LTL Reactive Synthesis with a Few Hints

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Synthesis

LTL Reactive Synthesis

$$\boxed{\text{Env}} \parallel \boxed{\ ?\ } \models \varphi$$

- Sys is constructed by an algorithm
- Sys is **correct** by construction
- Underlying theory: 2 player zero-sum **games** played on graphs
- Env is **adversarial** (worst-case assumption)
- Correct Sys = **Winning strategy**
- Main argument of Synthesis: **What** versus **How**

LTL Reactive Synthesis

$\forall \quad \bar{I} = i_0 i_1 \ldots i_n \ldots \in I^\omega \longrightarrow$ **Sys** $\longrightarrow \bar{O} = o_0 o_1 \ldots o_n \ldots \in O^\omega$

environment

synchronous response

$$i_0 o_0 i_1 o_1 \ldots i_n o_n \ldots \models \varphi$$

- Problem: given a LTL formula φ over $AP = I \uplus O$, decide if there exists a strategy $\sigma : I^* \cdot I \rightarrow O$ such that for **all** sequences of inputs (=env. is adversarial) $\bar{I} = i_0 i_1 \ldots i_n \ldots \in I^\omega$:

$$i_0 \cdot \sigma(i_0) \cdot i_1 \cdot \sigma(i_0 i_1) \cdot \ldots \cdot i_n \cdot \sigma(i_0 i_1 \ldots i_n) \cdot \ldots \models \varphi$$

and if so, construct such a strategy.

LTL Reactive Synthesis

An example - **Mutual exclusion**

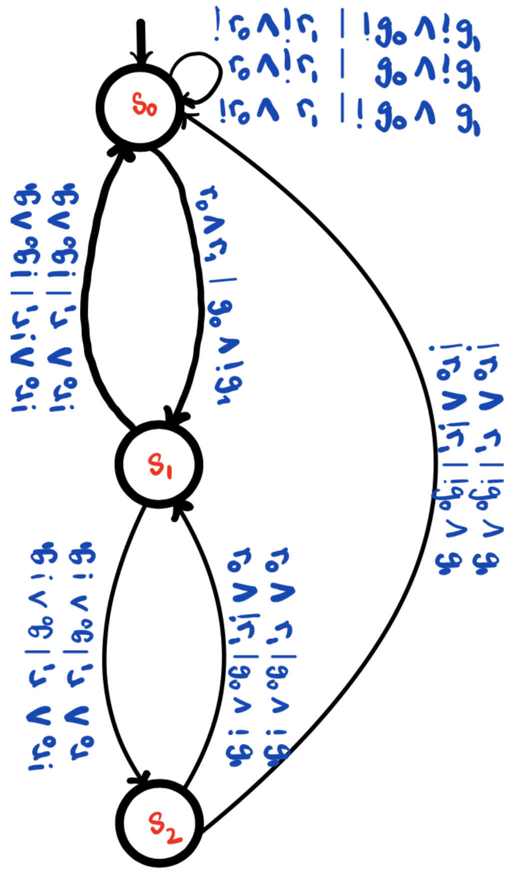
- **Input AP**: r_0, r_1
- **Output AP**: g_0, g_1
- **CORE Spec**:
  - $\varphi_{\text{CORE}} \equiv \square(\neg g_0 \vee \neg g_1) \wedge \square(r_0 \rightarrow \Diamond g_0) \wedge \square(r_1 \rightarrow \Diamond g_1)$

CORE Spec = properties that you would check on any solution to mutual exclusion (e.g. Peterson, Dedecker, etc.) - Remember “What vs. How”

Mealy Machine that Realizes Spec

$$\forall \quad \bar{I} = i_0 i_1 \dots i_n \dots \in I^\omega \longrightarrow \boxed{M} \longrightarrow \bar{O} = o_0 o_1 \dots o_n \dots \in O^\omega$$

$$i_0 o_0 i_1 o_1 \dots i_n o_n \dots \models \varphi_{\text{CORE}}$$



$$\models \varphi_{\text{CORE}}$$

- A Mealy machine is an **input-complete** deterministic automaton with **outputs** that encodes a **strategy**
- M realizes φ if $\forall \bar{I} \in I^\omega : M(\bar{I}) \models \varphi_{\text{CORE}}$, noted $M \models \varphi_{\text{CORE}}$

LTL Reactive Synthesis

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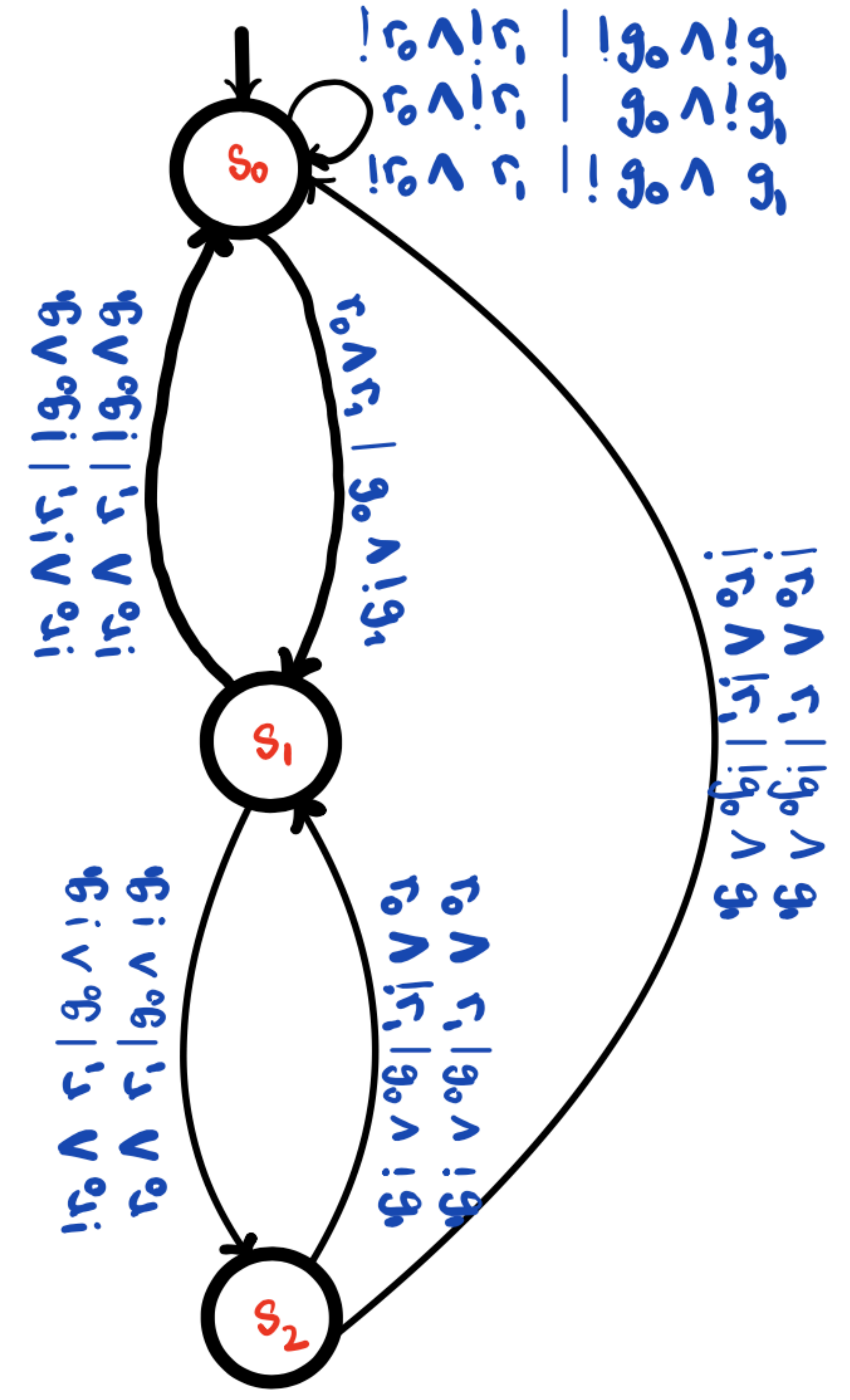
Synthesis algorithm

?



Solution=Winning Strategy=**Mealy Machine**

LTL Reactive Synthesis

- LTL reactive synthesis is **2ExpTime-C**
- Nevertheless, “efficient” implementations exist
  (medium size spec - $\approx$ one page) - e.g. Acacia (ULB-U Antwerpen) - STRIX (TUM)
- Demo: Mutual exclusion (STRIX)

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Assumptions:

```
true
```

Guarantees:

```
G (!grant_0 | !grant_1)
G (request_0 -> F grant_0)
G (request_1 -> F grant_1)
```

Input propositions:

```
request_0, request_1
```

Output propositions:

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grant_0, grant_1
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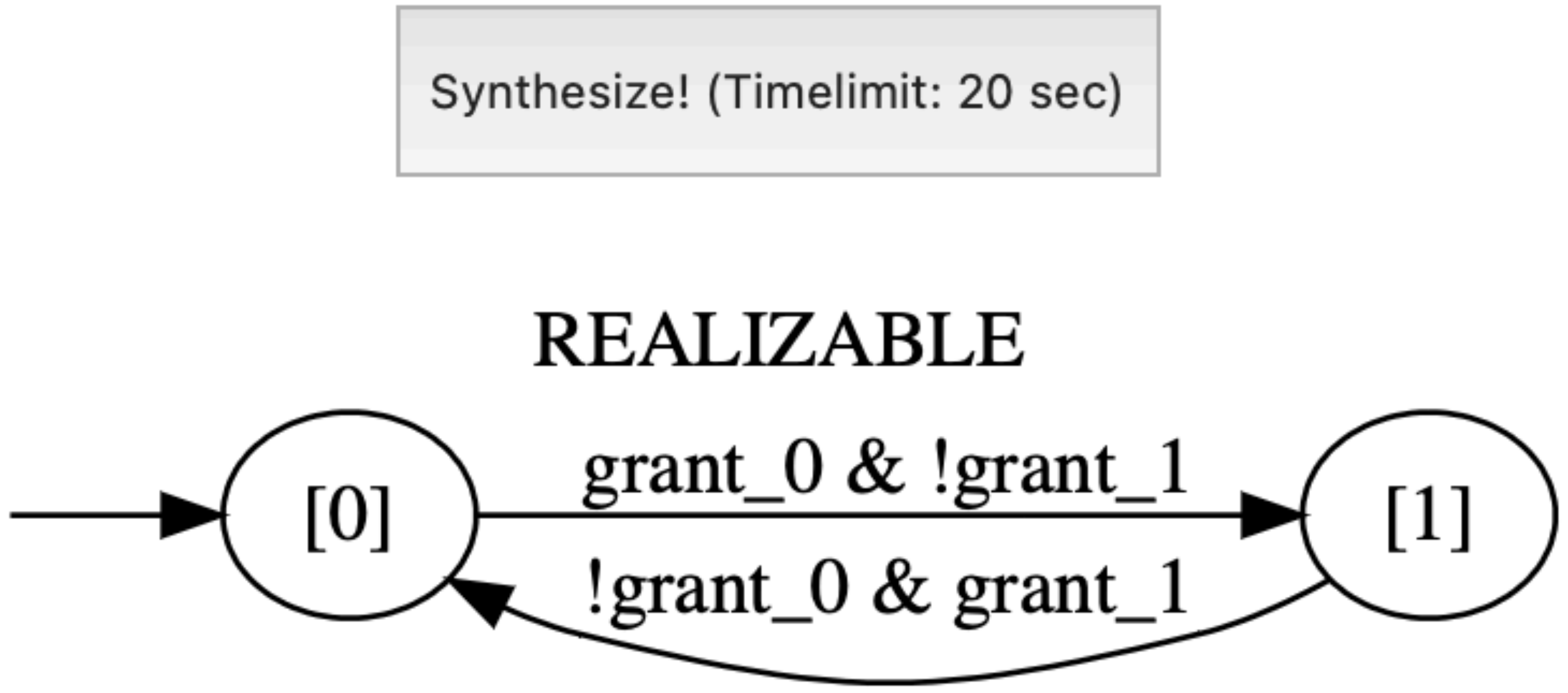

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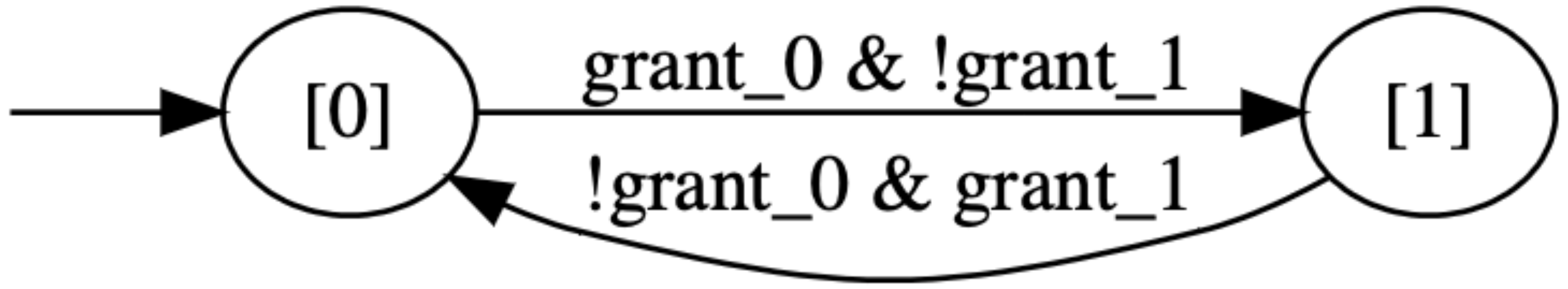
Synthesize! (Timelimit: 20 sec)

REALIZABLE



Quality of solution ?
Small is beautiful ?
is “What only” sufficient ?

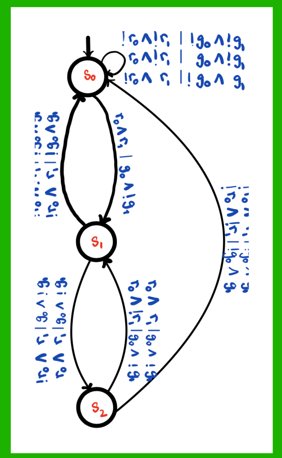
Space of solutions of φ_{CORE}

How to drive the synthesis procedure to good solutions?

Mealy Machines

Mealy machines that realizes φ_{CORE}

"Good"solutions



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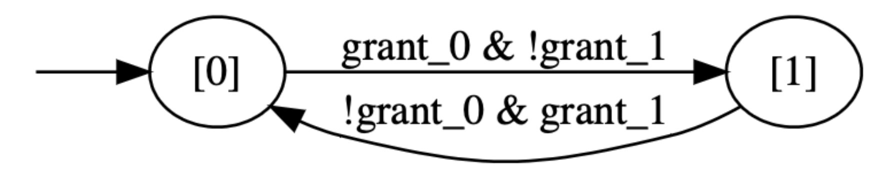

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Quality of solutions

- The **“What only”** may lead to solutions that are not of practical interest
  (e.g. unsolicited grants)
- Remedy ? Give a “**complete”** specification
- Example: Mutual exclusion without **unsolicited grants**

Quality of solutions

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Guarantees:

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G ((grant_0 & G !request_0) -> (F !grant_0))
G ((grant_1 & G !request_1) -> (F !grant_1))
G ((grant_0 & X (!request_0 & !grant_0)) -> X (request_0 R !grant_0))
G ((grant_1 & X (!request_1 & !grant_1)) -> X (request_1 R !grant_1))
G (!grant_0 | !grant_1)
request_0 R !grant_0
request_1 R !grant_1
G (request_0 -> F grant_0)
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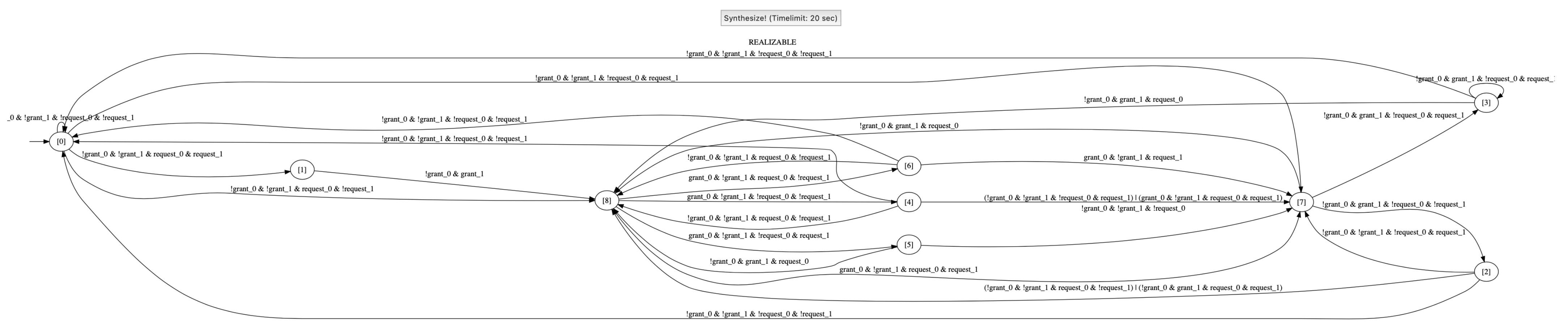


Quality of solutions

- The **"What only"** may lead to solutions that are not of practical interest
- A…
  s…
- Ex…
  exc…
  uns… …nts

Assumptions:

```
1 true
```

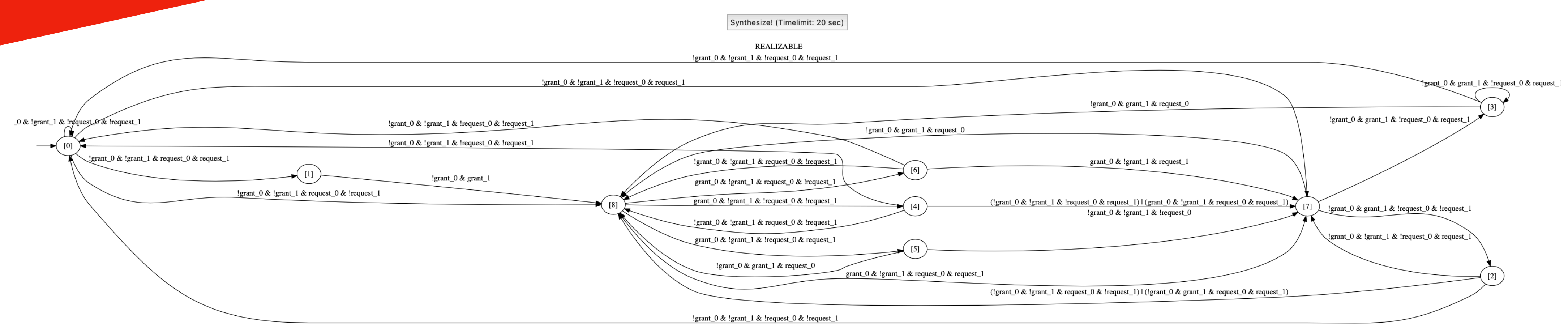
Guarant…

BUT

Specifying low level requirements in LTL may be difficult/cumbersome

It is not clear that it is the Mealy machine that we are looking for

Specifying lower level requirements clear goes against the very idea of synthesis!



Our proposal:
Add scenarios (examples)
to φ_{CORE}

$$\{\,!r_0, !r_1\,\}\,.\,\{\,!g_0, !g_1\,\}\#\{r_0, !r_1\,\}\,.\,\{g_0, !g_1\}\#\{\,!r_0, r_1\,\}\,.\,\{\,!g_0, g_1\,\}$$

Requirement engineering and scenarios

Formal spec and scenarios are complementary

- Scenarios are accepted in RE as an adequate tool to **elicit** requirements
- Scenarios are **easy** to produce: the designer controls **both** the inputs and the outputs
- … avoiding the main difficulty of reactive system design: having to cope with **all** possible environment inputs

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- Scenarios (=examples=hints) as an alternative to **guide** the search for “good” solutions: the synthesis algorithm must now produce solutions **compatible** with the examples

Another (more appealing) approach

LTL spec + Hints (scenarios)

- $\varphi_{\text{CORE}} \equiv \Box(\neg g_0 \vee \neg g_1) \wedge \Box(r_0 \rightarrow \Diamond g_0) \wedge \Box(r_1 \rightarrow \Diamond g_1)$
- + a few **Hints**:
    - $\{!r_0, !r_1\} . \{!g_0, !g_1\} \# \{r_0, !r_1\} . \{g_0, !g_1\} \# \{!r_0, r_1\} . \{!g_0, g_1\}$
    - $\{r_0, r_1\} . \{g_0, !g_1\} \# \{!r_0, !r_1\} . \{!g_0, g_1\}$

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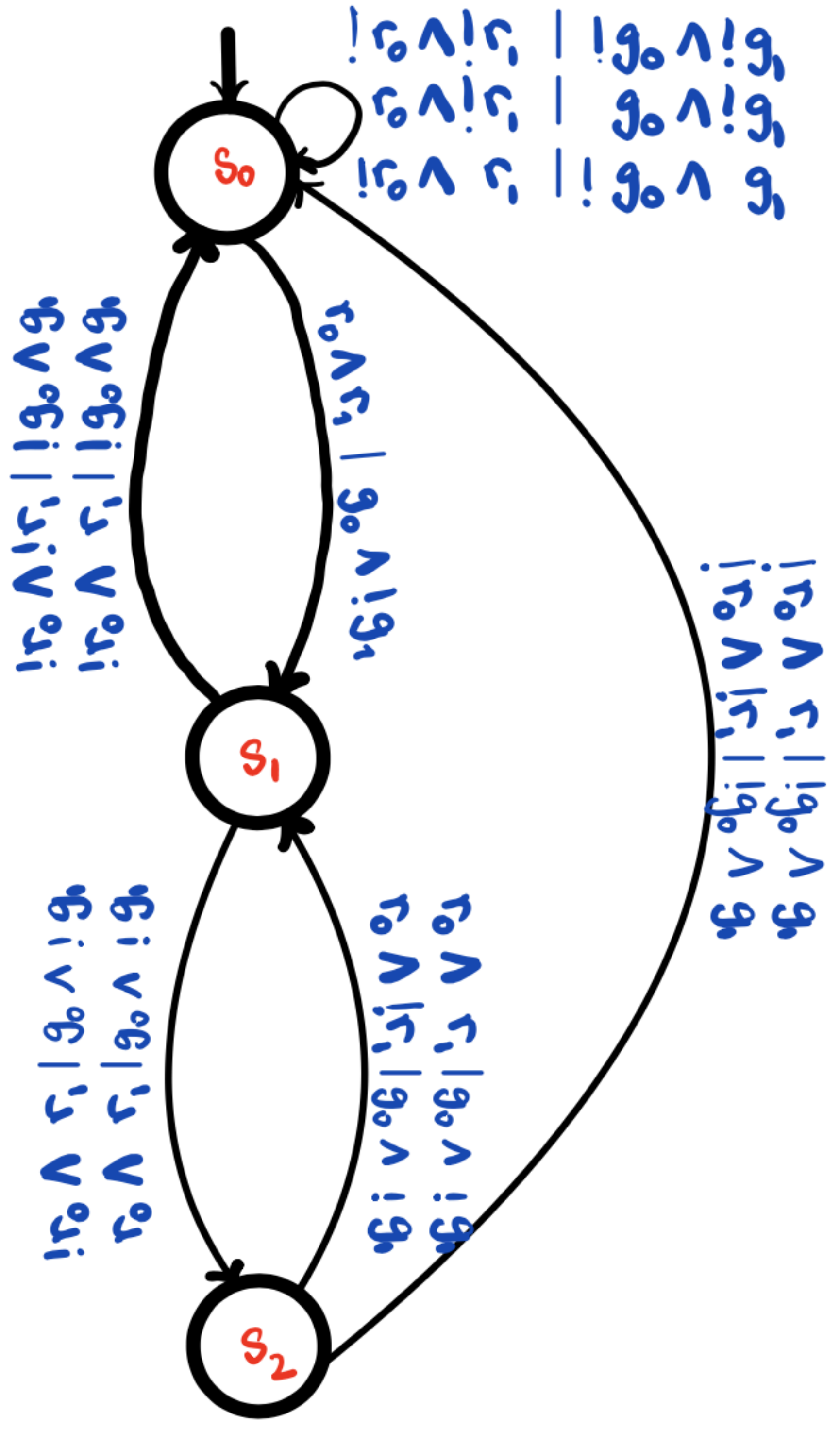
Learning+Synthesis



Another (more appealing) approach

LTL spec + Hints (scenarios)

- If you want a solution where g_1 comes before g_0 in case of concurrent request: **change the scenario** !
- **Hints**:
  - $\{!r_0, !r_1\} . \{!g_0, !g_1\}\#\{r_0, !r_1\} . \{g_0, !g_1\}\#\{!r_0, r_1\} . \{!g_0, g_1\}$
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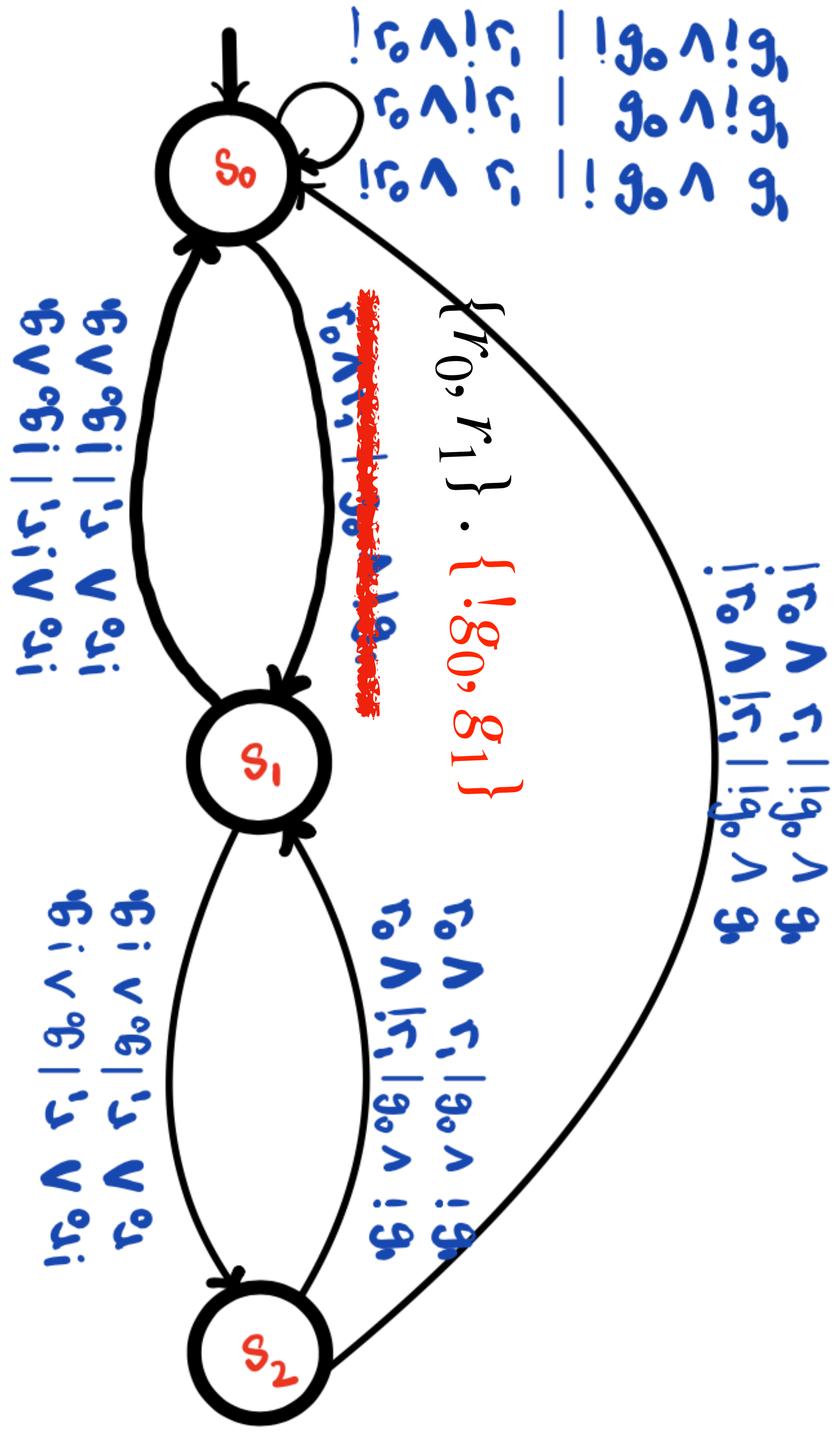

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LTL spec + Hints (scenarios)

- If you want a solution where $g_1$comes before g_0 in case of concurrent request: **change the scenario** !
- **Hints**:
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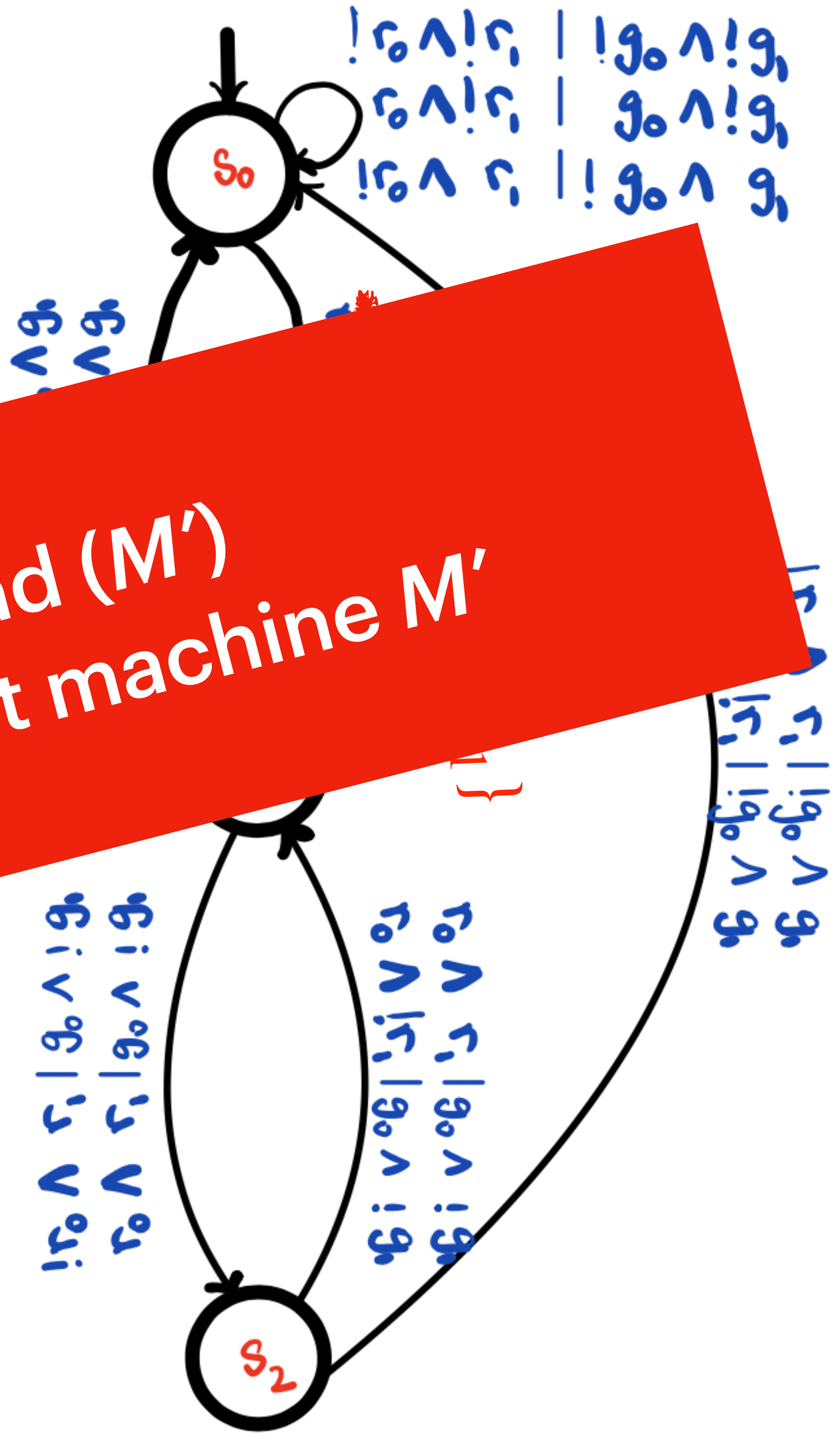
  - $\{r_0, r_1\} \,.\, \{\, !g_0, g$

If the solution M is not the one that User has in mind (M′)
then add an example that differentiates M and the target machine M′



$$(\varphi_{\mathsf{CORE}}, E_0) \longrightarrow M_0$$

$(\varphi_{\mathsf{CORE}}, E_0) \longrightarrow M_0$

If designer not happy

$(\varphi_{\mathsf{CORE}}, E_0 \cup \{e_1, e_2, \ldots, e_n\}) \longrightarrow M_1$

$(\varphi_{\mathsf{CORE}}, E_0) \longrightarrow M_0$

If designer not happy

$(\varphi_{\mathsf{CORE}}, E_0 \cup \{e_1, e_2, \ldots, e_n\}) \longrightarrow M_1$

If designer not happy

$(\varphi_{\mathsf{CORE}}, E_1 \cup \{e'_1, e'_2, \ldots, e'_m\}) \longrightarrow M_2$

...

Until designer is happy

LTL Reactive Synthesis with a Few Hints

The problem definition

- Given a (*i*) LTL formula φ and (*ii*) a prefix-closed set of examples (scenarios) $E \subseteq (I \cdot O)*$, construct a Mealy Machine M that is:
    - compatible with E and
    - such that for all $\forall \bar{I} = i_0 i_1 \dots i_n \dots \in I^\omega : M(\bar{I}) \models \varphi$

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+ informal requirement: generalize E

Our solution - two-phase algorithm

Mix formal methods and learning

- Phase 1: **learn** a pre-Mealy machine that **generalizes** the examples in ***E*** and which **maintains realizability** (checked using game-based synthesis) of φ_{CORE}

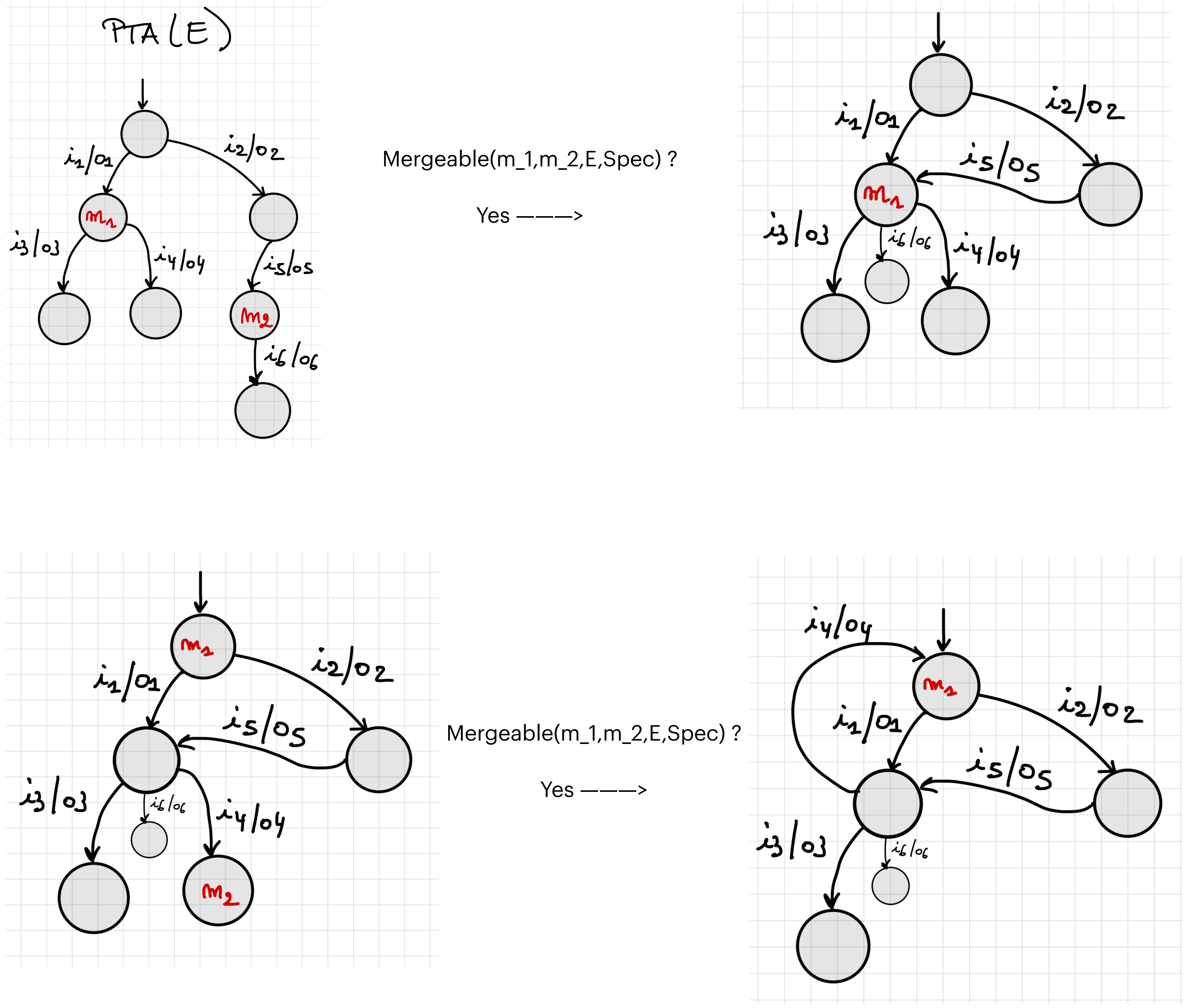
  intermediary output: a **pre**-Mealy machine (usually not input-complete)

- Phase 2: **complete** (using game-based synthesis) the pre-Mealy machine into a complete Mealy machine that realizes φ_{CORE} while maintaining compatibility with the examples in ***E***

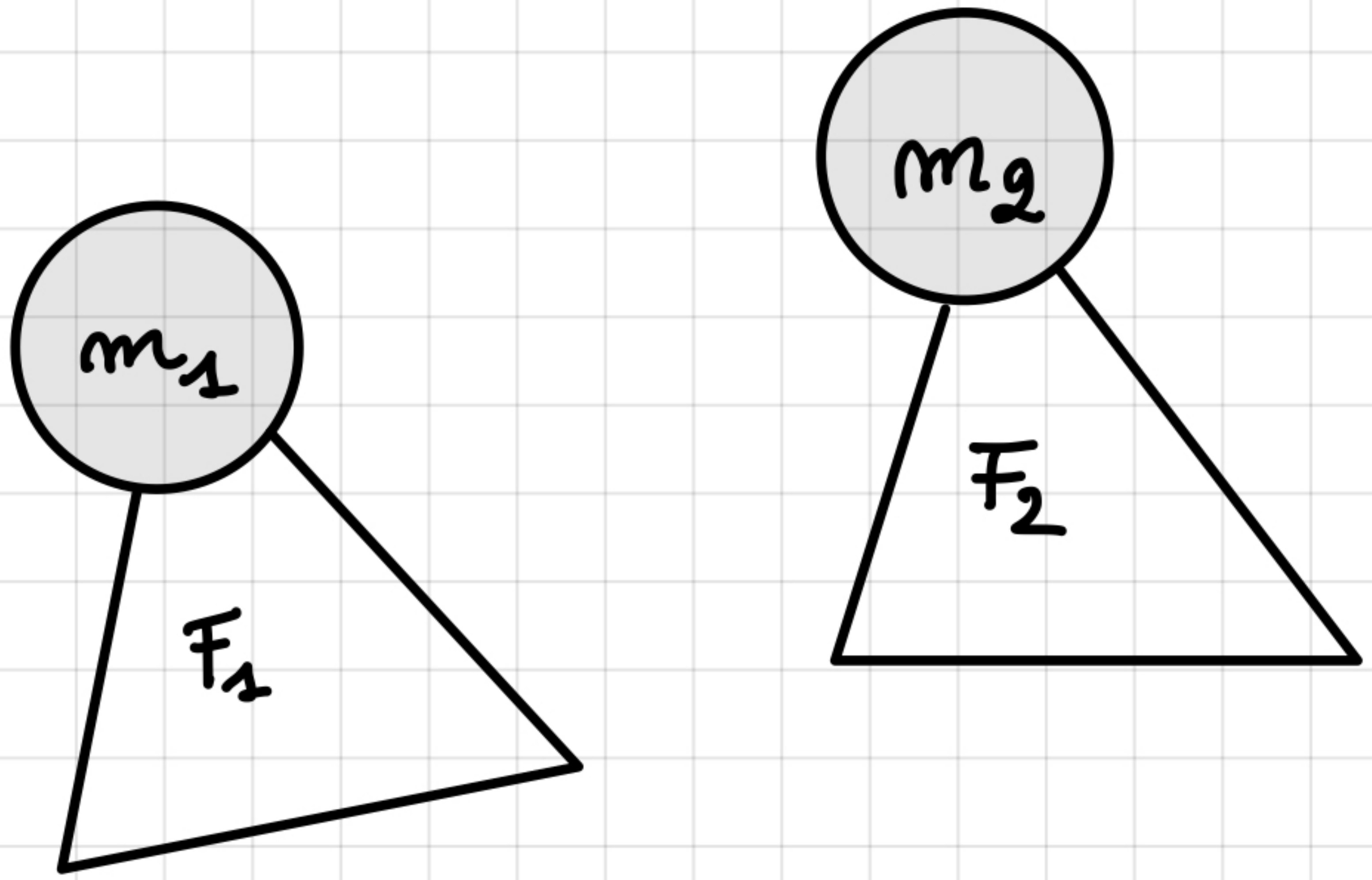
Phase 1 - Generalization

Learning automata from examples constrained by Spec realizability

- **RPNI** style learning:
  Start with PTA(E)=prefix tree automaton of the examples in ***E***
- **Merge** states when **possible** in order to **generalize** from the examples
- Mergeable?(m_1,m_2,***E***,φ_{CORE})
  - Yes, if the resulting pre-Mealy Machine is compatible with ***E*** and can be completed into a (full) Mealy Machine that realizes φ_{CORE}

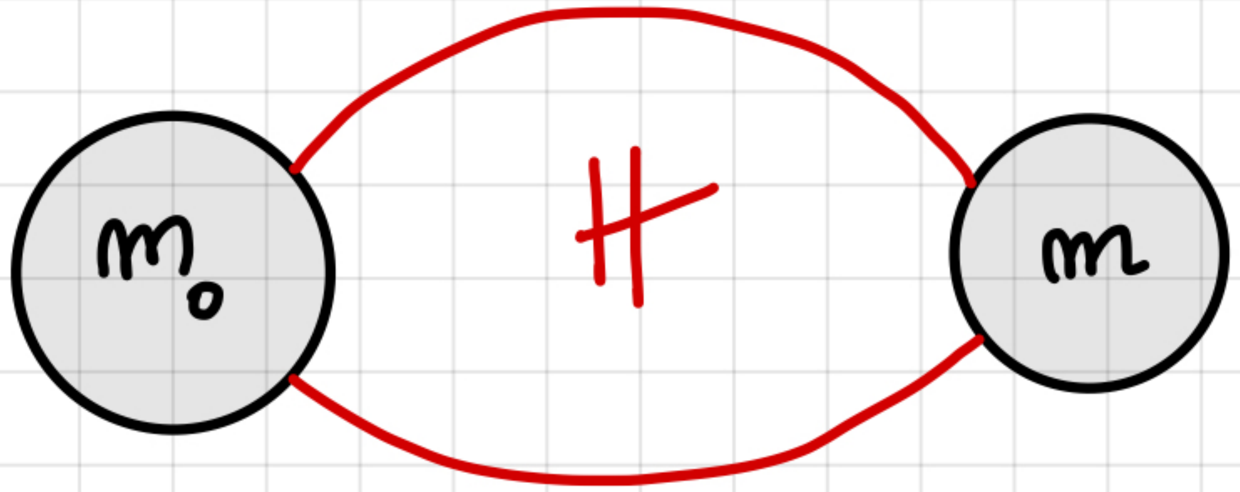


Mergeable?(m_1,m_2,*E*,Spec)



① F_1 and F_2 must agree on all $\bar{I} \in I^*$ for which F_1 and F_2 are defined

② $M \xrightarrow{\text{merge}(m_1, m_2)} M'$

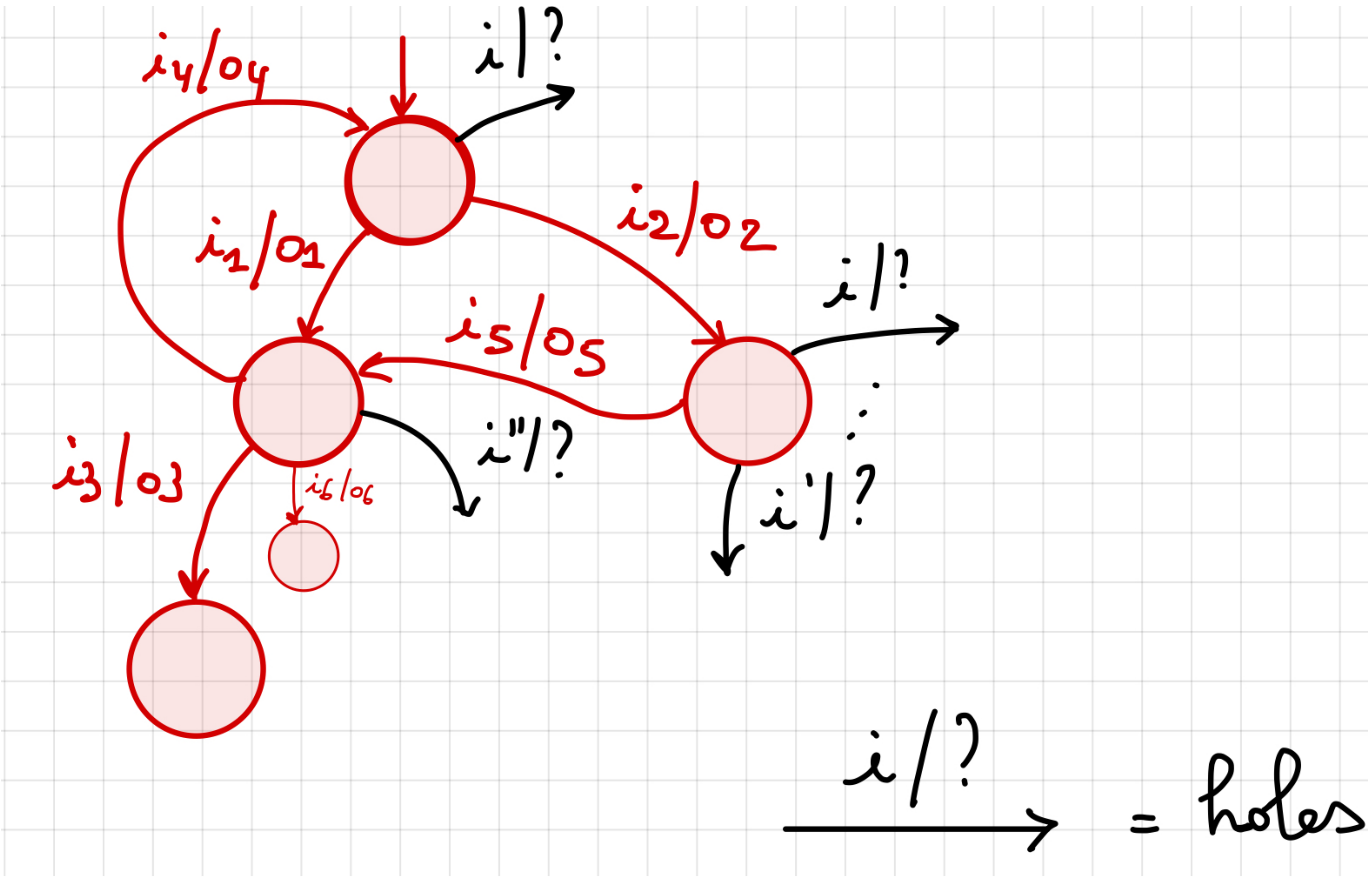


Check that from m, we can construct a strategy that works $\forall h \in H$!

Phase 2 - Completion

From pre-Mealy Machine to a (full) Mealy Machine that realizes Spec

- Given a pre Mealy machine ***M*** that generalizes the set of examples ***E*** and that can be completed into a Mealy machine ***M'*** that realizes φ



- Complete **holes** in the machine.
- Heuristics: try to avoid creating new states and **reuse** existing red ones (idea: **generalize** examples).

How to maintain efficiency

Exploit the most general strategy

- **Difficulty:**

  Theorem (**Mergeable complexity**): (Even) for a regular specification Spec φ_{CORE} given as a *deterministic Büchi automaton*, deciding Mergeable$(M, m, m', \varphi_{\text{CORE}}, E)$ is **ExpTime-C**. (a subset construction is needed)

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- … in the two-phase algorithm, we need to use $\mathsf{Mergeable}(M, m, m', \varphi_{\mathsf{CORE}}, E)$ multiple times, and in the worst-case the parity automaton A_φ associated to the LTL spec φ is already doubly exponential in $|\varphi|$.
- Can we avoid this complexity problem ? **YES**

How to maintain efficiency

Exploit the most general strategy

- **Theorem** Given (φ, E), SynthLearn(φ, E) returns a Mealy machine M such that $E \subseteq L(M)$ and $L_\omega(M) \subseteq [\![\varphi]\!]$ if it exists, in worst-case doubly exponential time in $|\varphi|$ and polynomial in $|E|$. Otherwise it returns UNREAL.

- More precisely, our algorithm is
  - polynomial in the size of E and
  - polynomial in a well-chosen symbolic representation the of set of Mealy machines that realize φ which is computed by Acacia-Bonzai for solving plain LTL synthesis for φ.
- So, generalizing from E comes at an additional **polynomial cost**.

Illustration

Phase 1: generalization of E

Specification:

$$\varphi_{\text{CORE}} \equiv \square(\neg g_0 \vee \neg g_1) \wedge \square(r_0 \rightarrow \lozenge g_0) \wedge \square(r_1 \rightarrow \lozenge g_1)$$

Examples E:

- $\{\,!r_0, !r_1\} \,.\, \{\,!g_0, !g_1\}\#\{r_0, !r_1\} \,.\, \{g_0, !g_1\}\#\{\,!r_0, r_1\} \,.\, \{\,!g_0, g_1\}$
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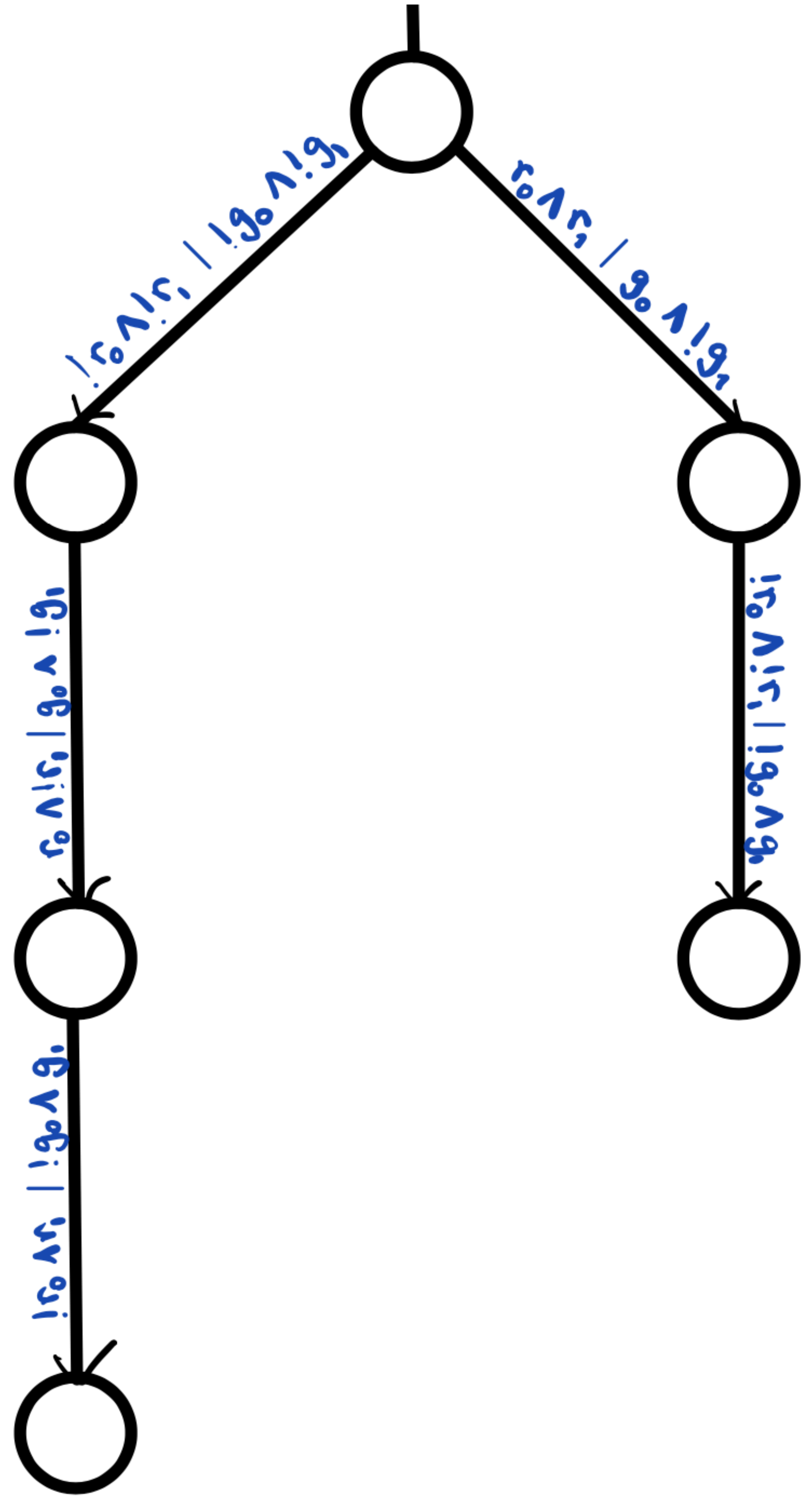
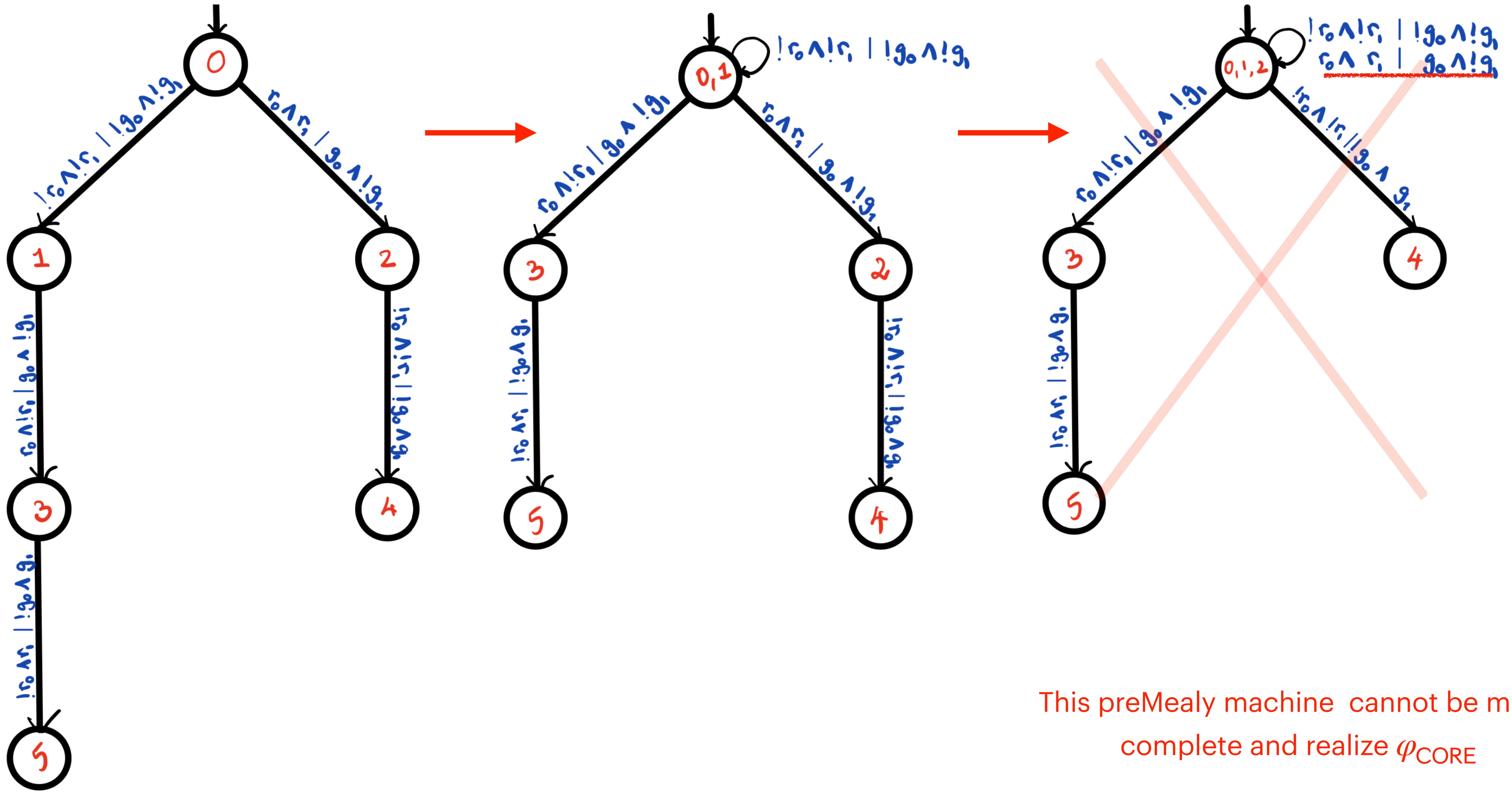
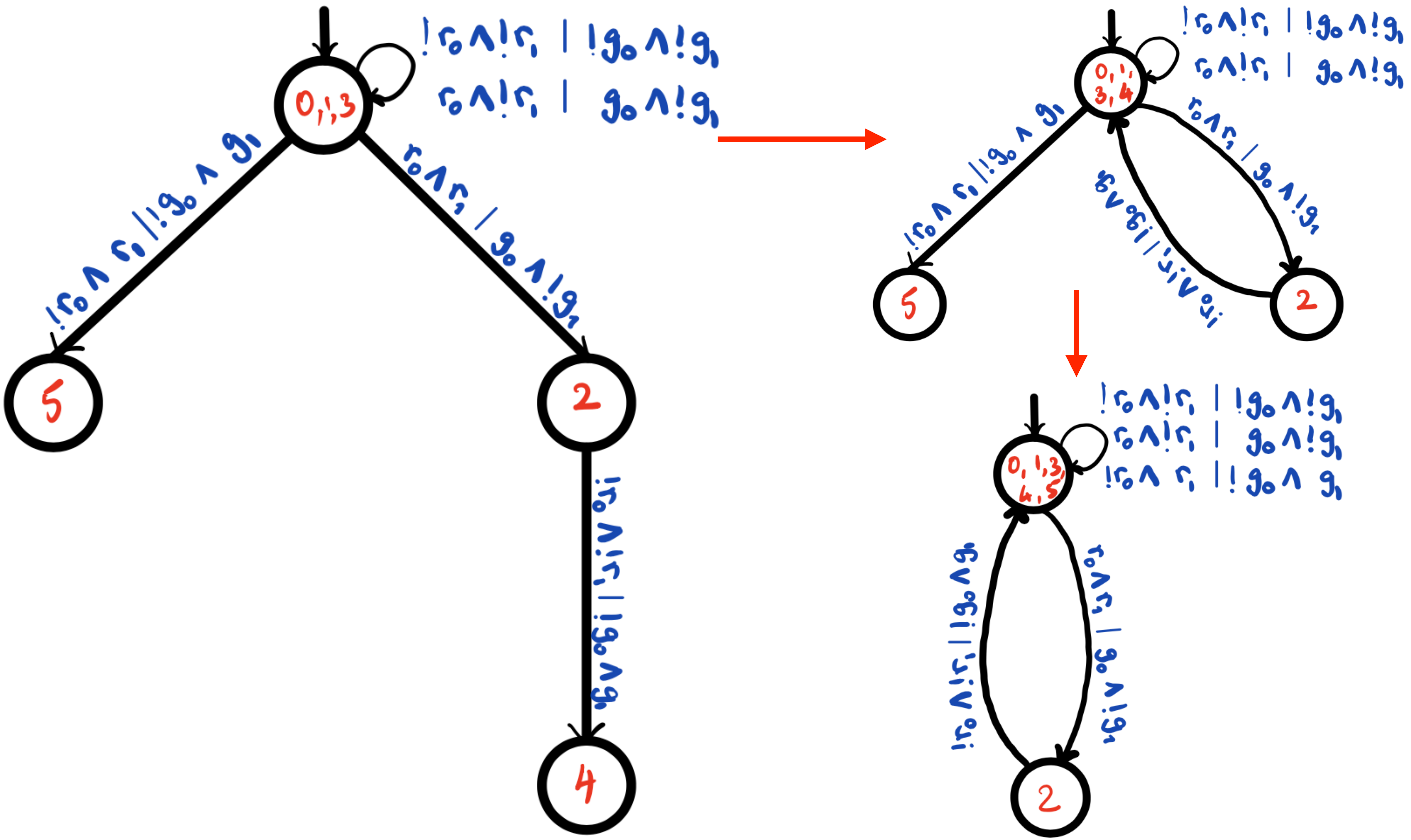




This preMealy machine cannot be made complete and realize φ_{CORE}


0,1,3
!r0 ∧ !r1 | !g0 ∧ !g1
r0 ∧ !r1 | g0 ∧ !g1
5
2
4
0,1,3,4
2
0,1,3,4,5
!r0 ∧ r1 | !g0 ∧ g1

Phase 2: complete the preMealy machine

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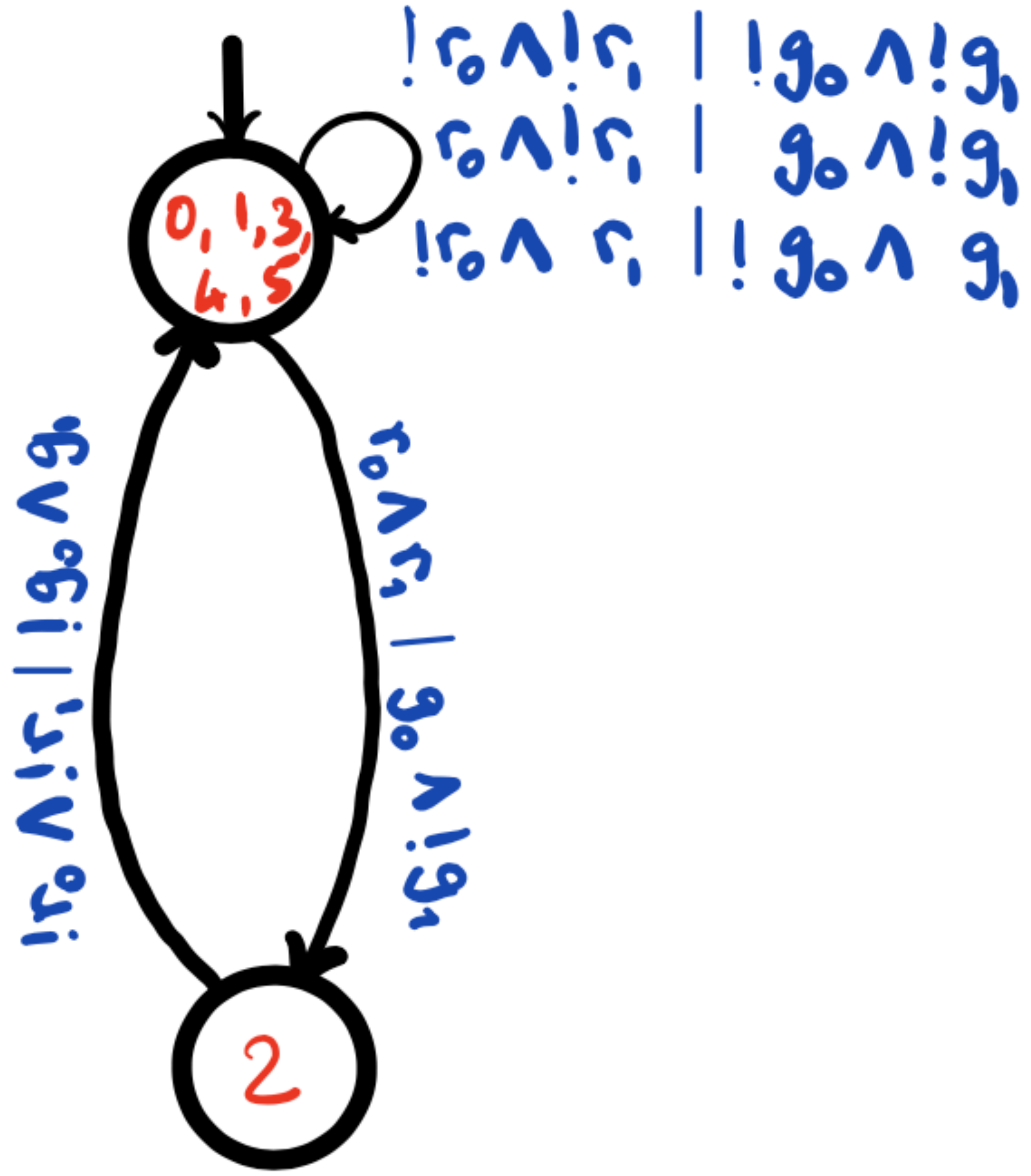
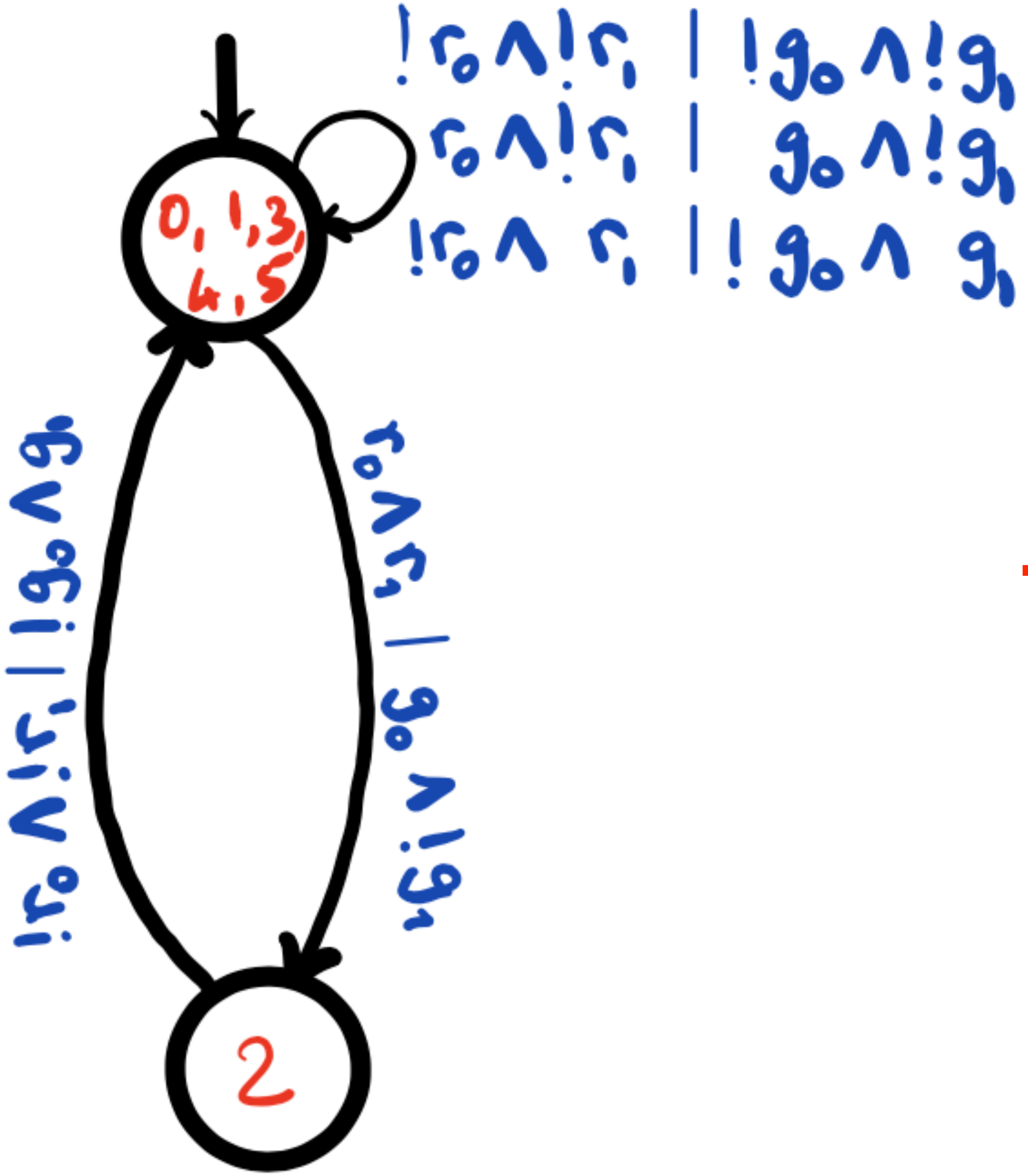
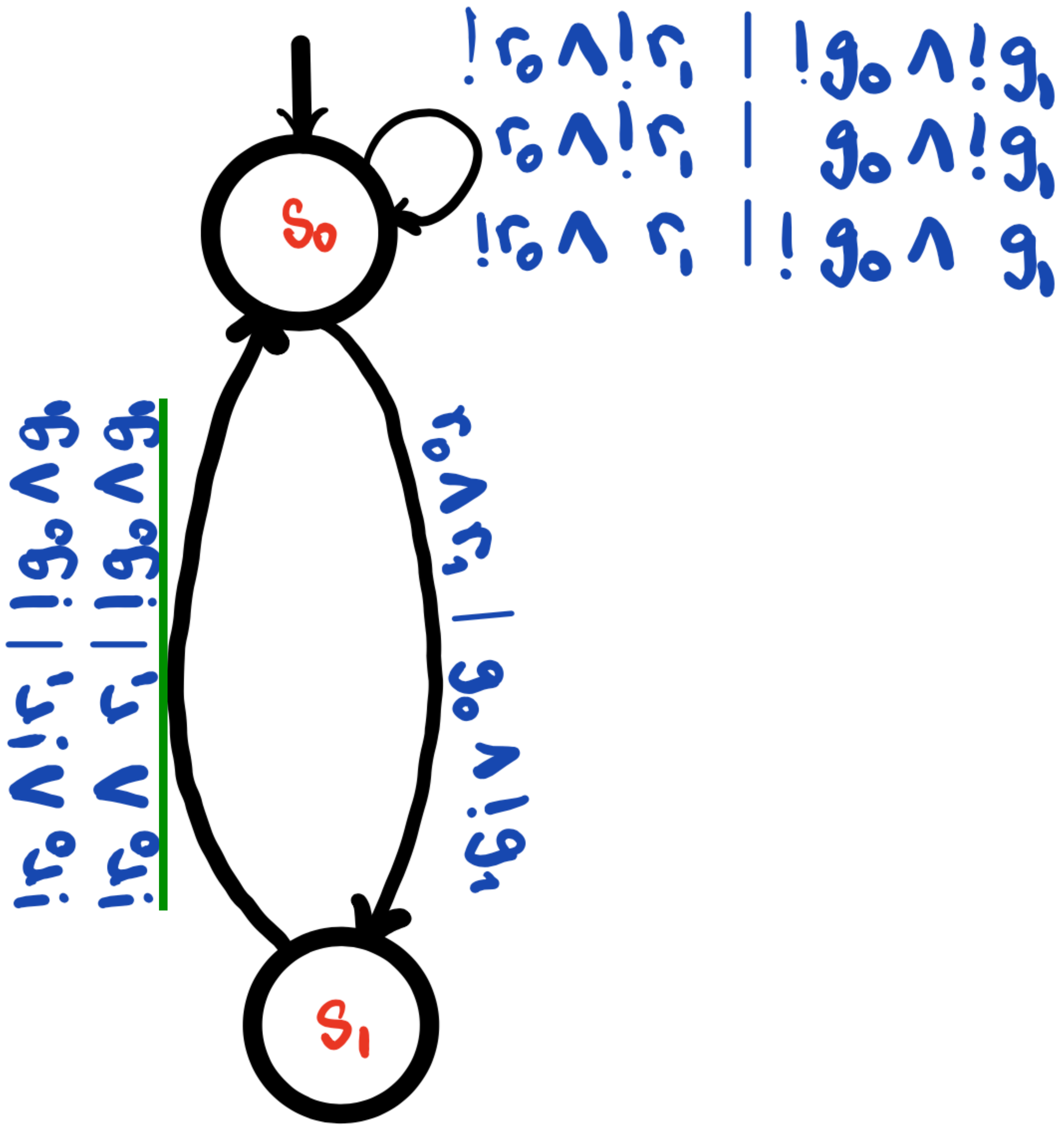
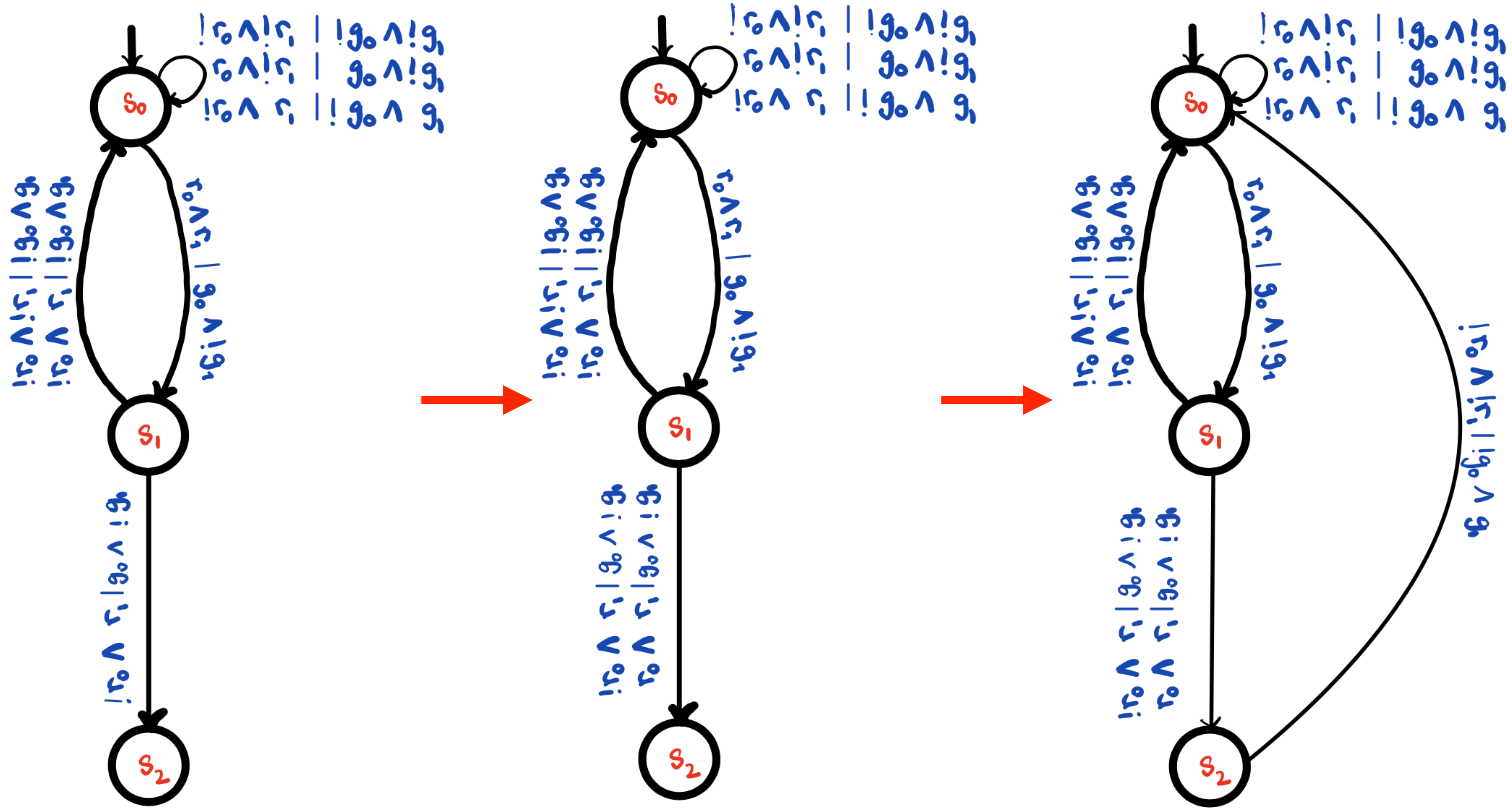


PreMealy machine obtained from the learning phase (generalization of the examples)

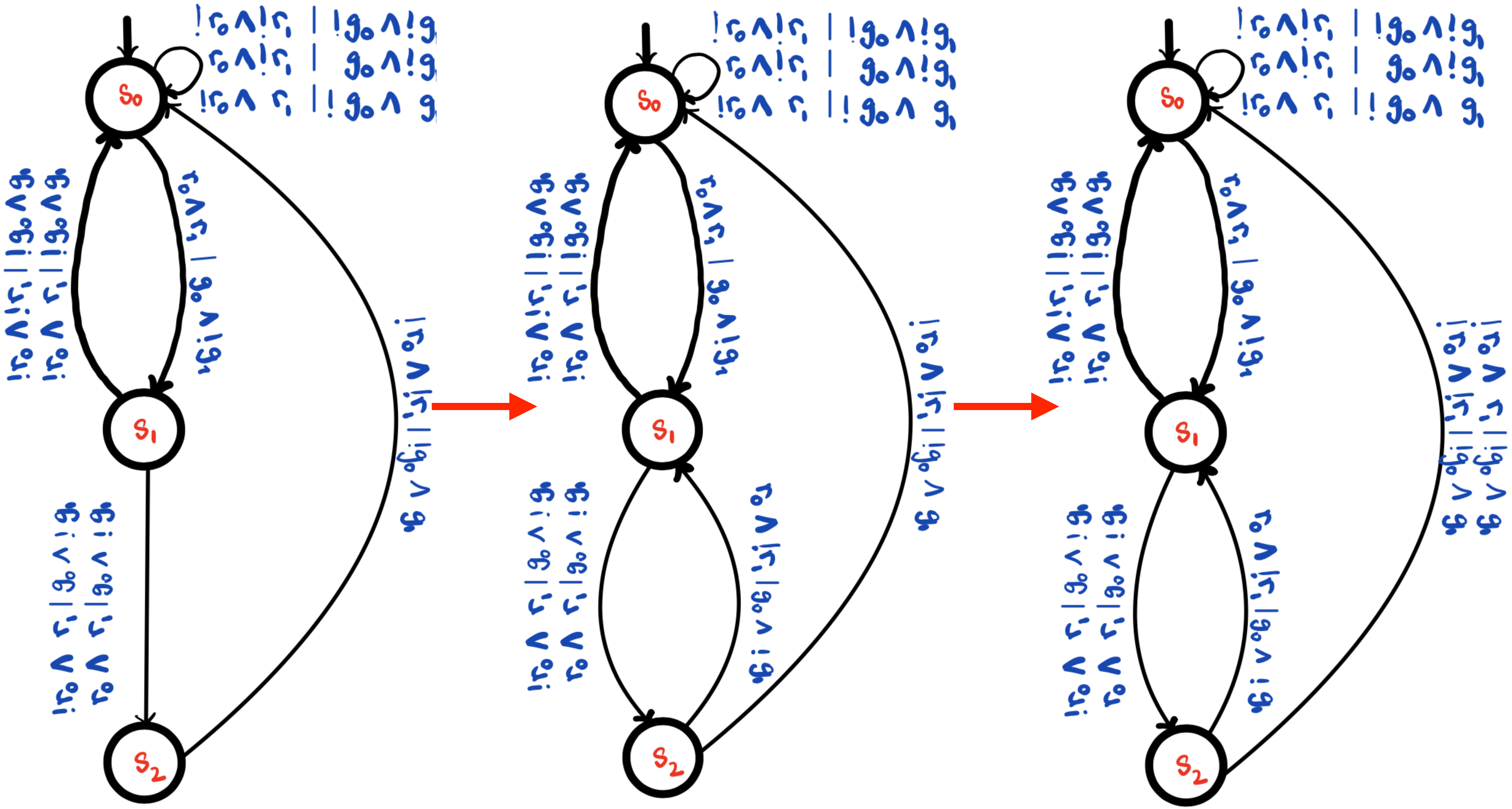


filling holes





Heuristic: try to reuse as much as possible states that were introduced by the learning phase
(try to imitate decisions that are illustrated by the examples)



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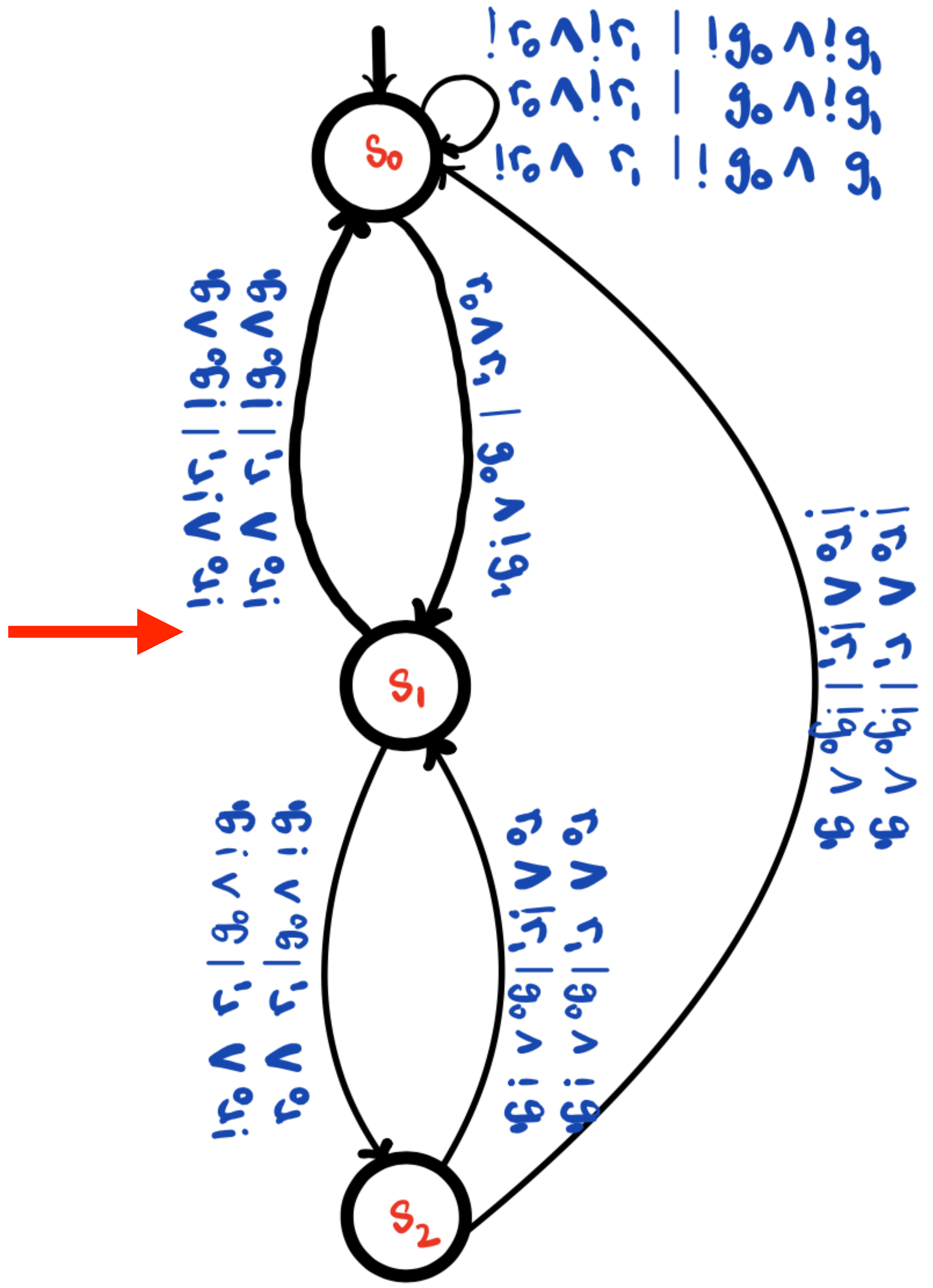
Final (complete) Mealy machine that enforces

φ_{CORE}



A few theorems

- Theorem 1 (**Termination** and **correctness**): For all (φ, E), SynthLearn(φ, E) terminates and returns a Mealy machine M such that (i) $M \models \varphi$ and (ii) is compatible with *E*, if it exists, and return **UNREAL** otherwise.

- Theorem 2 (Mealy **completeness**): For all specifications φ, for all Mealy machines M such that $M \models \varphi$, there is a set of examples E_M of size **polynomial** in M and such that SynthLearn$(\varphi, E_M) = M$.

- Theorem 3 (**Polynomial** additional cost): our algorithm is polynomial in the size of E and in a symbolic representation the of set of Mealy machines that realize φ (which is computed by Acacia-Bonzai for solving plain LTL synthesis for φ.)

Further works

Easy and more ambitious

- Negative examples: hist $\cdot\ i/\neg o$
- Infinite examples given as deterministic i/o $\omega-$regular expressions, e.g. $\left(\{!r_1,!r_2\}.\{!g_1,!g_2\}\#\{r_1,!r_2\}.\{g_1,!g_2\}\#\{!r_1,r_2\}.\{!g_1,g_2\}\right)^{\omega}$
- Symbolic examples and symbolic Mealy machines: $\psi_0^i \cdot \psi_0^o \sharp \psi_1^i \cdot \psi_1^o \sharp \ldots \sharp \psi_n^i / o$
- How to learn **programs** manipulating variables, queues, or stacks that realize a spec φ_{CORE} with a similar approach ?