Regular Separability of WSTS

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joint work with Wojciech Czerwiński, Sławomir Lasota, Sebastian Muskalla, K Narayan Kumar, and Prakash Saivasan

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Separability

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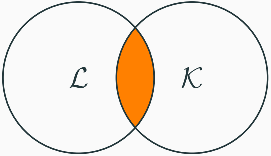
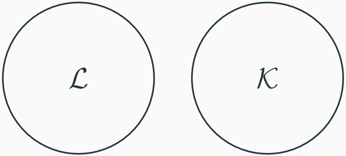
Case 1: $\mathcal{L} \cap \mathcal{K} \neq \varnothing$



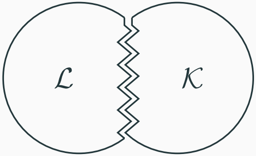
↳ Study $\mathcal{L} \cap \mathcal{K}$.

Separability

Case 2: $\mathcal{L} \cap \mathcal{K} = \varnothing$



vs.



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Consider separability.

Separability of $\mathcal{F}$ by $\mathcal{S}$

Given: Languages $\mathcal{L}, \mathcal{K} \subseteq \Sigma^*$ from $\mathcal{F}$

Decide: Is there $\mathcal{R} \subseteq \Sigma^*$ from $\mathcal{S}$ such that

$$\mathcal{L} \subseteq \mathcal{R}, \quad \mathcal{K} \cap \mathcal{R} = \varnothing?$$

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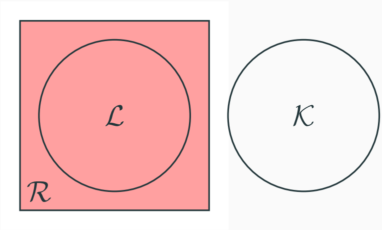

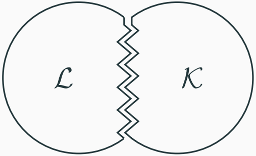

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- $\mathcal{S} \subsetneq \mathcal{F} = REG$

  e.g. $\mathcal{S}$ = Star-free languages

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Given: Languages $\mathcal{L}, \mathcal{K} \subseteq \Sigma^*$ from $\mathcal{F}$

Decide: Is there $\mathcal{R} \subseteq \Sigma^*$ regular such that

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Observation:

Problem is symmetric in the input:

If $\quad \mathcal{L} \subseteq \mathcal{R}, \quad \mathcal{K} \cap \mathcal{R} = \varnothing$

then $\quad \mathcal{K} \subseteq \overline{\mathcal{R}}, \quad \mathcal{L} \cap \overline{\mathcal{R}} = \varnothing$.

↳ Call $\mathcal{L}, \mathcal{K}$ regularly separable if separator $\mathcal{R}$ exists.

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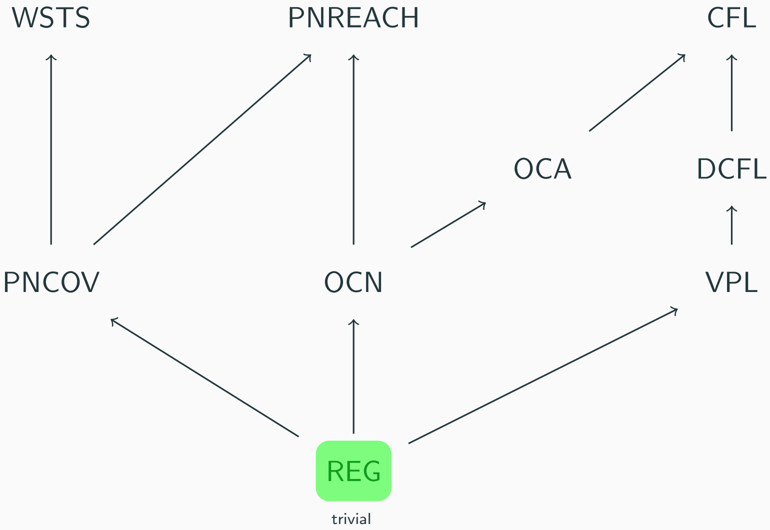
Disjointness is always necessary for (any kind of) separability.

It is not always sufficient:

$$\mathcal{L} = a^n b^n, \quad \mathcal{K} = \overline{\mathcal{L}} \ .$$

Regular separability — related work



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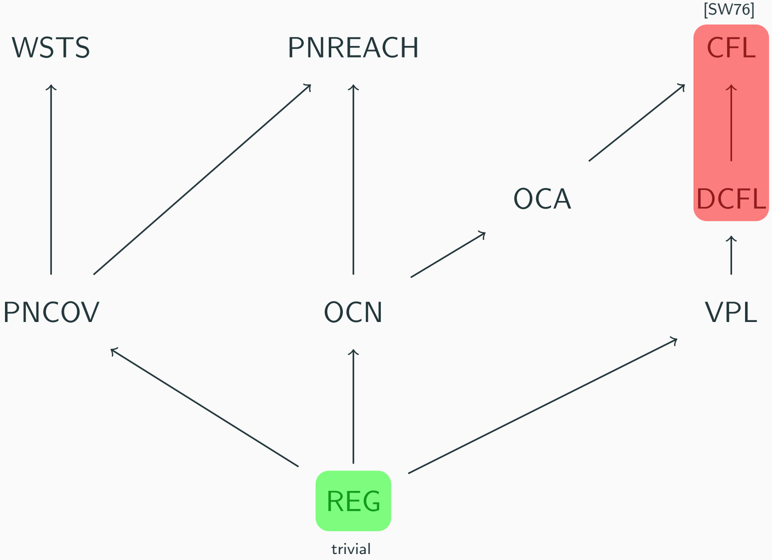

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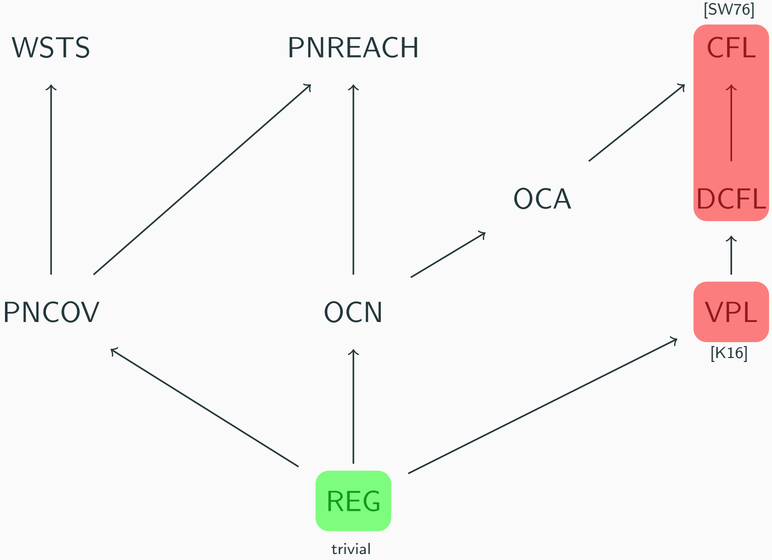

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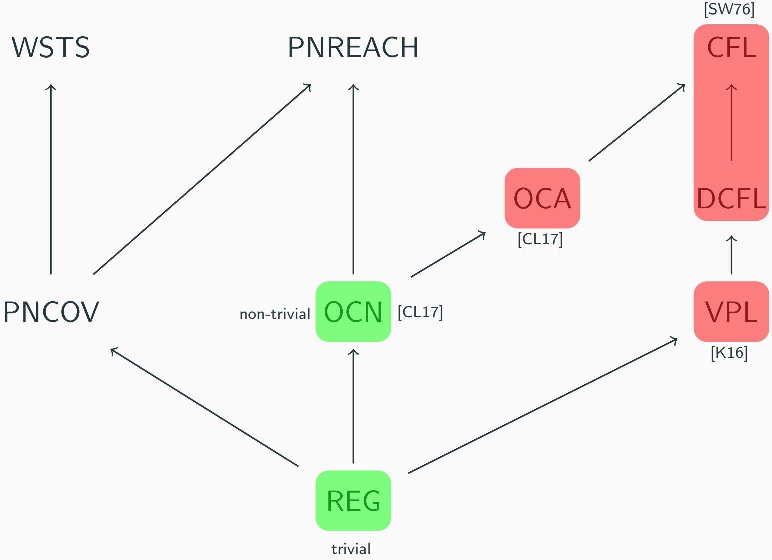

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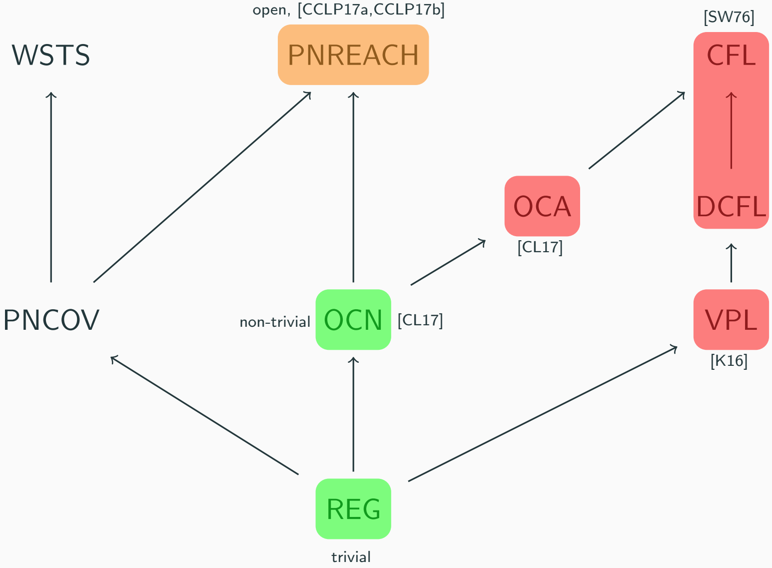

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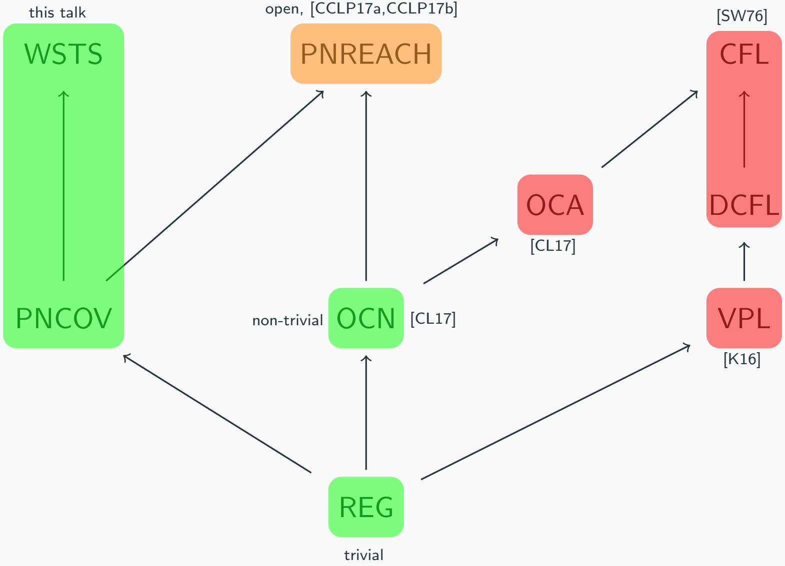

The result

Well-structured transiton systems [F87,AJ93,ACJT96,FS01]

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$$\mathcal{W} = (S, \leqslant, T, I, F).$$

- $(S, \leqslant)$ states well quasi ordering
- $T \subseteq S \times \Sigma \times S$ labeled transitions
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- Monotonicity / Simulation property:

$$\begin{array}{ccc} s' & \overset{a}{\dashrightarrow} & r' \ (\exists) \\ \curlyvee\!| & & \curlyvee\!| \\ s & \overset{a}{\longrightarrow} & r \end{array}$$

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Coverability language

$$\mathcal{L}(\mathcal{W}) = \left\{ w \in \Sigma^* \;\middle|\; c_i \xrightarrow{w} c_f \text{ for some } c_i \in I, c_f \in F \right\}.$$

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Labeled Petri nets with covering acceptance condition yield WSTS

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Example 2:

Labeled lossy channel systems (LCS) [AJ93] yield WSTS.

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If two WSTS languages, one of them finitely branching, are disjoint, then they are regularly separable.

Applications and speculation

Compositional Safety Verification

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	iff	$\exists\ \mathcal{L}_1, \mathcal{L}_2$ regular with $\mathcal{L}(P) \subseteq \mathcal{L}_1$, $\mathcal{L}(Q) \subseteq \mathcal{L}_2$, and $\mathcal{L}_1 \cap \mathcal{L}_2 = \varnothing$.

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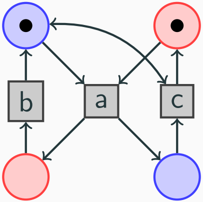

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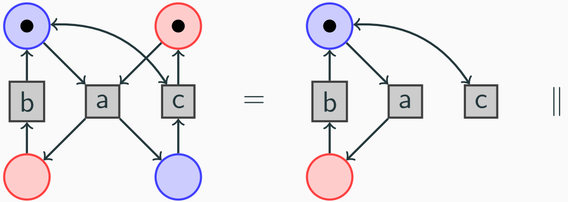

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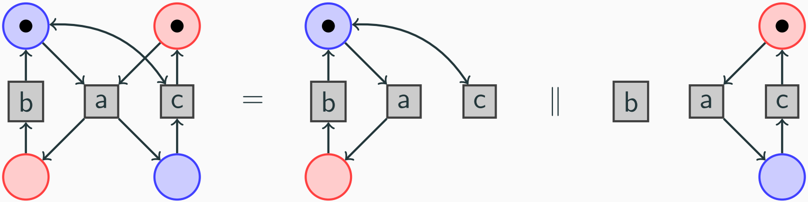

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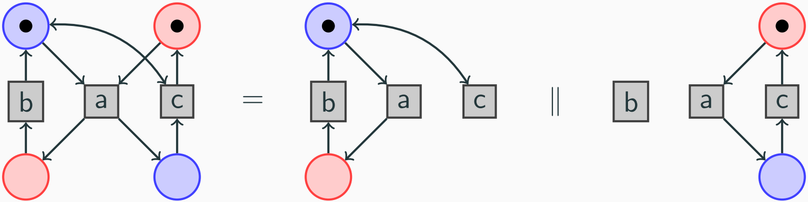

$$(ab + c)^*.a$$

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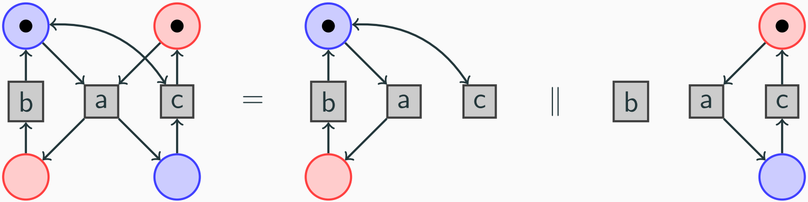

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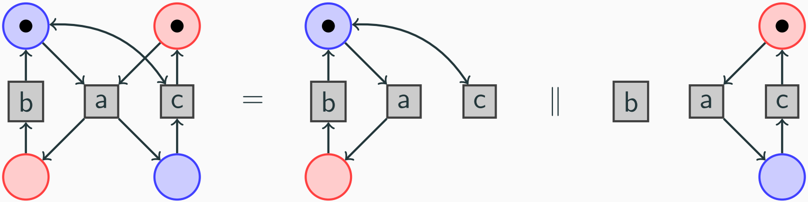

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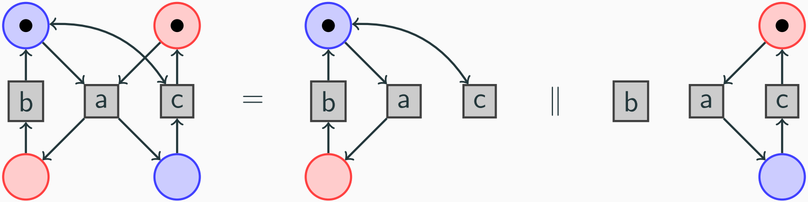

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Petri nets seem to have a regular type.

Learning-based verification without ICE

Learning invariants [Madhusudan, Neider et al. since 2014]

Given: Configurations G reachable from init, B leading to bad.

Learn: Separator S of G and B.

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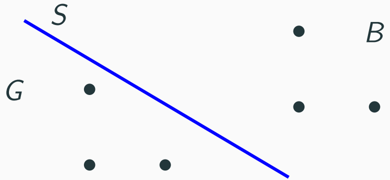

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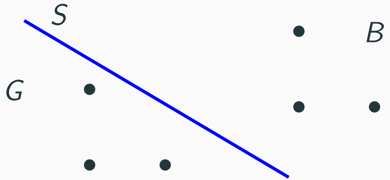

Inductiveness problem: What if $x \in S$ but $y = post(x) \notin S$?

Should x be outside S or y be in S?

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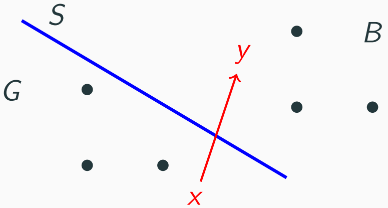

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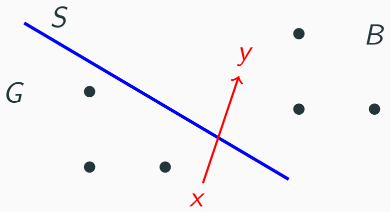

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Solution [Madhusudan, Neider et al.]:
Generalize learning algorithms to take into account pairs (x, y).

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But: No new framework needed!

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$$G := \varnothing =: B$$

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$$\downarrow$$

Learn $\mathcal{R}$
separating G from B

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$$\mathcal{L}(P) \subseteq \mathcal{R}$$

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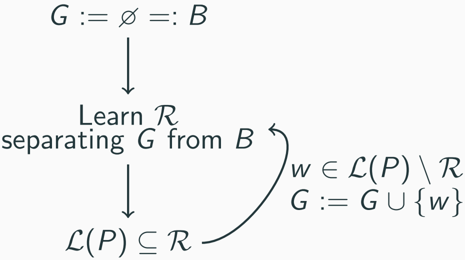

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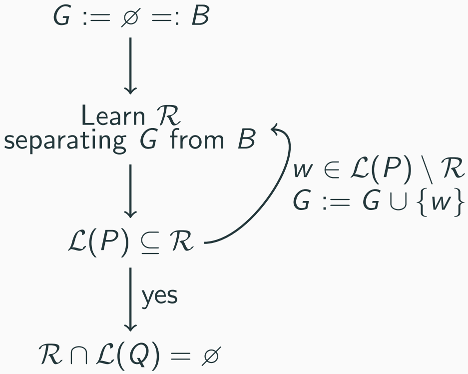

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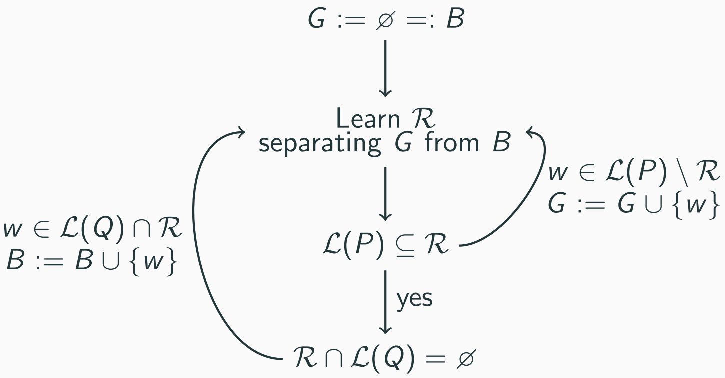

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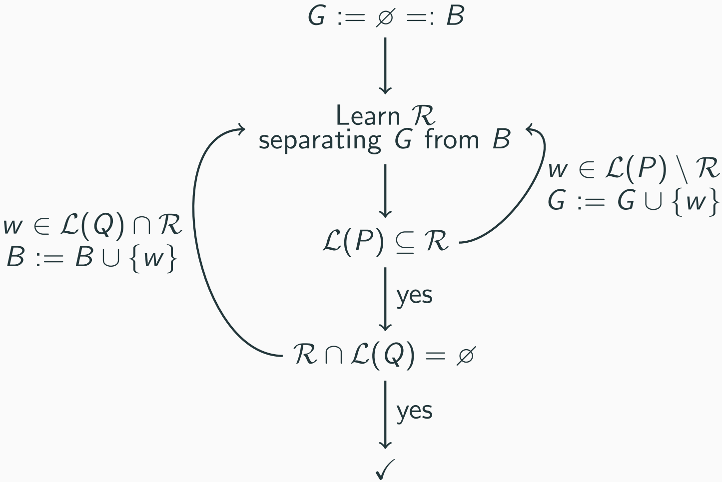

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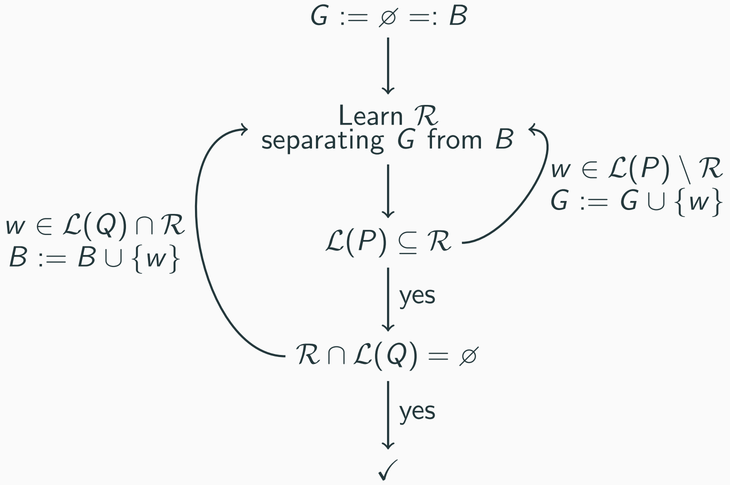

There is a dual algorithm learning $\mathcal{L}_1$ and $\mathcal{L}_2$ from above.

Interpolation-based regular model checking

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Given: Formulas $F = init \vee post(init)$, $G = pre^{\leqslant k}(bad)$.

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Craig's theorem 1957: First-order logic has interpolants.

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Regular model checking [Abdulla et al. since 1997]

Analyze programs where configurations are words:

Interpolation-based regular model checking

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Regular model checking [Abdulla et al. since 1997]

Analyze programs where configurations are words:

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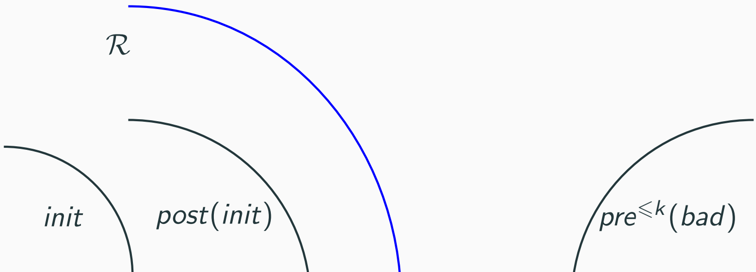

Interpolation of string-manipulating programs

Again: Separators may be the right thing!

Language-theoretic consequences

Theorem

If two WSTS languages, one of them finitely branching, are disjoint, then they are regularly separable.

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Generalizes results for Petri nets [Kumar et al. 1998].

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Generalizes results for Petri nets [Kumar et al. 1998].

Corollary

No subclass of finitely branching WSTS beyond REG is closed under complement.

Expressiveness results:
Languages of finitely branching WSTS

Our result - Recall

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$\mathcal{W}$ finitely branching: I finite, $\mathrm{Post}_\Sigma(c)$ finite for all c.

How much of a restriction is it to assume finite branching?

What do we gain by assuming finite branching?

Expressibility I

Proposition

Languages of ω^2-WSTS
$\subseteq$ Languages of finitely branching WSTS.

$$\begin{aligned} & (S, \leqslant)\ \omega^2\text{-wqo} \\ \text{iff} \quad & \left(\mathcal{P}^{\downarrow}(S), \subseteq\right) \text{ wqo} \\ \text{iff} \quad & (S, \leqslant) \text{ does not embed the Rado order.} \end{aligned}$$

Our result applies to all WSTS of practical interest!

Expressibility II

Proposition

Languages of finitely branching WSTS
= Languages of deterministic WSTS.

Sufficient to show:

Theorem

If two WSTS languages, one of them deterministic, are disjoint, then they are regularly separable.

Proof sketch

Proof approach

Theorem

If two WSTS languages, one of them deterministic, are disjoint, then they are regularly separable.

Proof approach:

Relate separability to the existence of certain invariants.

Separability talks about the languages,

invariants talk about the state space!

Inductive invariant [Manna, Pnueli 1995]

Inductive invariant X

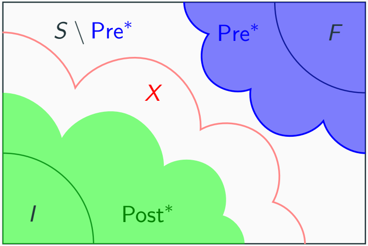
for WSTS $\mathcal{W}$:

(1) $X \subseteq S$ downward-closed

(2) $I \subseteq X$

(3) $F \cap X = \varnothing$

(4) $\mathrm{Post}_\Sigma(X) \subseteq X$



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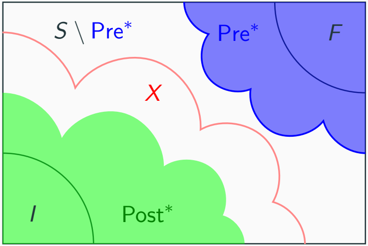

Lemma

$\mathcal{L}(\mathcal{W}) = \varnothing$ iff inductive invariant for $\mathcal{W}$ exists.

Proof approach

$$\mathcal{L}(\mathcal{W}_1), \mathcal{L}(\mathcal{W}_2) \text{ reg. sep} \overset{!}{\iff} \mathcal{L}(\mathcal{W}_1) \cap \mathcal{L}(\mathcal{W}_2) = \mathcal{L}(\mathcal{W}_1 \times \mathcal{W}_2) = \varnothing$$

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$$\Updownarrow$$

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Finitely represented invariants

The desired implication does not hold.

Call an invariant X finitely represented if $X = Q\downarrow$ for Q finite.

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Call an invariant X finitely represented if $X = Q\downarrow$ for Q finite.

Recall:

$(S, \leqslant)$ well quasi order (wqo)

iff upward-closed sets have finitely many minimal elements.

No such statement for downward-closed sets and maximal elements!

Finitely represented invariants

The desired implication does not hold.

Call an invariant X finitely represented if $X = Q\downarrow$ for Q finite.

We can show:

Theorem

Let $\mathcal{W}_1, \mathcal{W}_2$ WSTS, $\mathcal{W}_2$ deterministic.

If $\mathcal{W}_1 \times \mathcal{W}_2$ admits a finitely represented inductive invariant, then $\mathcal{L}(\mathcal{W}_1)$ and $\mathcal{L}(\mathcal{W}_2)$ are regularly separable.

Proof approach II

$$\mathcal{L}(\mathcal{W}_1), \mathcal{L}(\mathcal{W}_2) \text{ reg. sep} \overset{!}{\iff} \mathcal{L}(\mathcal{W}_1) \cap \mathcal{L}(\mathcal{W}_2) = \mathcal{L}(\mathcal{W}_1 \times \mathcal{W}_2) = \varnothing$$

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Finitely represented invariants do not necessarily exist.

Solution: Ideals

Definition

For WSTS $\mathcal{W}$, let $\widehat{\mathcal{W}}$ be its ideal completion [KP92,BFM14,FG12].

Lemma

$\mathcal{L}(\mathcal{W}) = \mathcal{L}(\widehat{\mathcal{W}})$.
$\widehat{\mathcal{W}}$ is deterministic if so is $\mathcal{W}$.

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If X *is an inductive invariant for* $\mathcal{W}$,
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is a finitely represented inductive invariant for $\widehat{\mathcal{W}}$.

Proof

Putting everything together:

If $\mathcal{W}_1, \mathcal{W}_2$ are disjoint, $\mathcal{W}_1 \times \mathcal{W}_2$ admits an invariant X.

Then $\textsc{Idec}(X)\downarrow$ is a finitely represented invariant for $\widehat{\mathcal{W}_1 \times \mathcal{W}_2} \cong \widehat{\mathcal{W}_1} \times \widehat{\mathcal{W}_2}$.

This finitely represented invariant gives rise to a regular separator.

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This finitely represented invariant gives rise to a regular separator.

We have shown:

Theorem

If two WSTS languages are disjoint,
one of them finitely branching or deterministic or ω^2,
then they are regularly separable.

Proof details:
From fin. rep. invariants to regular separators

From invariants to separability

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Assume $Q\downarrow$ is an invariant.

Idea: Construct separating NFA with Q as states.

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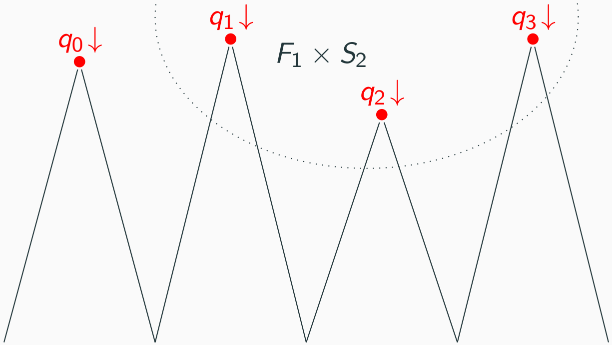
$$Q \ni (s, s') \xrightarrow[\text{in } \mathcal{A}]{a} (r, r') \in Q$$

$$(r, r') \geqslant (t, t')$$

$$Q \ni (s, s') \xrightarrow[\text{in } \mathcal{W}_1 \times \mathcal{W}_2]{a} (t, t') \in S_1 \times S_2$$

Behavior of $\mathcal{A}$



$\mathcal{A}$ over-approximates the behavior of the product system using the configurations from Q.

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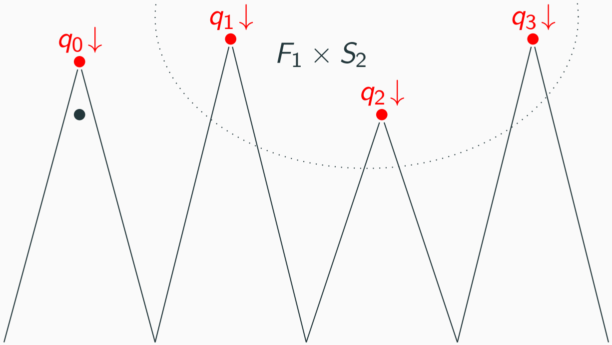

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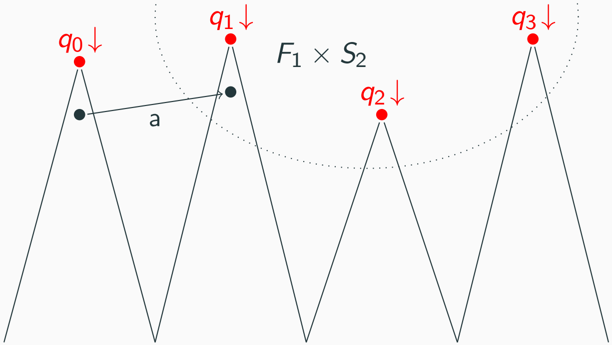

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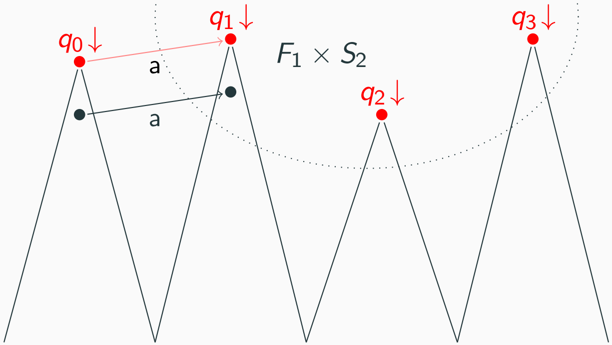

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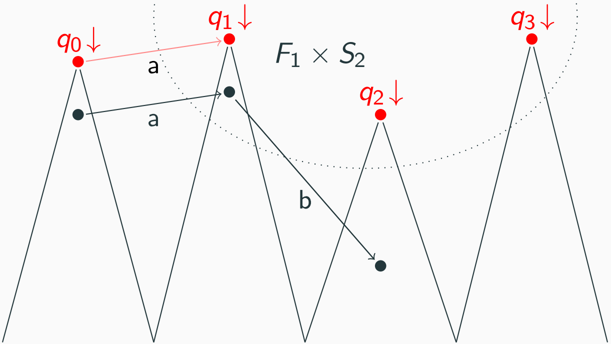

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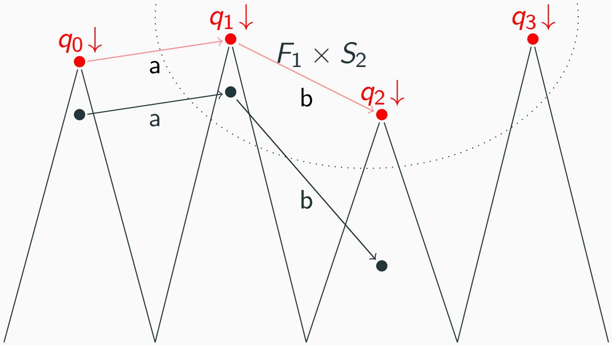

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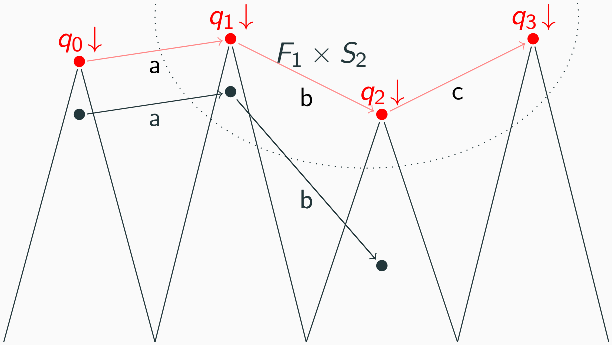

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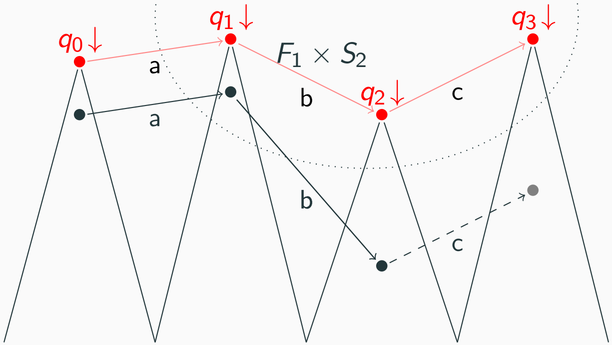

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Proving separability: Inclusion

Lemma

$\mathcal{L}(\mathcal{W}_1) \subseteq \mathcal{L}(\mathcal{A})$.

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Proof.

Any run $c \xrightarrow{w} d$ of $\mathcal{W}_1$

synchronizes with *the* run of $\mathcal{W}_2$ for w

in the run $(c, c') \xrightarrow{w} (d, d')$ of $\mathcal{W}_1 \times \mathcal{W}_2$.

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If d is final in $\mathcal{W}_1$,

the over-approximation of (d, d') is final in $\mathcal{A}$. □

Proving separability: Disjointness

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If $w \in \mathcal{L}(\mathcal{W}_2) \cap \mathcal{L}(\mathcal{A})$

then some run of $\mathcal{A}$ reaches a state (q, q') with

- q final in $\mathcal{W}_1$ (def. of Q_F)
- q' final in $\mathcal{W}_2$ ($w \in \mathcal{L}(\mathcal{W}_2)$ + argument above).

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Contradiction to $(F_1 \times F_2) \cap Q{\downarrow} = \varnothing$! □

Proof details:
The ideal completion and fin. rep. invariants

Finitely represented invariants

Lemma

Let $U \subseteq S$ be an upward-closed set in a wqo.

There is a finite set U_{min} such that $U = U_{min}\uparrow$.

A similar result for downward-closed subsets and maximal elements does not hold.

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Example:

Consider $\mathbb{N}$ in $(\mathbb{N}, \leqslant)$

Intuitively, $\mathbb{N} = \omega\downarrow$.

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A similar result for downward-closed subsets and maximal elements does not hold.

Consequence:

Finitely represented invariants may not exist!

Solution:

Move to a language-equivalent system for which they always exist.

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Example 1:

For each $c \in S$, $c{\downarrow}$ is an ideal.

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Example 2:

Consider $(\mathbb{N}^k, \leqslant)$

The ideals are the sets $u\!\downarrow$ for $u \in (\mathbb{N} \cup \{\omega\})^k$.

Ideal decomposition

Lemma ([Kabil, Pouzet 1992])

Let $(S, \leqslant)$ *be a wqo.*

For $D \subseteq S$ *downward closed, let* $\textsc{Idec}(D)$ *be the set of* *inclusion-maximal ideals in* D*.*

$\textsc{Idec}(D)$ *is unique, finite, and we have*

$$D = \bigcup \textsc{Idec}(D) \ .$$

Ideal completion

Definition ([FG12,BFM14])

Let $\mathcal{W} = (S, \leqslant, T, I, F)$ WSTS.

Its ideal completion is
$\widehat{\mathcal{W}} = (\{\mathcal{I} \subseteq S \mid \mathcal{I} \text{ ideal}\}, \subseteq, \widehat{T}, \textsc{IDec}(I\downarrow), \widehat{F})$ with

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Lemma

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- $\mathcal{L}(\widehat{\mathcal{W}}) = \mathcal{L}(\mathcal{W})$.

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Proof.

Property of being an inductive invariant carries over.

Any set of the shape $\textsc{Idec}(Y)\downarrow$ is finitely-represented in $\widehat{\mathcal{W}}$. $\square$

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Proof.

Property of being an inductive invariant carries over.

Any set of the shape $\textsc{Idec}(Y)\downarrow$ is finitely-represented in $\widehat{\mathcal{W}}$. $\square$

Result in particular applies to $\mathsf{Cover} = \mathsf{Post}^*(I_1 \times I_2)\downarrow$.

Using the ideal completion

Proposition

If X is an inductive invariant for $\mathcal{W}$,
then its ideal decomposition $\textsc{Idec}(X)\downarrow$
is a finitely represented inductive invariant for $\widehat{\mathcal{W}}$.

Proof.

Property of being an inductive invariant carries over.

Any set of the shape $\textsc{Idec}(Y)\downarrow$ is finitely-represented in $\widehat{\mathcal{W}}$. □

Result in particular applies to $\mathsf{Cover} = \mathsf{Post}^*(I_1 \times I_2)\downarrow$.

Remark: $\widehat{\mathcal{W}}$ is not necessarily a WSTS.

Separator size: The case of Petri nets

Separator size

Question:

Number of states of the separating automaton?

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1. Determinism.

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Number of states of the separating automaton?

Consider Petri nets.

Problems:

1. Determinism.
2. Size estimation on the ideal decomposition of an invariant.

Enforcing determinism

Given: Labeled Petri nets over Σ

$$N_A = (P_A, T_A, \lambda_A, \mathrm{in}_A, \mathrm{out}_A, M_{0A}, M_{fA})$$

$$N_B = (P_B, T_B, \lambda, \mathrm{in}_B, \mathrm{out}_B, M_{0B}, M_{fB}) \ .$$

See board.

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$$N_B = (P_B, T_B, \lambda, \mathrm{in}_B, \mathrm{out}_B, M_{0B}, M_{fB})\ .$$

Construct: Labeled Petri nets over T_B

$$N_A^{-\lambda} = (P_A, T_A^{-\lambda}, \ell, \mathrm{in}_A^{-\lambda}, \mathrm{out}_A^{-\lambda}, M_{0A}, M_{fA})$$

$$N_B^{det} = (P_B, T_B, \mathrm{id}, \mathrm{in}_B, \mathrm{out}_B, , M_{0B}, M_{fB})\ .$$

See board.

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$$\mathcal{L}(N_A \times N_B) = \lambda\Big(\mathcal{L}\Big(N_A^{-\lambda} \times N_B^{det}\Big)\Big)$$

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If $\mathcal{R}$ separates $\mathcal{L}\left(N_A^{-\lambda}\right)$ and $\mathcal{L}(N_B^{det})$,
then $\lambda(\overline{\mathcal{R}})$ separates $\mathcal{L}(N_A)$ and $\mathcal{L}(N_B)$.

Obtaining an ideal decomposition of an invariant

First idea:

Coverability graph provides ideal decomposition of Cover.

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Use ideal decomposition of $\mathbb{N}^k \setminus \mathrm{Pre}^*(M_{fA}\uparrow \times M_{fB}\uparrow)$.

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Theorem ([Bozzelli, Ganty 2011])

$Pre^*(M_f\uparrow) = \{v_1, \ldots, v_k\}$ *with* k *and* $||v_i||_\infty$ *doubly exponential.*

The upper bound

Theorem (BG11)

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Theorem (Upper bound)

Given two disjoint Petri nets, we can construct an NFA separating their coverability languages of triply-exponential size.

Upper vs. lower bound

Theorem (Upper bound)

Given two disjoint Petri nets, we can construct an NFA separating their coverability languages of triply-exponential size.

Theorem (Lower bound)

The disjoint Petri net coverability languages

$$\mathcal{L}_{0@2^{2^k}} \text{ and } \mathcal{L}_{1@2^{2^k}} \text{ over } \{0, 1\}$$

cannot be separated by a DFA of less than triply-exponential size.

Conclusion

Regular separability for WSTS languages

Theorem

If two WSTS languages are disjoint,
one of them finitely branching or deterministic or ω^2,
then they are regularly separable.

Open problems: Expressiveness

Non-Determinism:

Does non-determinism add to the expressiveness of WSTS:

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$$\text{deterministic WSTS languages} \quad \subsetneq \quad \text{all WSTS languages} \quad ?$$

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Complexity:

Tight bound on the separator size for Petri nets.

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Complexity:

Tight bound on the separator size for Petri nets.

Replace homomorphism trick or show combinatorial magic.

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Regular separability result:

Are disjoint WSTS languages always regularly separable?

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A similarly general result would be surprising

given the negative results for LCS [Abdulla, Jonsson 1996].

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There are not yet practical algorithms
for and based on separability :)

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Compute separators from automata or WMSO formulas.

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Iterated decomposition in the Petri net case open.
Learning would benefit from extrapolation.

Open problems

Beyond regular separability?

Open problems

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Beyond WSTS?

Thank you!

Questions?