a Killing-Yano tensor of valence 2 with itself is rederived for a Riemannian manifold. This condition is applied to generalized Euclidean Taub-NUT metrics which admit a Kepler-type symmetry. It is shown that, in general, the Staeckel-Killing tensors involved in the Runge-Lenz vector cannot be expressed as a product of Killing-Yano tensors. The only exception is the original Taub-NUT metric. The non-existence of the Killing-Yano tensors makes the study of "hidden" symmetries and supersymmetries more laborious in models of relativisticparticles with spin 1/2 involving anticommuting Grassmann variables.

J.Phys.A:Math.Gen. 33, 4383 (2000); hep-th/ 9911126.

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NON-ABELIAN GAUGE THEORIES ON NON-COMMUTATIVE SPACES

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The structure of non-commutative spaces will be discussed for the canonical structure, the lie algebra structure and the quantum space structure. As far as the quantum space structure is concerned, a more detailed discussion of these spaces and their connection to quantum groups will be given. Weyl quantization will be introduced as a tool to connect non-commutative spaces to deformation quantization. The elements of the algebra will be

characterized by functions of commuting variables, the non-commutative structure will be expressed by the star product. In this formalism a gauge theory will be developed, which, to start with, requires an enveloping algebra-valued connection. With the help of the Seiberg-Witten map it will be shown that this enveloping algebra-valued connection can be build up by a finite number of gauge fields. These are exactly the gauge fields of the lie algebra-valued connection.

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SUPERPARTICLES WITH TENSORIAL OR SCALAR CENTRAL CHARGES AND SPINNING PARTICLES

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Models of extended superparticles with tensorial and or scalar central charges in superspace whose set of coordinates is enlarged by bosonic index spinors and which are classically equivalent to the corresponding models of spinning particle possessing world-line supersymmetry in presence additional index spinor coordinates of space-time are constructed and examined. Such models are interesting in connection with fast development of branes theory where appearing of different types of central charges for pointlike and extended objects, i. e. particles, strings and branes, is natural due to the existence of the D=11 supergravity solutions preserving various fractions of the underlying supersymmetry and having interpretations in terms of intersecting branes. We take the minimal complete set of bosonic coordinates for arbitrary spin and isospin description and adding it to the space-time coordinates of spinning particle and extended superparticle we arrived at classical equivalence of these models when space-time supersymmetry is suitably broken. Last phenomena is described formally by introducing of Lorentz tensorial central charges and internal symmetry ones. It is interesting that, at least for particles, the obtained correspondence between supersymmetries of space-time and world-line does not need in proper central charge coordinates. It is sufficient to have the central charges constructed with the index spinors. It should be stressed that our consideration is, in a sense, insensitive to the mass of the particle and space-time dimension because we used the index spinor formalism which in massless case is equivalent to the twistorial one. By fixing of gauging to remain with physical degree of freedom only we are able to regard quantum equivalence of the particle models with partially broken target-space supersymmetry and world-line supersymmetry. The same result is obtained by the Dirac-Gupta-Bleuler first quantization of the models. Partial gauge-fixing for kappa-invariance, which is possible, in constructed superparticle models creates some series of superparticle models with more low preserving supersymmetry.

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FINITE UNIFIED THEORIES — THEORY AND PREDICTIONS

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All-loop Finite Unified Theories (FUTs) are very interesting N=1 supersymmetric GUTs in which a complete reduction of couplings has been achieved. FUTs realise an old field theoretical dream and have remarkable predictive power. Reduction of dimensionless couplings in N=1 GUTs is achieved by searching for renormalization group invariant (RGI) relations among them holding beyond the unification scale. Finiteness results from the fact that there exist RGI relations among dimensionless couplings that guarantee the vanishing of the beta-functions in certain N=1 GUTs even to all orders. Recent developments in the soft supersymmetry breaking (SSB) sector of N=1 GUTs and FUTs lead to exact RGI relations also in this sector of the theories. Of particular interest is a RGI sum rule for the soft scalar masses holding to all orders. The characteristic features of SU(5) models that have been constructed based on the above tools are: a) the old agreement of the top quark mass prediction with the measured value remains unchanged; b) the lightest Higgs boson is predicted to be around 120 GeV c) the s-spectrum starts above several hundreds of GeV.

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