CONTEXTUAL EQUIVALENCE FOR STATE AND CONTROL VIA NESTED DATA

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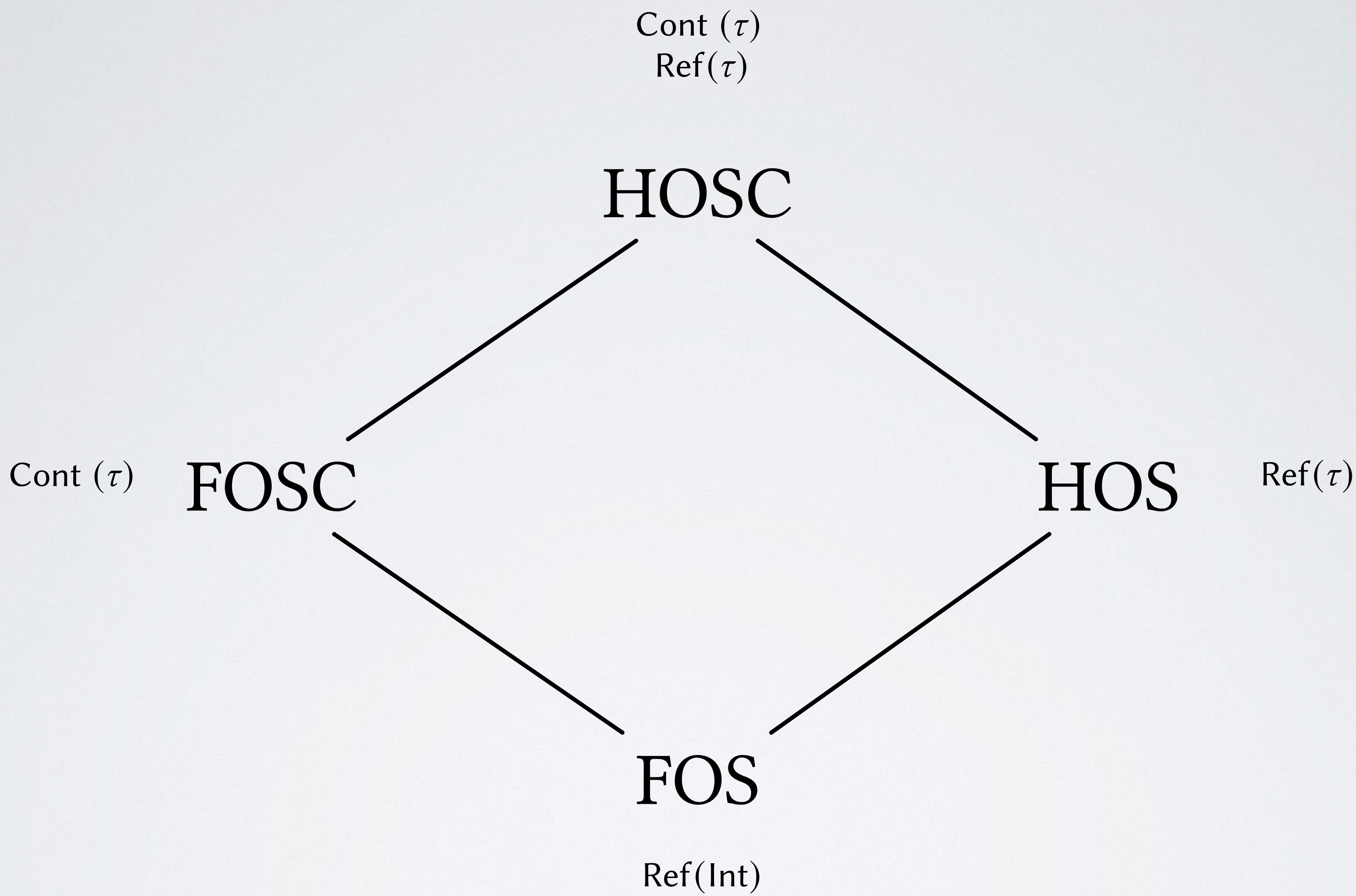


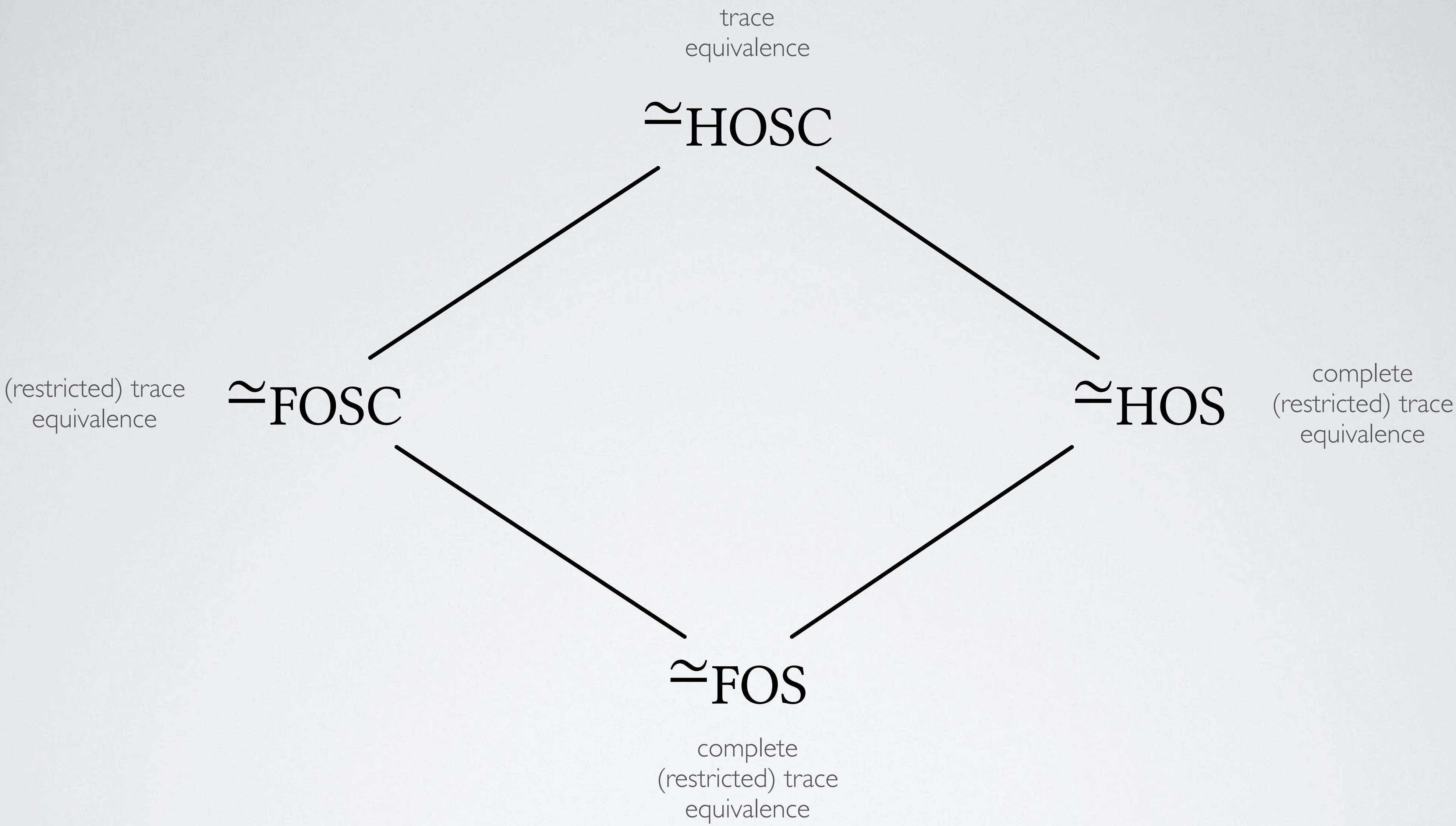
Cont (τ)
Ref(τ)
HOSC
Cont (τ)
FOSC
HOS
Ref(τ)
FOS
Ref(Int)

CONTEXTUAL APPROXIMATION AND EQUIVALENCE

Let $\Gamma \vdash M_1, M_2 : \tau$ be HOSC terms and $X \in \{\text{HOSC, FOSC, HOS, FOS}\}$.

We define $\Gamma \vdash M_1 \lesssim_X M_2$ to hold, when for all X contexts $C[\tau]$ such that $\vdash C[M_1], C[M_2] : \tau'$ for some τ', if $(C[M_1], \emptyset) \Downarrow$ then $(C[M_2], \emptyset) \Downarrow$.

The terms $\Gamma \vdash M_1, M_2 : \tau$ are called *contextually equivalent* (wrt X contexts), written $\Gamma \vdash M_1 \simeq_X M_2$, when $\Gamma \vdash M_1 \lesssim_X M_2$ and $\Gamma \vdash M_2 \lesssim_X M_1$.

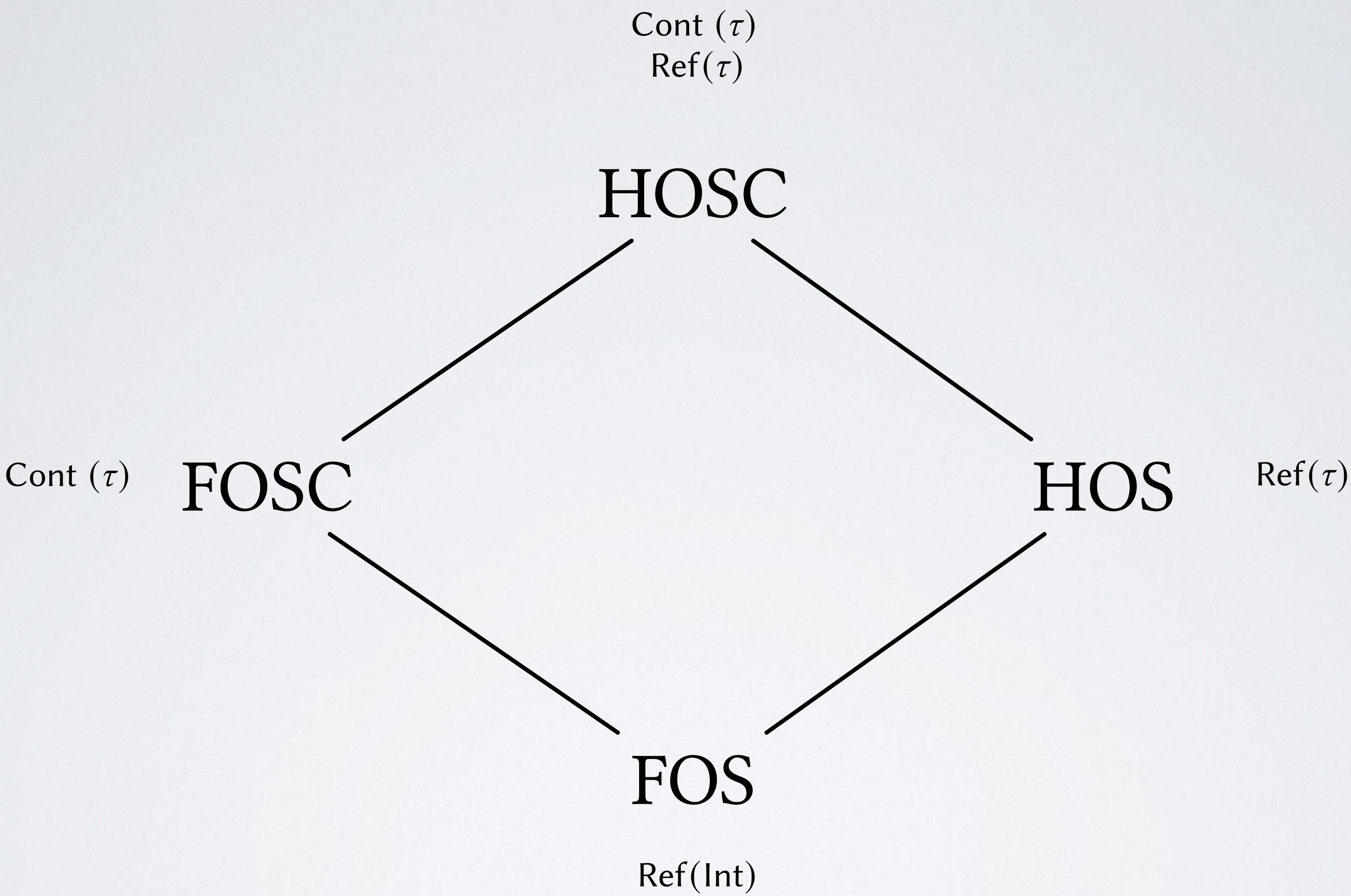


Complete trace models of state and control*

Guilhem Jaber[1] (✉) and Andrzej S. Murawski[2] (✉)

SOURCE OF UNDECIDABILITY

- integer arithmetic
- higher-order recursion (with state)
- general references


Cont (τ)
Ref(τ)
HOSC
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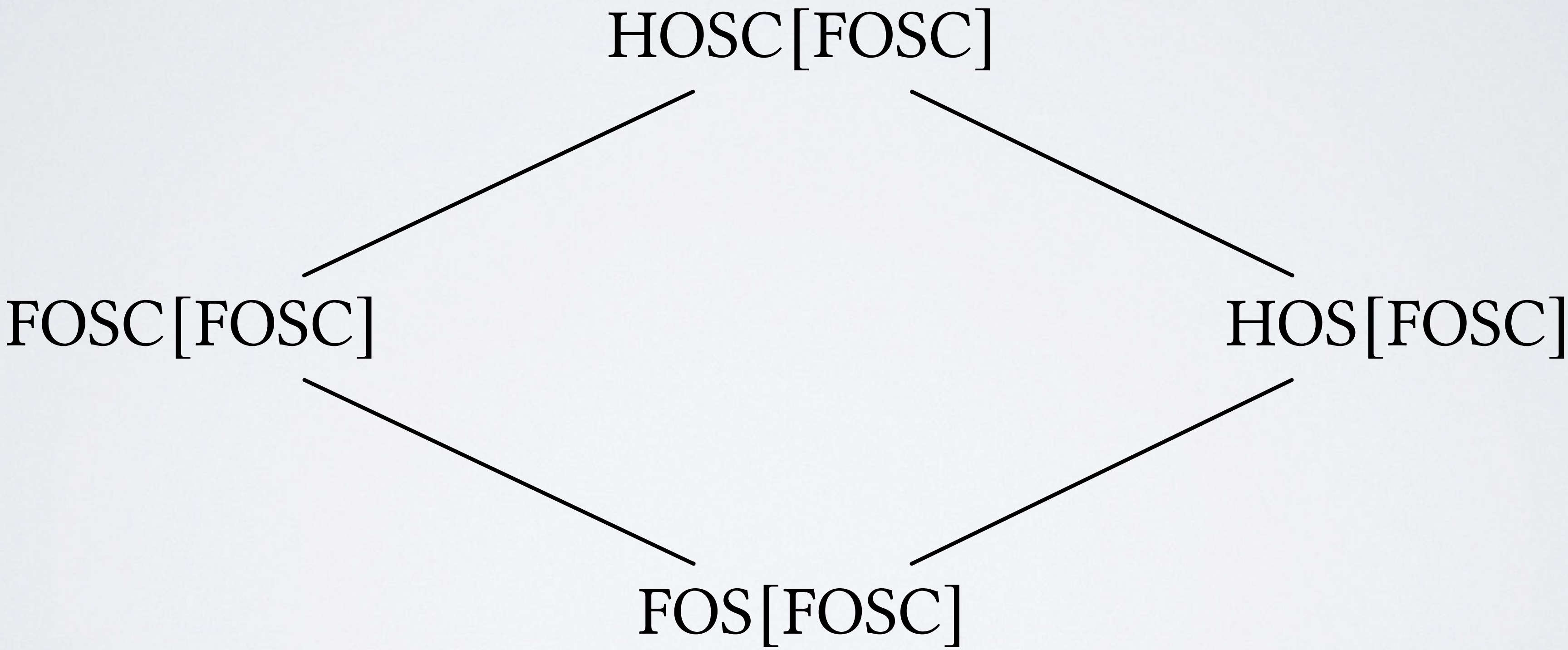
DECIDABILITY STATUS OF X[FOSC]
(FINITE BASE TYPES, NO RECURSION)



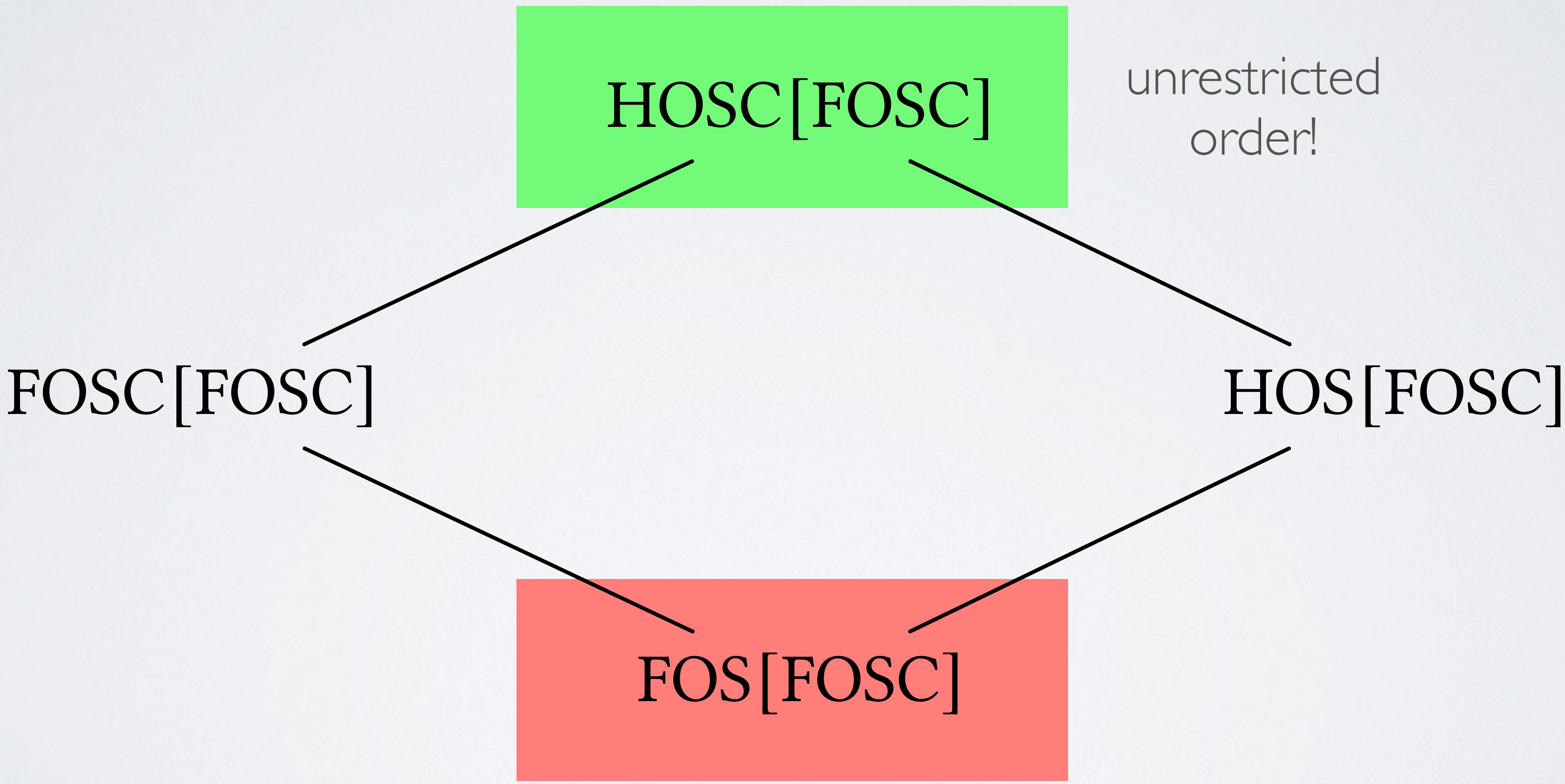
RELATED UNDECIDABILITY RESULTS

- FOS[FOS] is undecidable [M., LICS'03]

$$\mathrm{GOS} = \mathrm{FOS} + \mathrm{Ref}^{i}(\mathrm{Int})$$

- GOS[GOS] is undecidable [M. & Tzevelekos, ICALP'12]
- The undecidability of GOS[GOS] implies that X[GOS] is undecidable for any X.

DECIDABILITY STATUS OF X[FOSC]
(FINITE BASE TYPES, NO RECURSION)



TRACE SEMANTICS

$$
\begin{array}{l|ll}
(P\tau) & \langle M, c, \gamma, \xi, \phi, h\rangle \xrightarrow{\tau} \langle N, c', \gamma, \xi, \phi, h'\rangle \\
& \text{when } (M, c, h) \to (N, c', h') \\
(PA) & \langle V, c, \gamma, \xi, \phi, h\rangle \xrightarrow{\bar{c}(A)} \langle \gamma \cdot \gamma', \xi, \phi \uplus \nu(A), h\rangle \\
& \text{when } c : \sigma,\ (A, \gamma') \in \mathbf{AVal}_\sigma(V) \\
(PQ) & \langle K[fV], c, \gamma, \xi, \phi, h\rangle \xrightarrow{\bar{f}(A,c')} \langle \gamma \cdot \gamma' \cdot [c' \mapsto K], \xi \cdot [c' \mapsto c], \\
& \qquad \phi \uplus \nu(A) \uplus \{c'\}, h\rangle \\
& \text{when } f : \sigma \to \sigma',\ (A, \gamma') \in \mathbf{AVal}_\sigma(V),\ c' : \sigma' \\
(OA) & \langle \gamma, \xi, \phi, h\rangle \xrightarrow{c(A)} \langle K[A], c', \gamma, \xi, \phi \uplus \nu(A), h\rangle \\
& \text{when } c : \sigma,\ A : \sigma,\ \gamma(c) = K,\ \xi(c) = c' \\
(OQ) & \langle \gamma, \xi, \phi, h\rangle \xrightarrow{f(A,c)} \langle VA, c, \gamma, \xi, \phi \uplus \nu(A) \uplus \{c\}, h\rangle \\
& \text{when } f : \sigma \to \sigma',\ A : \sigma,\ c : \sigma',\ \gamma(f) = V
\end{array}
$$

AUTOMATA OVER INFINITE ALPHABETS

- Σ is a finite alphabet of *tags*.
- $\mathcal{D}$ is an infinite alphabet of *data values*.
- Automata accept *data words* from $(\Sigma \times \mathcal{D})^*$.

$$(t_1, d_1)(t_2, d_2) \cdots (t_k, d_k)$$

Class memory automata [Björklund & Schwentick, FCT 2007] maintain finite state as well as class memory (a map from a finite subset of $\mathcal{D}$ to a finite set of memories).

$$(t_1, d_1)(t_2, d_2) \cdots (t_k, d_k)(t_{k+1}, d_2)$$

FROM LTS TO CMA

- Lack of recursion means terms do not grow unboundedly and there are finitely many "skeletons".
- But terms also contain function names (introduced by the environment) as well as location names!
- It is not immediate to accommodate γ as class memory.

FROM LTS TO CMA

- Fortunately, traces of FOSC satisfy a visibility condition: function and location names are always introduced in the current scope (P-view).
- P-views for recursion-free FOSC terms are bounded, so we can enumerate the names and refer to their position in the P-view.
- This makes it possible to represent γ as a class memory function!

HEAP

- However, location names must also be updated. For this we need access to whole "scope" (P-view).
- Idea: use a tree-shaped set $\mathcal{D}$ and allow the automaton to access memory not only for one data value but for its ancestors too.
- To achieve this, we use nested data and class memory automata over nested data [Cotton-Barratt, M. & Ong, LATA 2015].

NDCMA

$$(t_1, d_1)(t_1, d_2)(t_2, d_3)(t_3, d_4)(t_4, d_3)$$

$$(q_0, \emptyset) \qquad (q_0,\ d_1^{m_1}) \qquad (q_0,\ d_1^{m_1}\ \ d_2^{m_1}) \qquad (q_1,\ \underset{\underset{\displaystyle d_3^{m_3}}{|}}{d_1^{m_2}}\ \ d_2^{m_1}\ )$$

$$(q_1,\ \ \underset{d_3^{m_3}\ \ \ \ \ \ d_4^{m_4}}{\overset{}{d_1^{m_2}}}\ \ \ d_2^{m_1}\ ) \qquad (q_2,\ \ \underset{d_3^{m_6}\ \ \ \ \ \ d_4^{m_4}}{\overset{}{d_1^{m_5}}}\ \ \ d_2^{m_1}\ )$$

SUMMARY

- The trace semantics of FOSC terms can be faithfully represented using deterministic NDCMA.
- The containment problem for deterministic NDCMA is decidable.
- HOSC[FOSC] is decidable.

DECIDABILITY STATUS OF X[FOSC]
(FINITE BASE TYPES, NO RECURSION)