Dynamic Partial Order Reduction under the Release-Acquire Semantics

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OOPSLA'2018

Outline

- SC (Sequential Consistency) Semantics
  - Runs and Traces
- Dynamic Partial-Order Reduction (DPOR)
- Release-Acquire Semantics
- Experiments
- Conclusions

Concurrent Programs (under SC)

Initially $x = 0$

th_1	$\parallel$	th_2
$x := 1$		$x := 2$
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- Threads
  - Concurrent
  - Loop-Free
  - Deterministic
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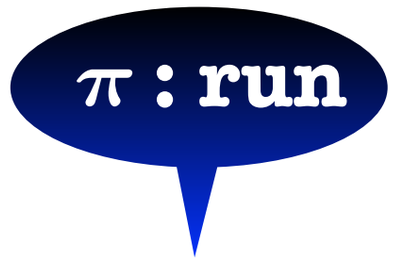

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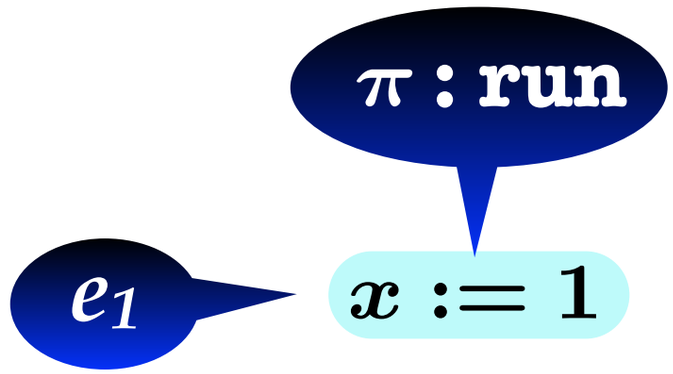

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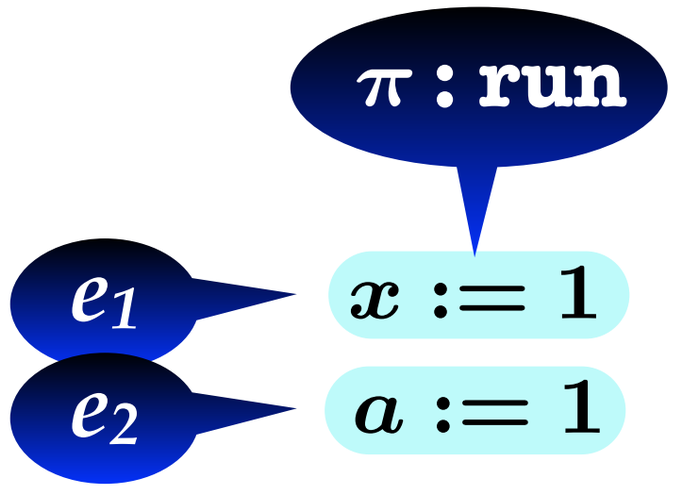

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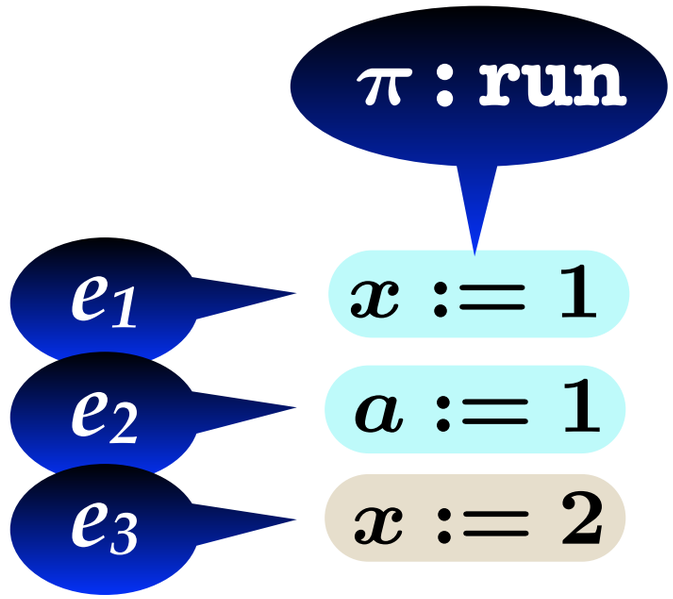

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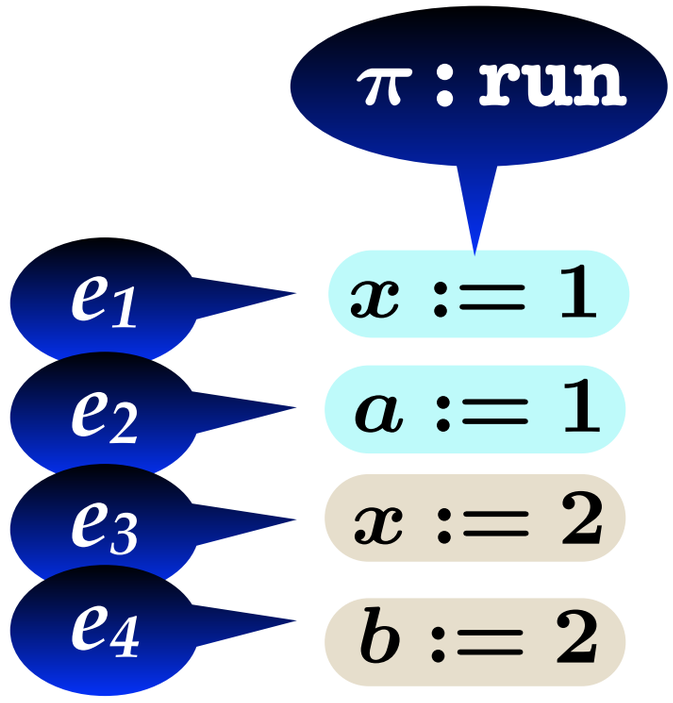

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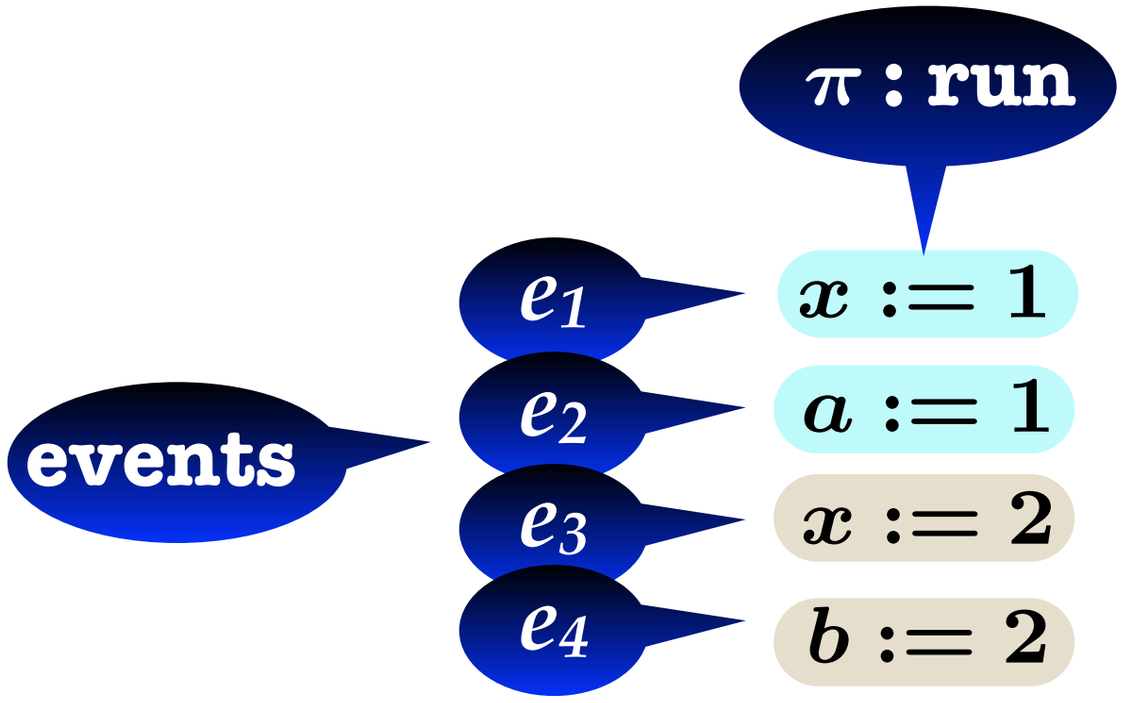

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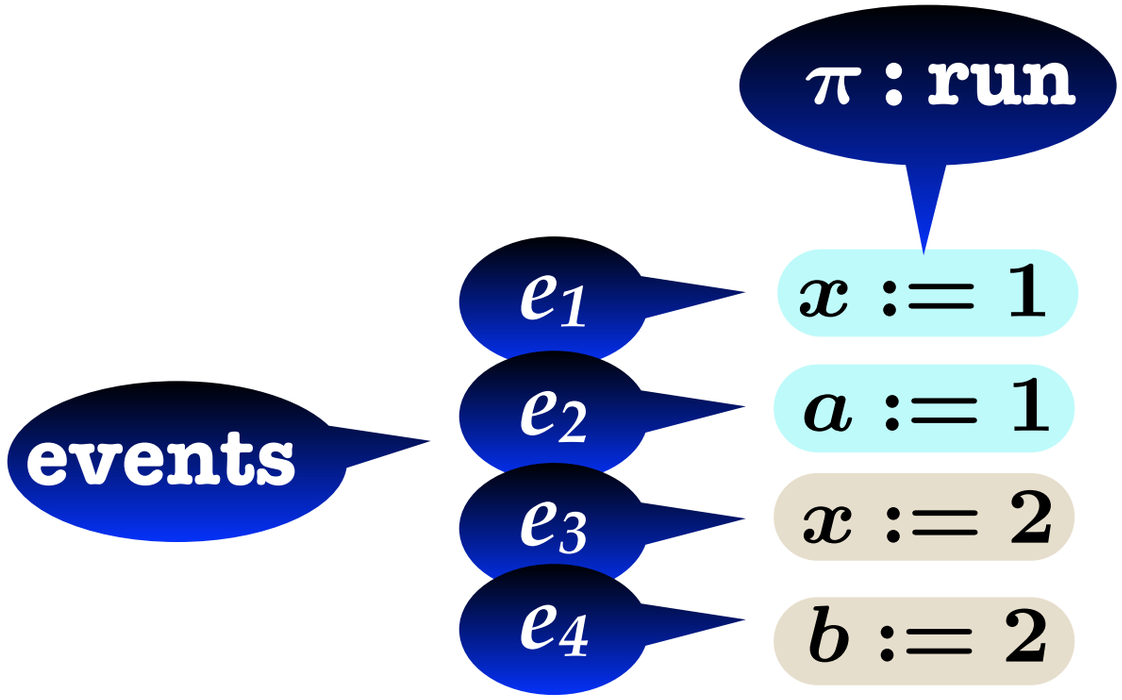

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- interleave instructions of different threads
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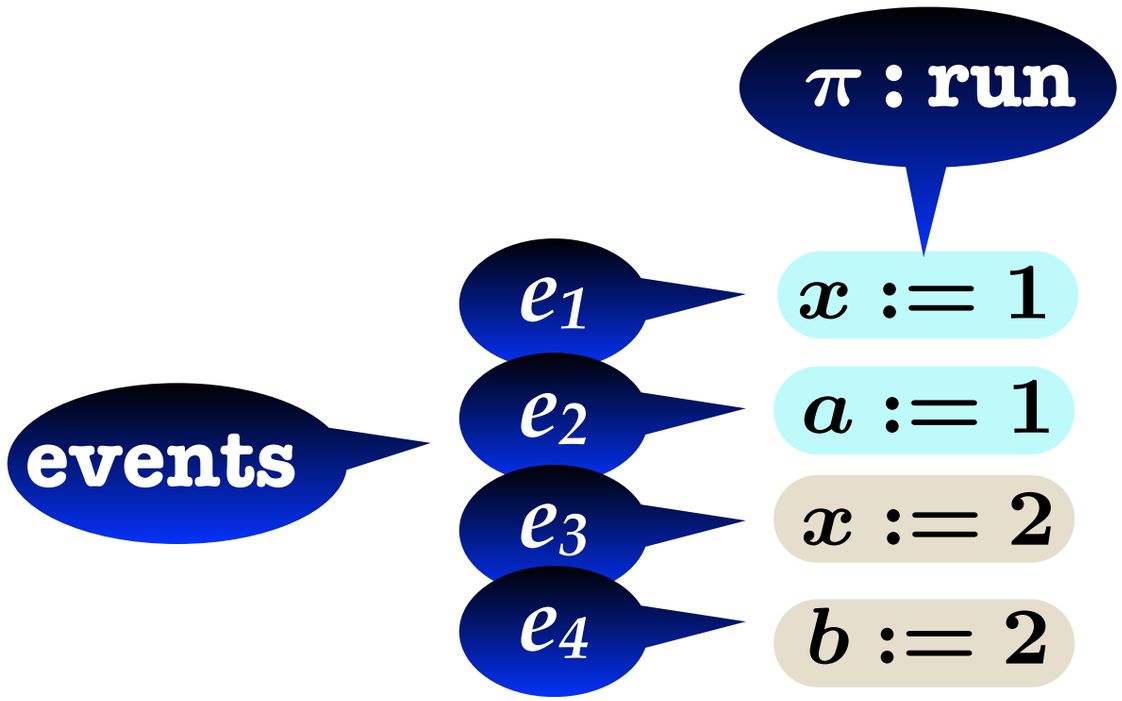

assertion

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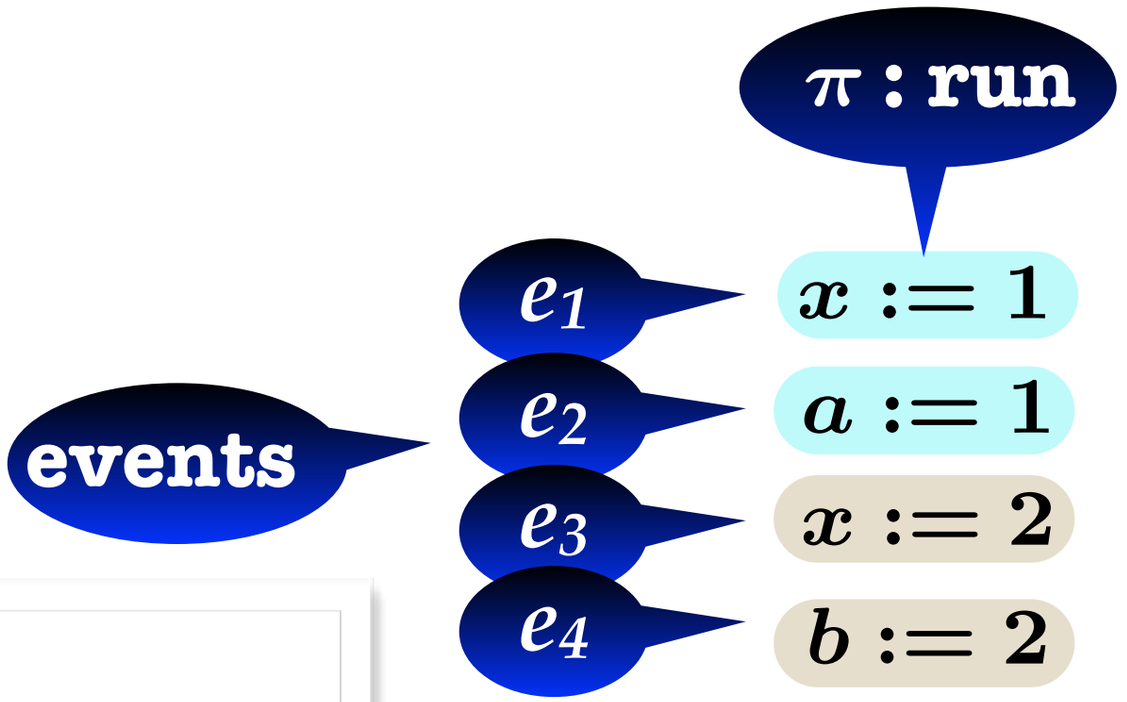

- **Traces**
  - **efficient:**
    - **more compact than runs**
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    - **allow different memory models**

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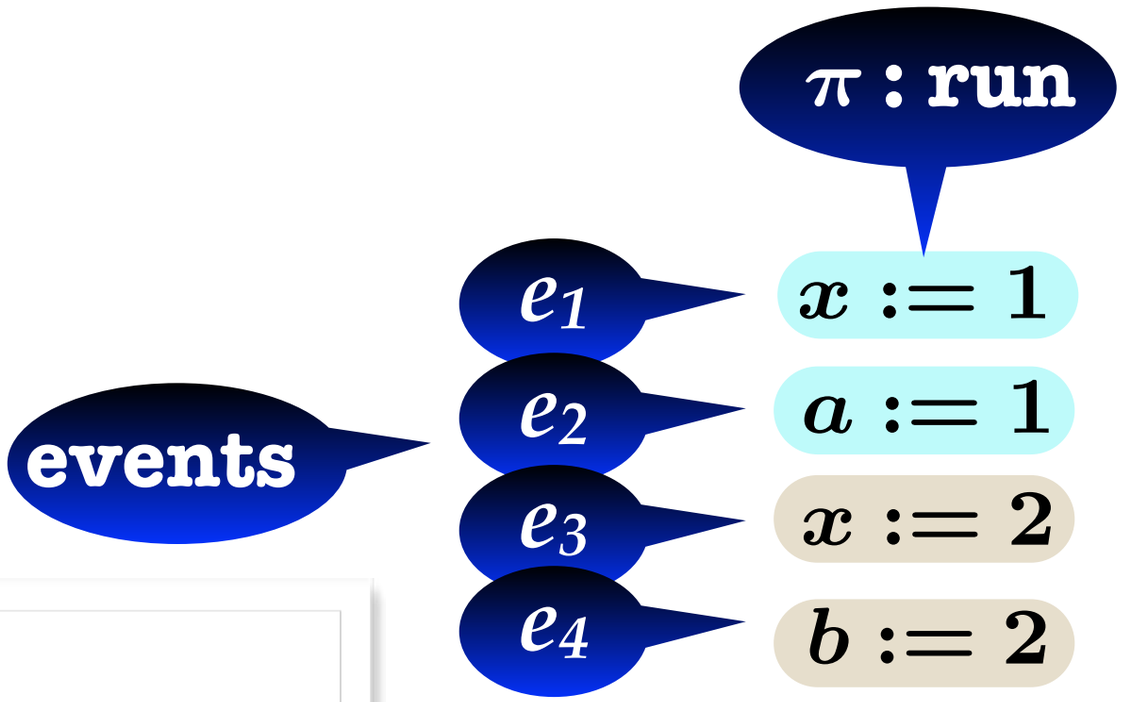

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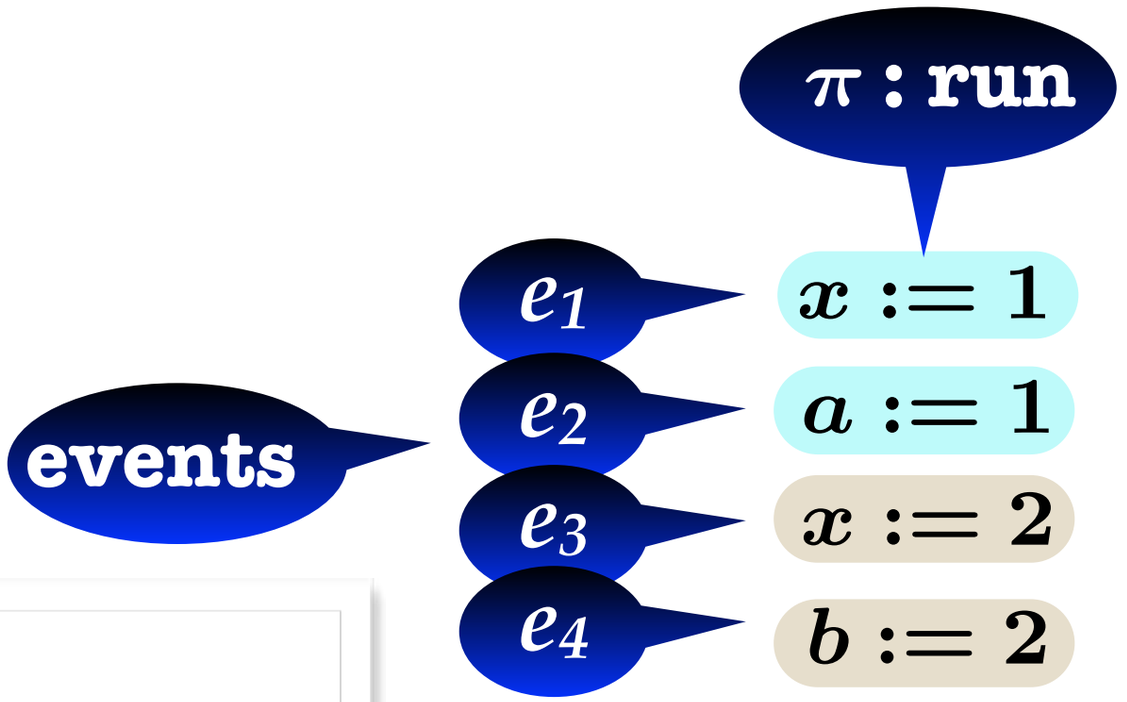

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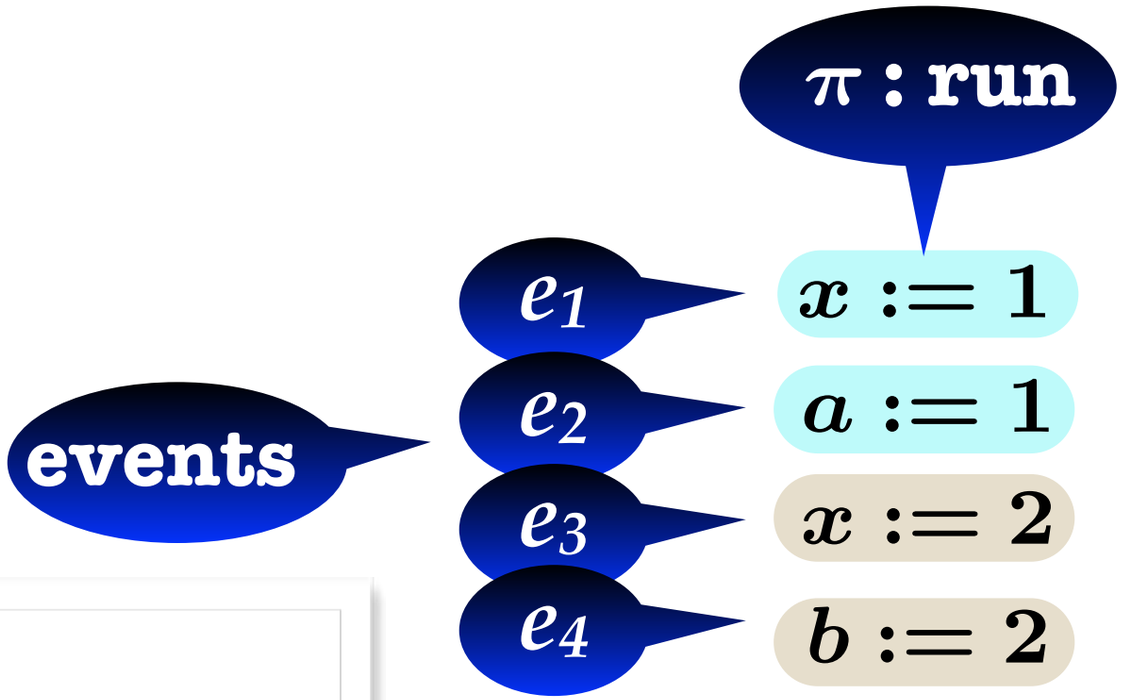

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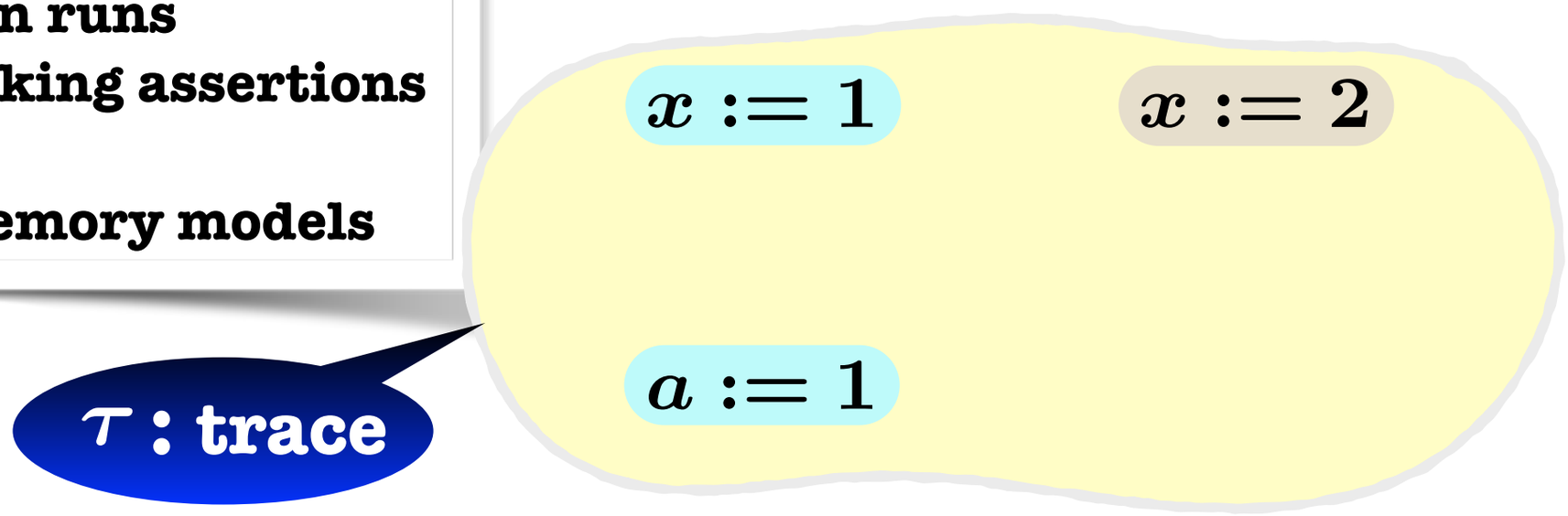

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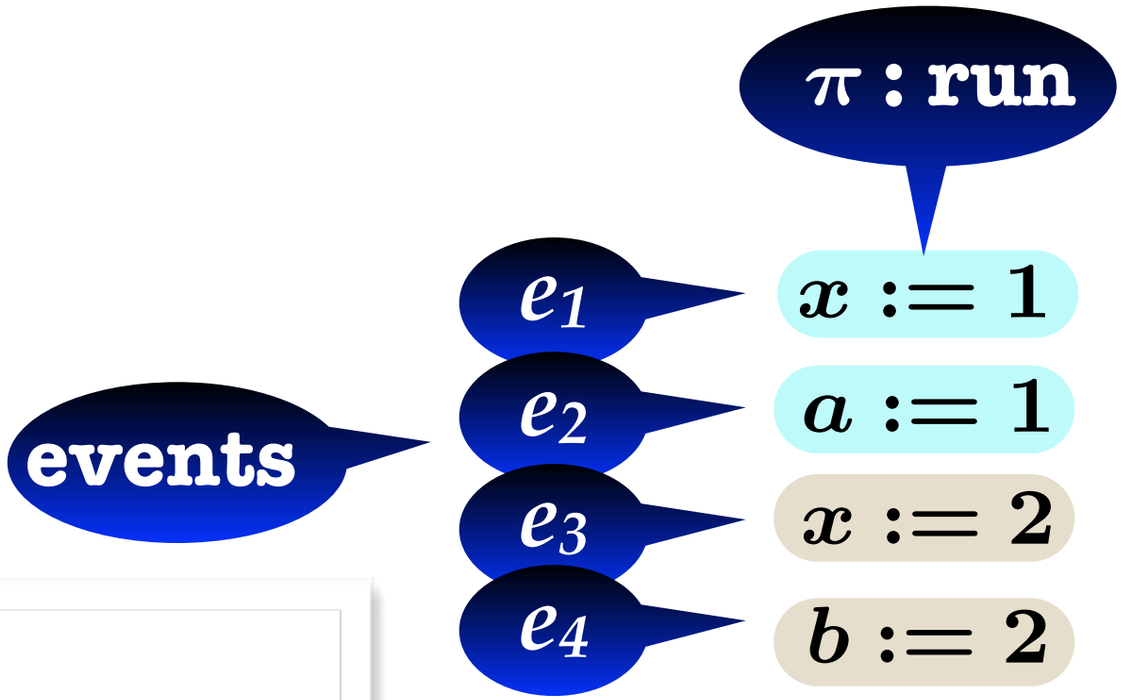

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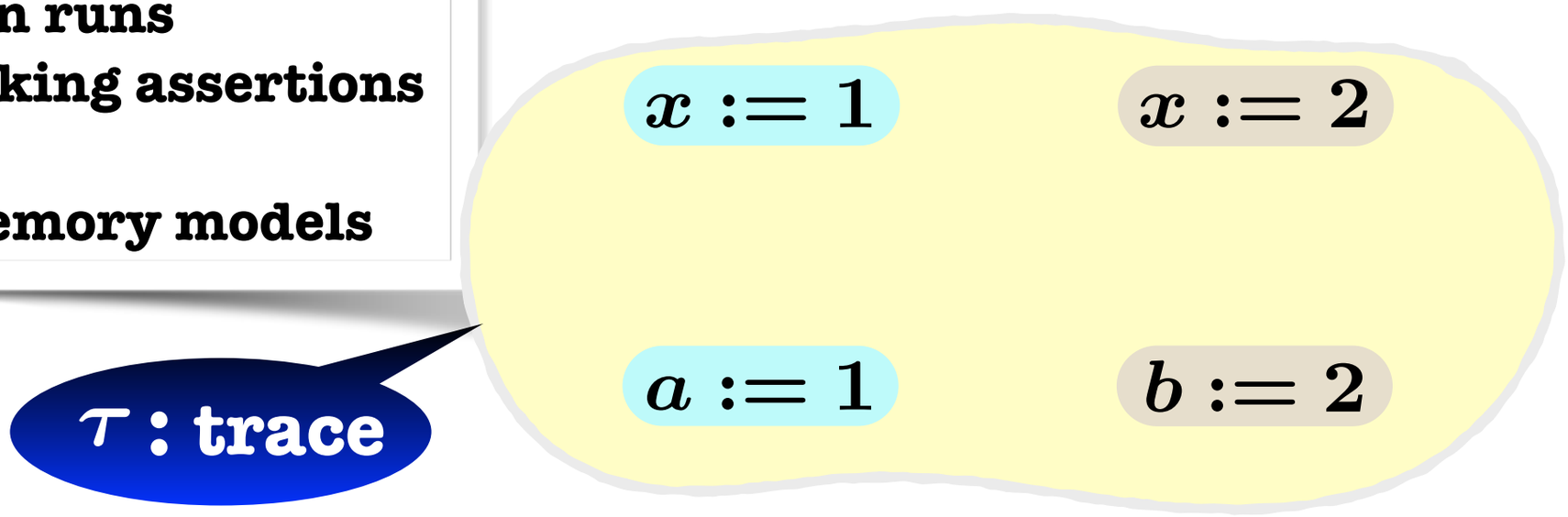

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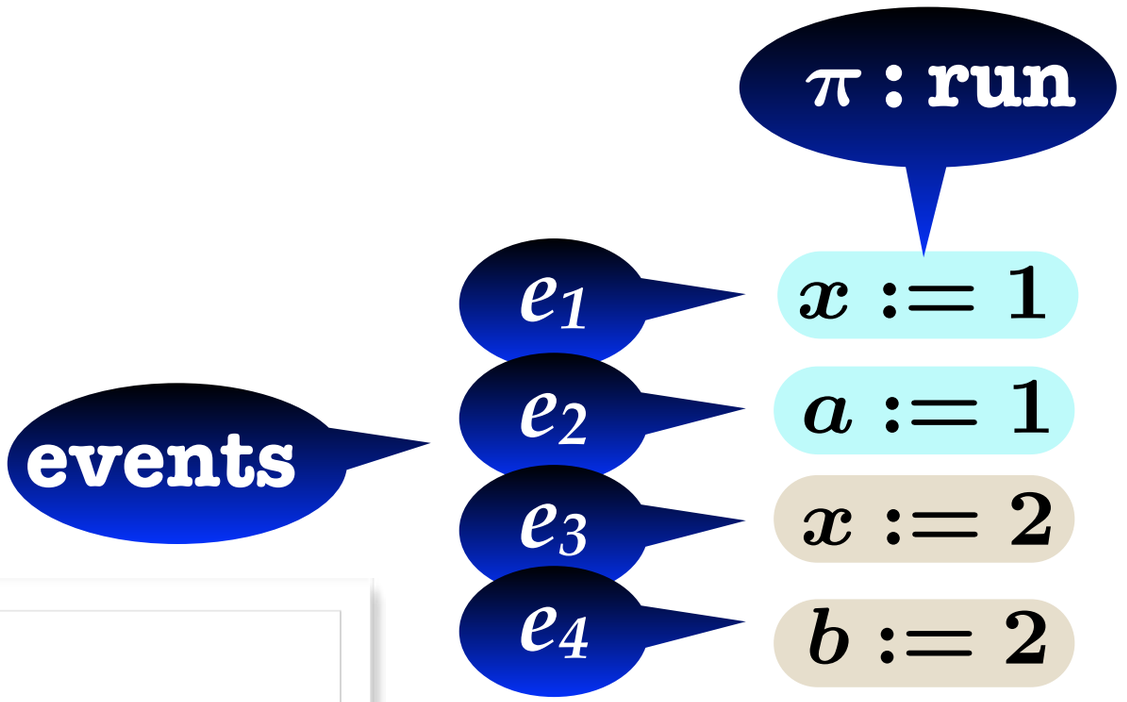

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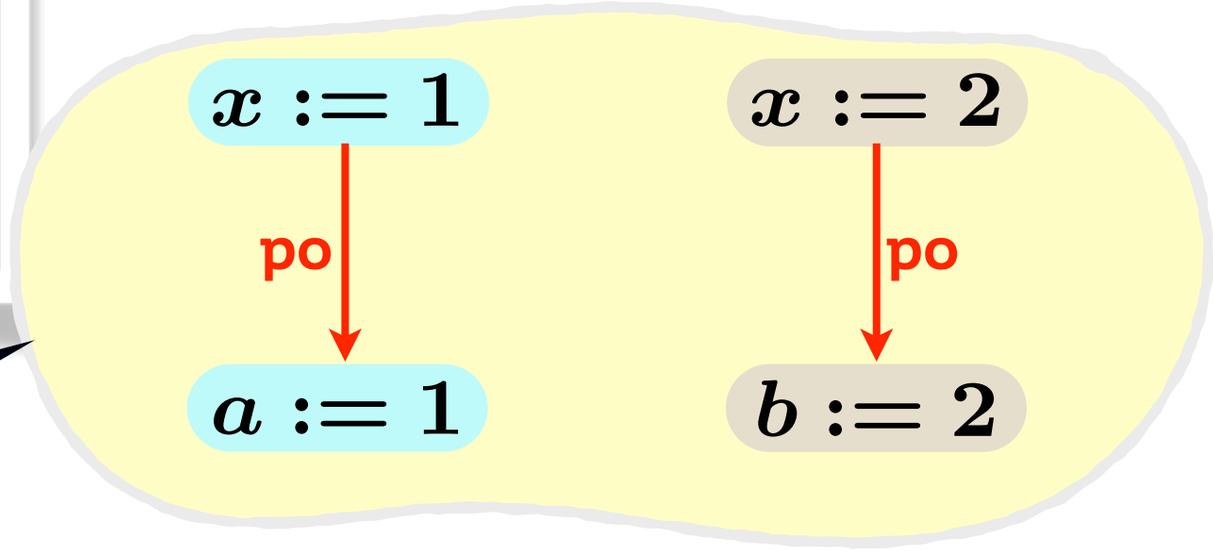

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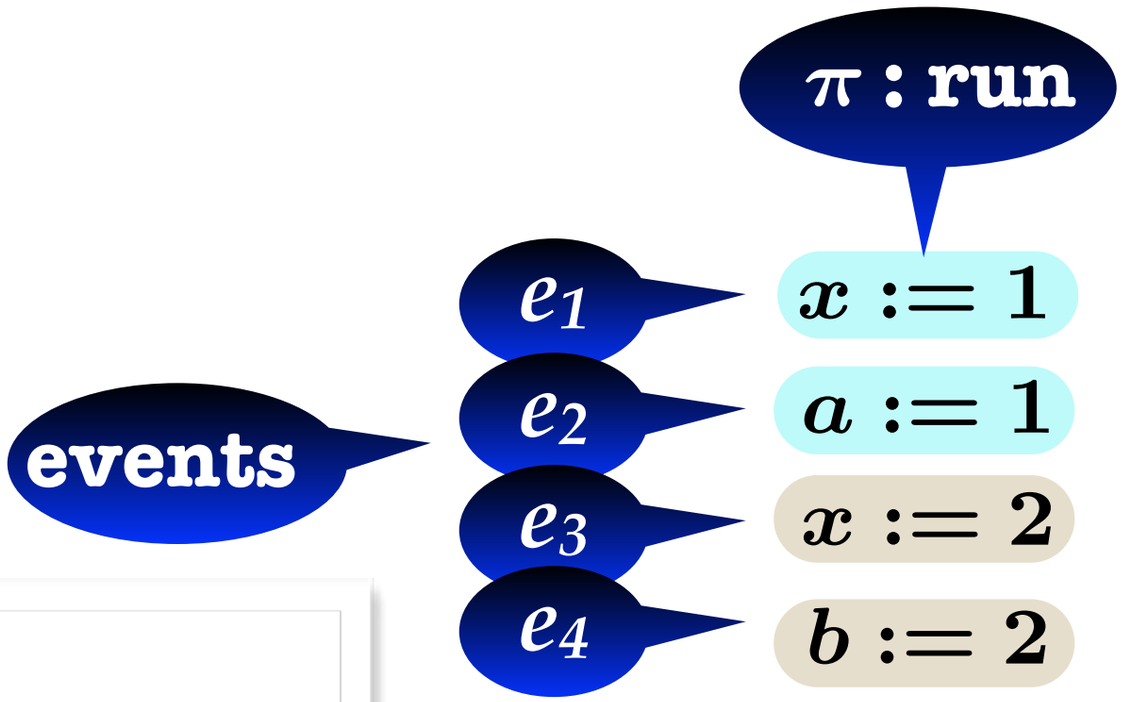

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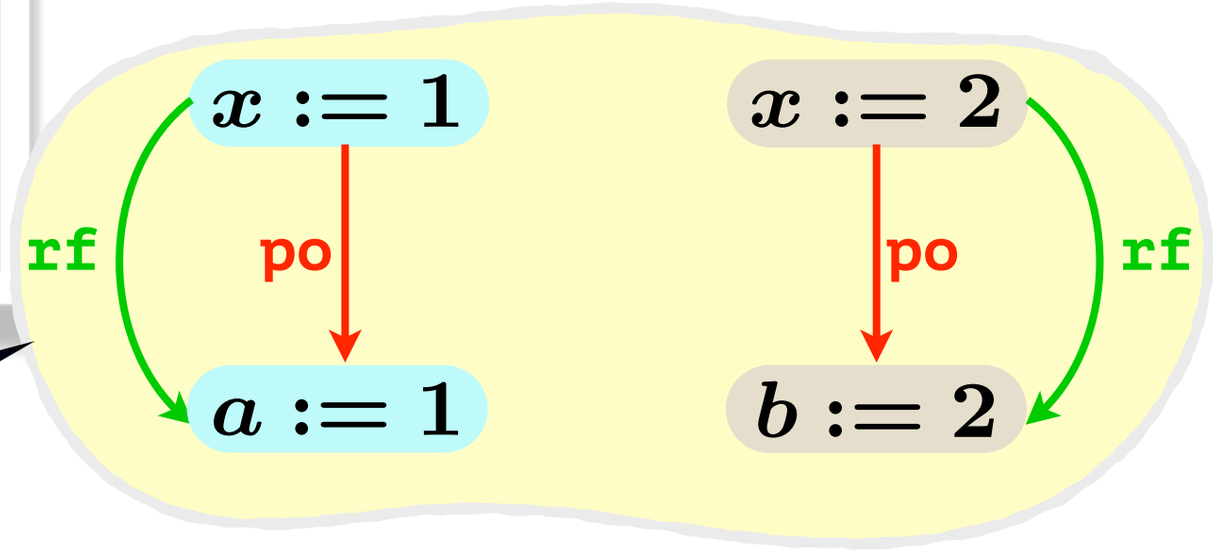

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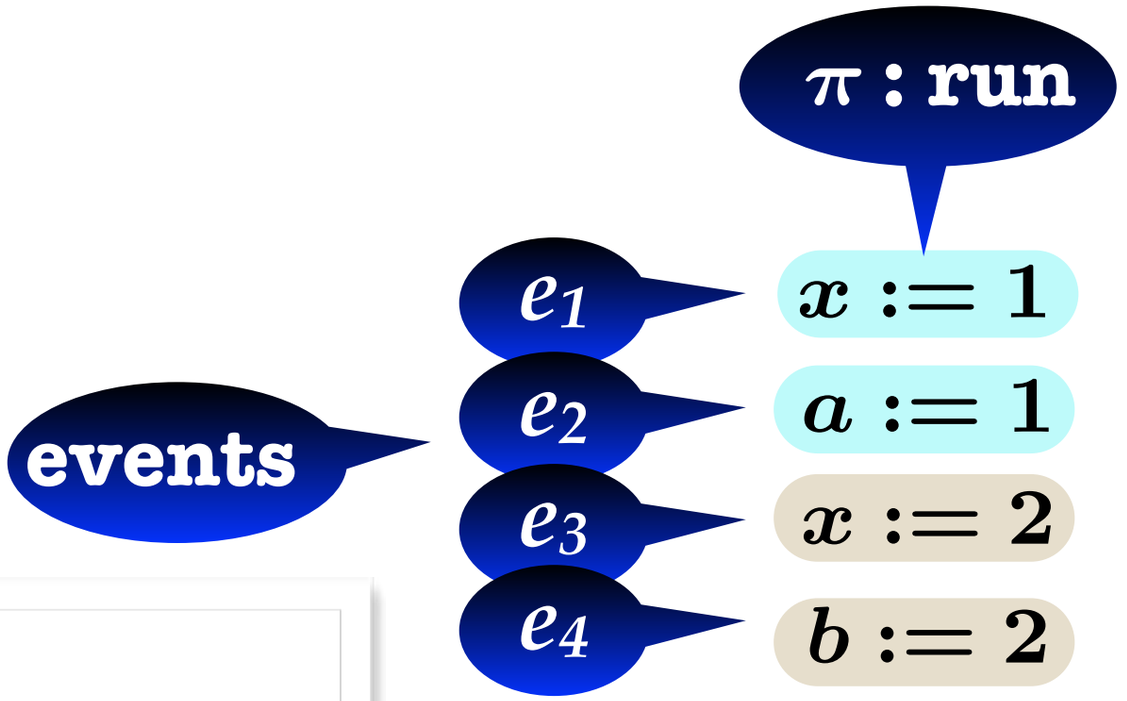

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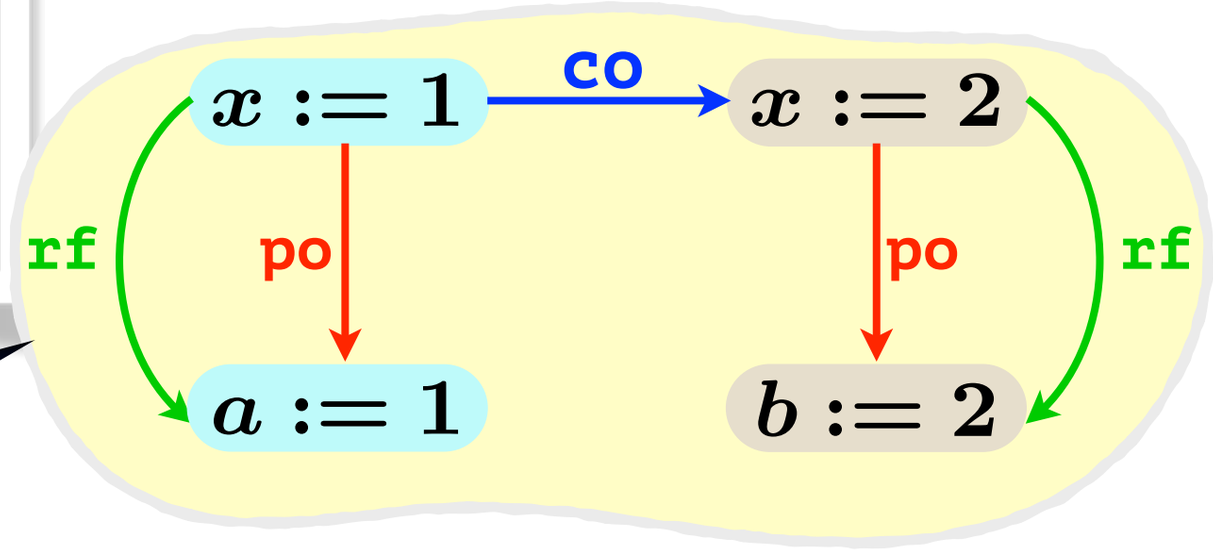

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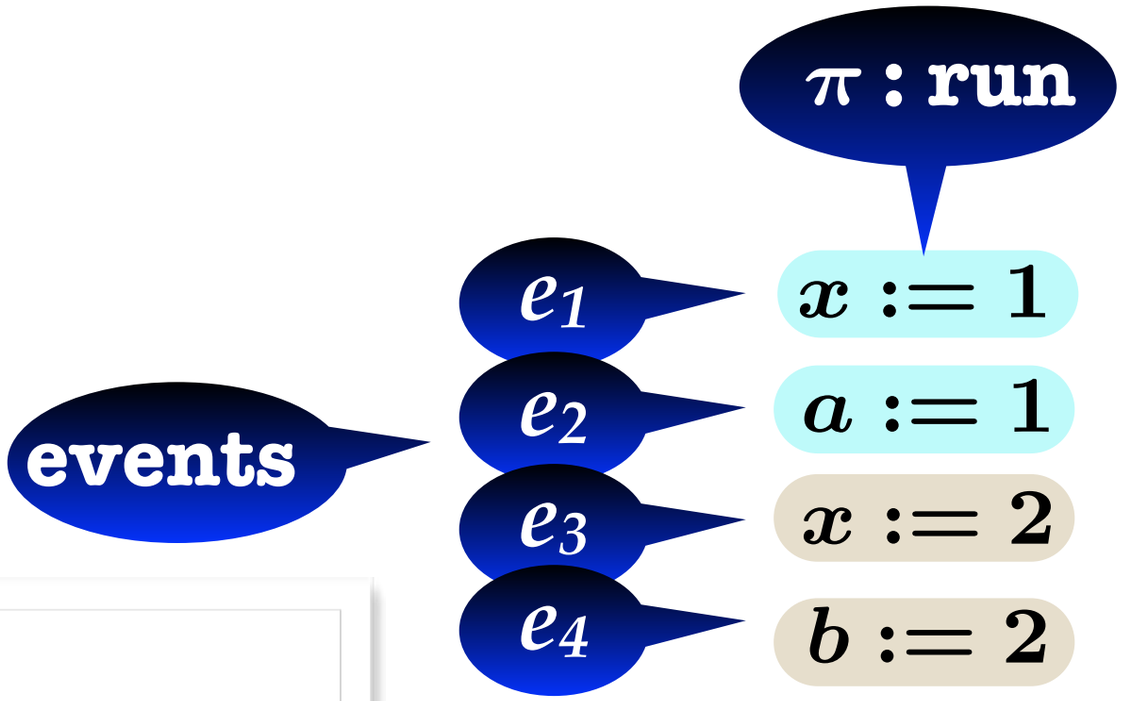

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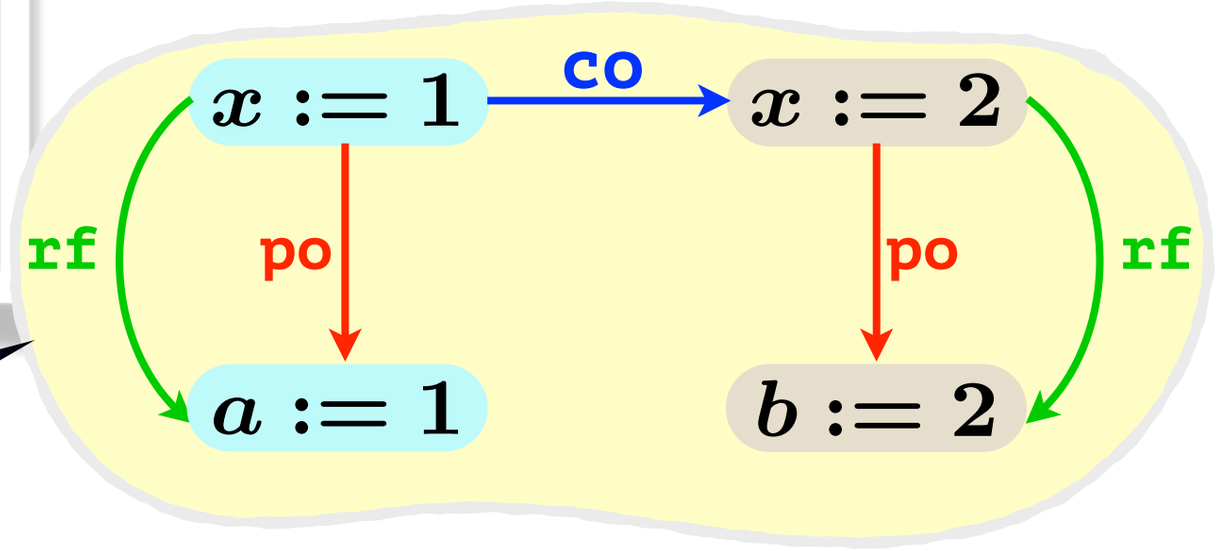

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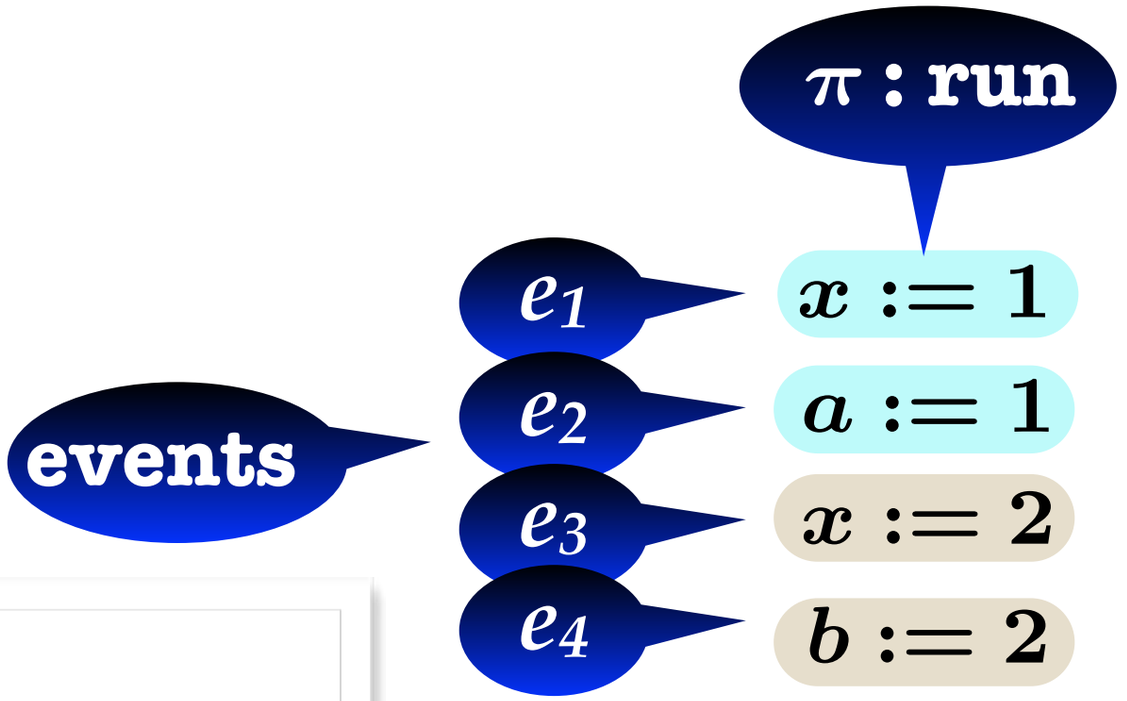

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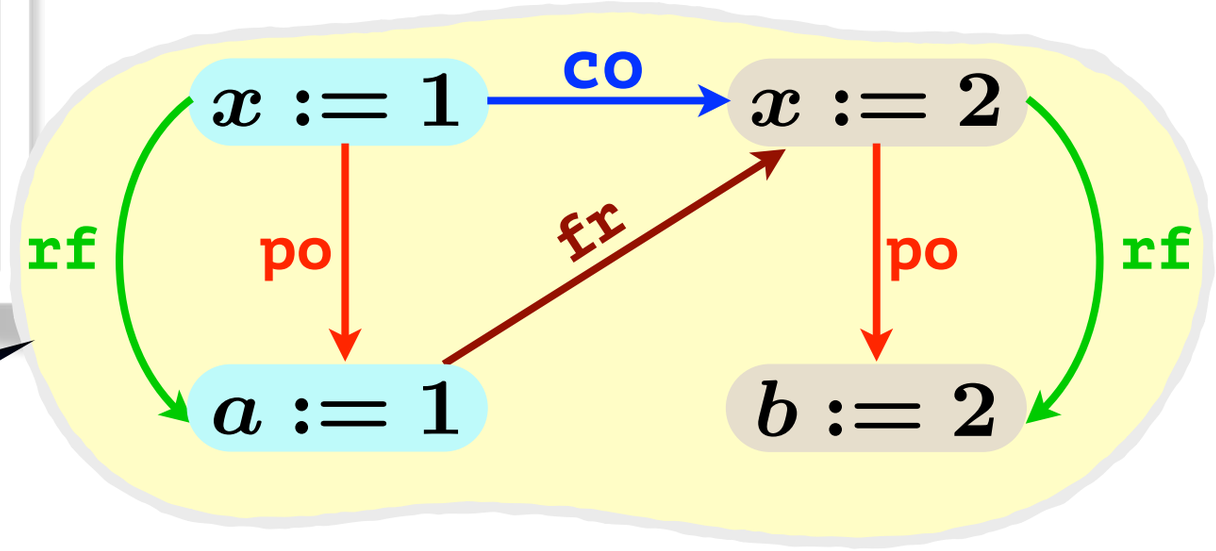

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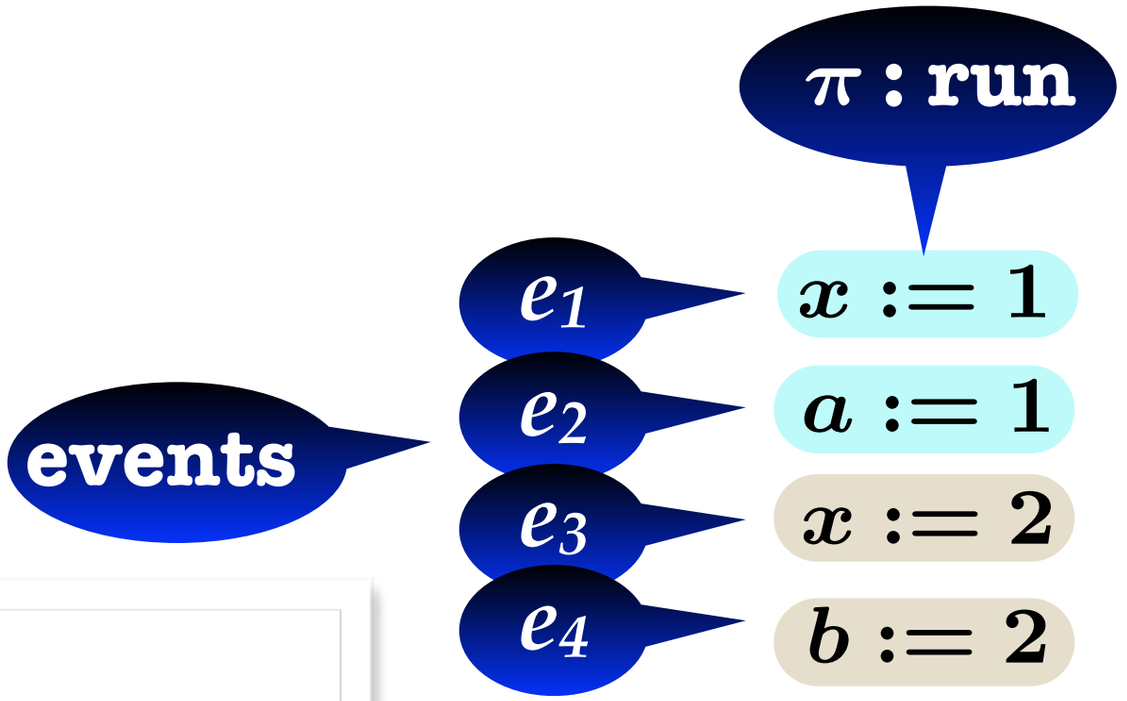

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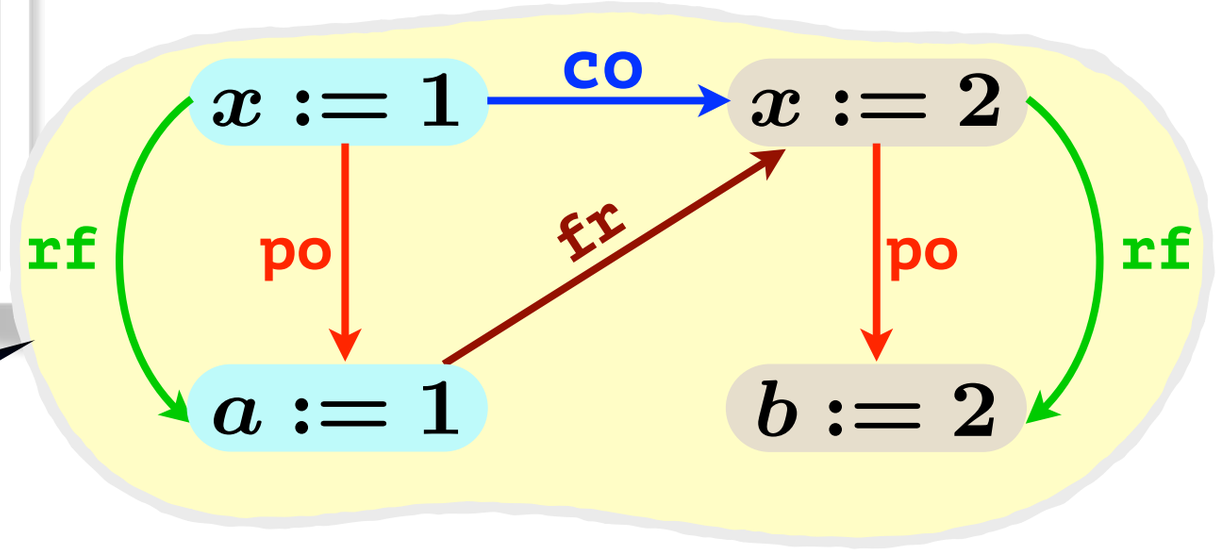

SC: $acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co} \cup \mathtt{fr})$

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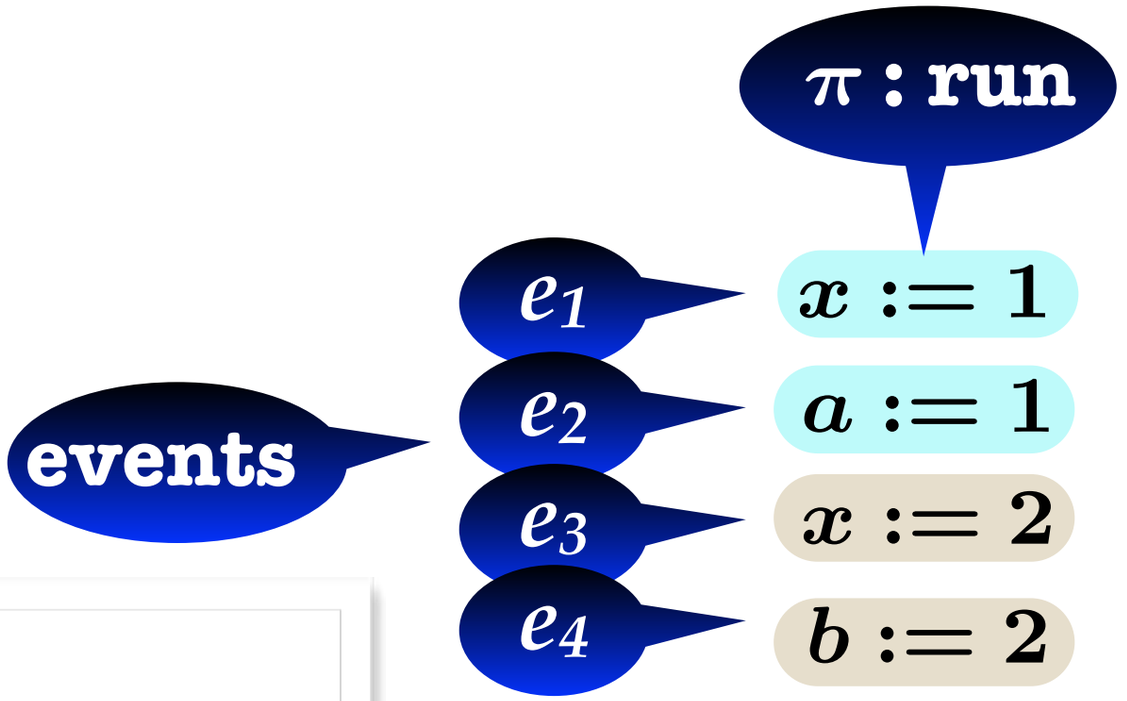

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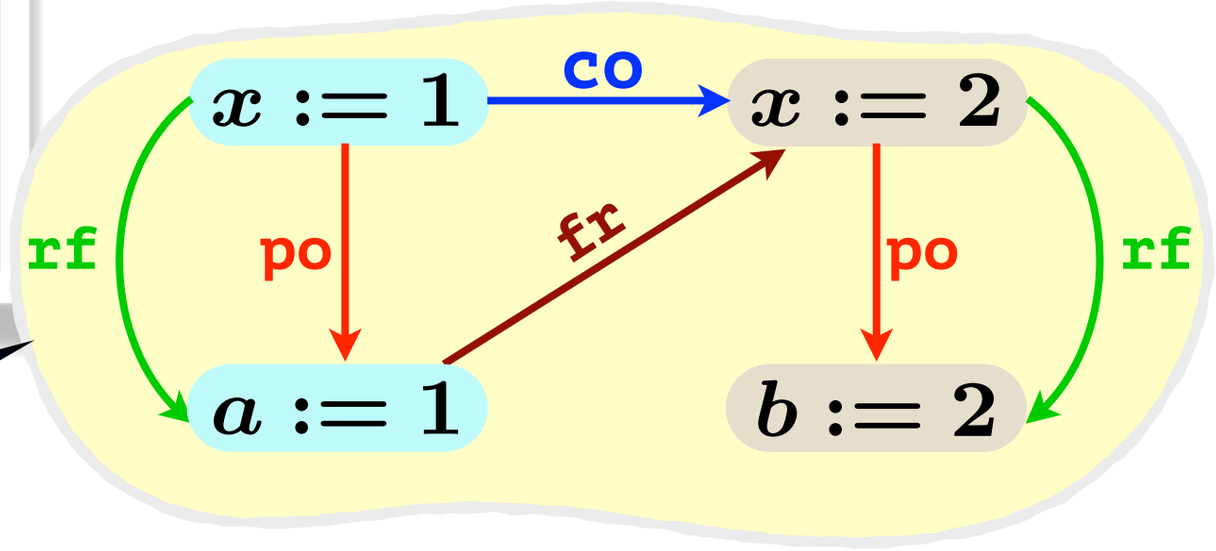

$$\text{SC: } acyclic(\texttt{po} \cup \texttt{rf} \cup \texttt{co} \cup \texttt{fr})$$

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$$\tau \models \text{SC}$$

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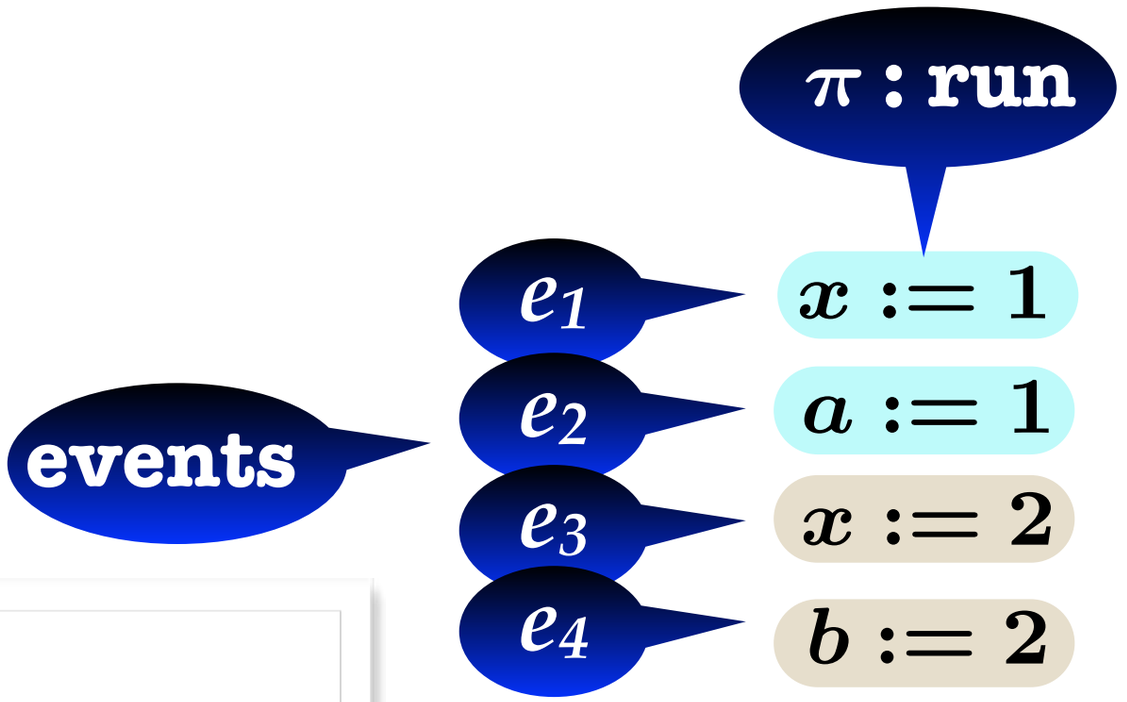

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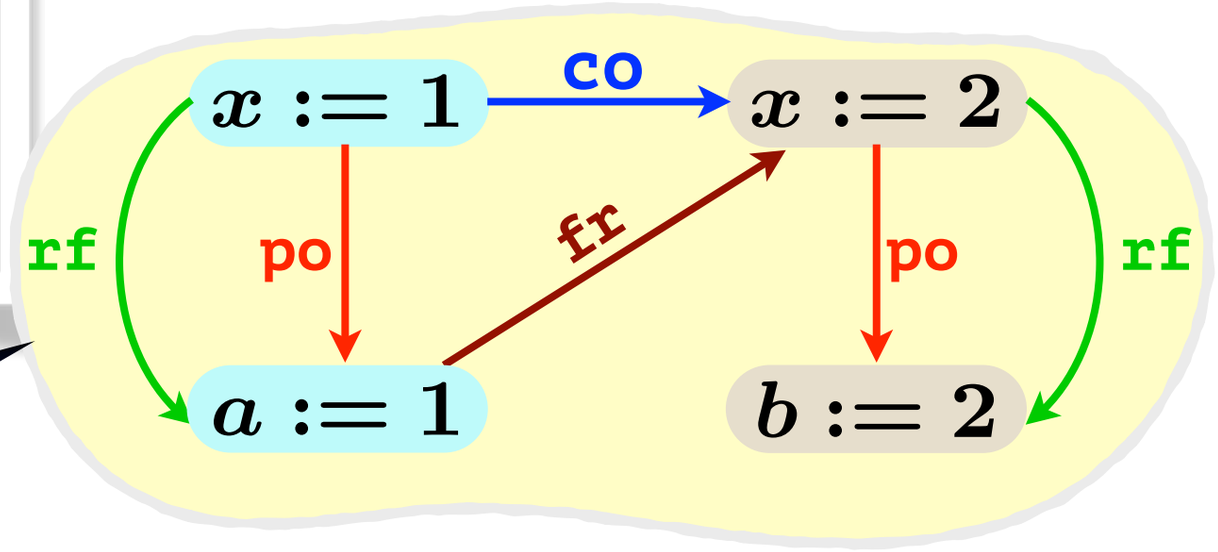

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SC: $acyclic(\mathbf{po} \cup \mathbf{rf} \cup \mathbf{co} \cup \mathbf{fr})$

$\tau \models \mathrm{SC}$

$\pi \models \tau$

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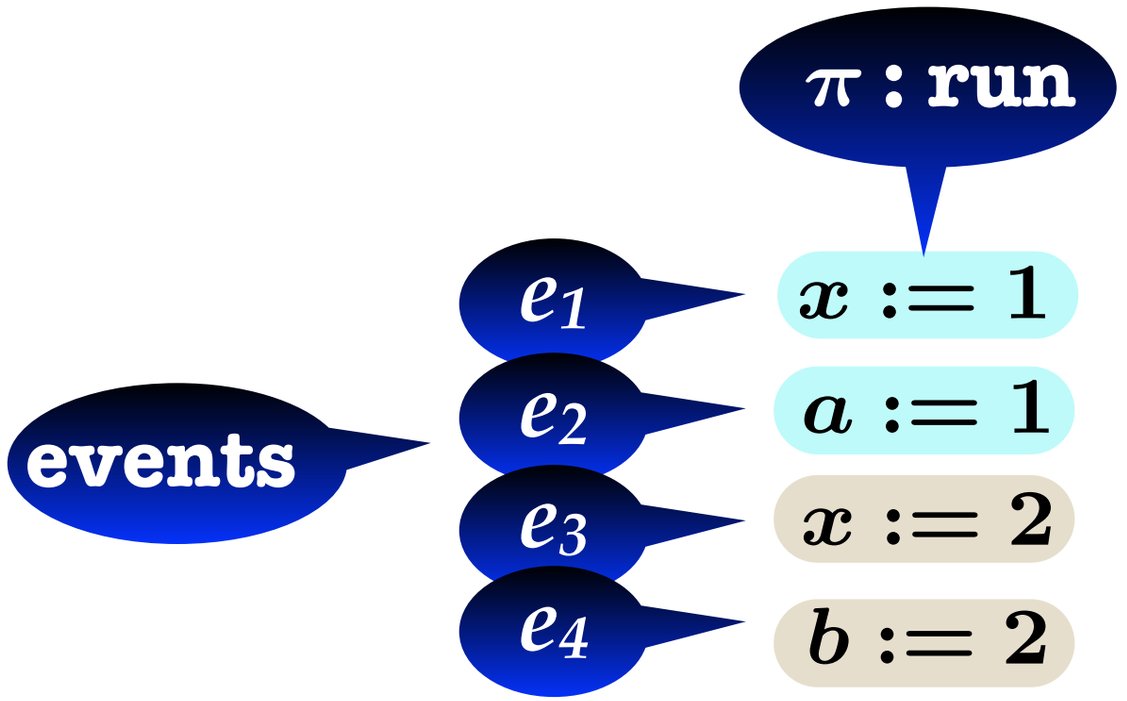

assertion

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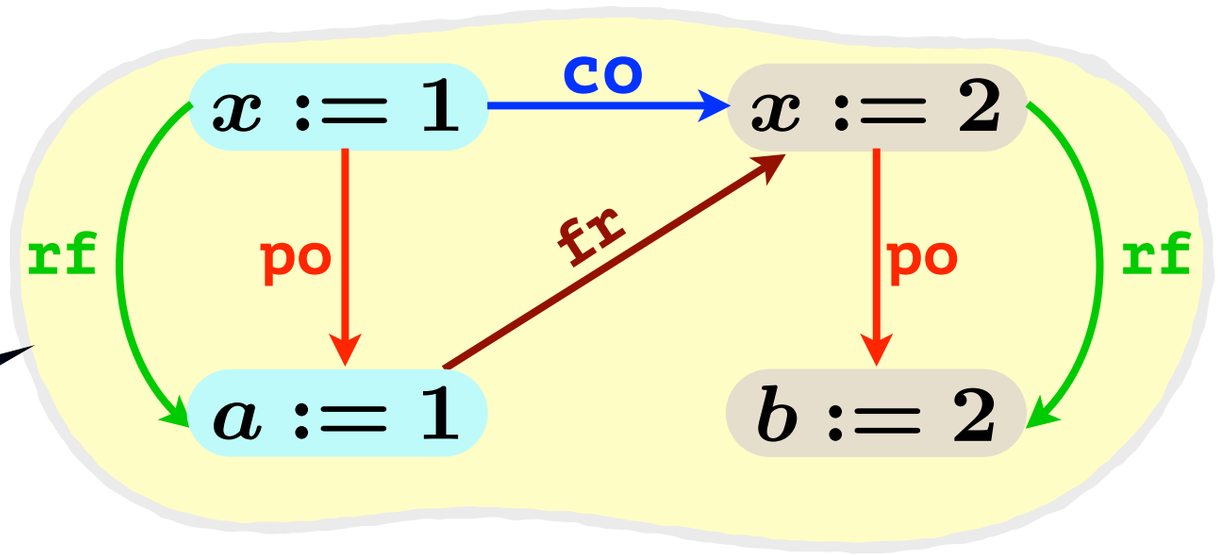
- **rf**: read-from
- **co**: coherence order
- **fr**: from-read



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π_1

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π_2

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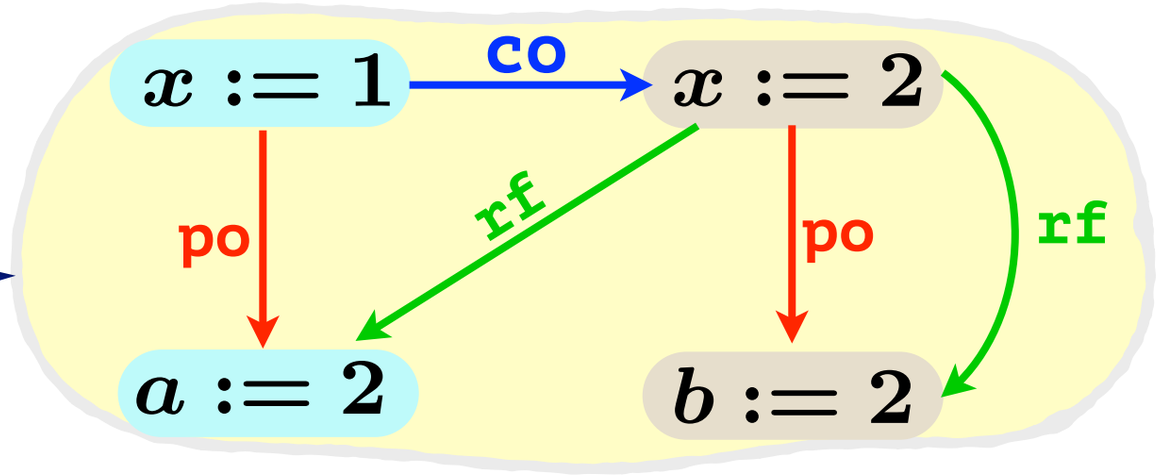
τ



- po: program order
- rf: read-from
- co: coherence order
- fr: from-read

Concurrent Programs (under SC)

Initially $x = 0$

th_1	$\|\|$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$

π_1

$x := 1$

$x := 2$

$b := 2$

$a := 2$

π_2

$x := 1$

$x := 2$

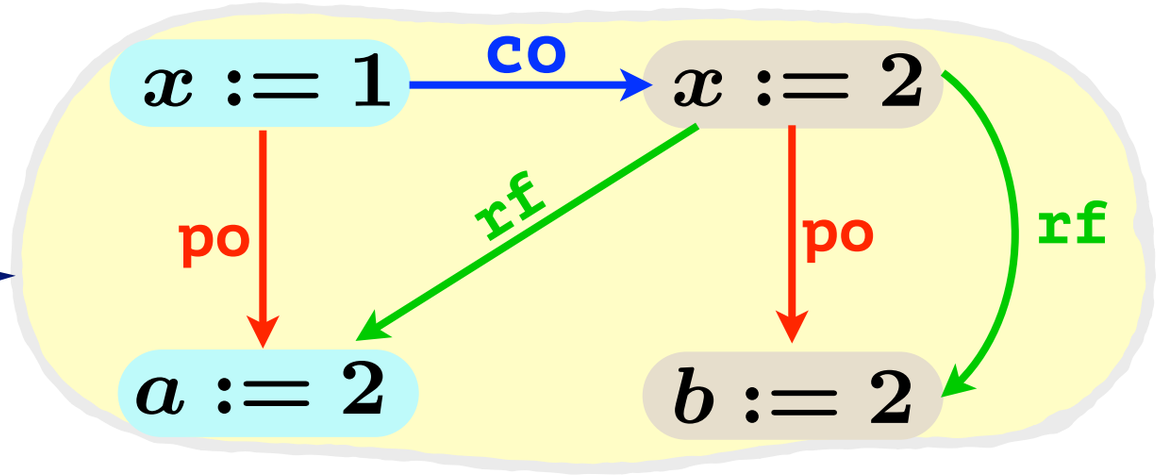
$a := 2$

$b := 2$

- **Traces**
  - **efficient:**
    - **more compact than runs**
    - **sufficient for checking assertions**
  - **abstract:**
    - **allows different memory models**

- po: program order
- rf: read-from
- co: coherence order
- fr: from-read

τ



$\pi_1 \models \tau$

$\pi_2 \models \tau$

DPOR: Dynamic Partial Order Reduction

Initially $x = 0$

th_1	$\parallel$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$

(optimal) DPOR

DPOR: Dynamic Partial Order Reduction

Initially $x = 0$

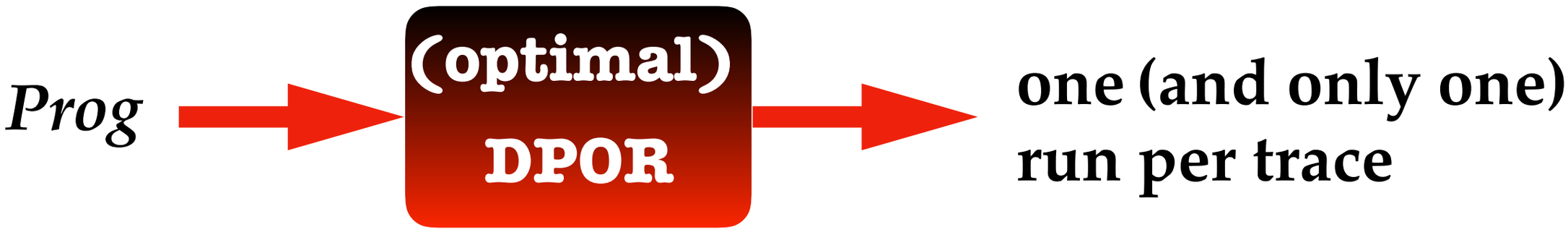
th_1	$\|\|$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$

Prog → **(optimal) DPOR**

DPOR: Dynamic Partial Order Reduction

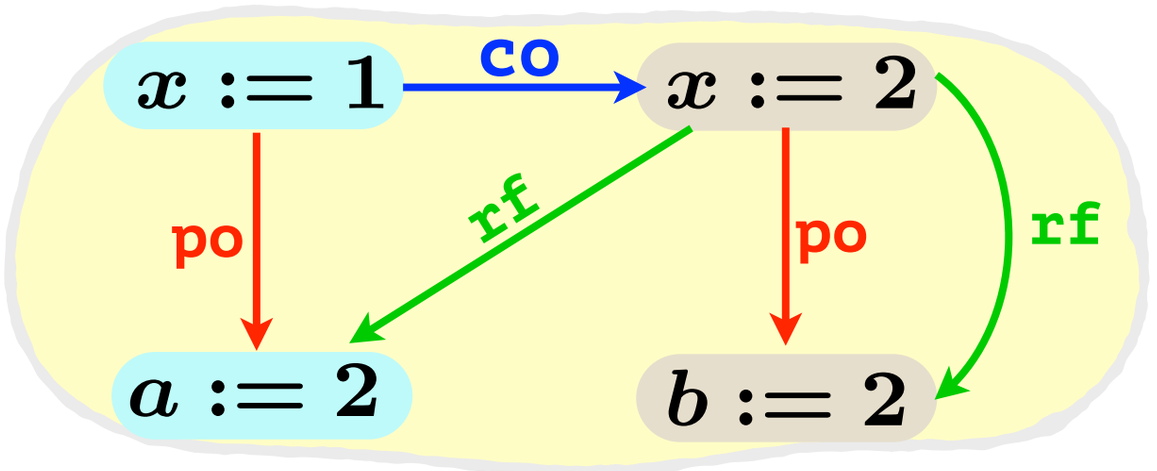
Initially $x = 0$

th_1	$\|\|$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$



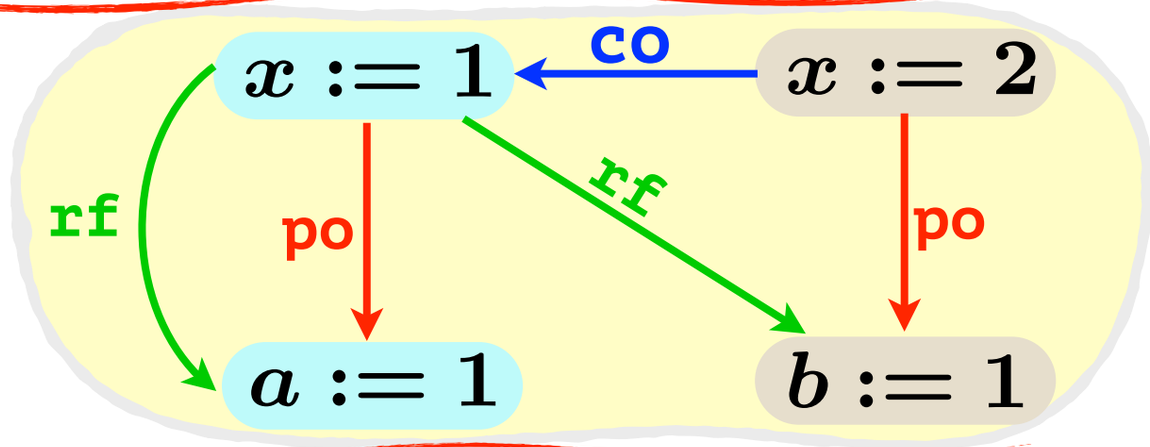
$x := 1$
$x := 2$
$a := 2$
$b := 2$

$x := 1$
$x := 2$
$b := 2$
$a := 2$

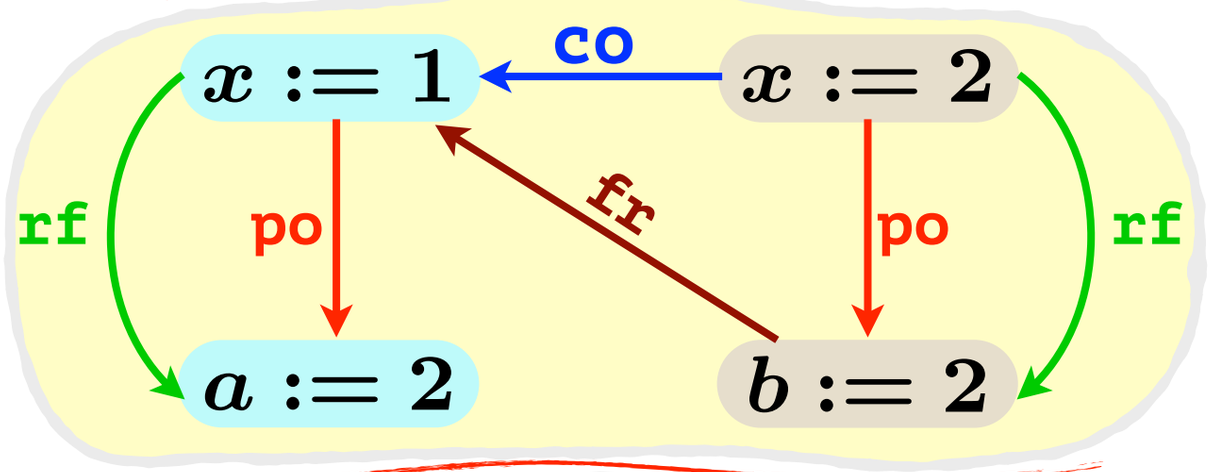


$x := 2$
$x := 1$
$b := 1$
$a := 1$

$x := 2$
$x := 1$
$a := 1$
$b := 1$

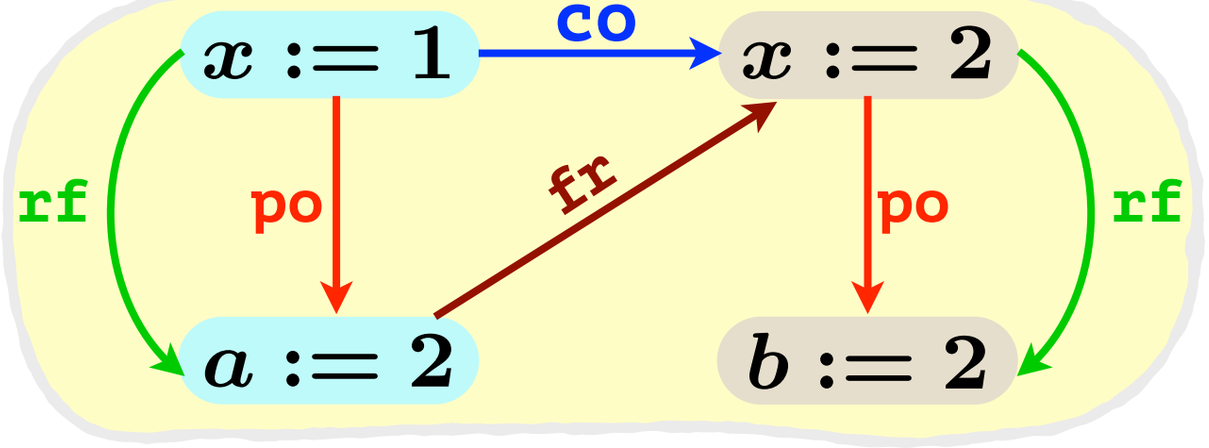


$x := 1$
$a := 1$
$x := 2$
$b := 2$

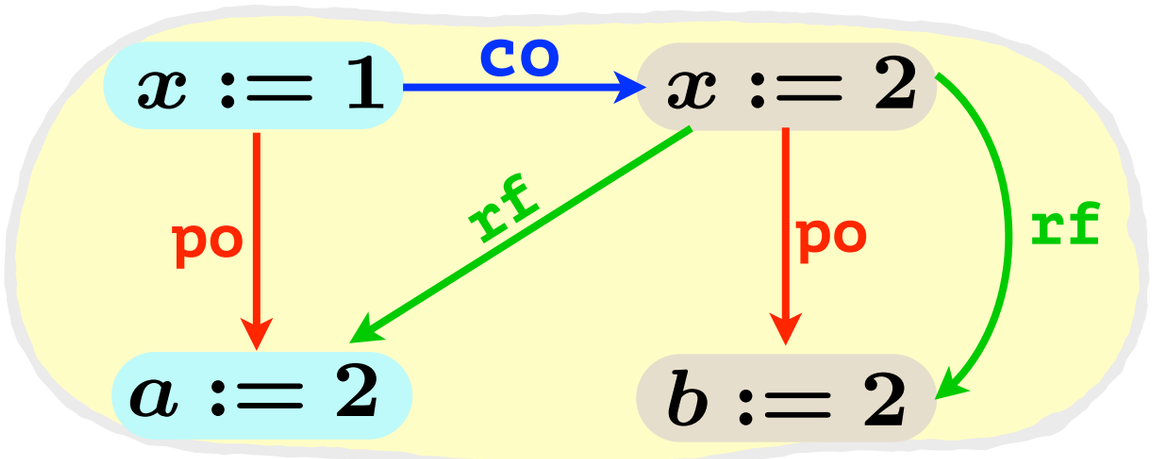


(optimal) DPOR

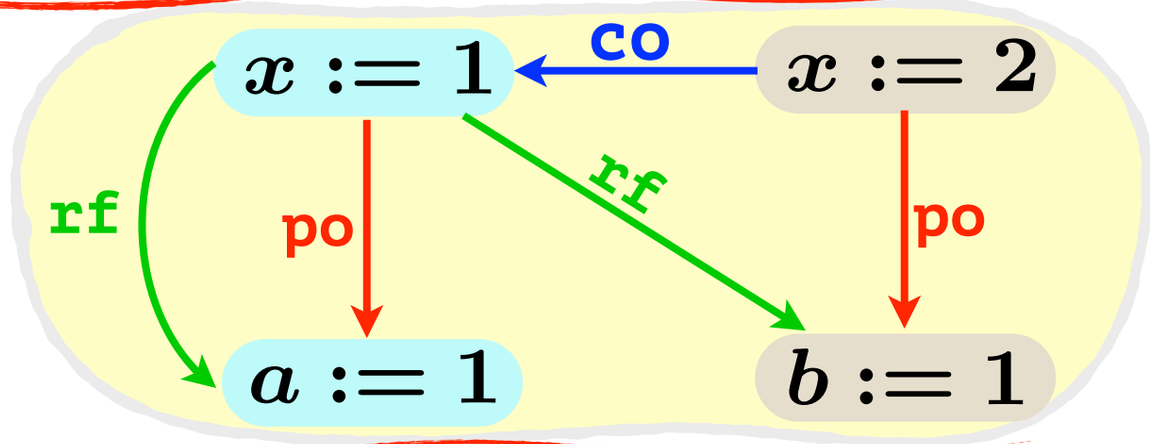
$x := 2$
$b := 2$
$x := 1$
$a := 1$



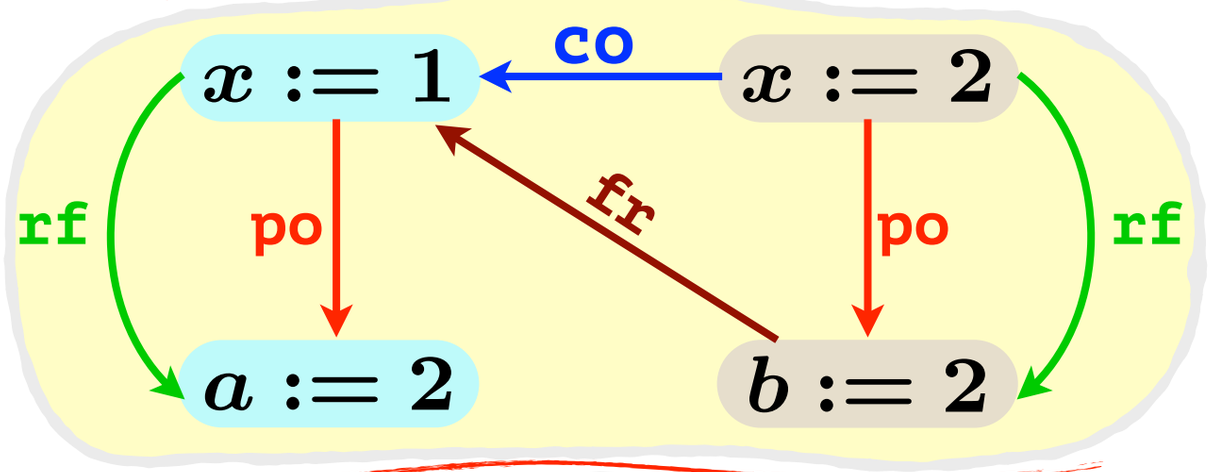
$x := 1$
$x := 2$
$a := 2$
$b := 2$



$x := 2$
$x := 1$
$b := 1$
$a := 1$

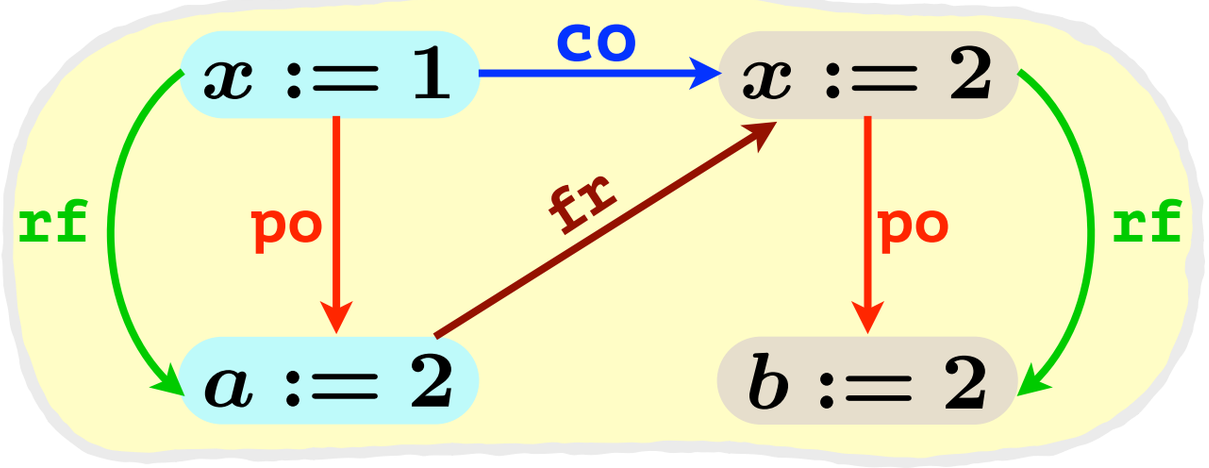


$x := 1$
$a := 1$
$x := 2$
$b := 2$

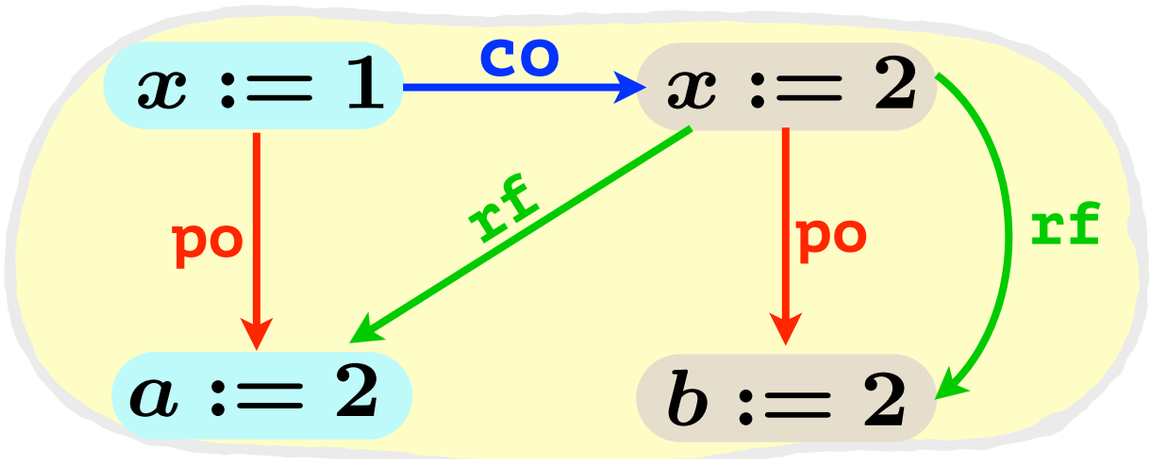


(optimal) DPOR

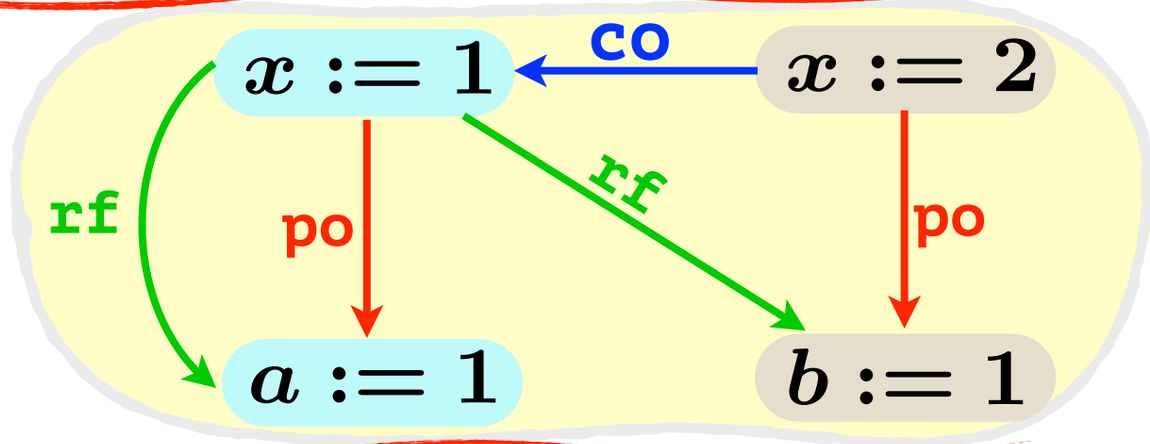
$x := 2$
$b := 2$
$x := 1$
$a := 1$



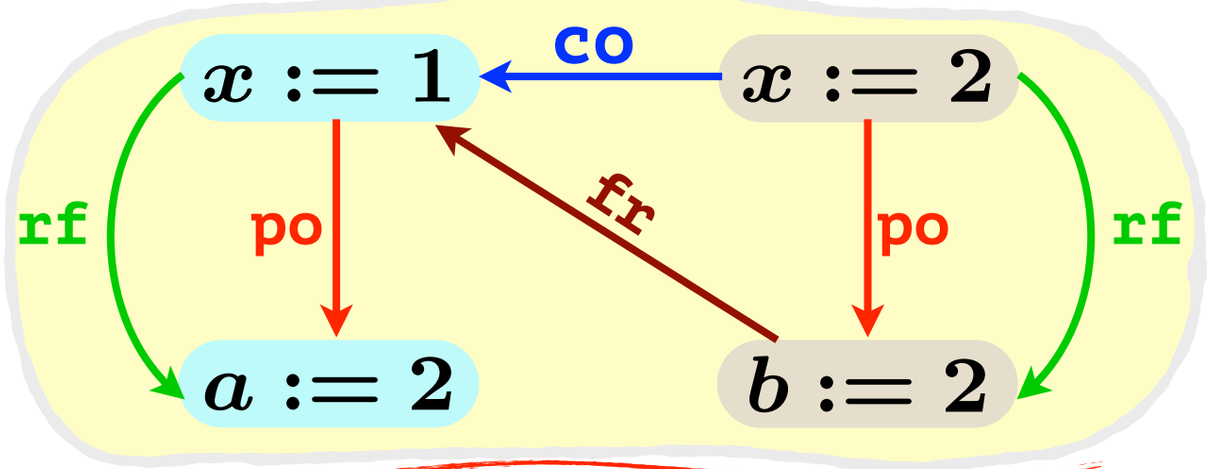
$x := 1$
$x := 2$
$a := 2$
$b := 2$



$x := 2$
$x := 1$
$b := 1$
$a := 1$

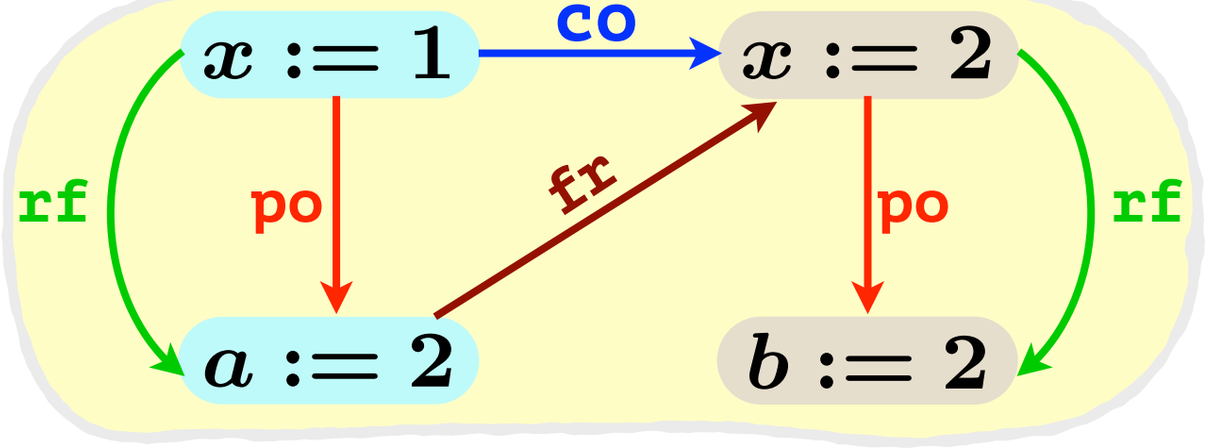


$x := 1$
$a := 1$
$x := 2$
$b := 2$

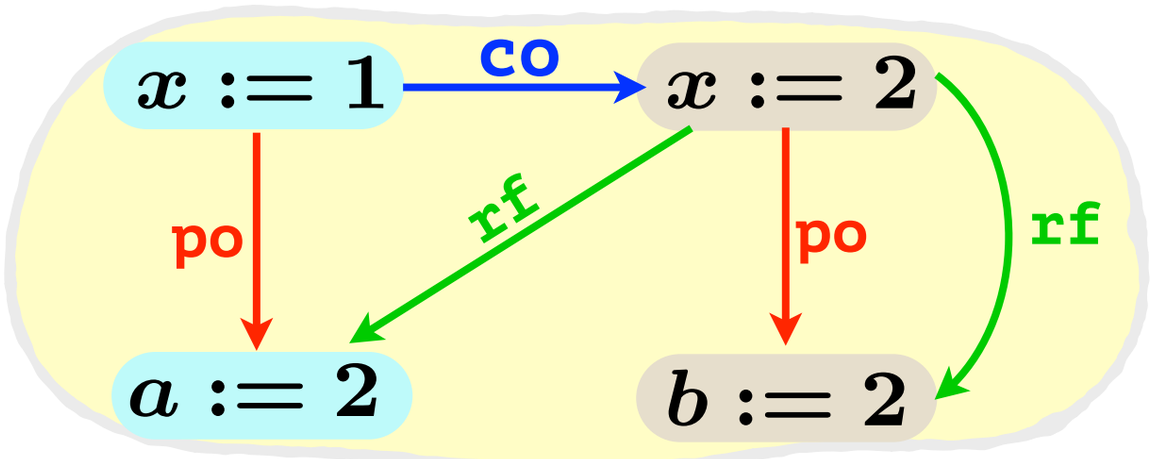


(optimal) DPOR

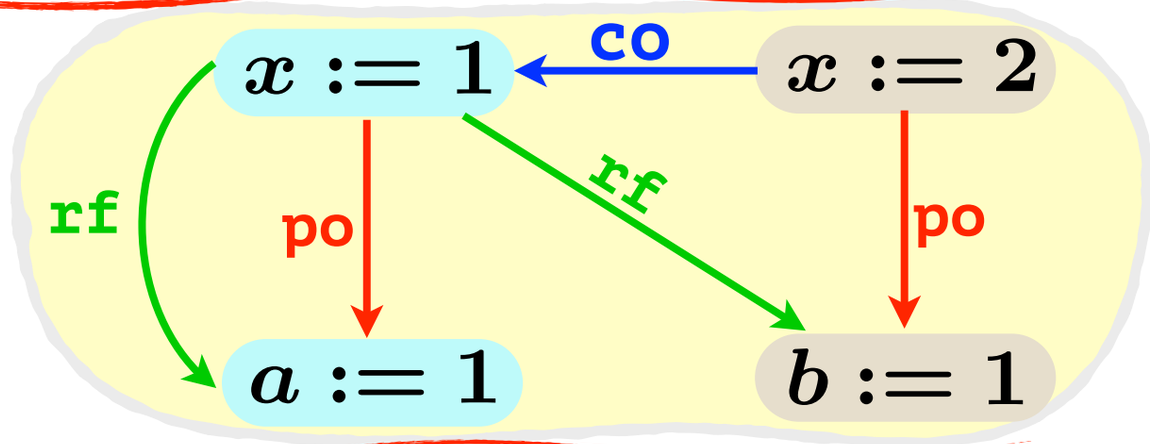
$x := 2$
$b := 2$
$x := 1$
$a := 1$



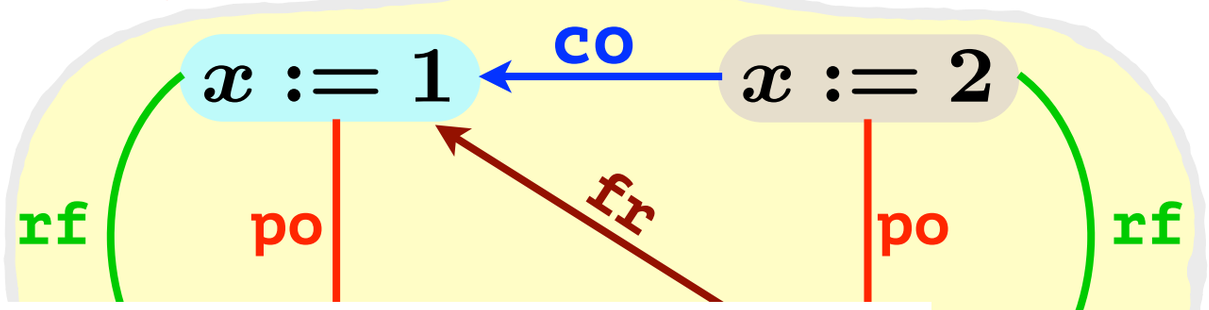
$x := 1$
$x := 2$
$a := 2$
$b := 2$



$x := 2$
$x := 1$
$b := 1$
$a := 1$



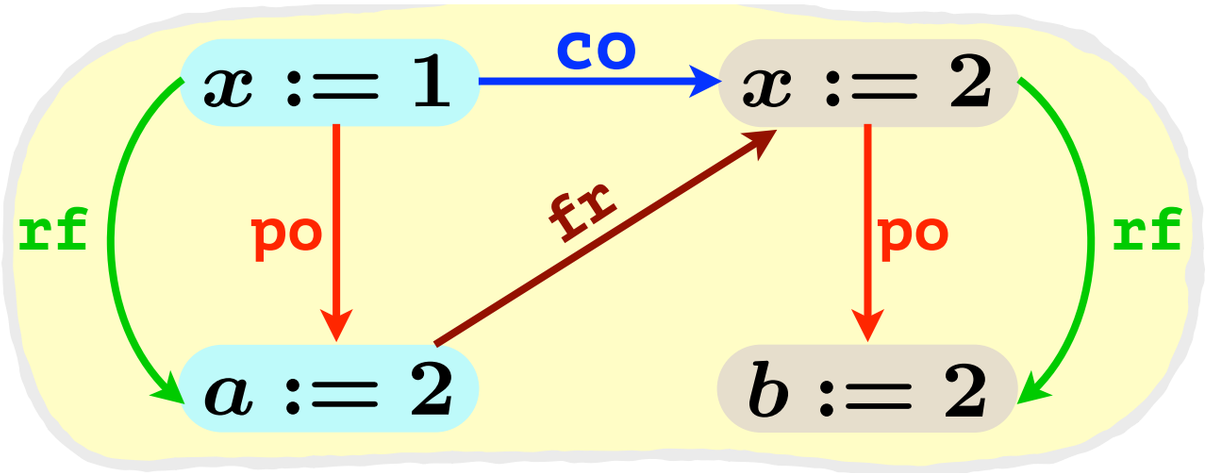
$x := 1$
$a := 1$
$x := 2$



Parosh Aziz Abdulla, Stavros Aronis, Bengt Jonsson, Konstantinos Sagonas:
Source Sets: A Foundation for Optimal Dynamic Partial Order Reduction. J. ACM 64(4): 25:1-25:49 (2017)

(optimal) DPOR

$x := 2$
$b := 2$
$x := 1$
$a := 1$

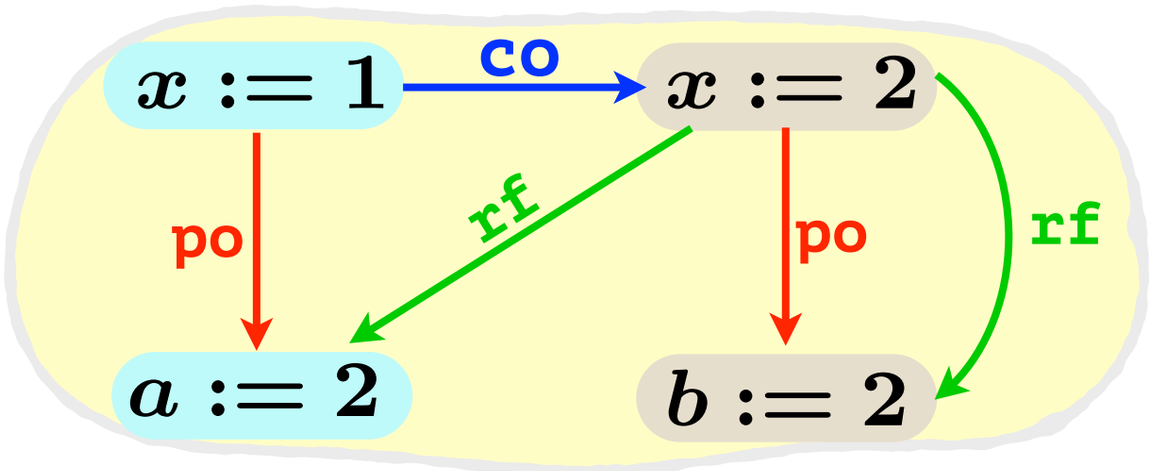


$x := 1$

$x := 2$

$a := 2$

$b := 2$

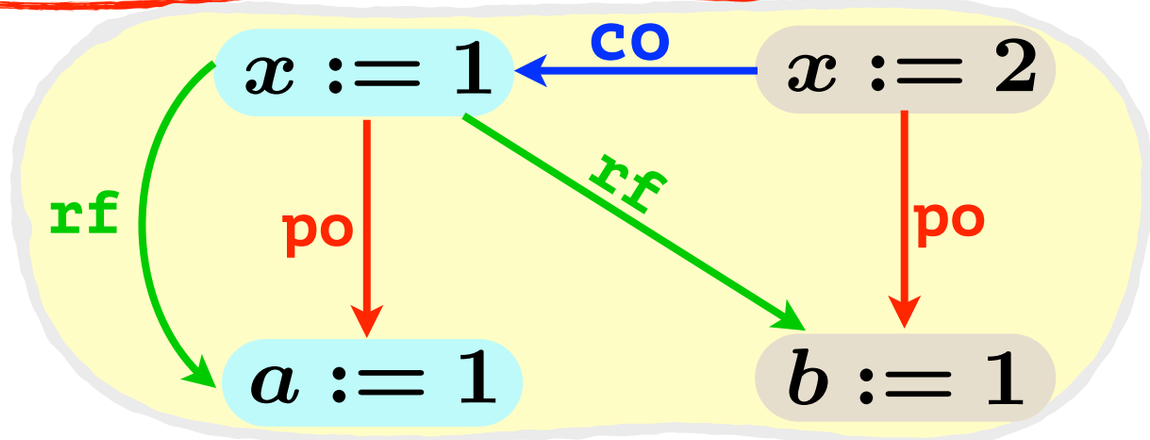


$x := 2$

$x := 1$

$b := 1$

$a := 1$

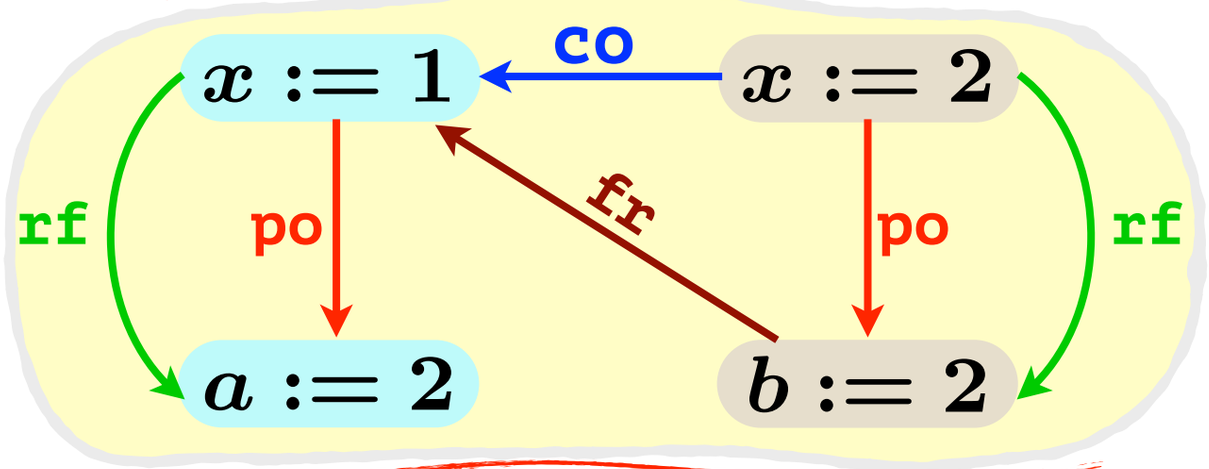


$x := 1$

$a := 1$

$x := 2$

$b := 2$



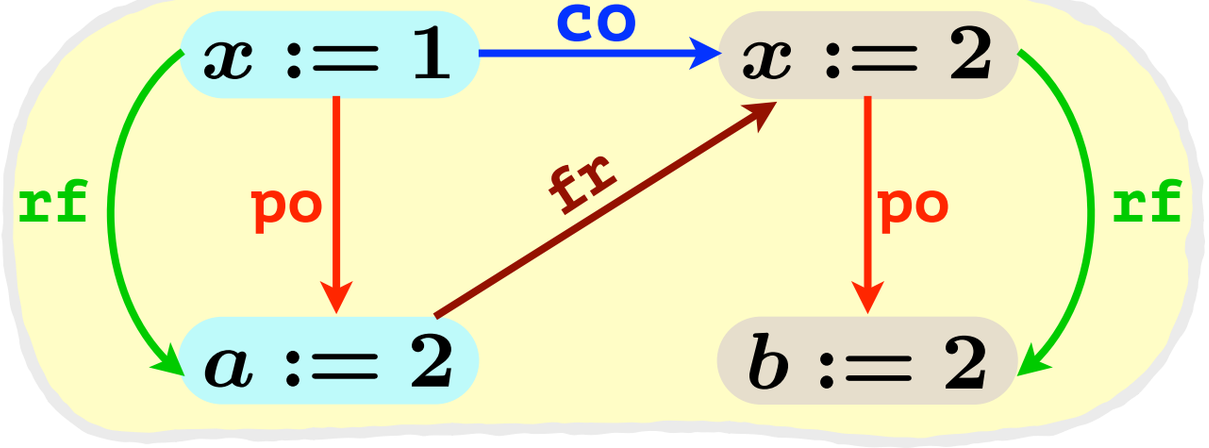
(super-optimal) DPOR

$x := 2$

$b := 2$

$x := 1$

$a := 1$

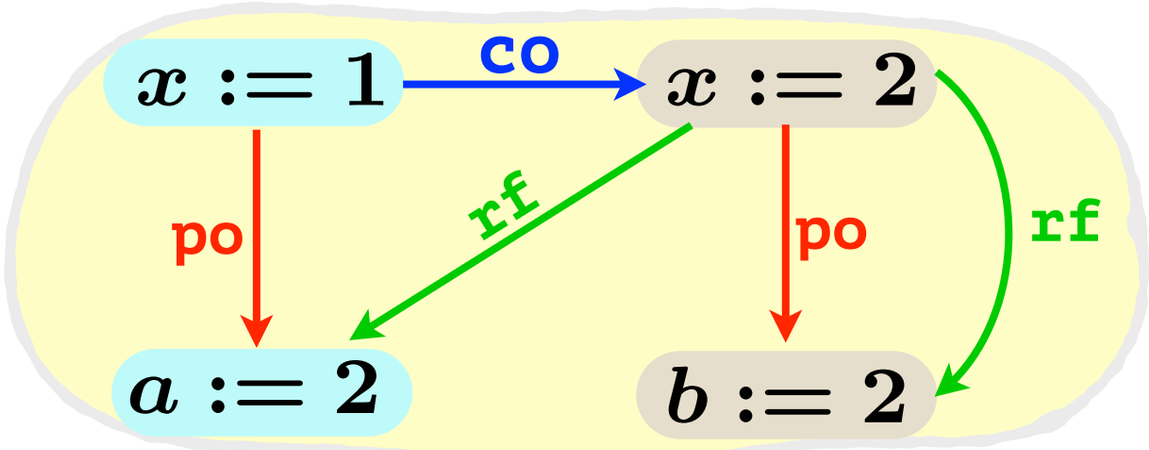


$x := 1$

$x := 2$

$a := 2$

$b := 2$

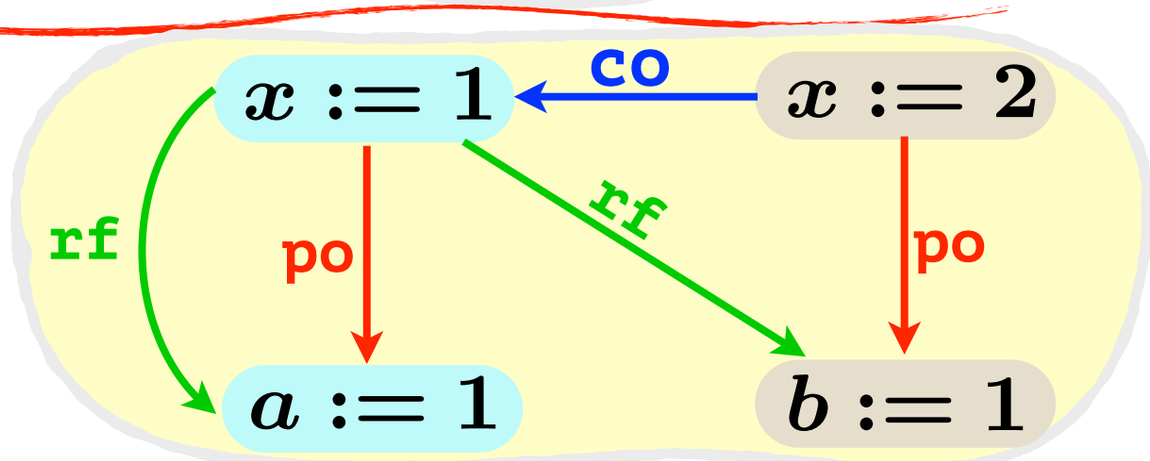


$x := 2$

$x := 1$

$b := 1$

$a := 1$

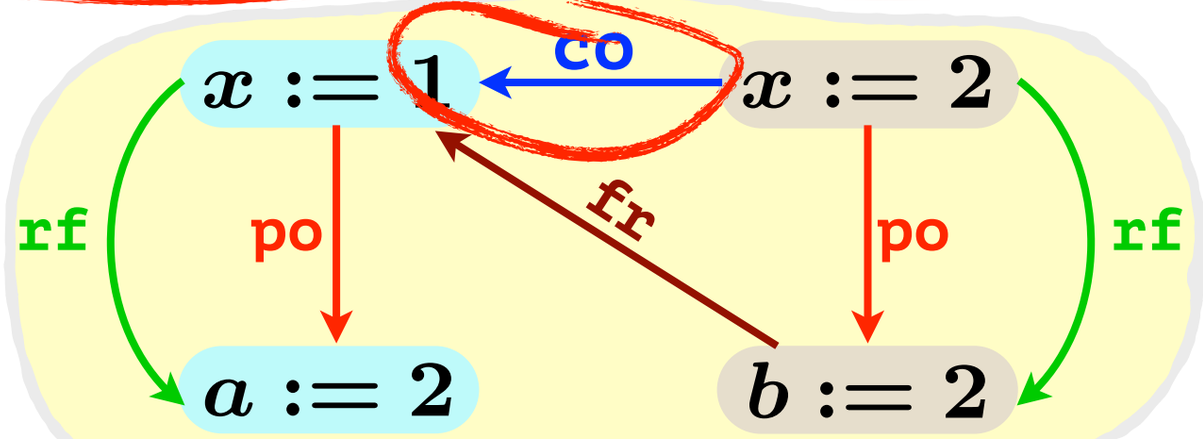


$x := 1$

$a := 1$

$x := 2$

$b := 2$



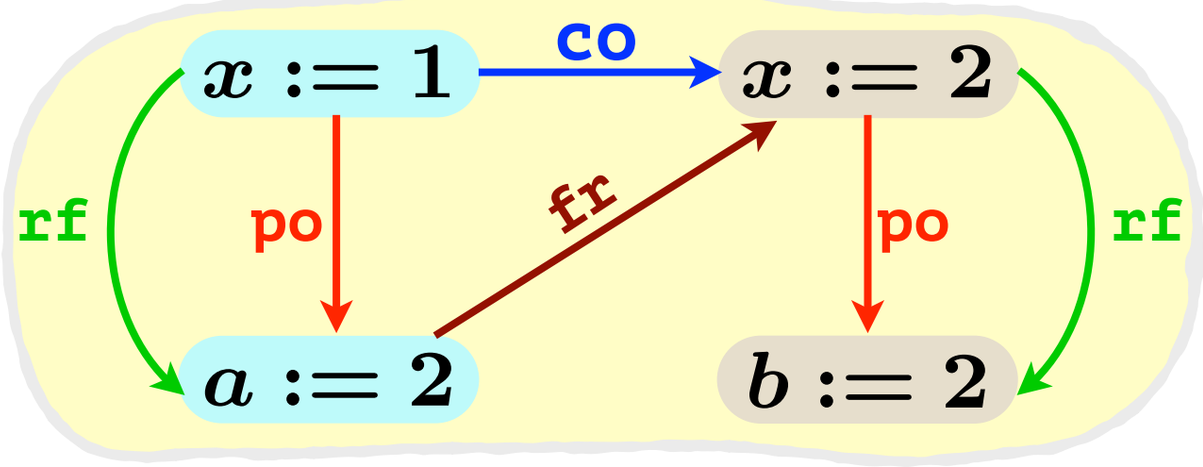
(super-optimal) DPOR

$x := 2$

$b := 2$

$x := 1$

$a := 1$

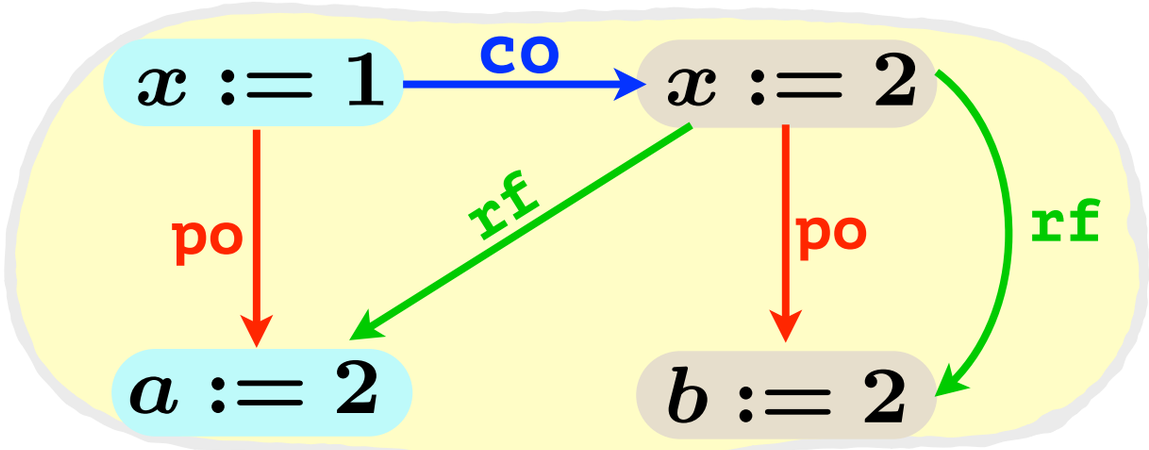


$x := 1$

$x := 2$

$a := 2$

$b := 2$

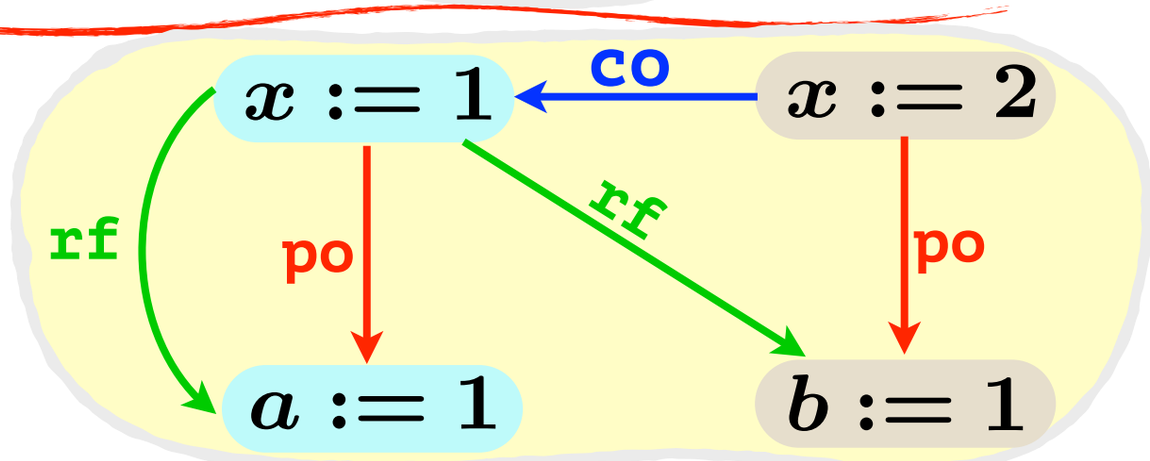


$x := 2$

$x := 1$

$b := 1$

$a := 1$

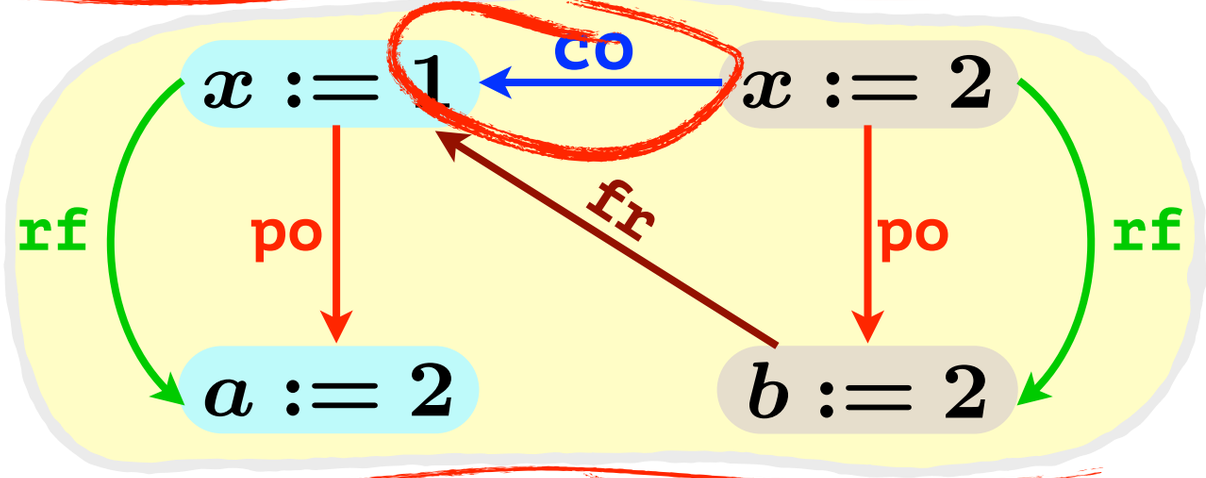


$x := 1$

$a := 1$

$x := 2$

$b := 2$



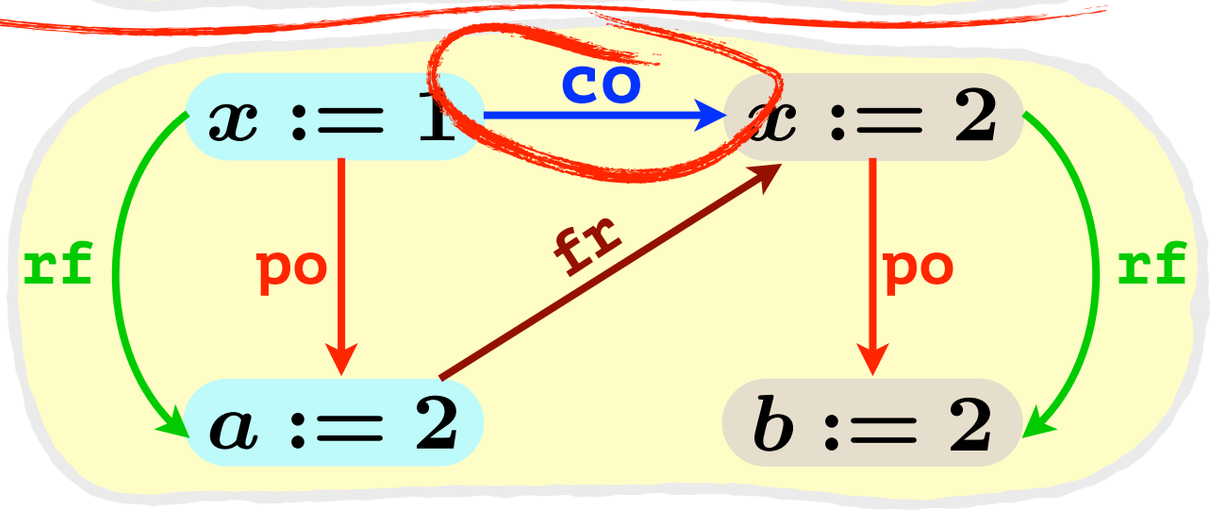
(super-optimal) DPOR

$x := 2$

$b := 2$

$x := 1$

$a := 1$

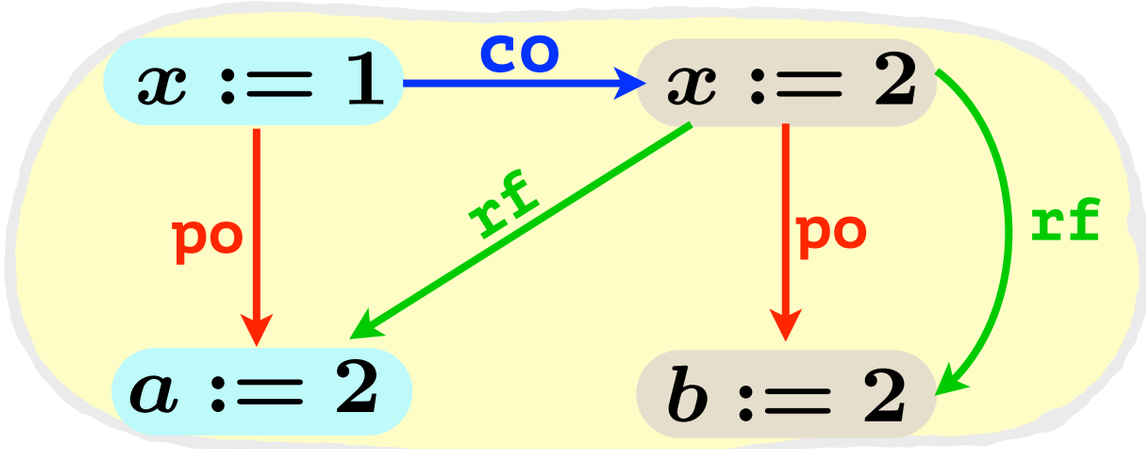


$x := 1$

$x := 2$

$a := 2$

$b := 2$

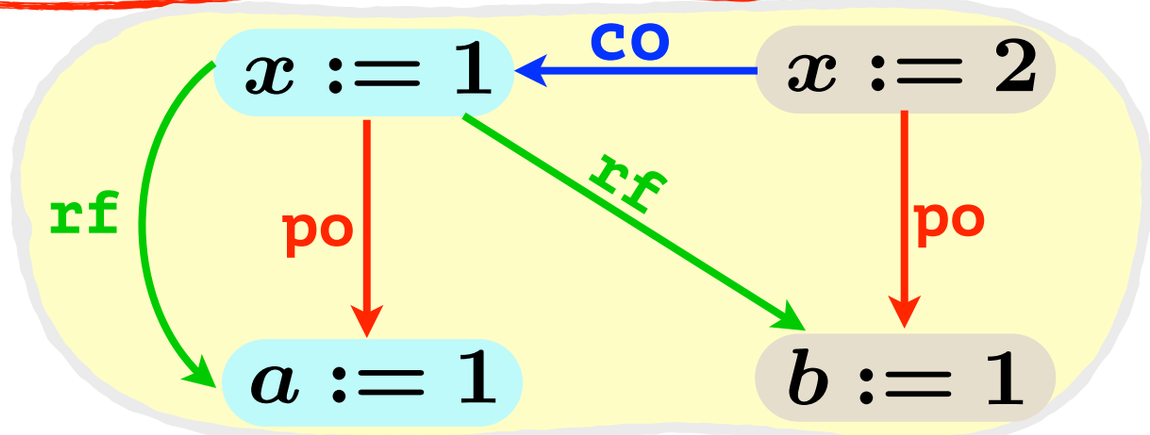


$x := 2$

$x := 1$

$b := 1$

$a := 1$

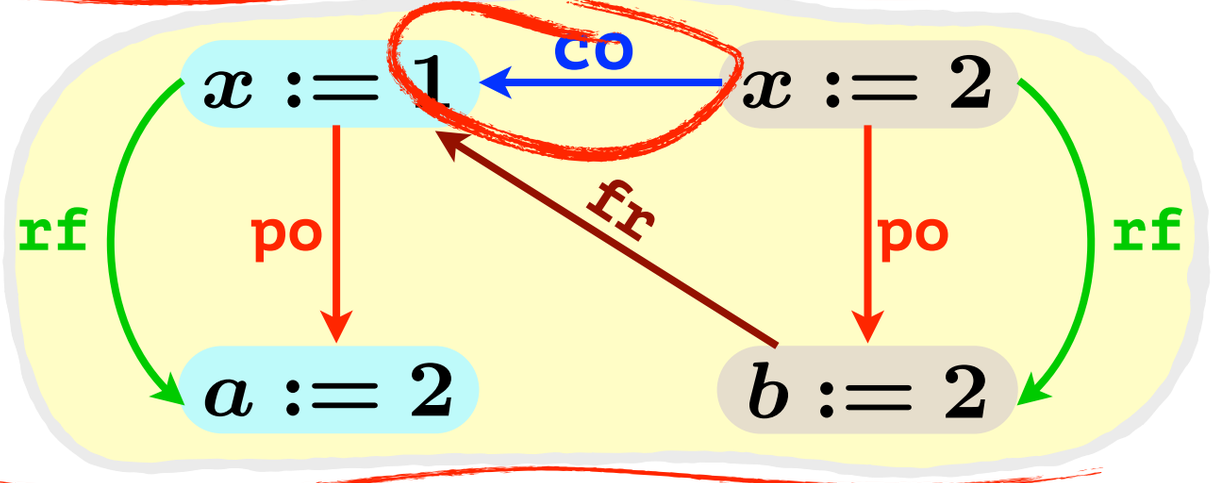


$x := 1$

$a := 1$

$x := 2$

$b := 2$



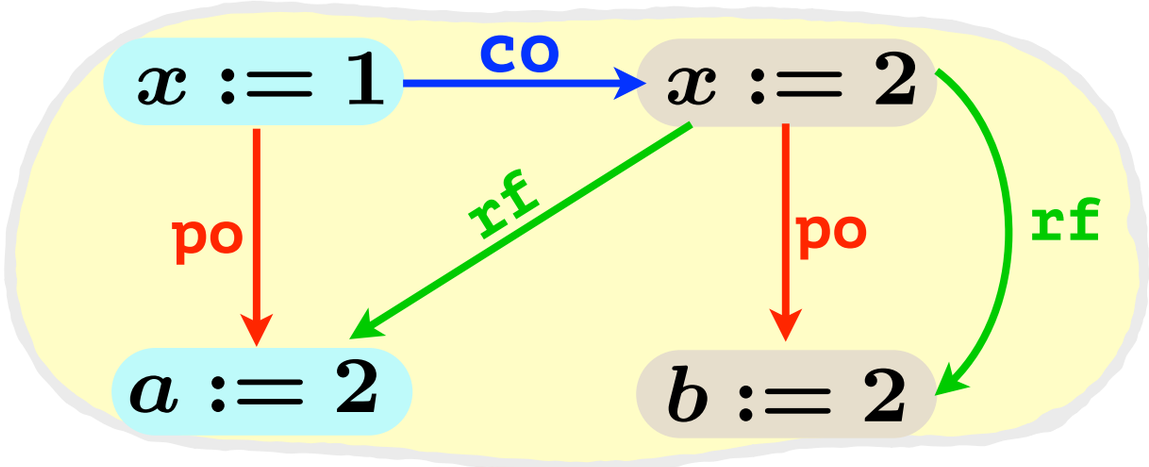
(super-optimal) DPOR

$x := 1$

$x := 2$

$a := 2$

$b := 2$

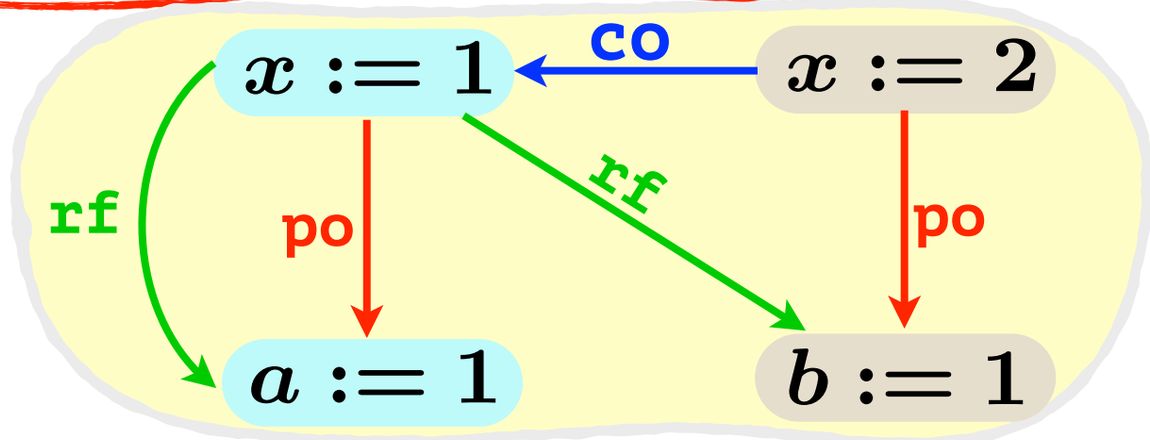


$x := 2$

$x := 1$

$b := 1$

$a := 1$

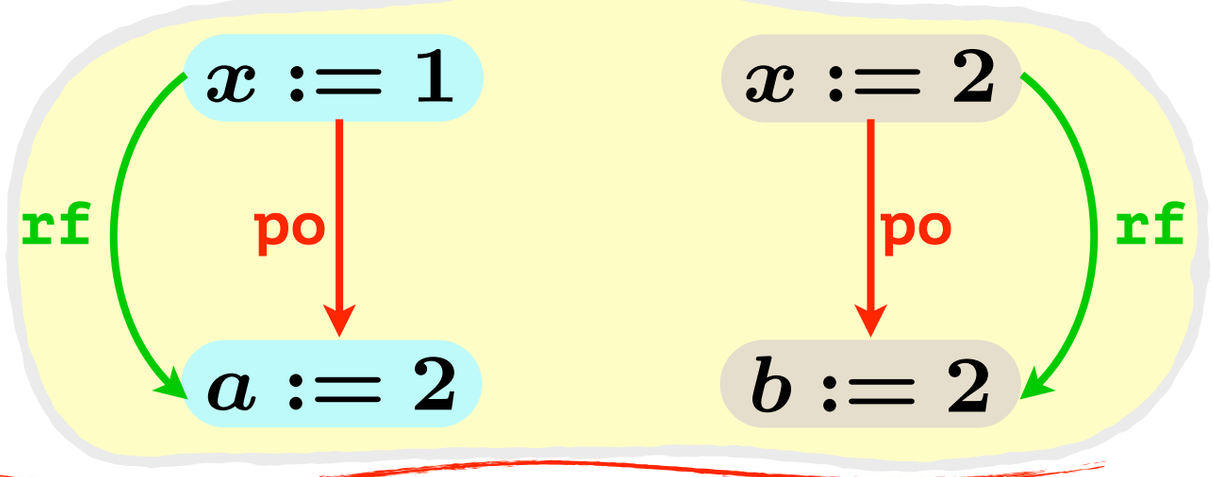


$x := 1$

$a := 1$

$x := 2$

$b := 2$



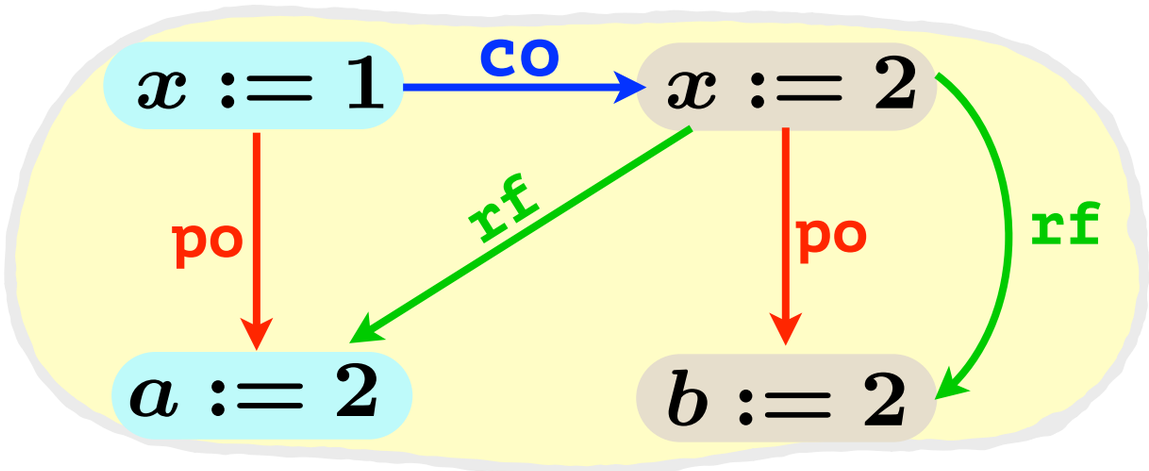
(super-optimal) DPOR

$x := 1$

$x := 2$

$a := 2$

$b := 2$

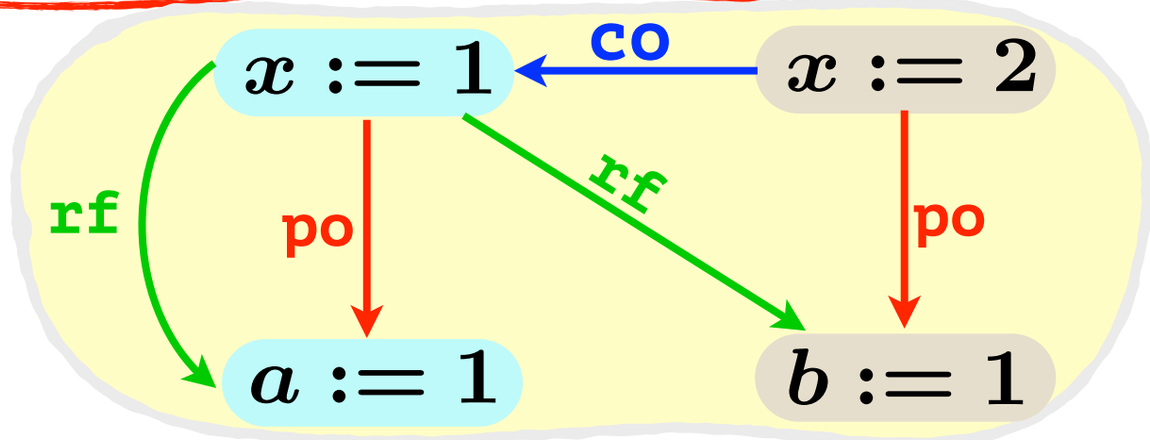


$x := 2$

$x := 1$

$b := 1$

$a := 1$



$x := 1$

$a := 1$

$x := 2$

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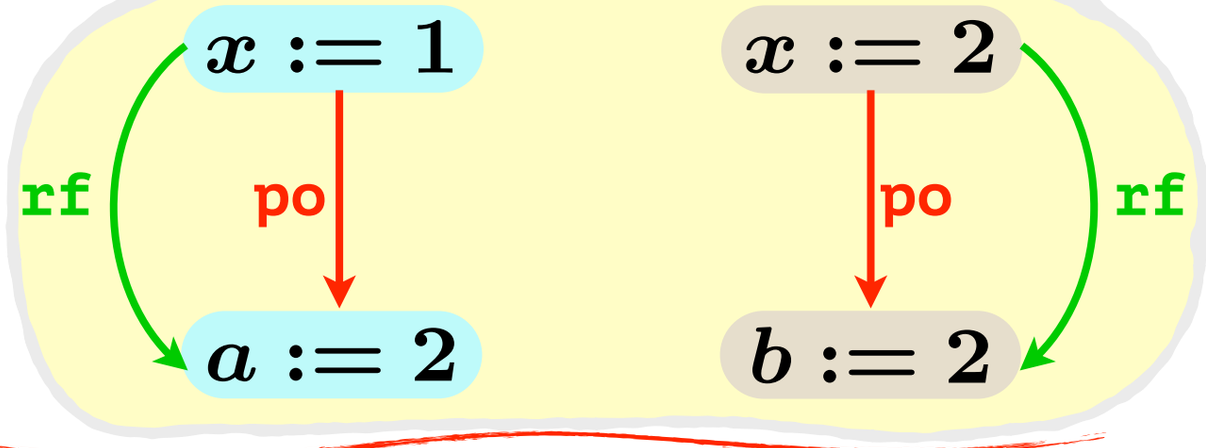
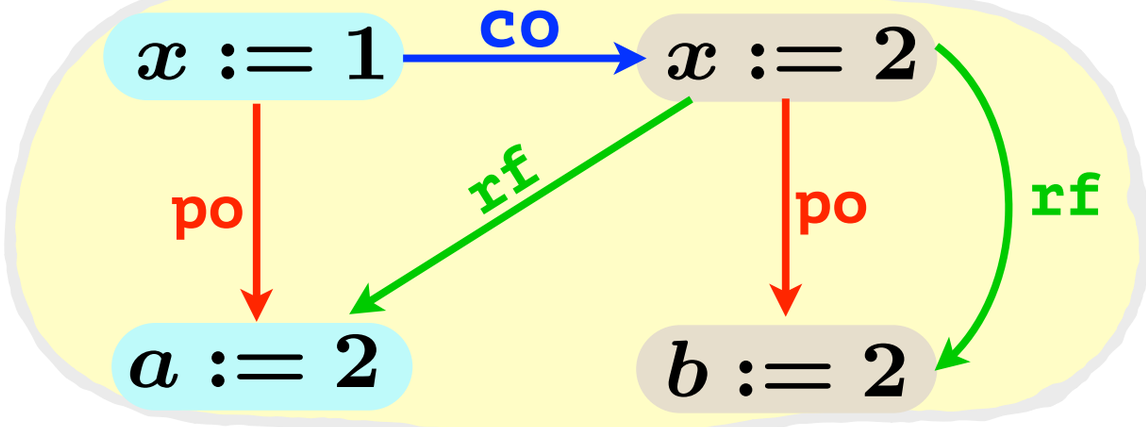


$x := 1$

$x := 2$

$a := 2$

$b := 2$

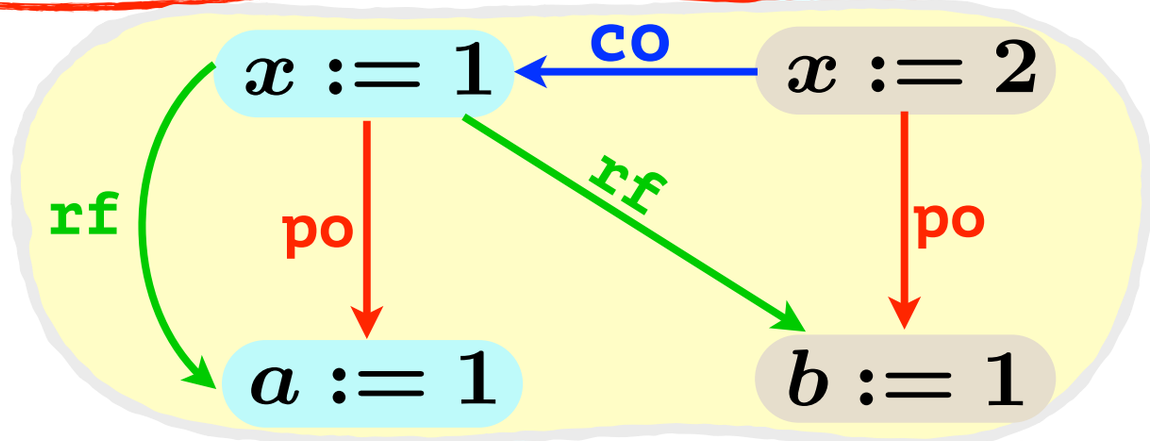


$x := 2$

$x := 1$

$b := 1$

$a := 1$

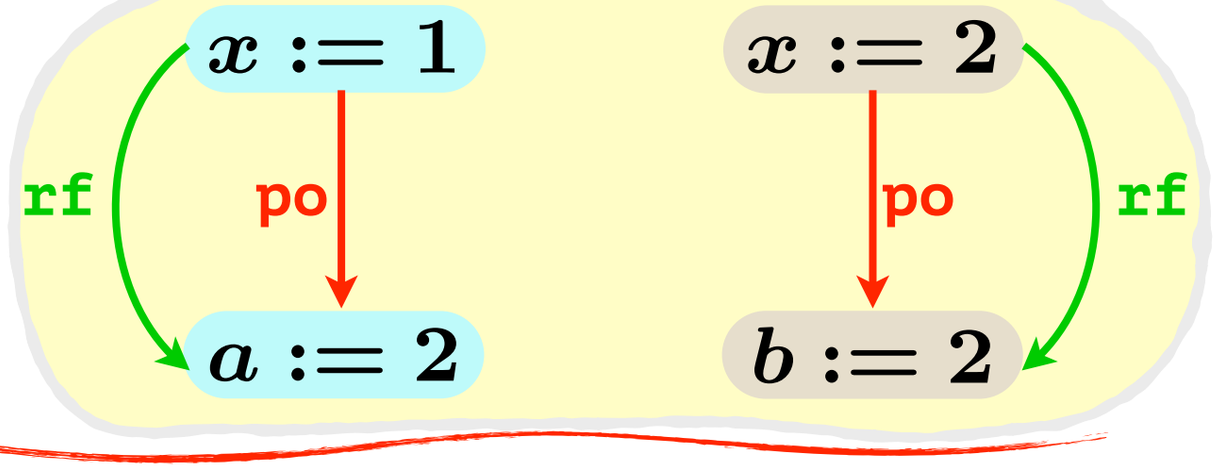


$x := 1$

$a := 1$

$x := 2$

$b := 2$



Prog → **(super-optimal) DPOR** → **only traces with different po and rf**

- **SC**
  - **Shared memory**
  - **Processes: atomic read/write**
  - **interleaving of the operations**
  - **Simple and intuitive**
  - **Too strong**

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  - **Efficient**
  - **All levels of systems design**
    - **microprocessors: TSO, POWER, ARM**
    - **cache protocols**
    - **programming languages: C11, Java**
    - **geo-replicated systems: Amazon, Facebook**
  - **Unexpected behaviours**

- **SC**
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    - **programming languages: C11, Java**
    - **geo-replicated systems: Amazon, Facebook**
  - **Unexpected behaviours**

- **SC**
  - **Shared memory**
  - **Processes: atomic read/write**
  - **interleaving of the operations**
  - **Simple and intuitive**
  - **Too strong**

- **Weak memory models**
  - **Efficient**
  - **All levels of systems desig**
    - **microprocessors: TSO, POWER, ARM**
    - **cache protocols**
    - **programming languages: C11, Java**
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  - **Unexpected behaviours**

Release-Acquire Semantics

Release-Acquire (RA)

$$a := x \;\|\; b := x \;\|\; x := 1 \;\|\; y := 2 \;\|\; y := 1 \;\|\; x := 2 \;\|\; c := y \;\|\; d := y$$

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RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

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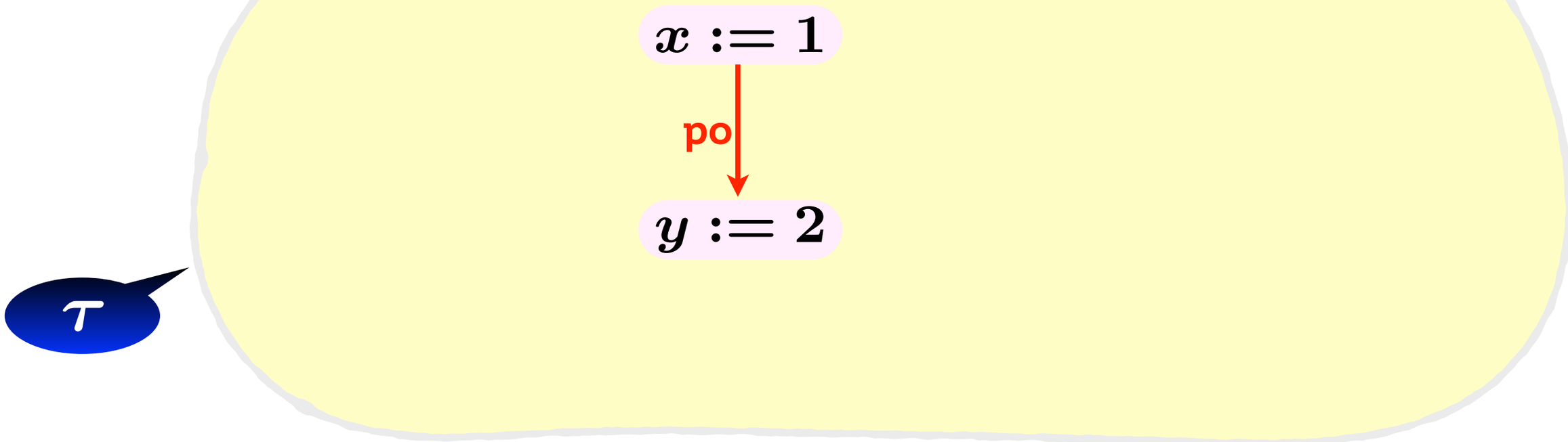

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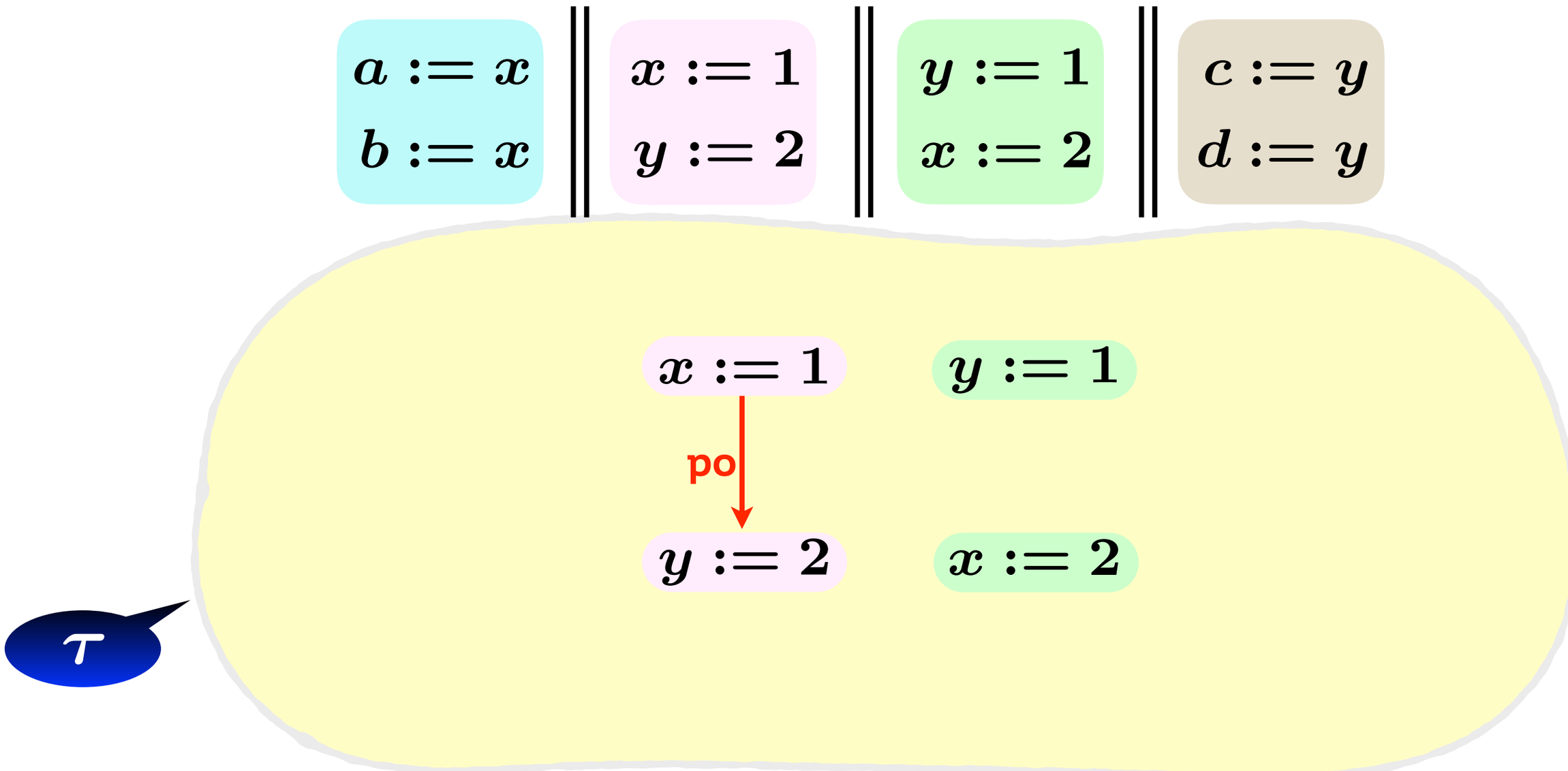

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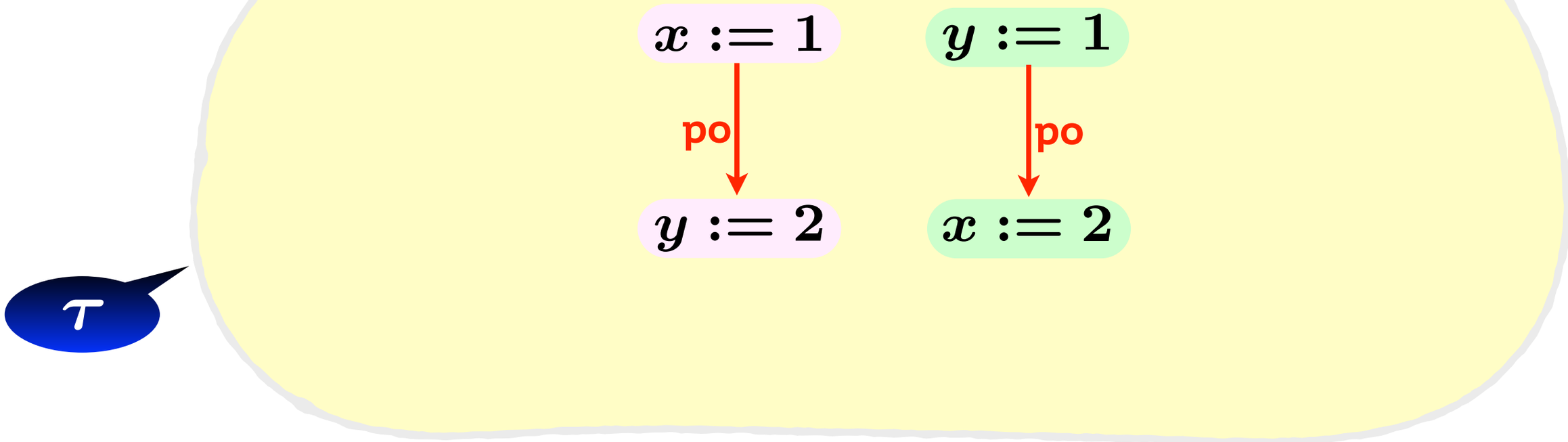

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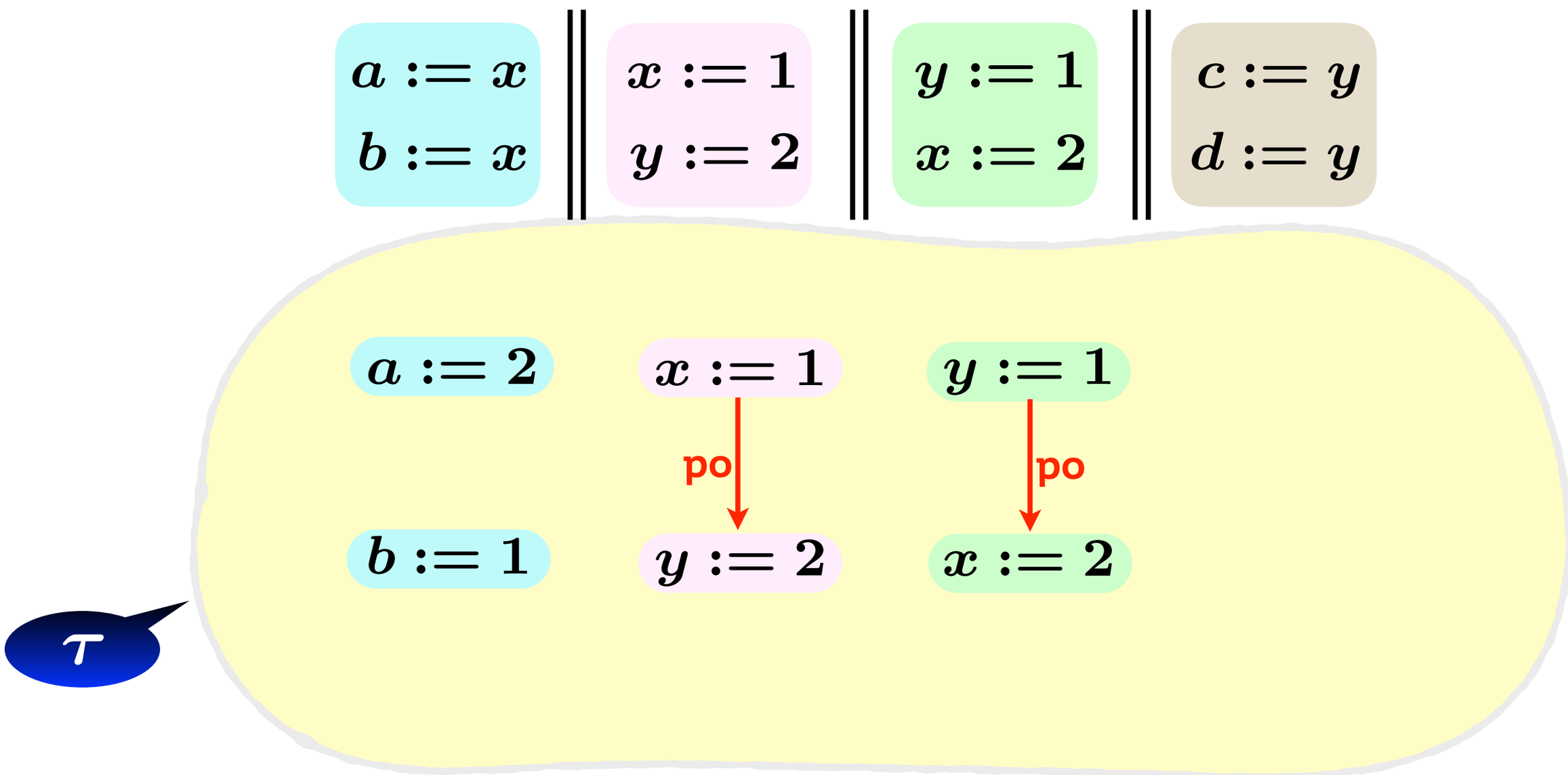

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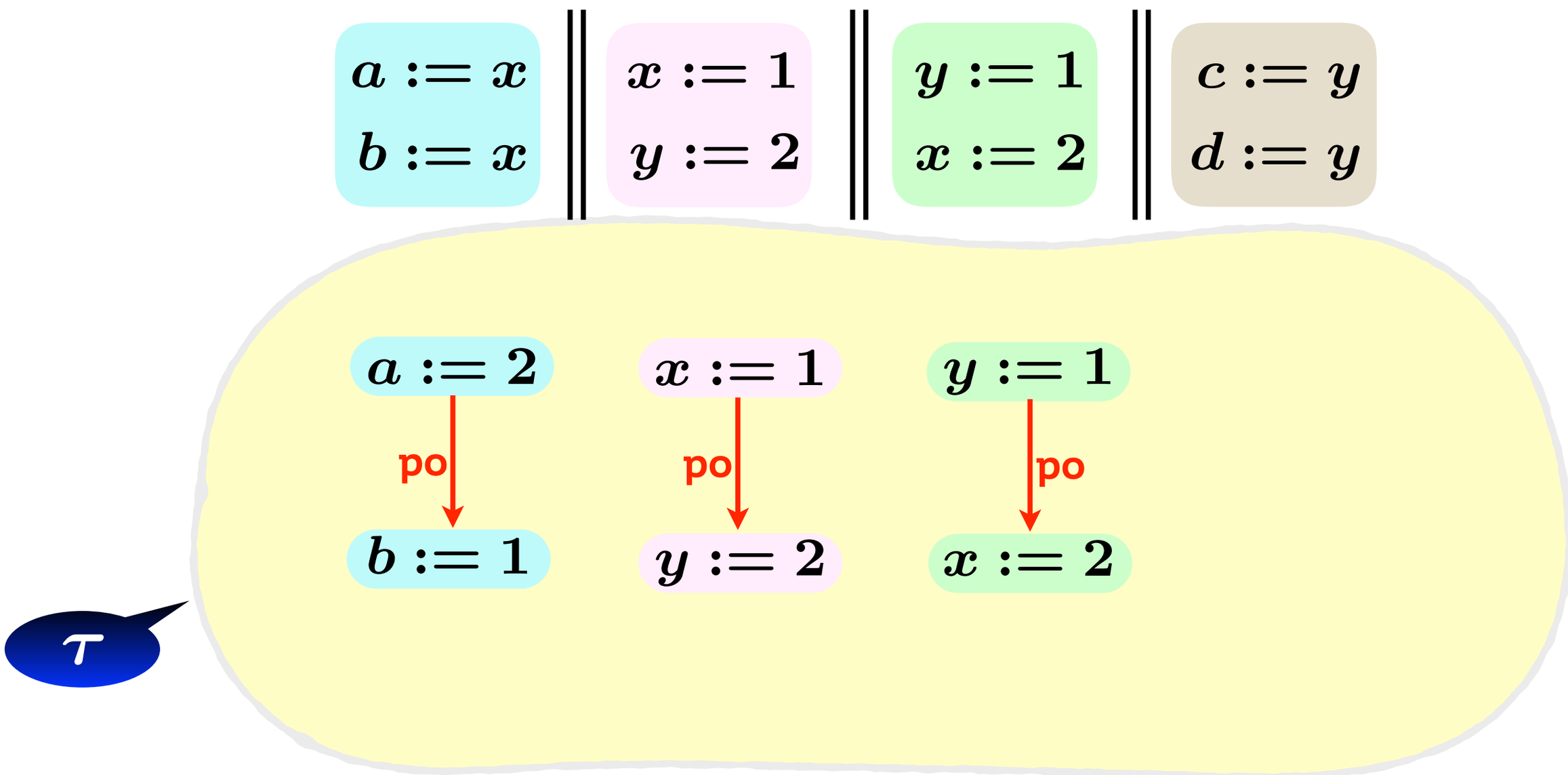

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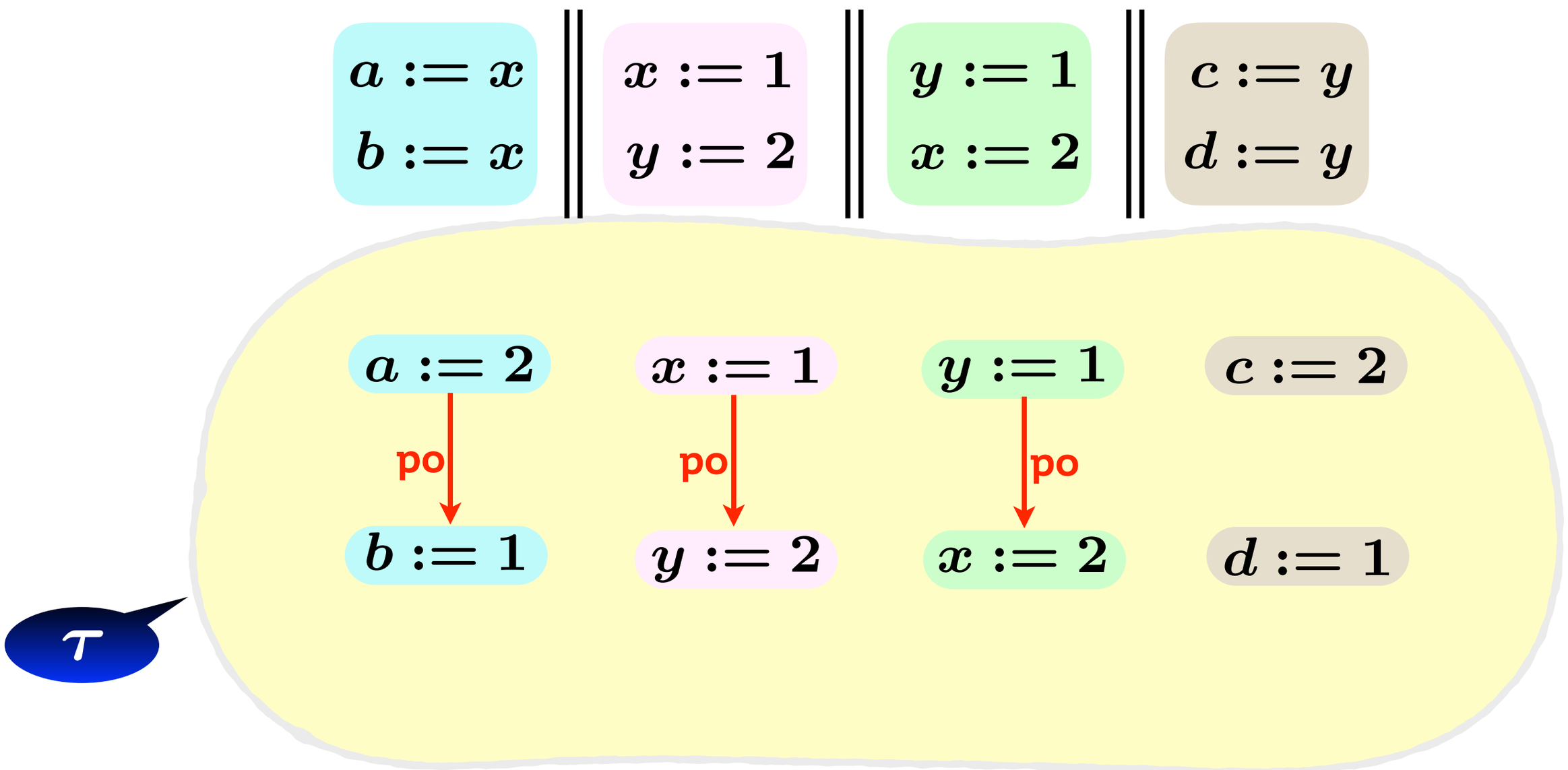

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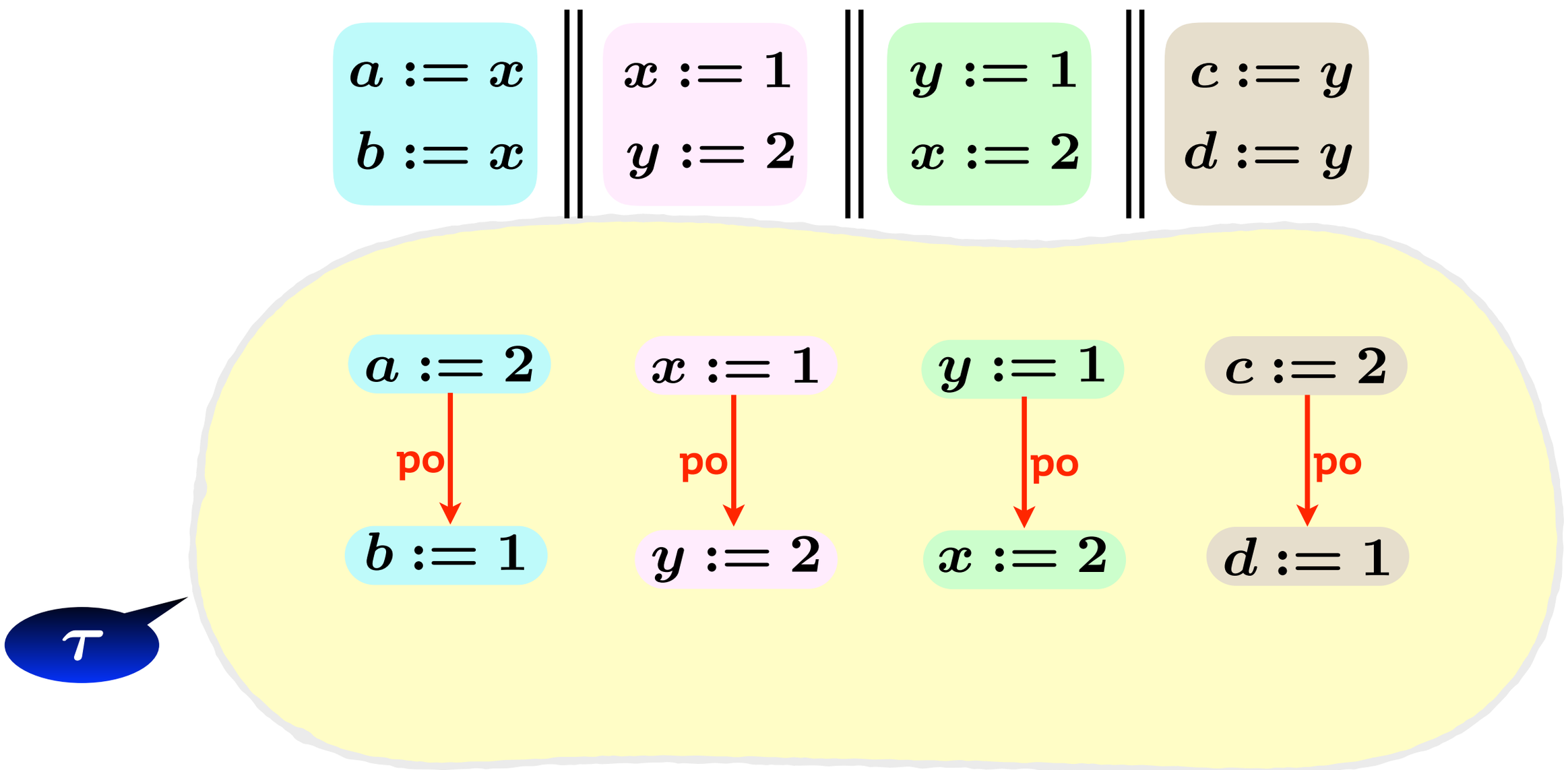

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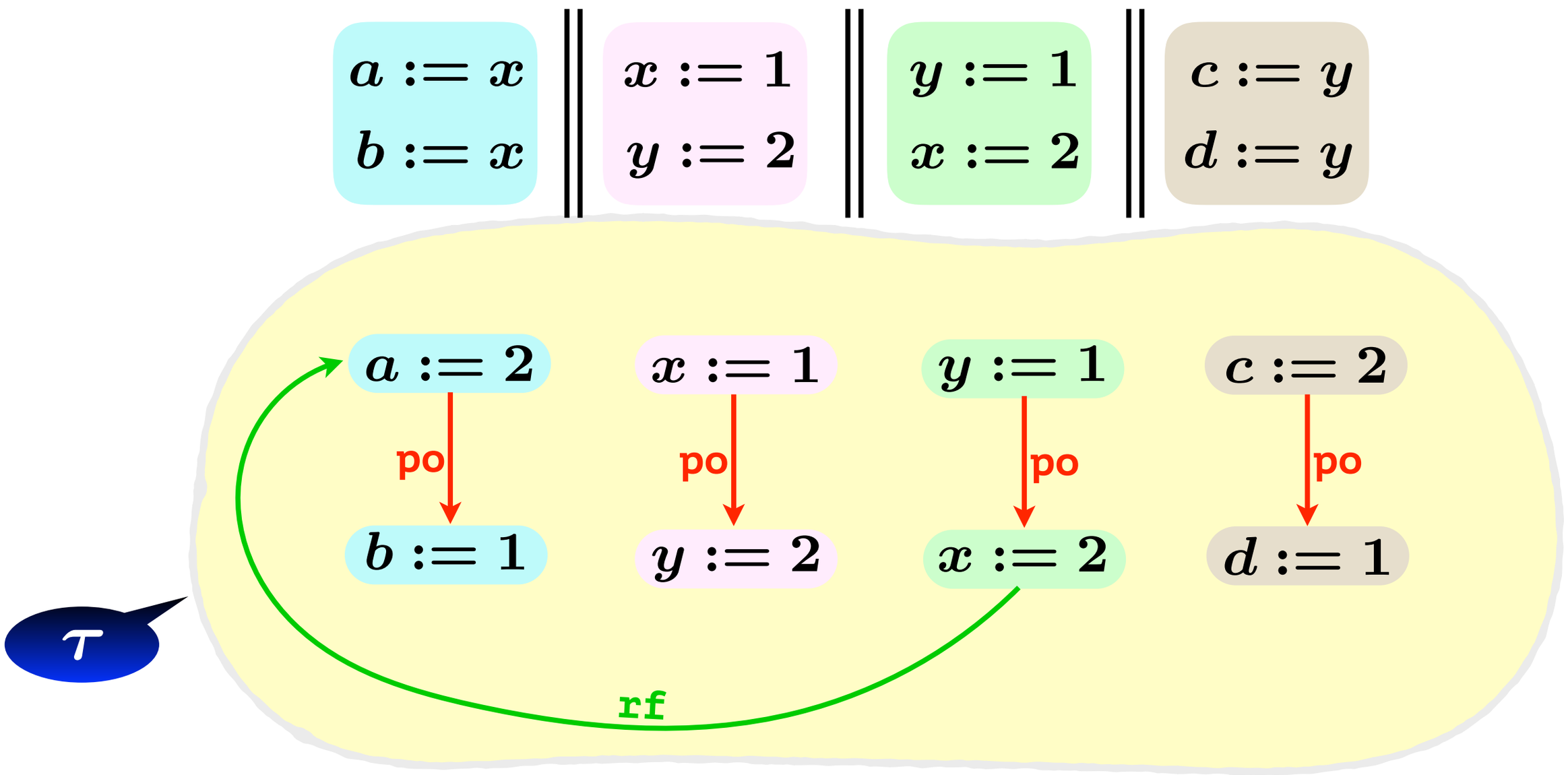

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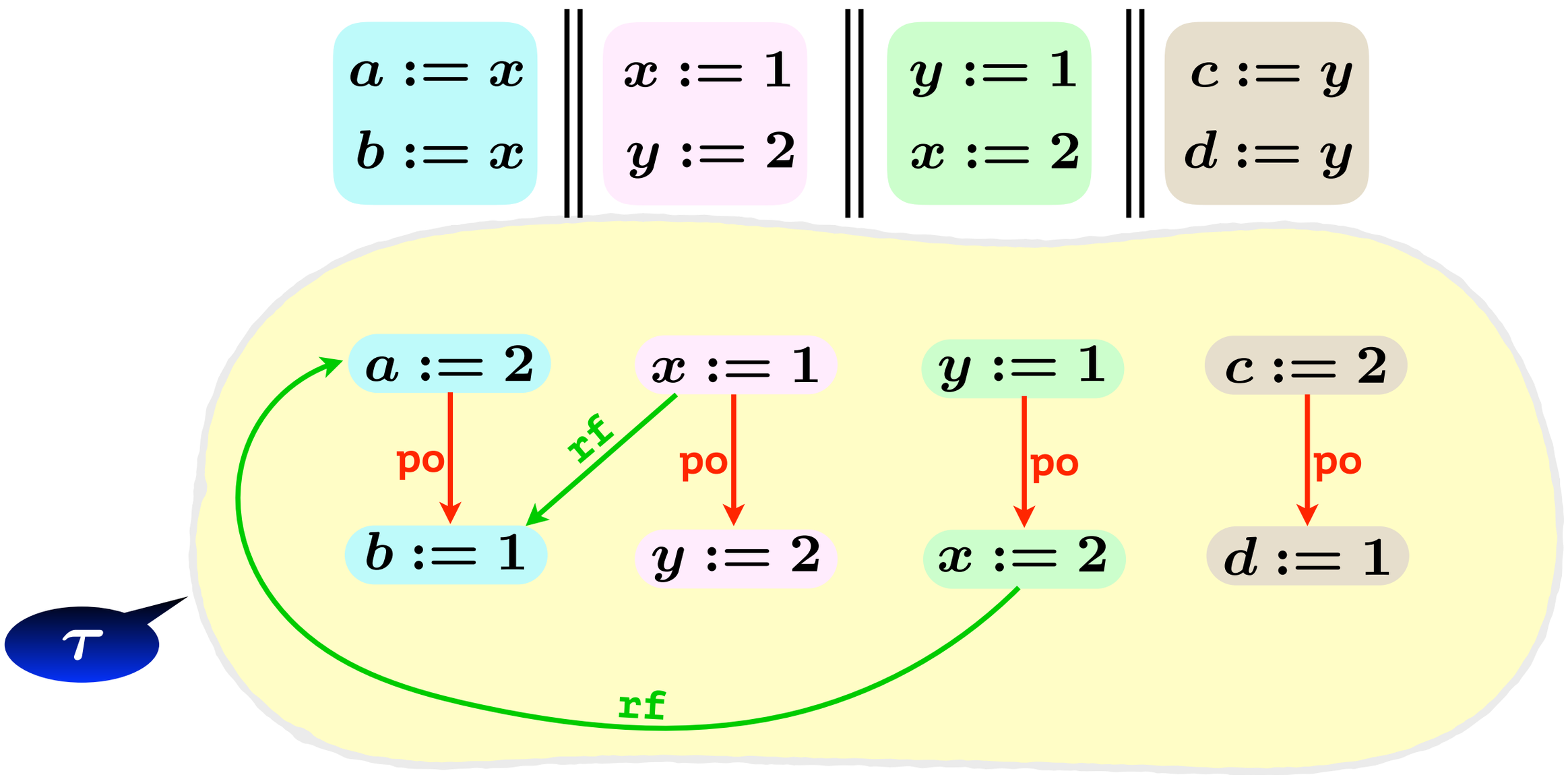

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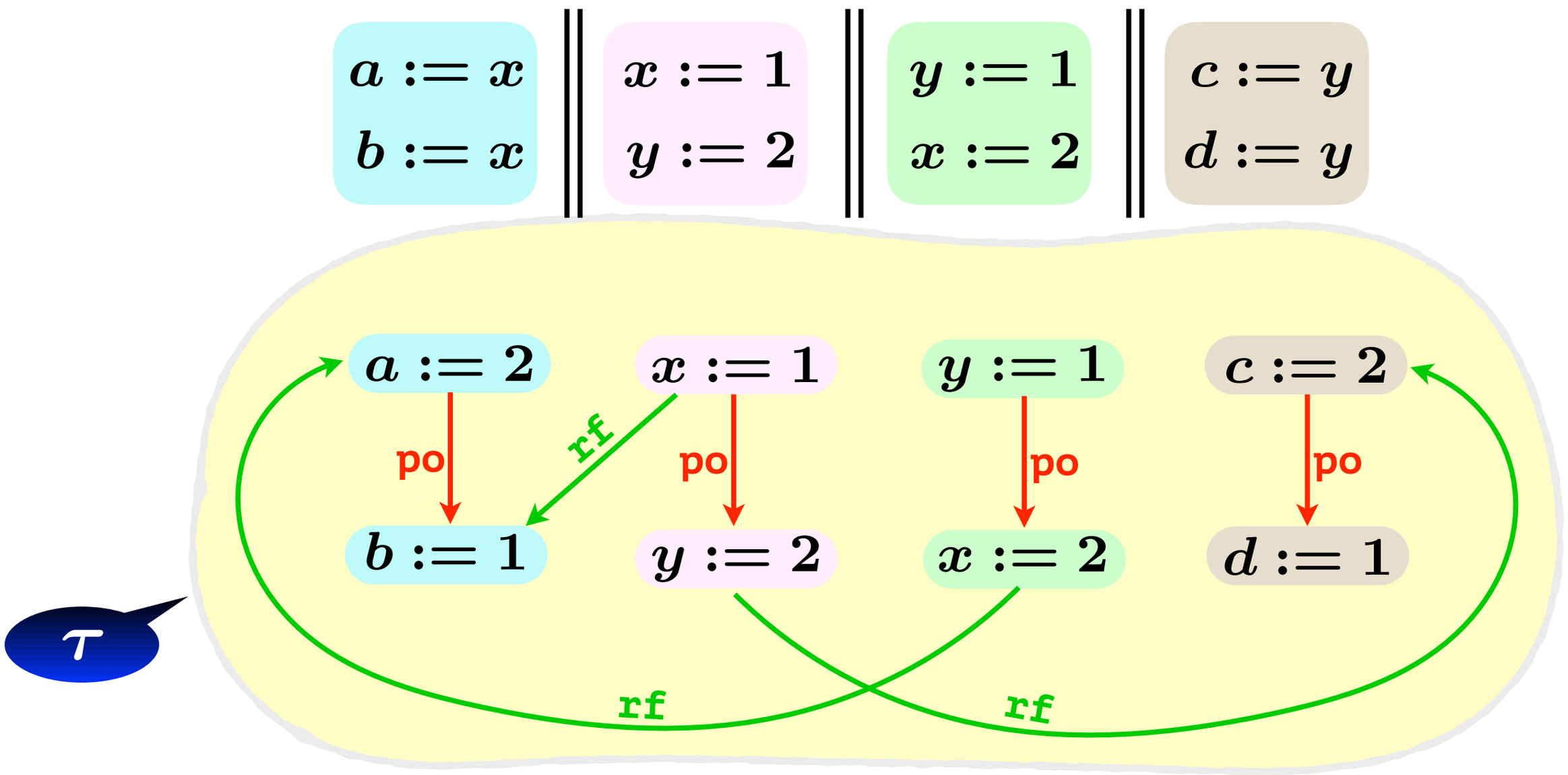

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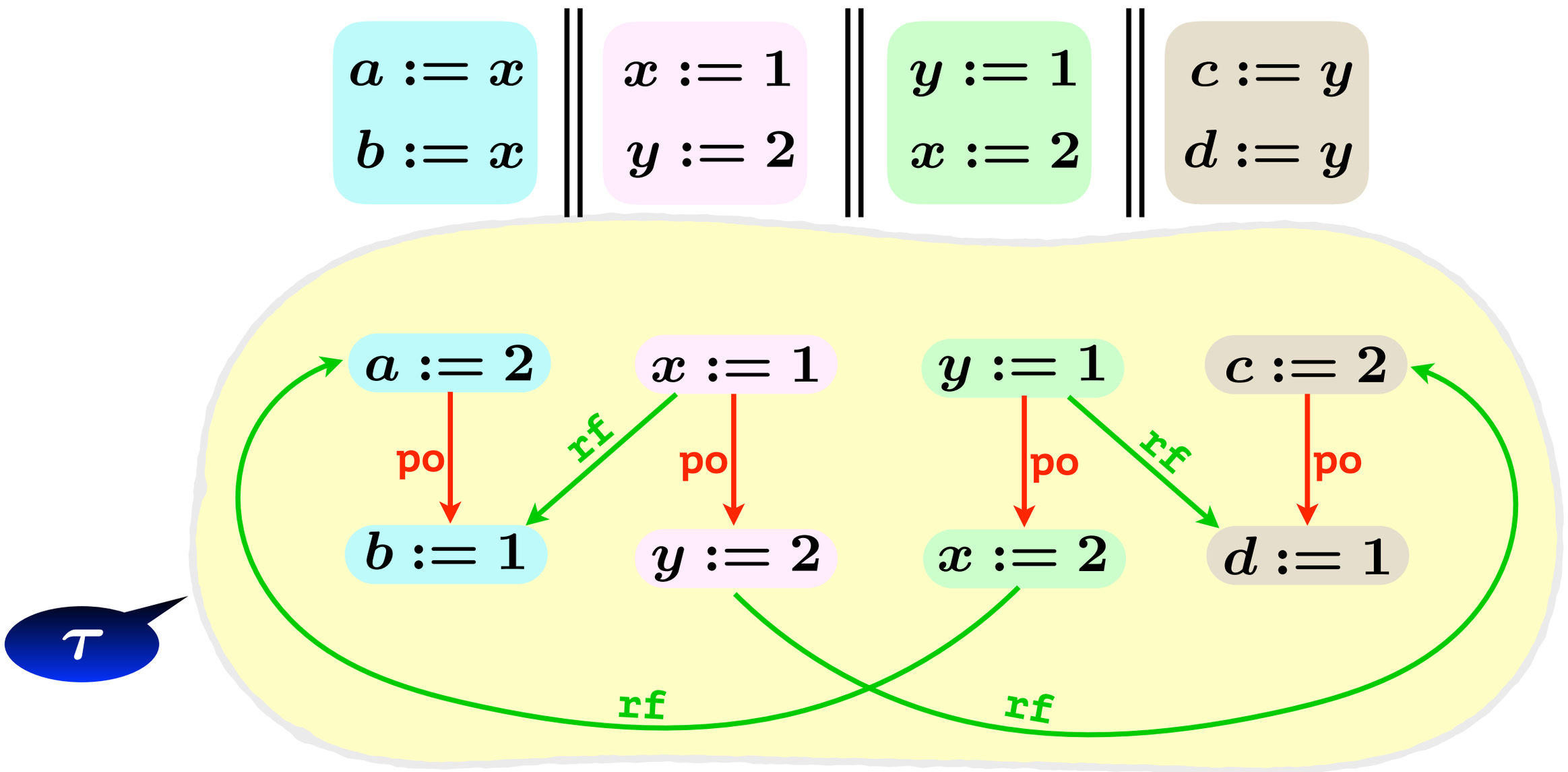

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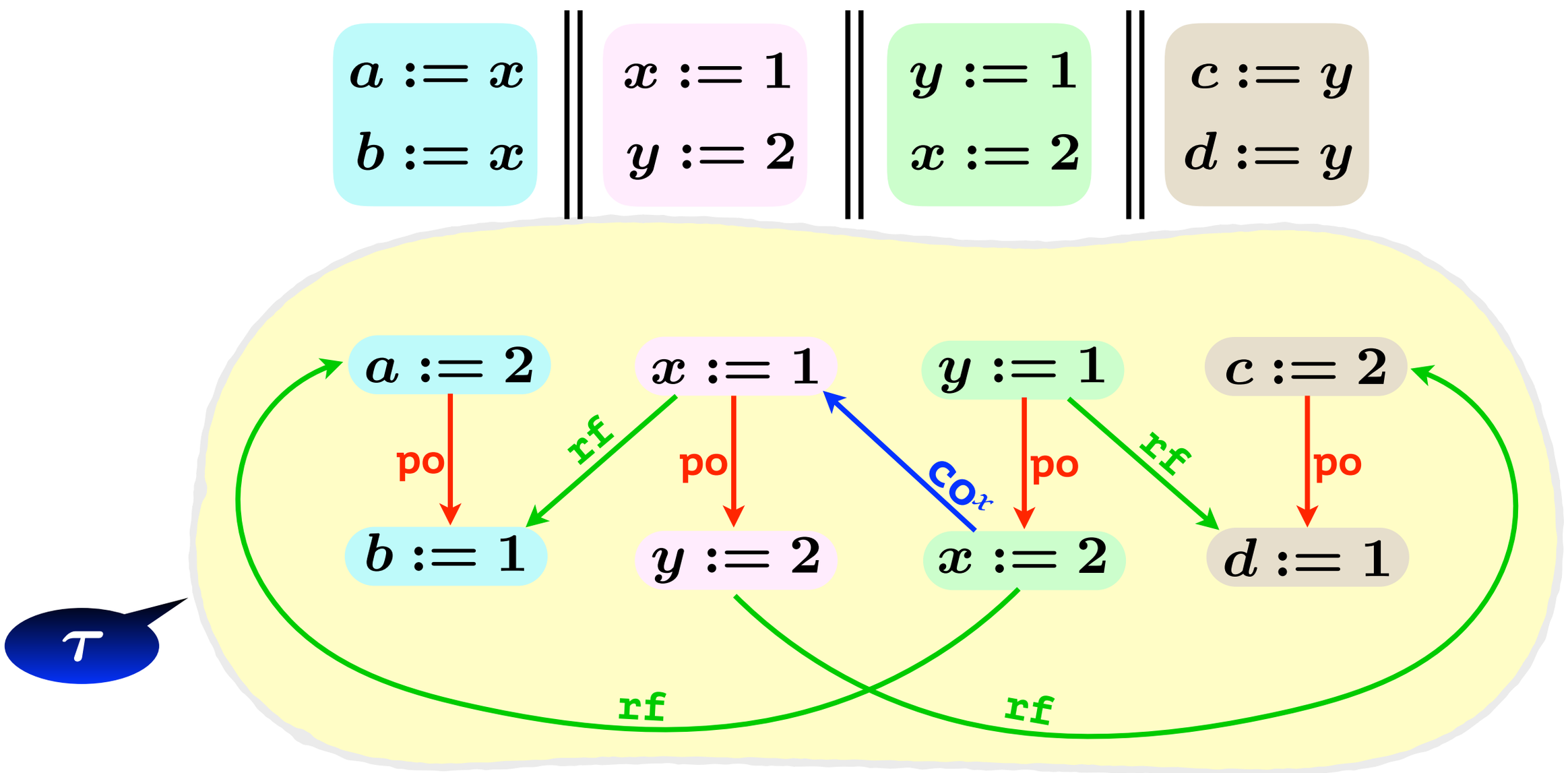

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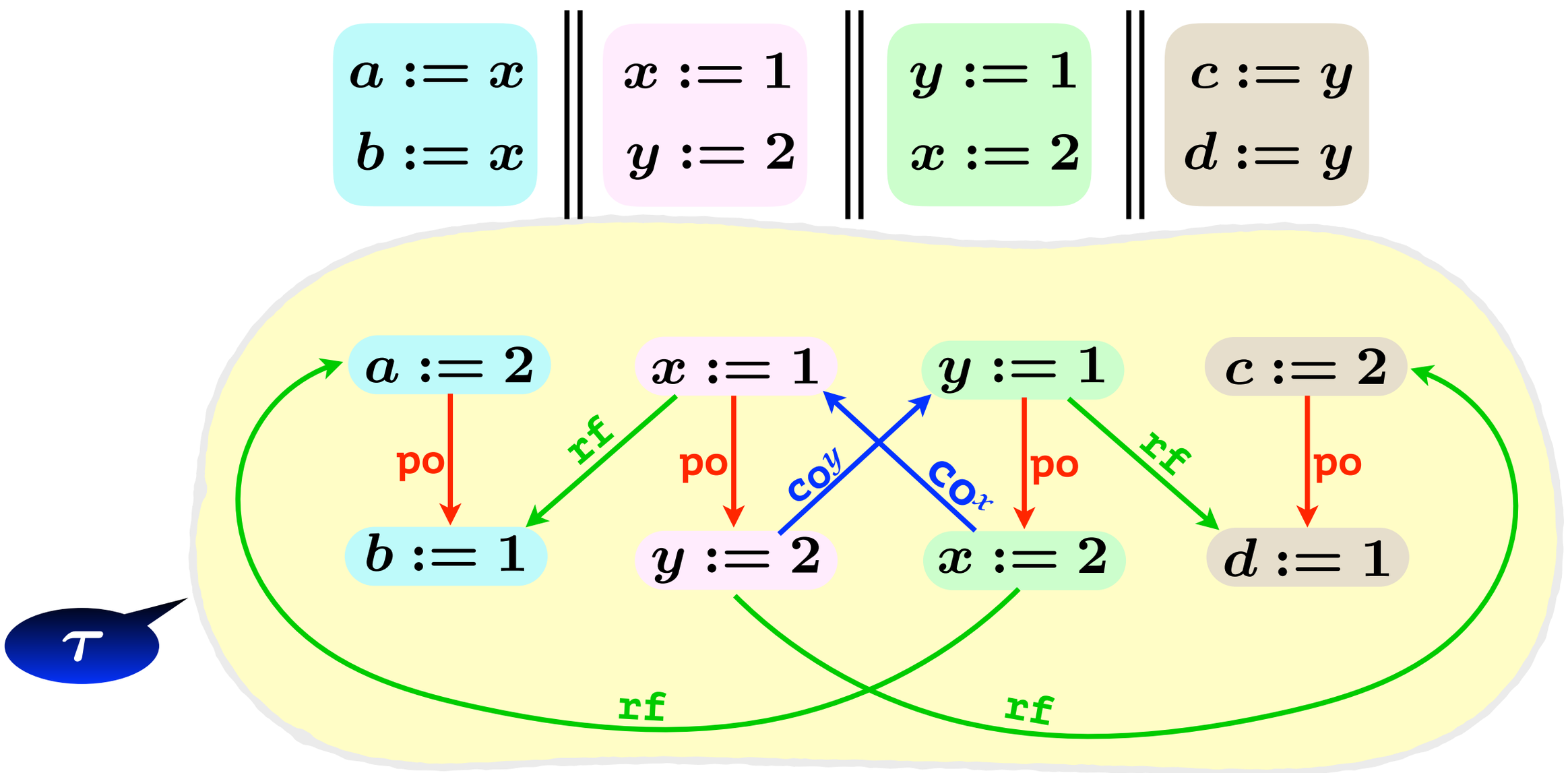

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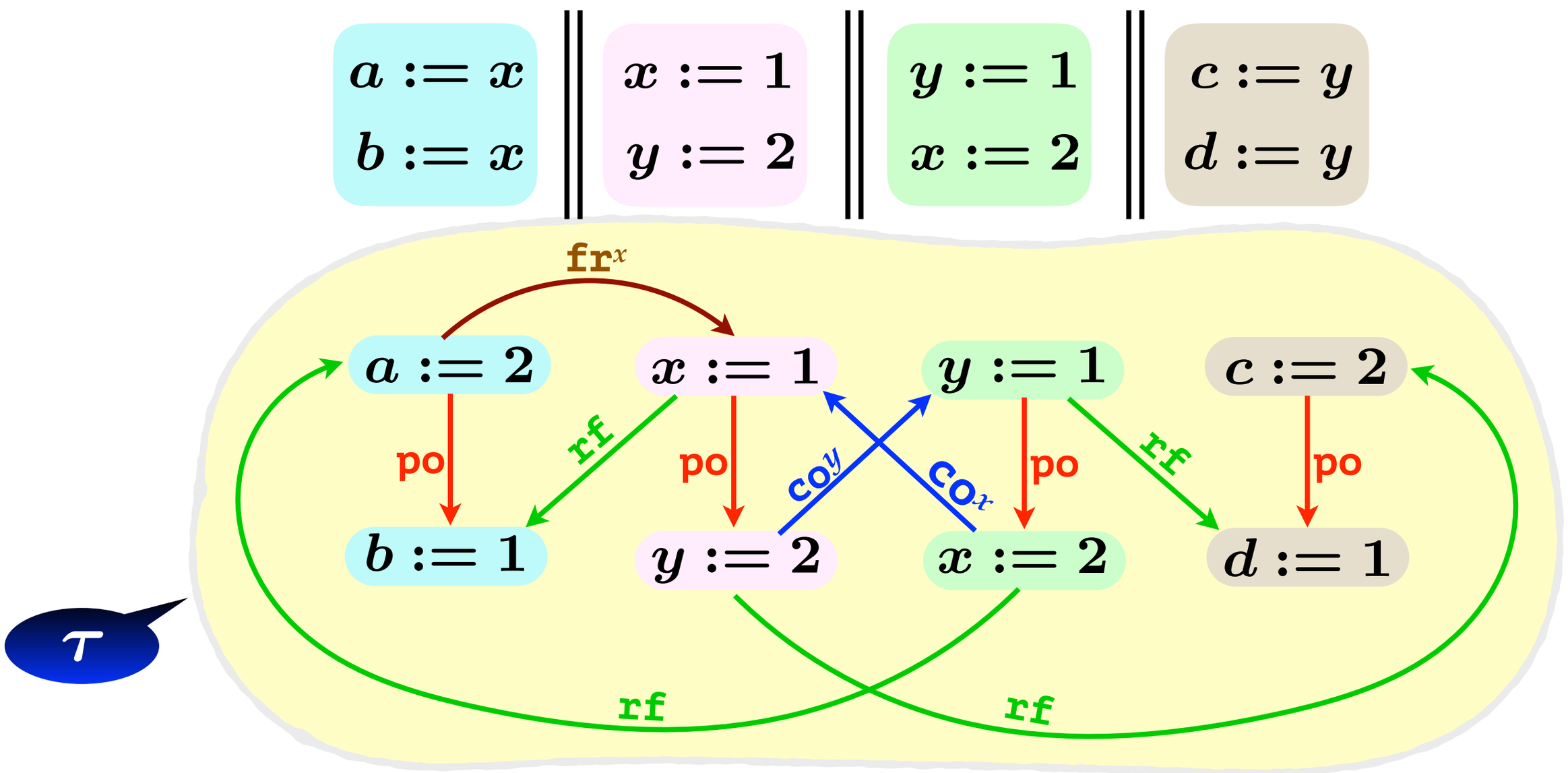

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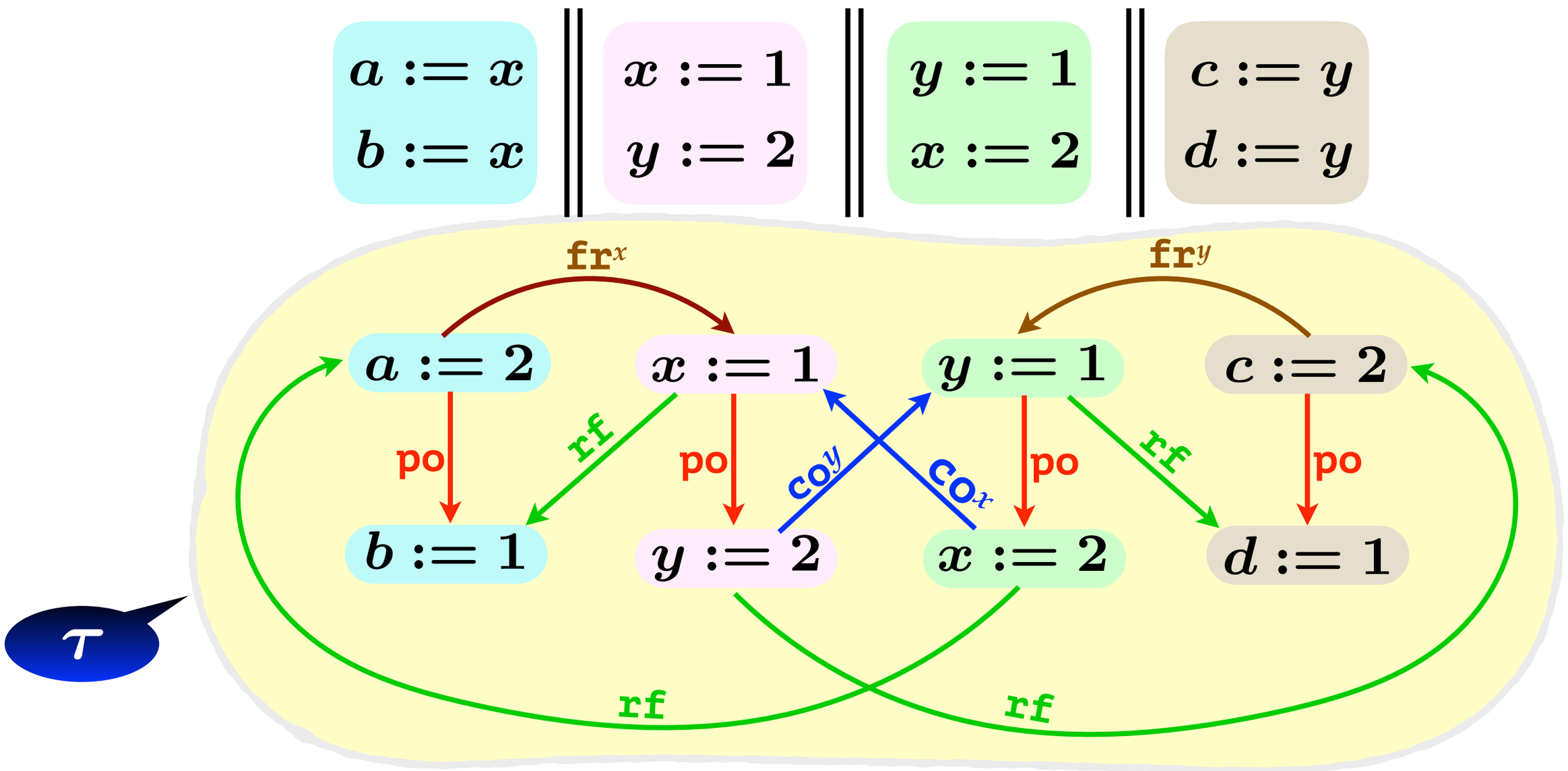

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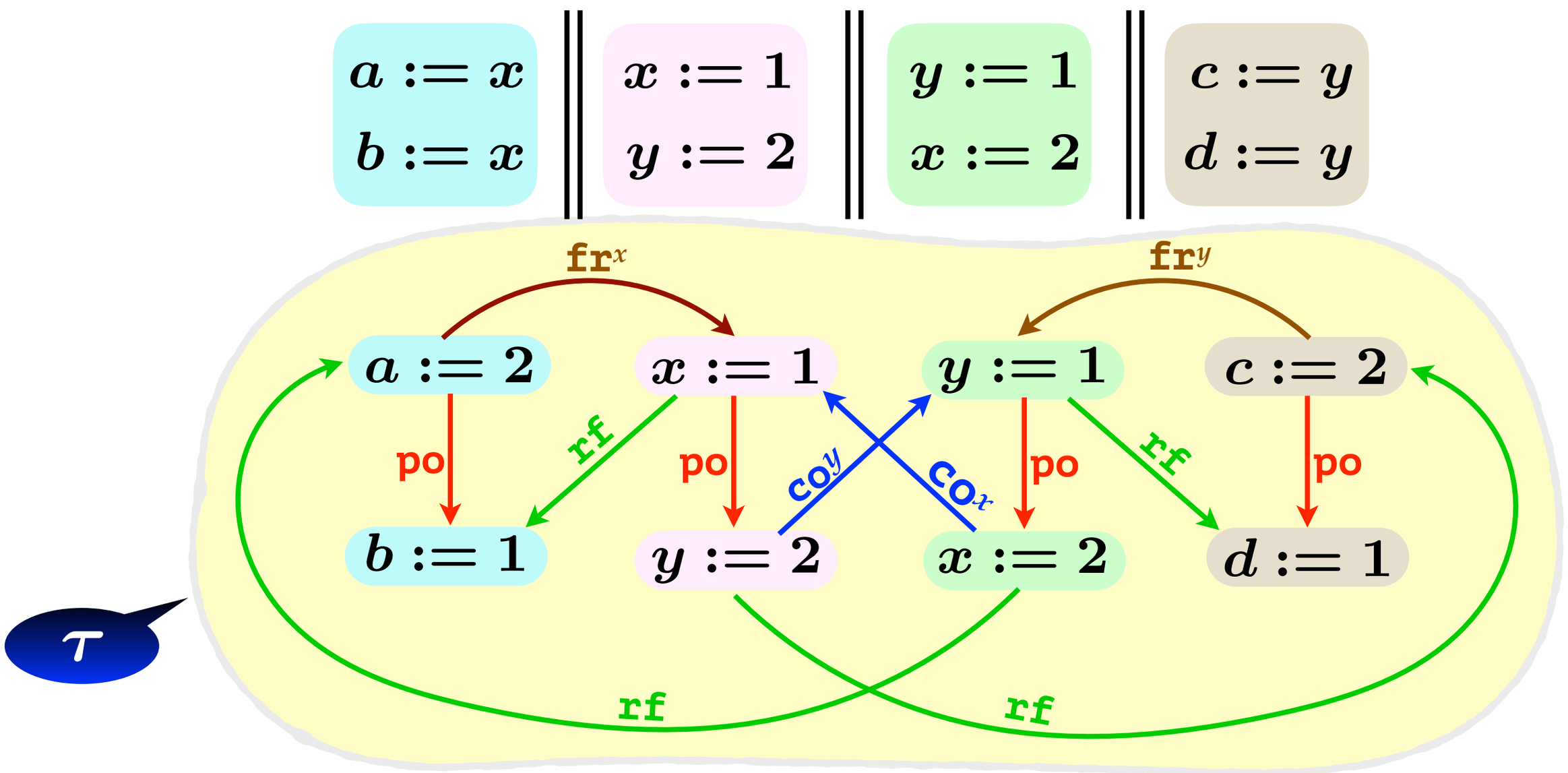

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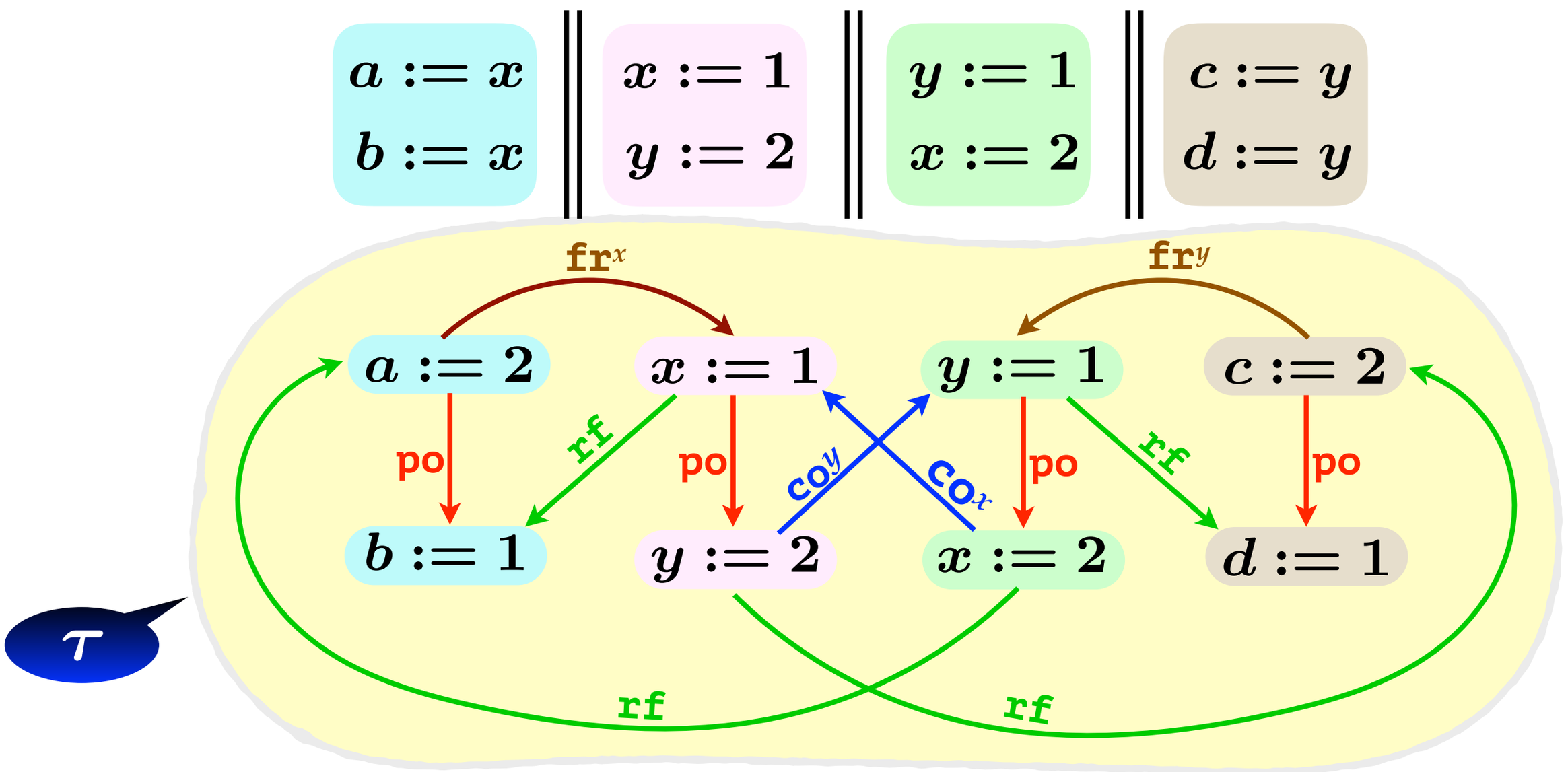

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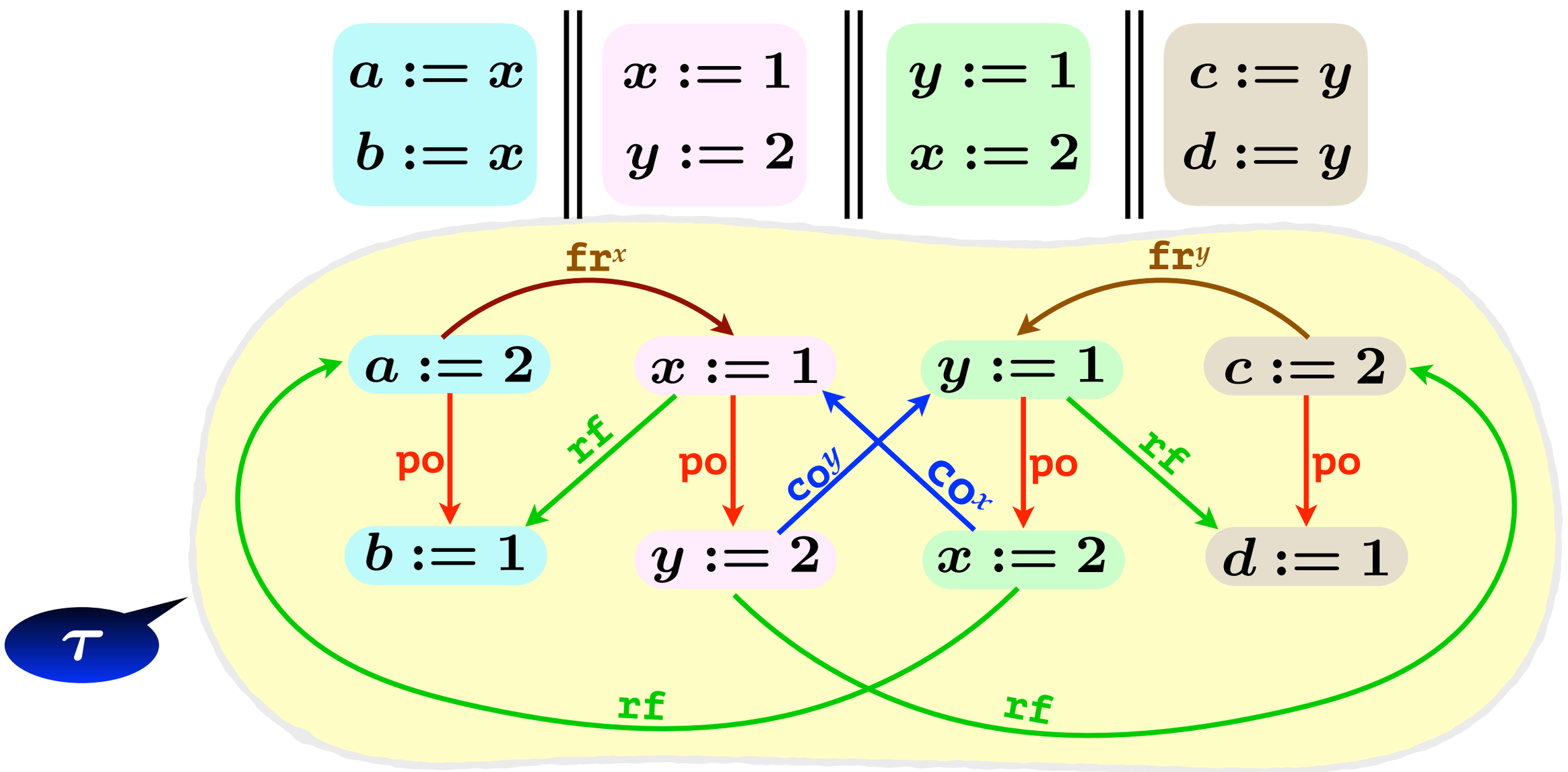

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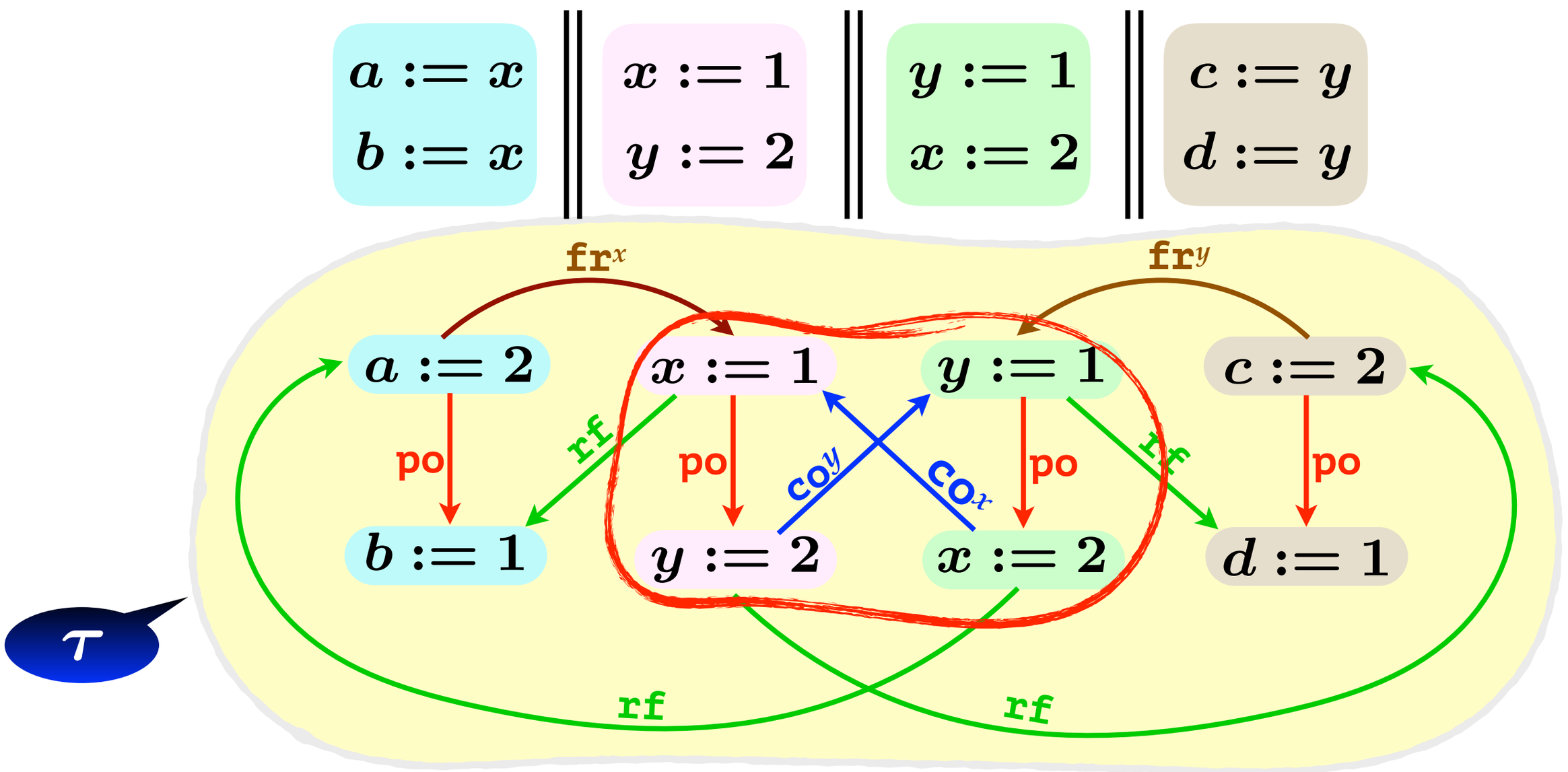

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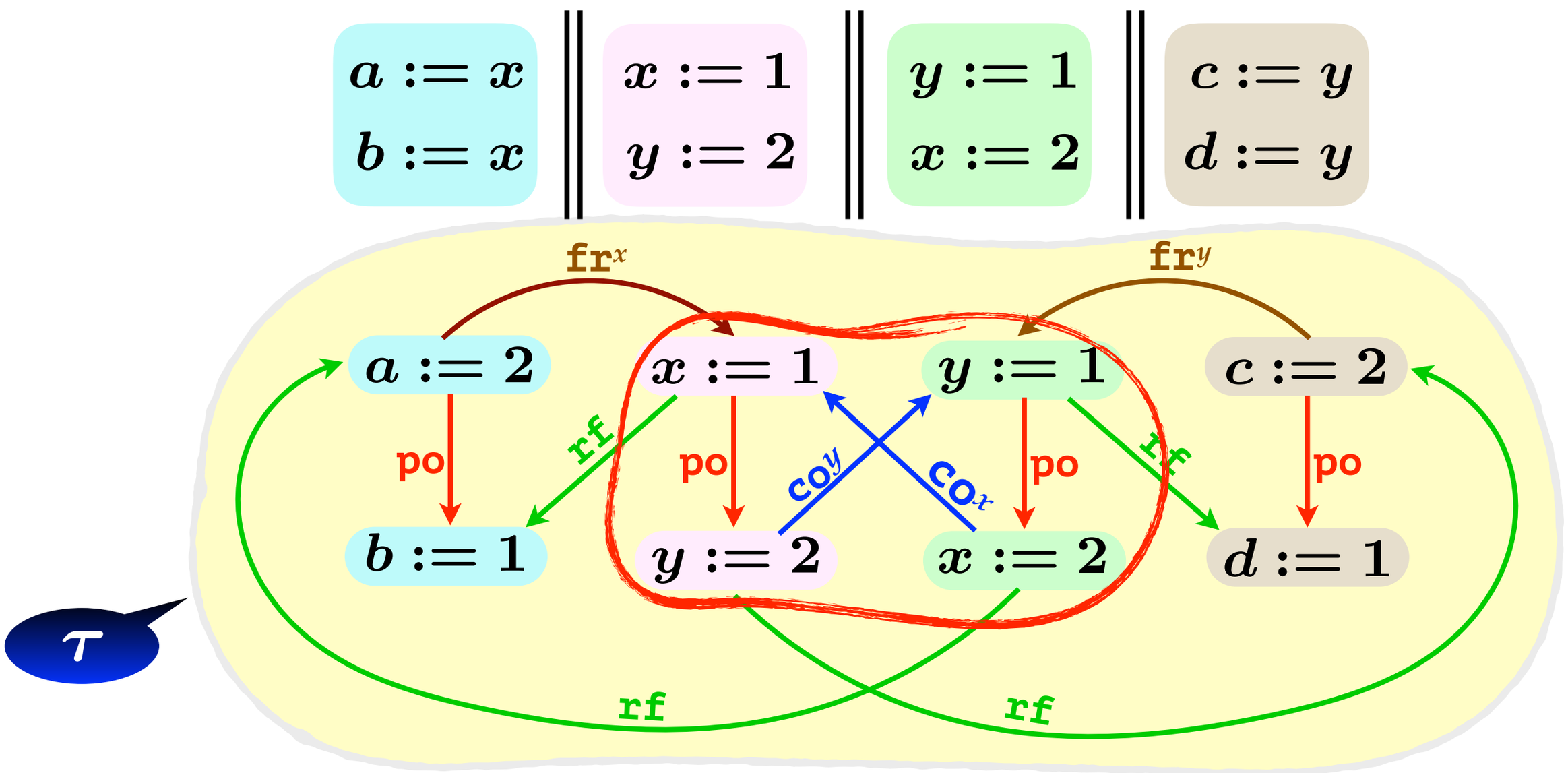

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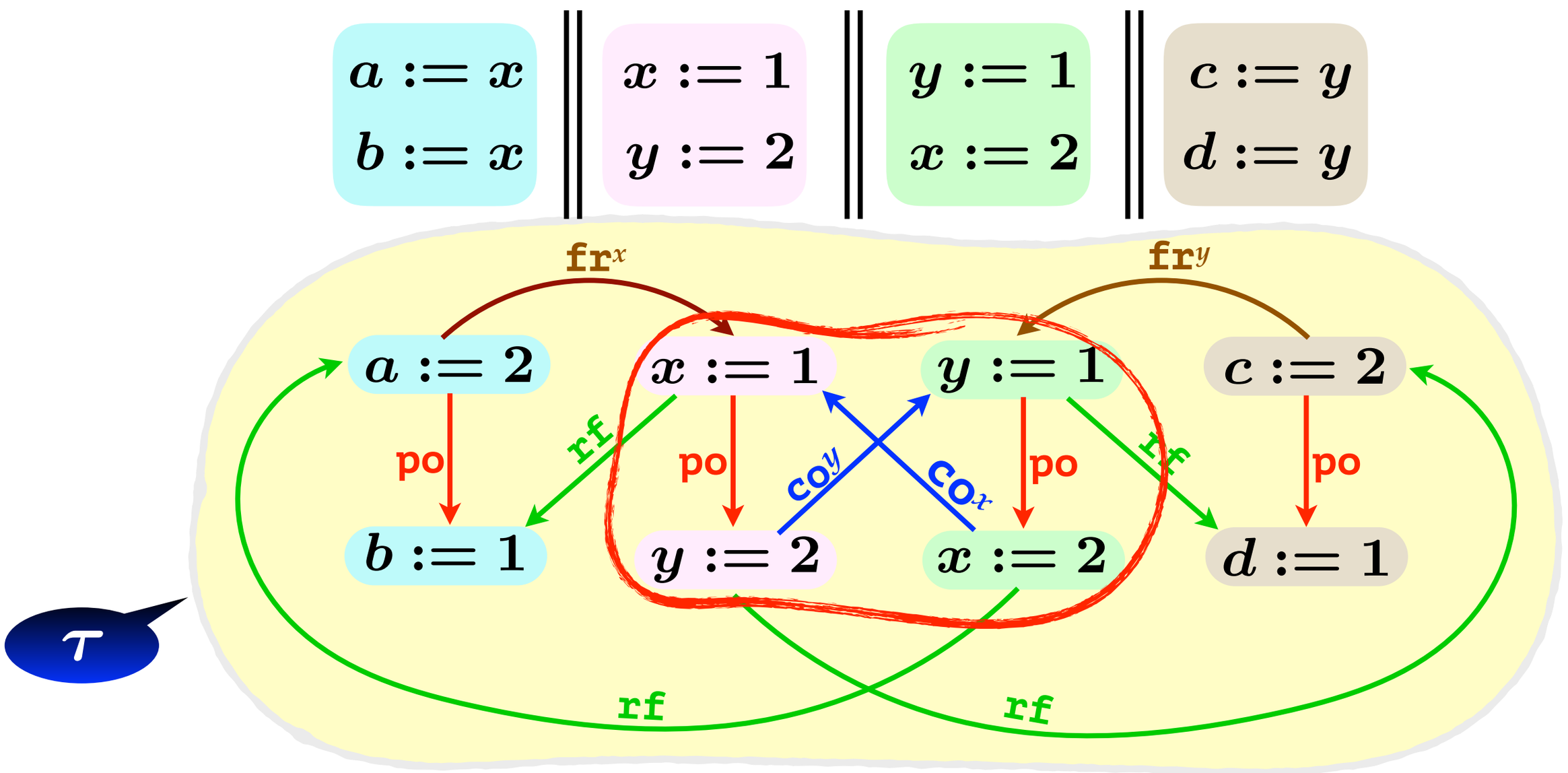

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  - only valid traces
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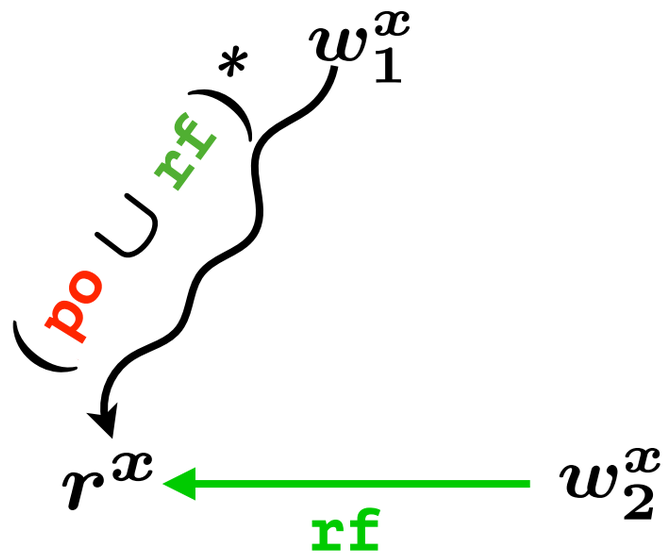
Saturation



- Add co-edges by demand
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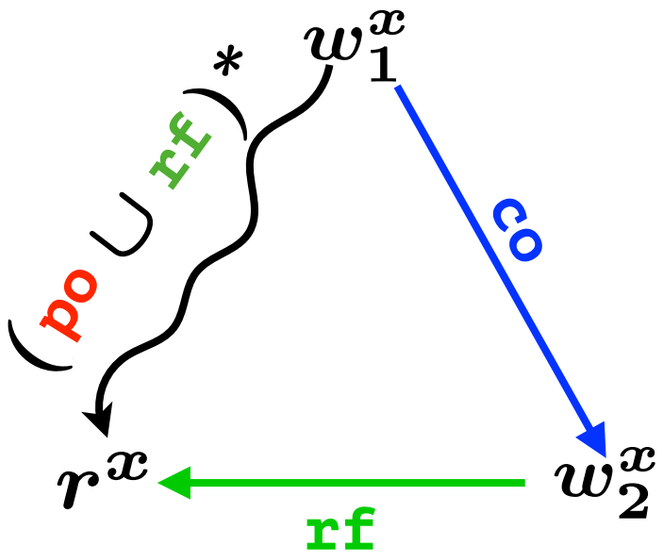

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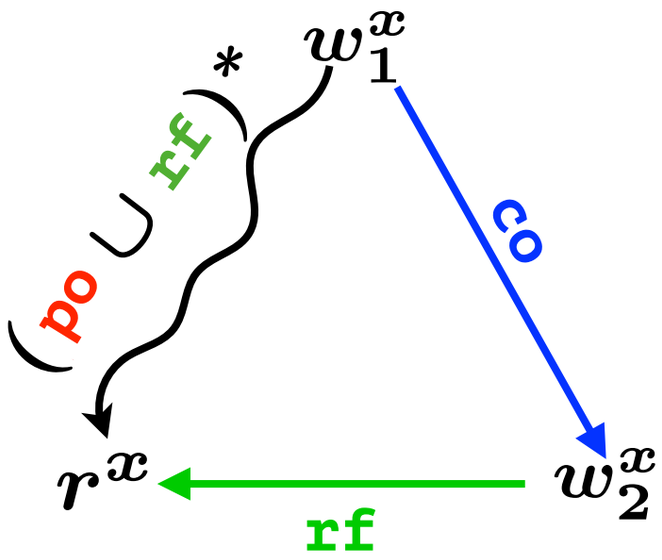

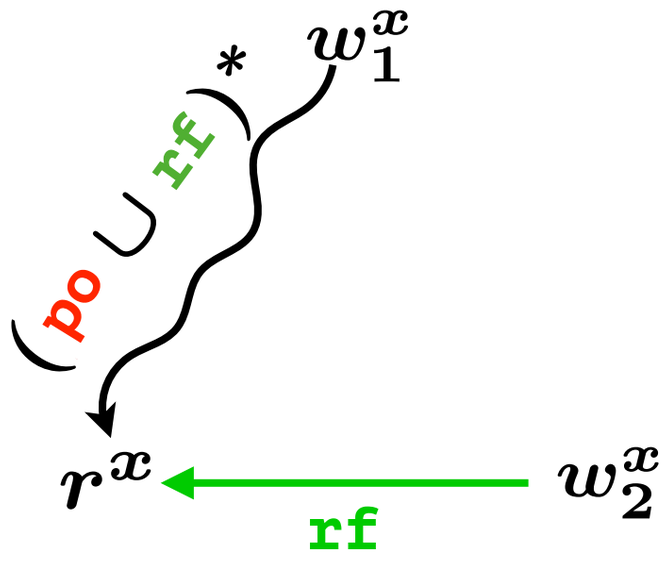

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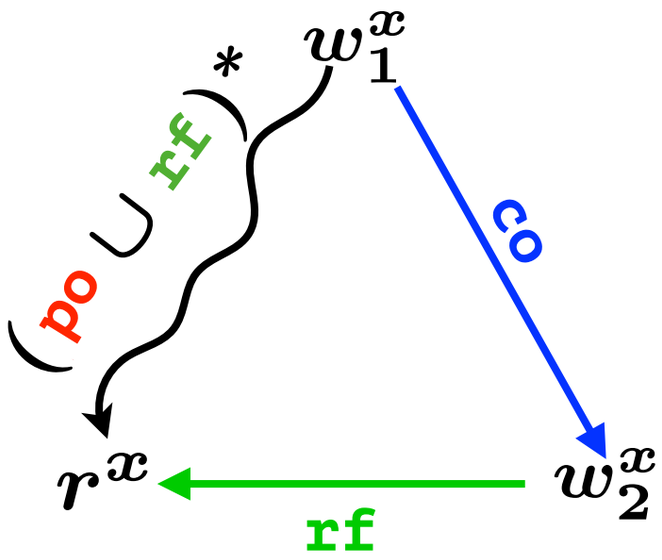

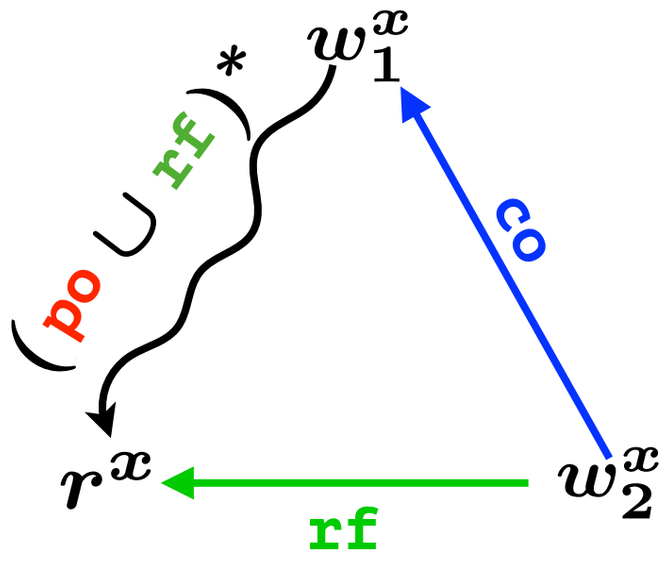

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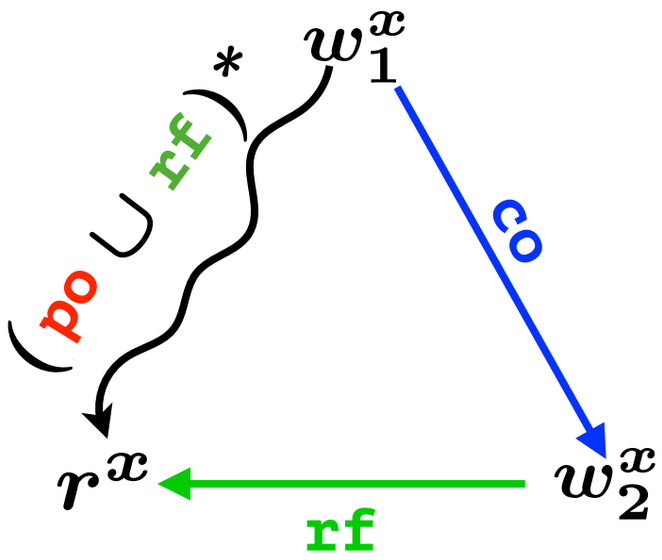

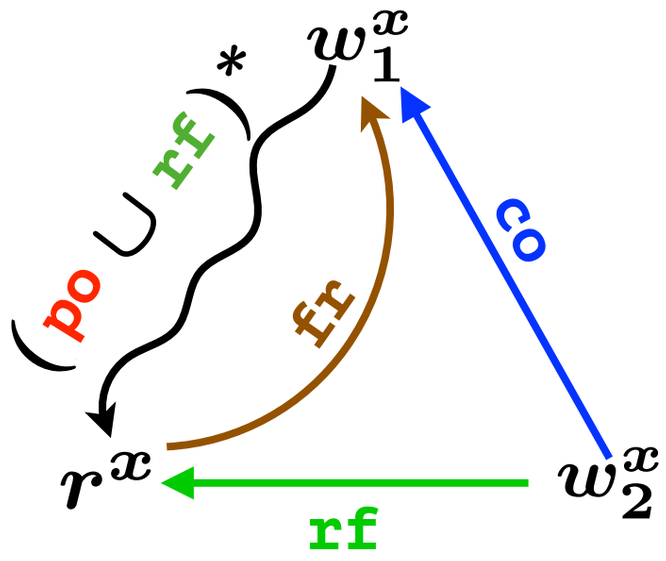

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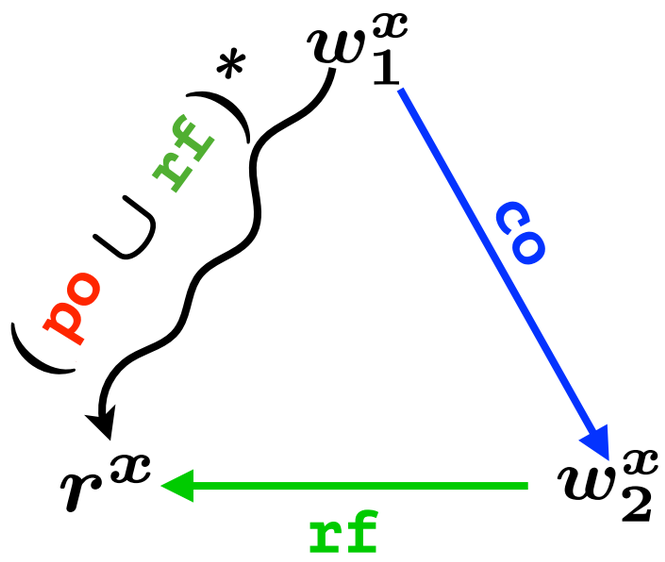

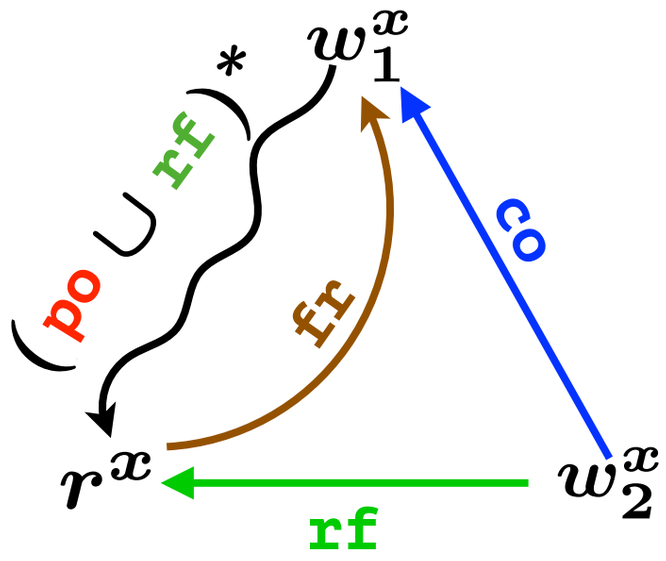

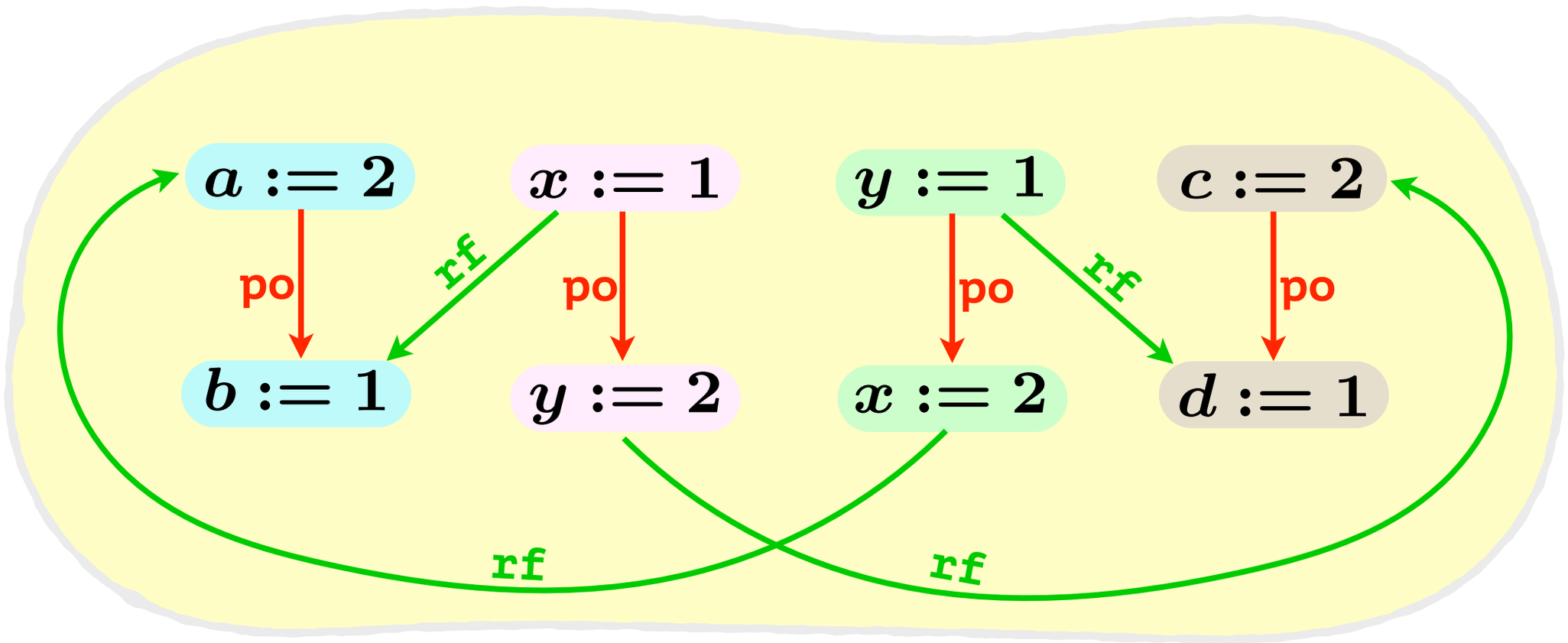

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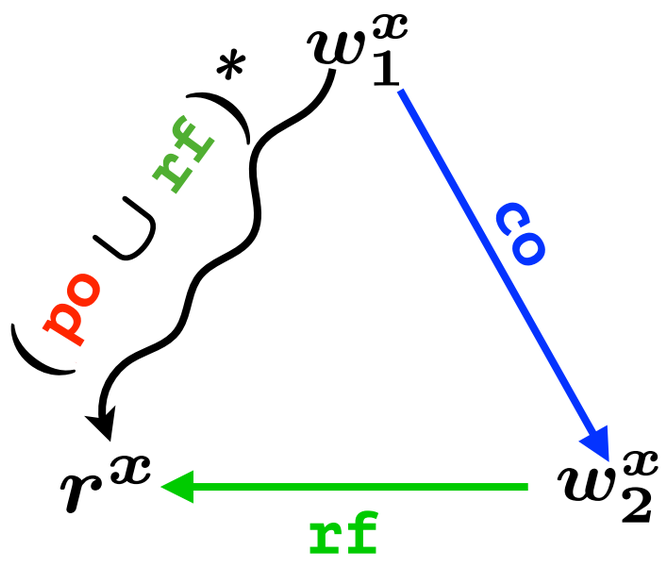

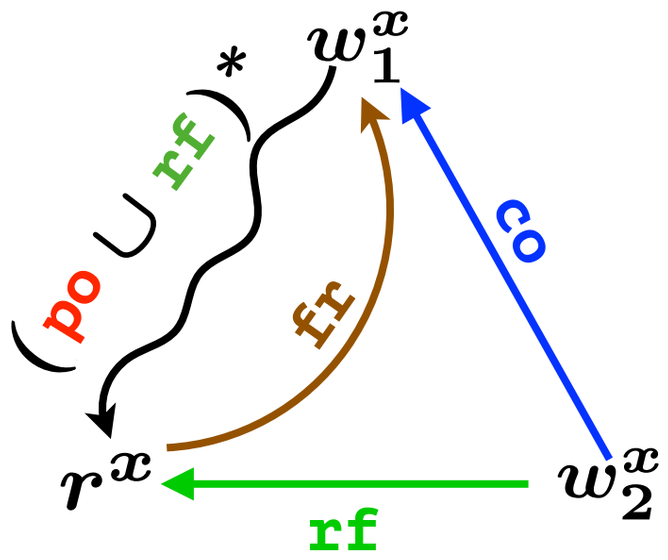

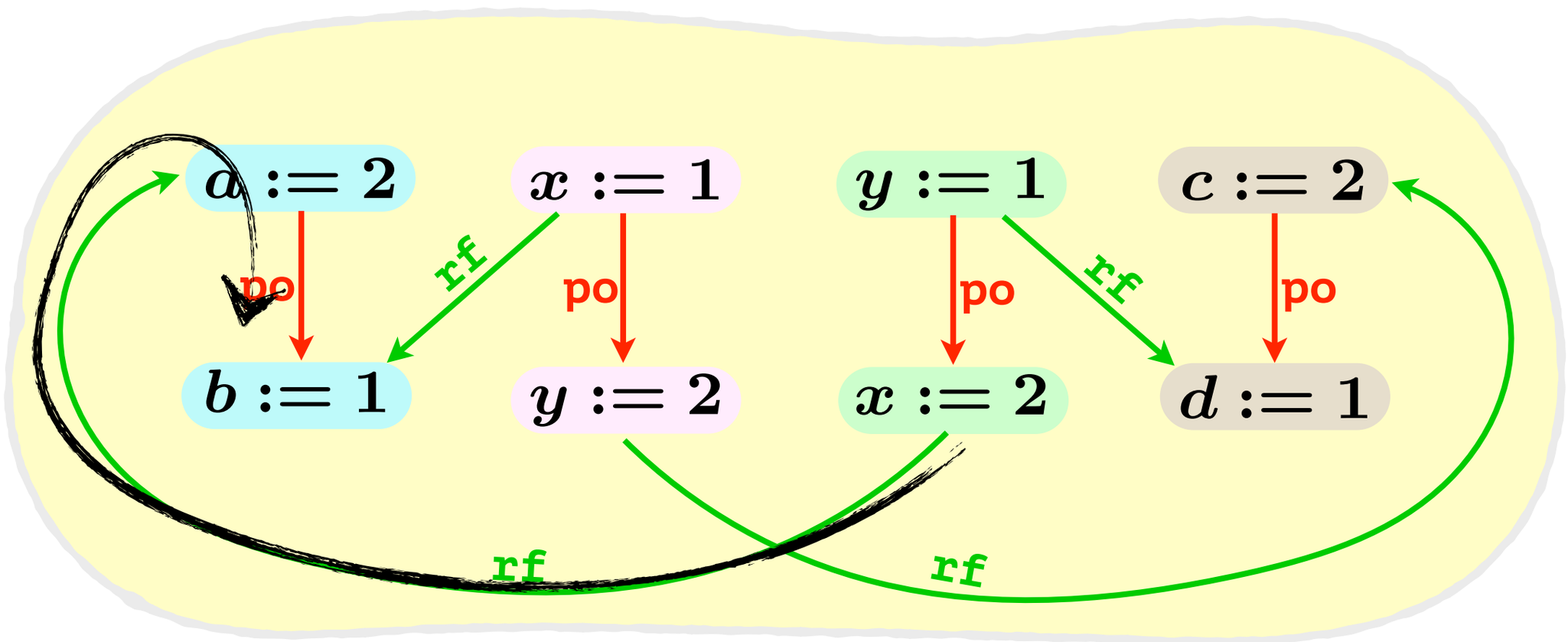

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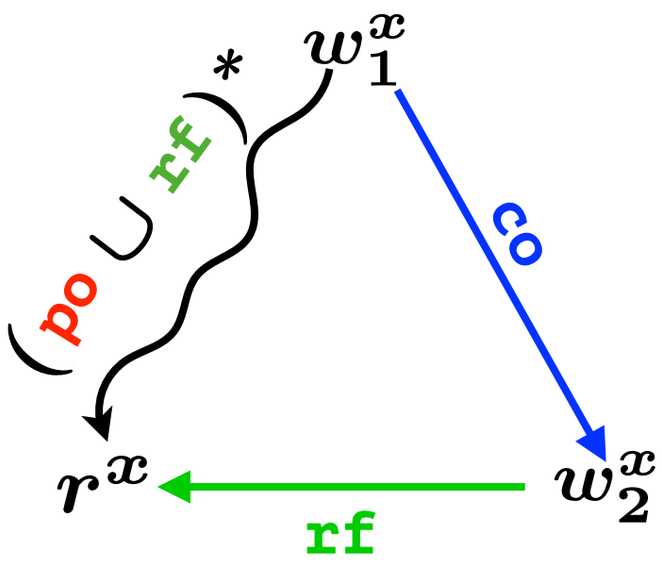

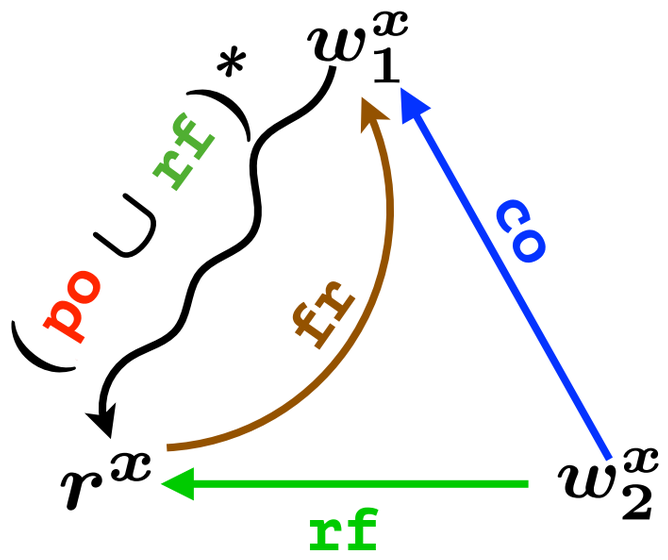

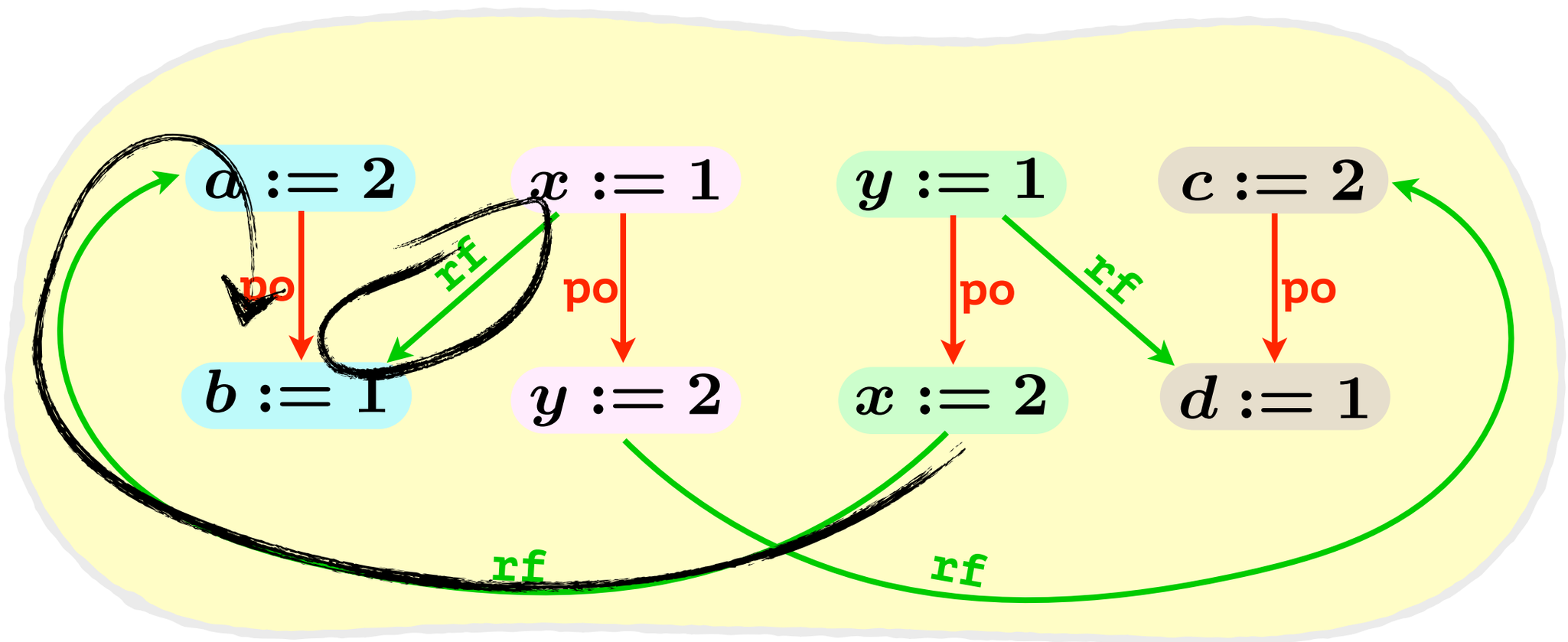

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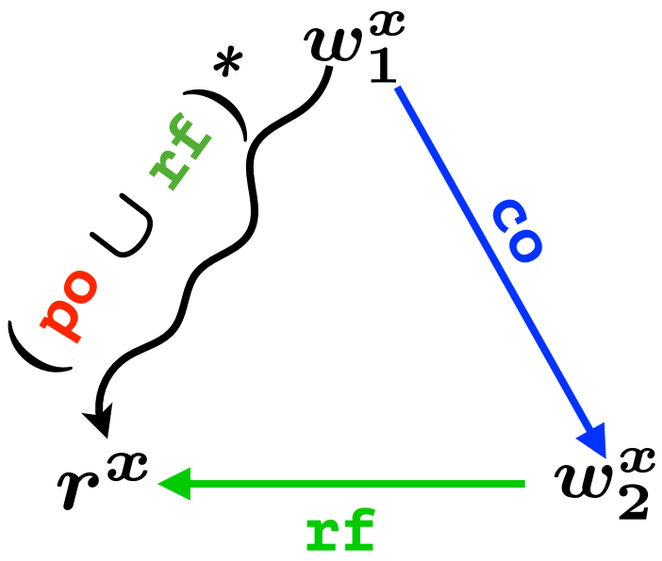
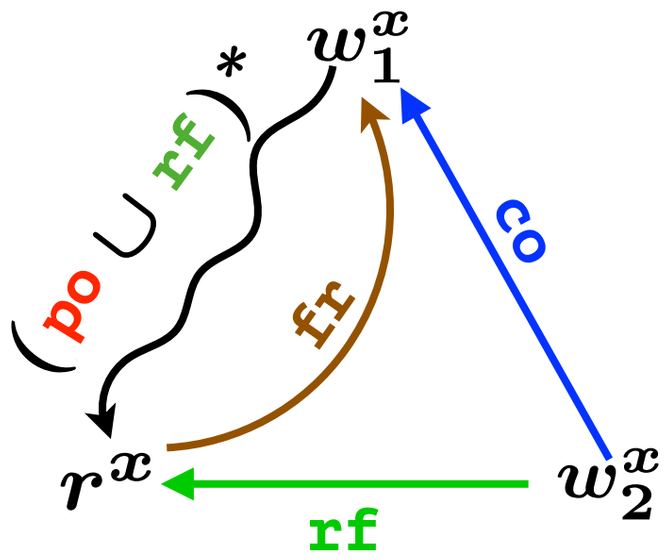
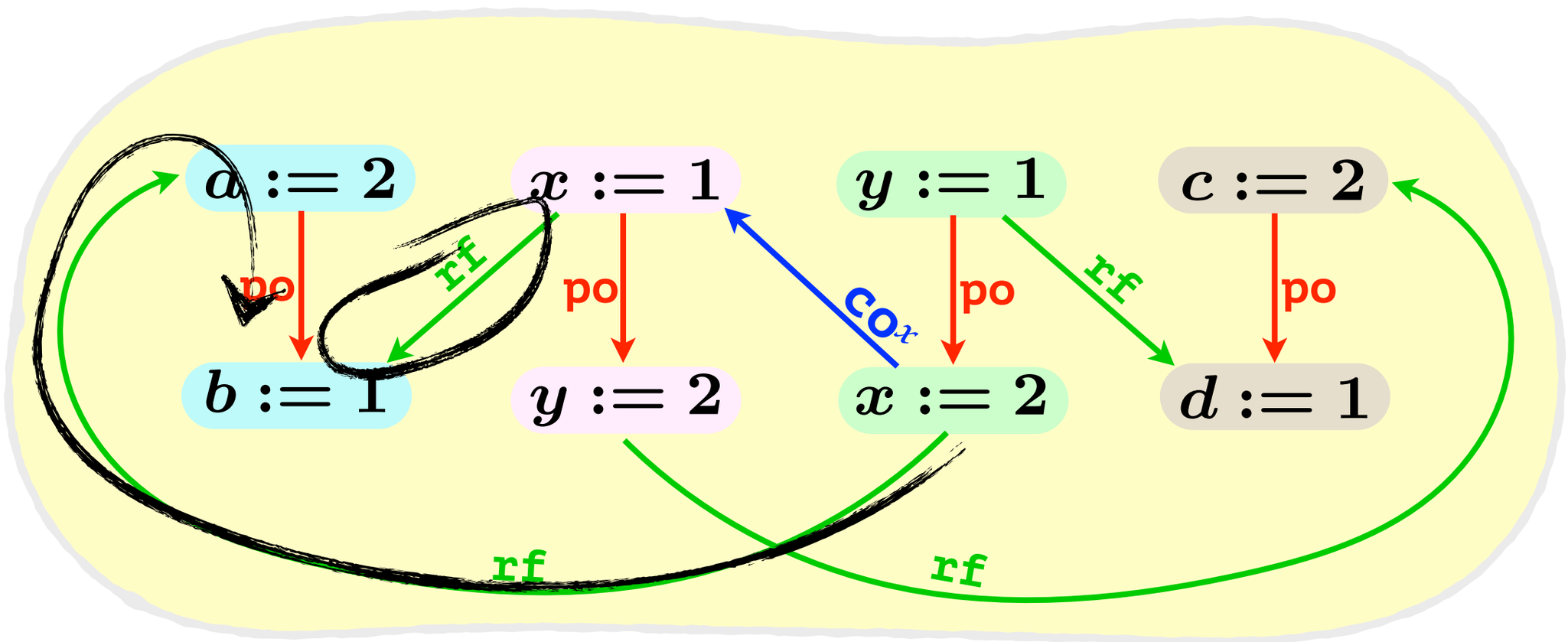
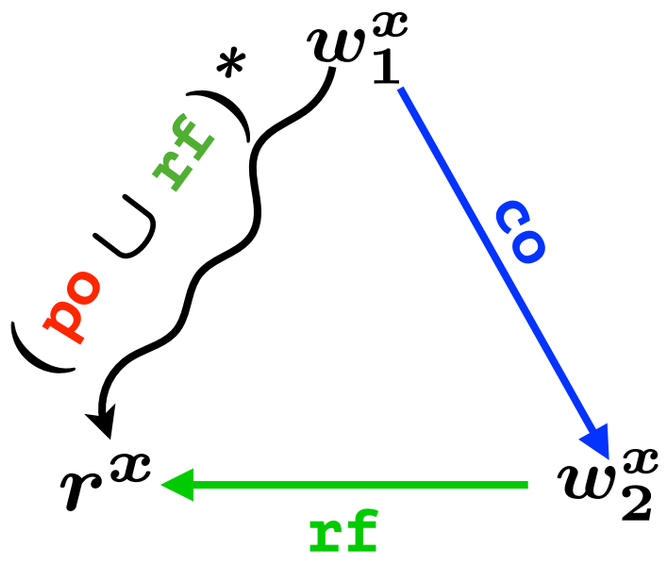


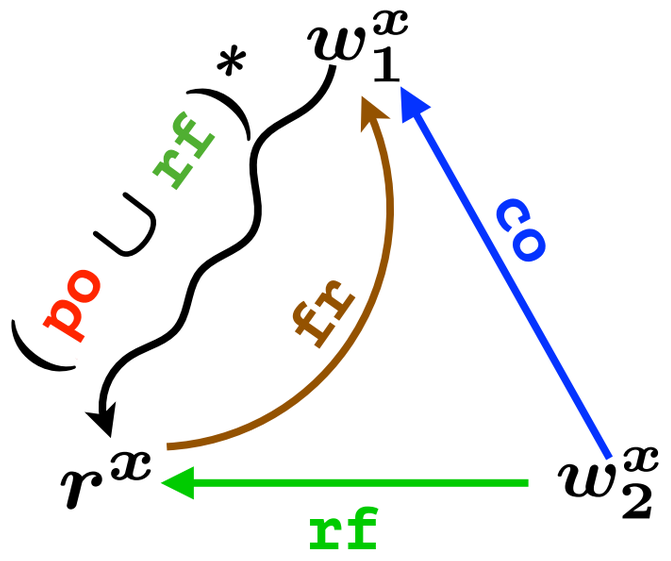


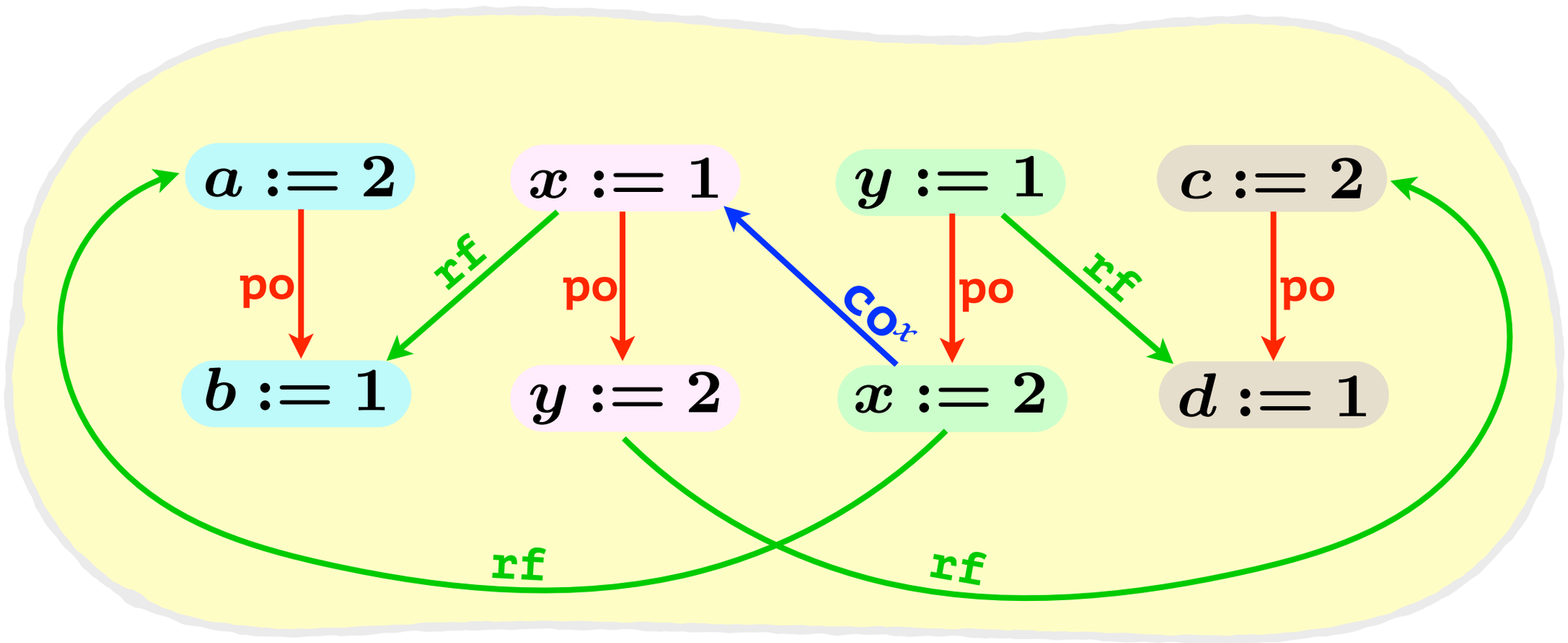


RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Saturation







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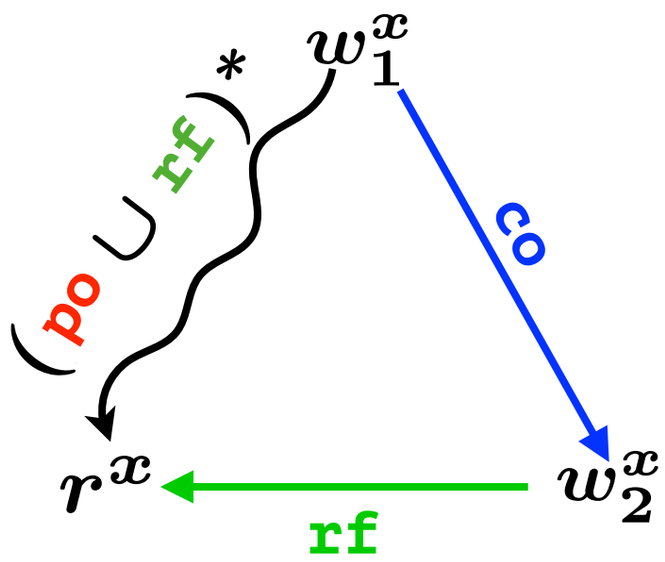

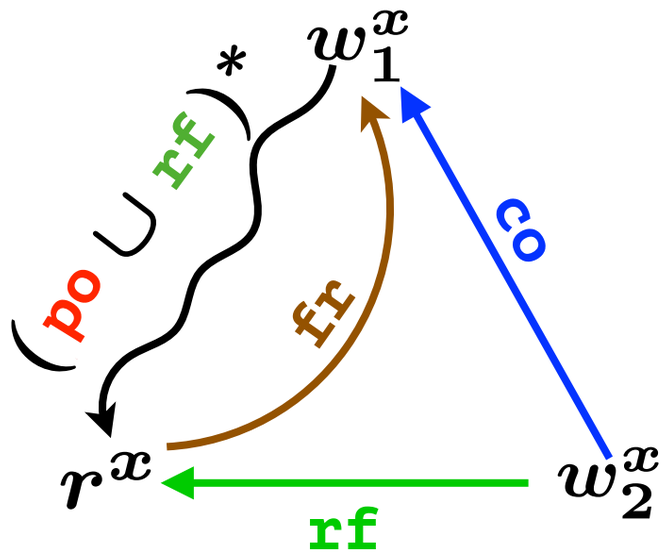

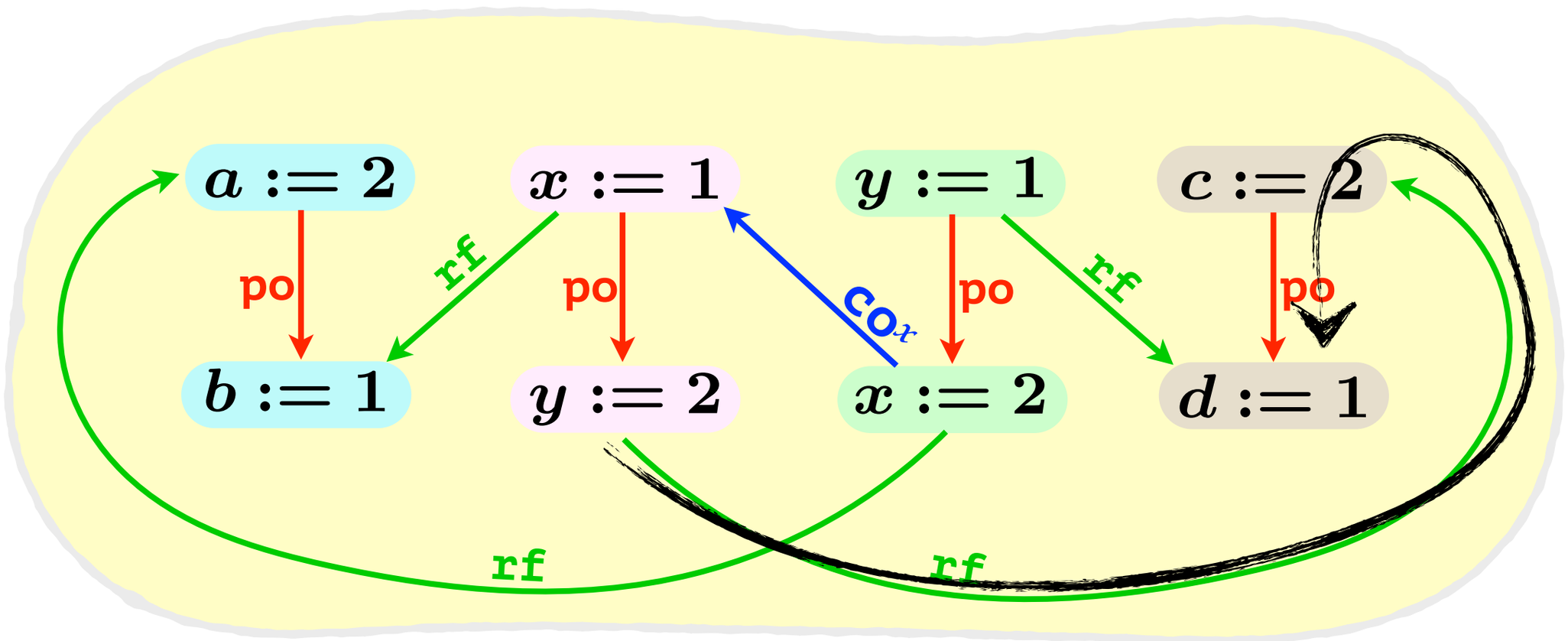

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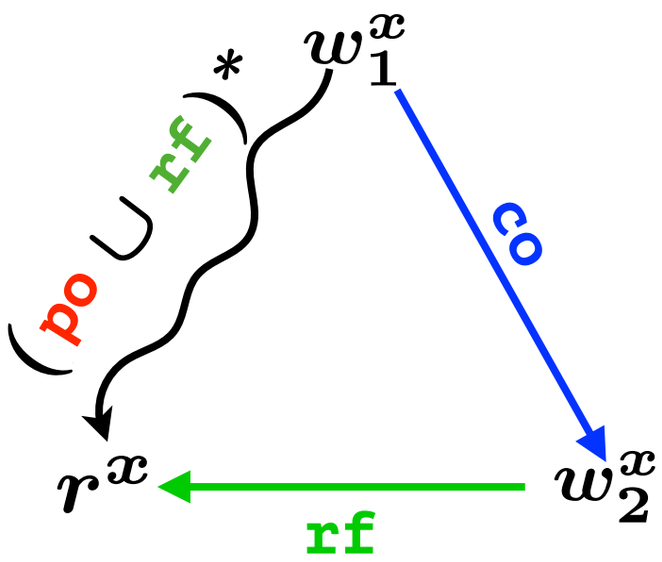

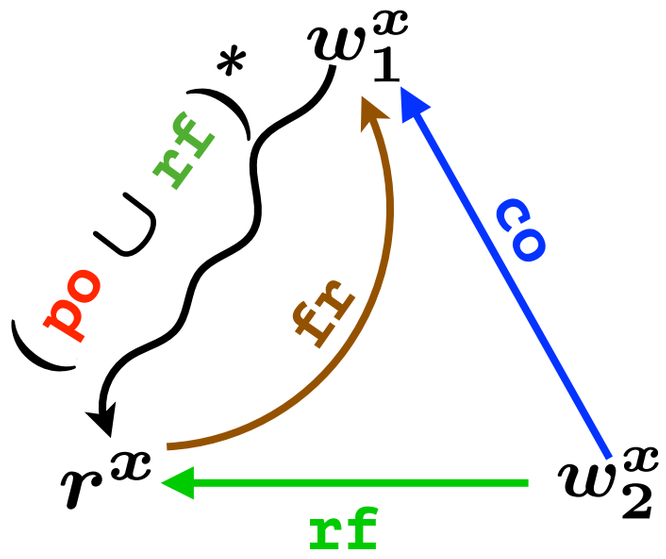

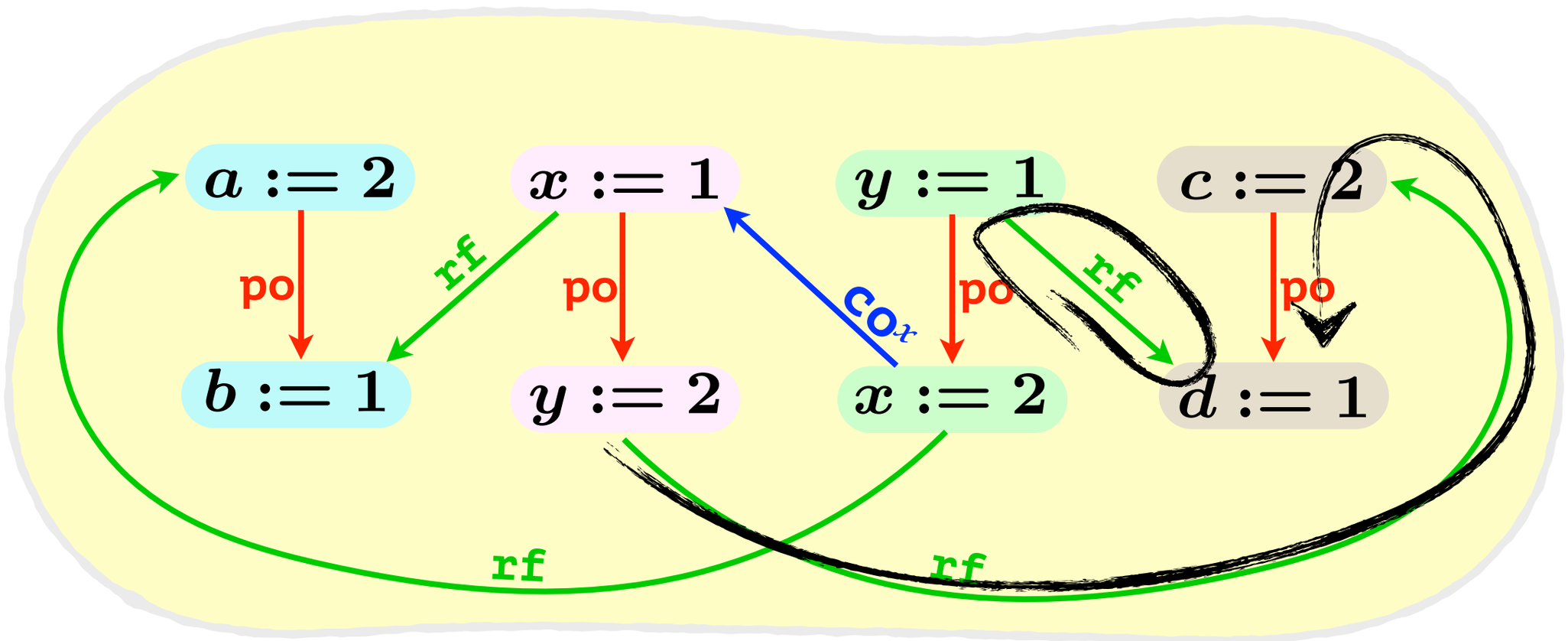

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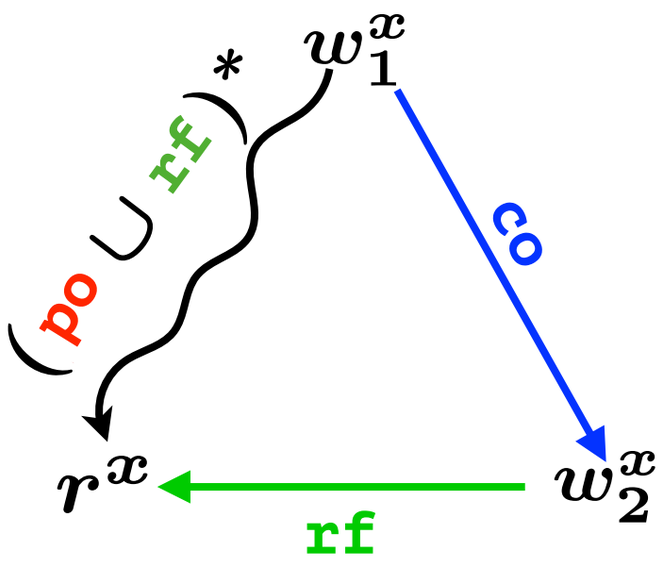

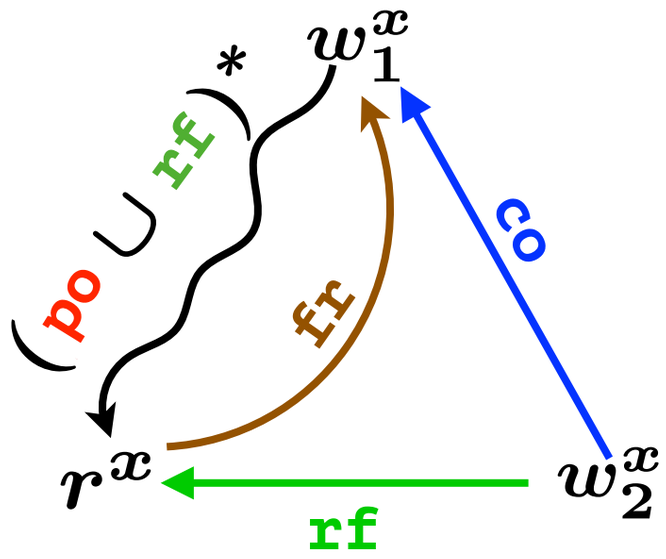

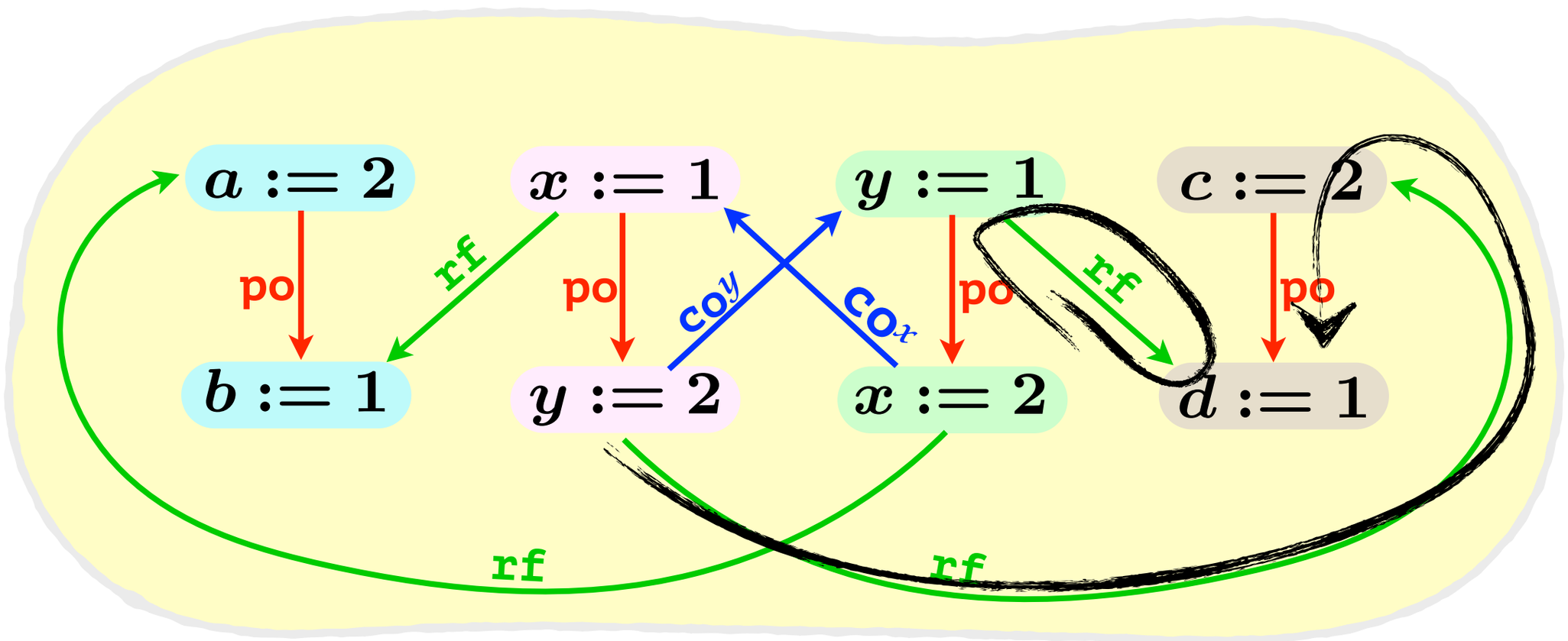

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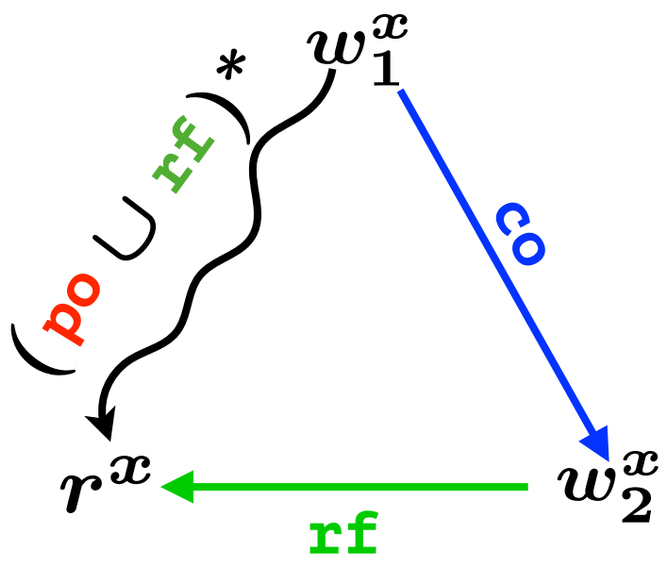

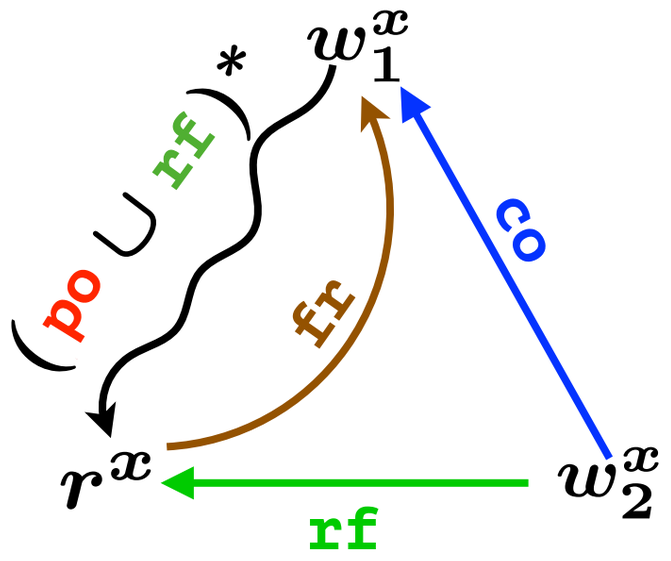

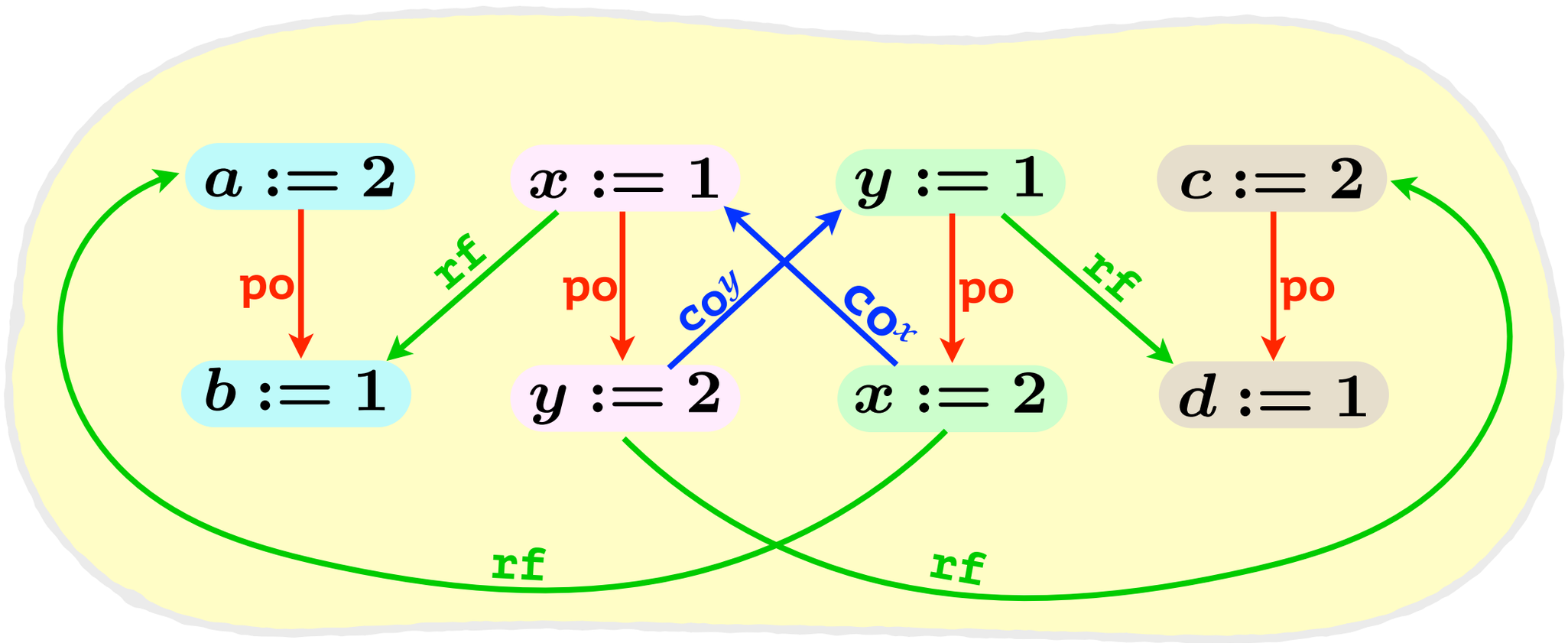

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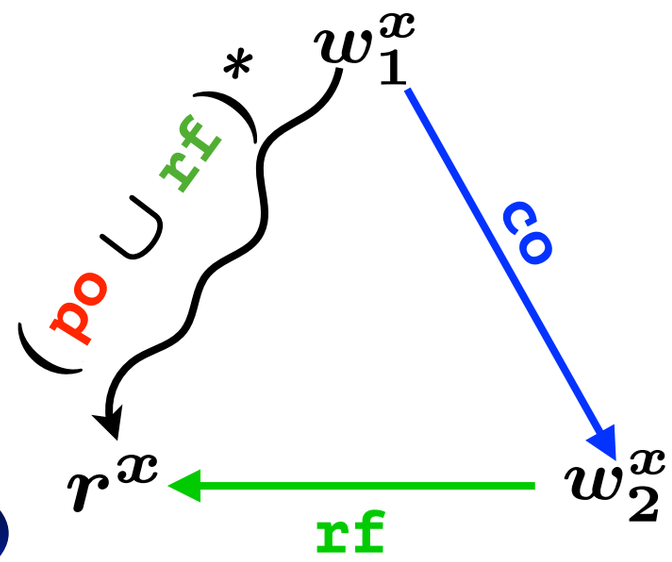

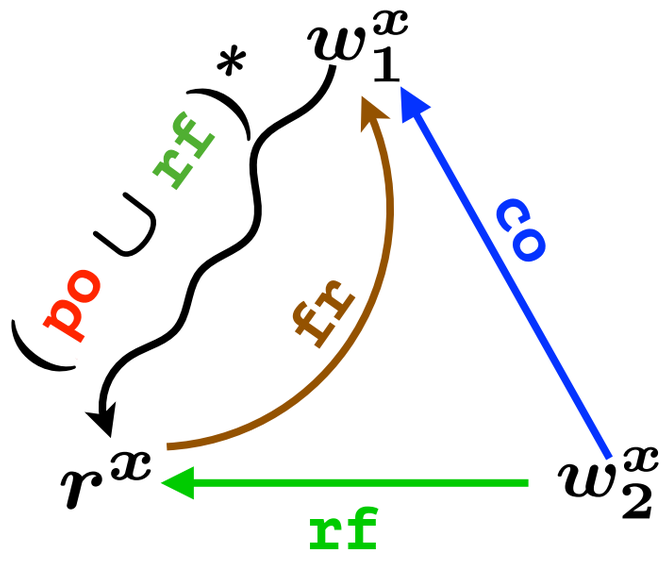

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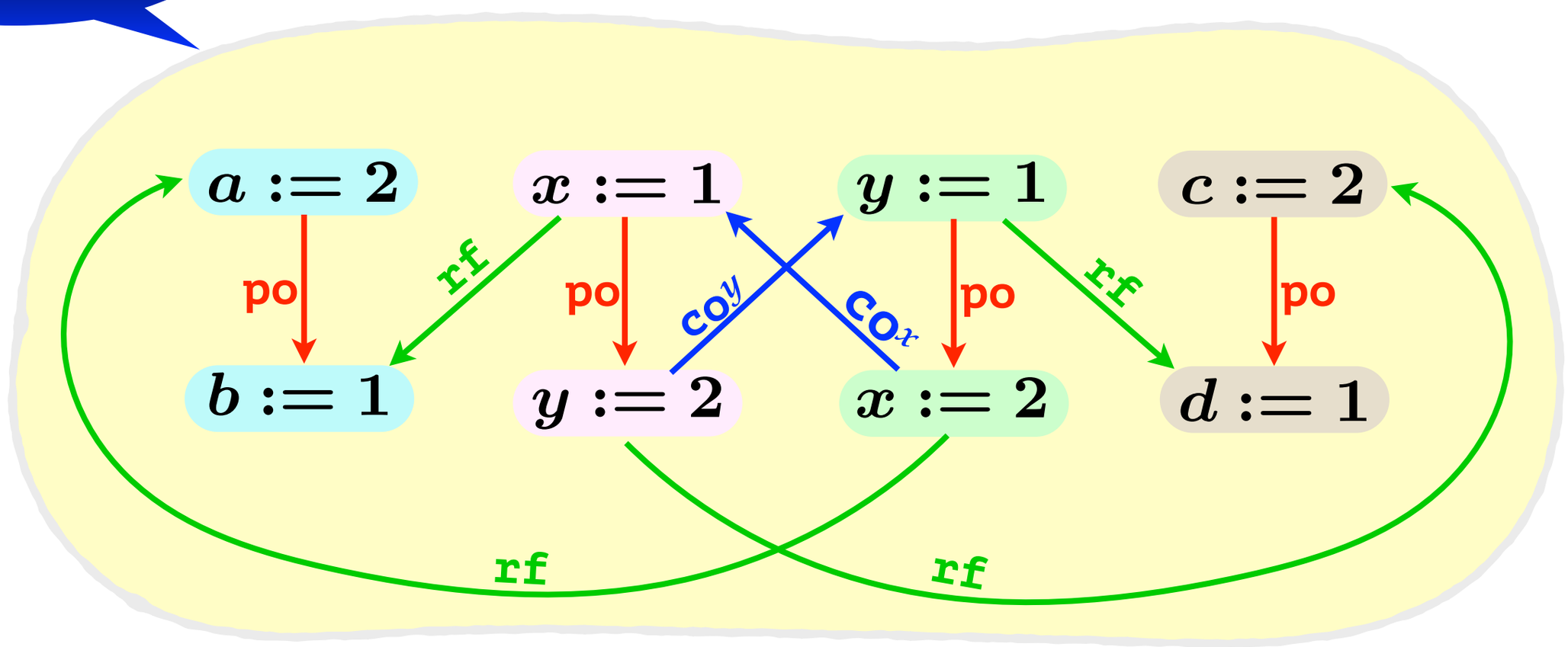

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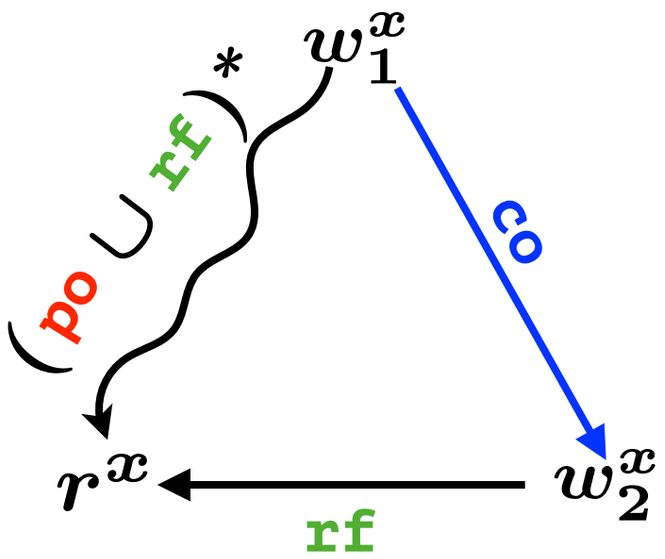

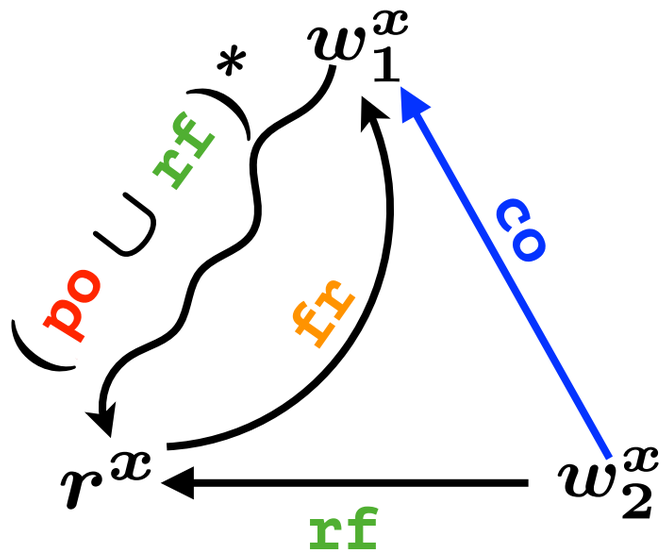

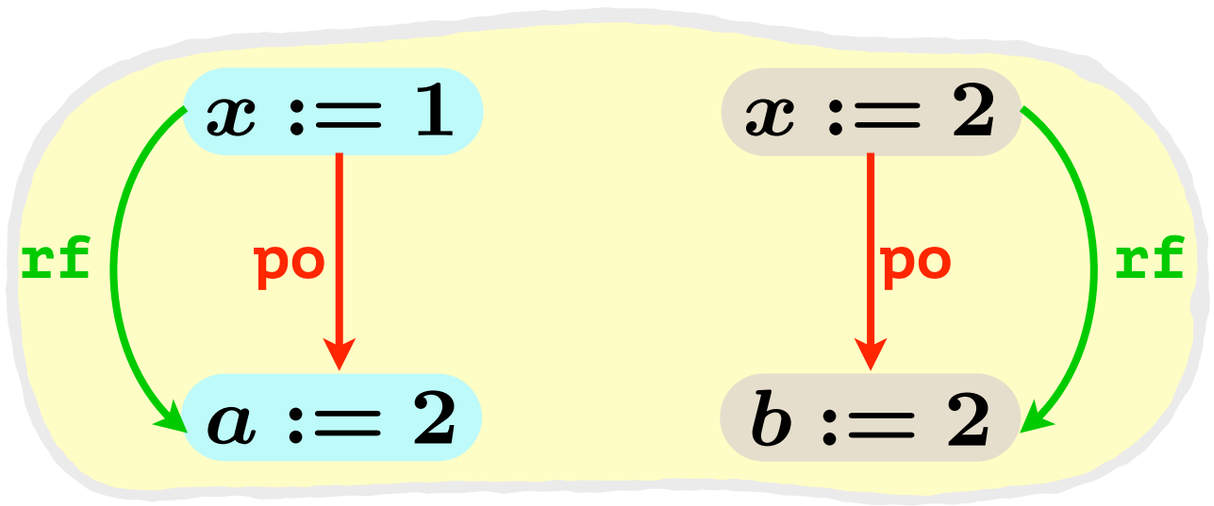

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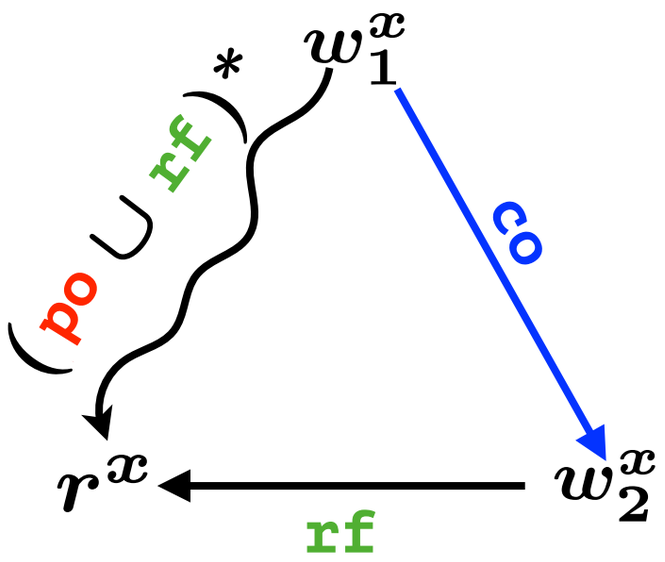

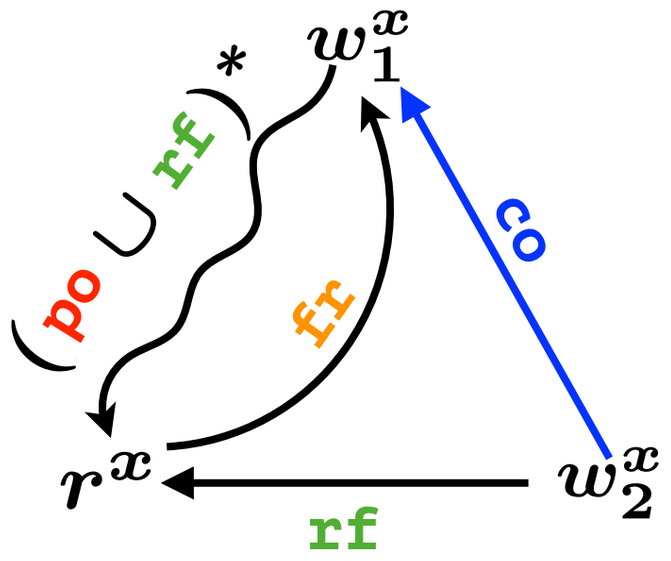

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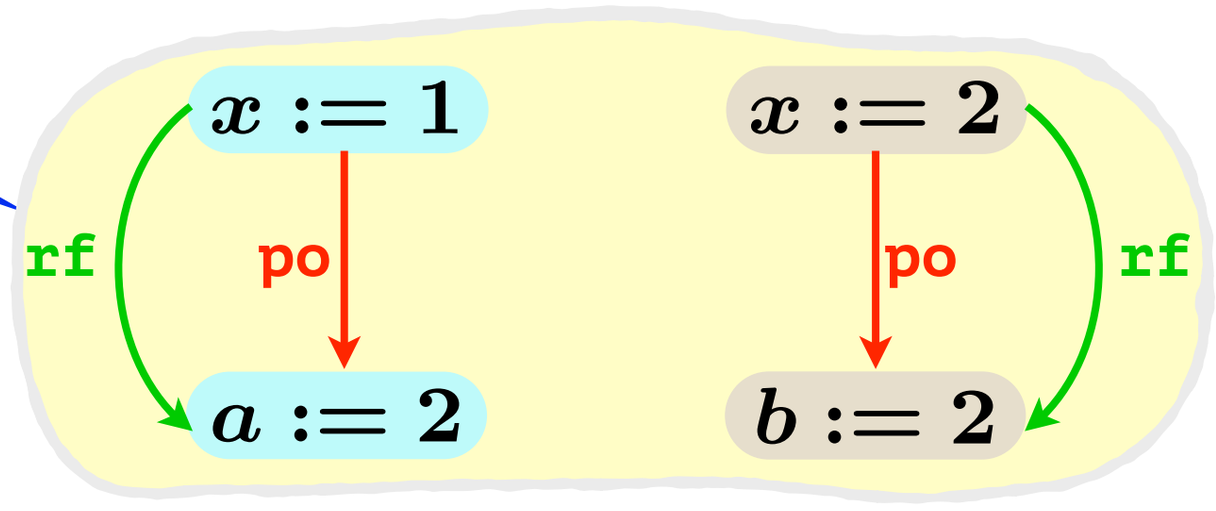

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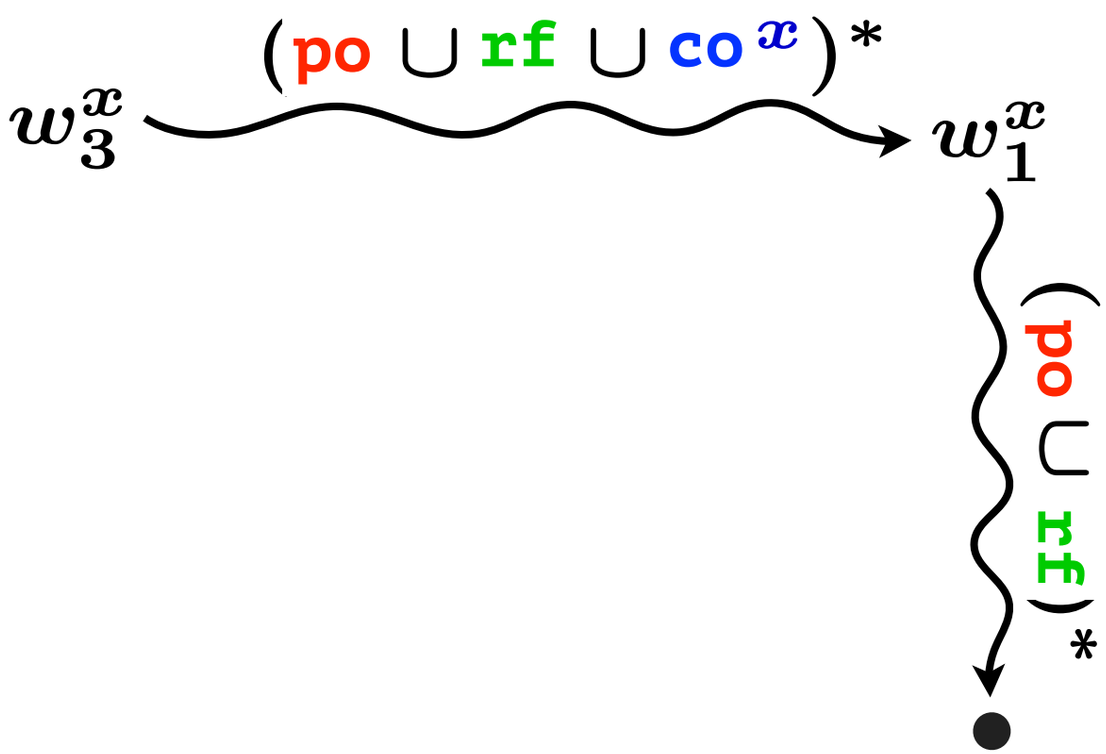
Adding Events to Traces



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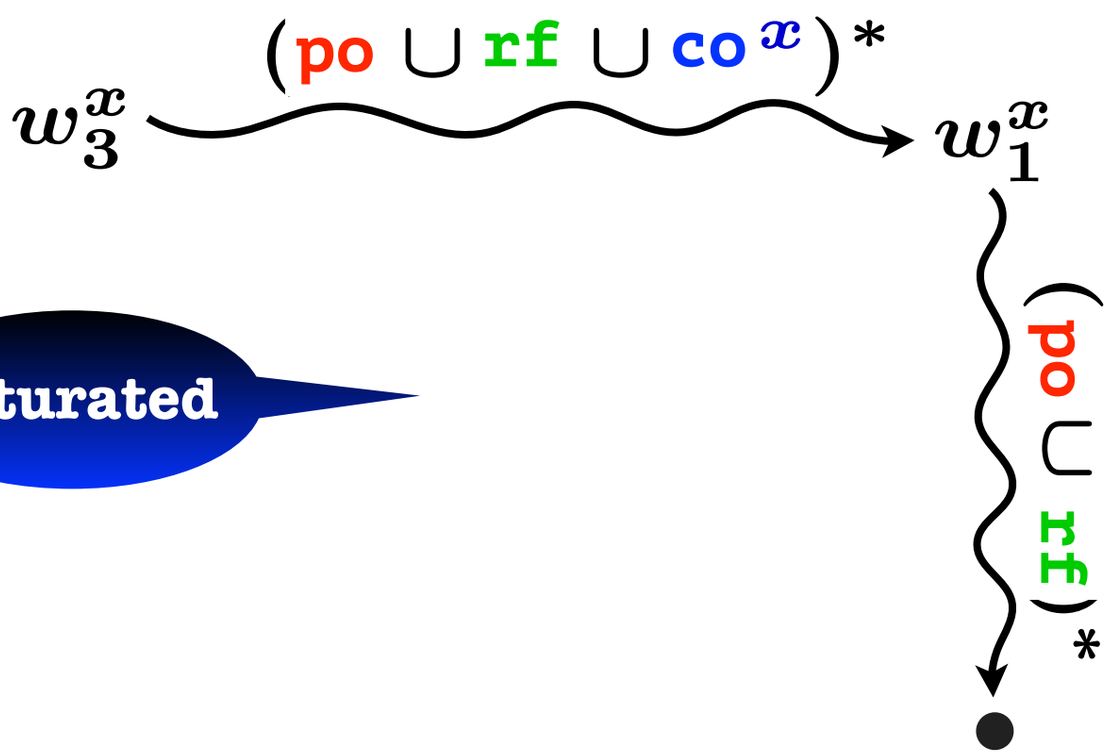

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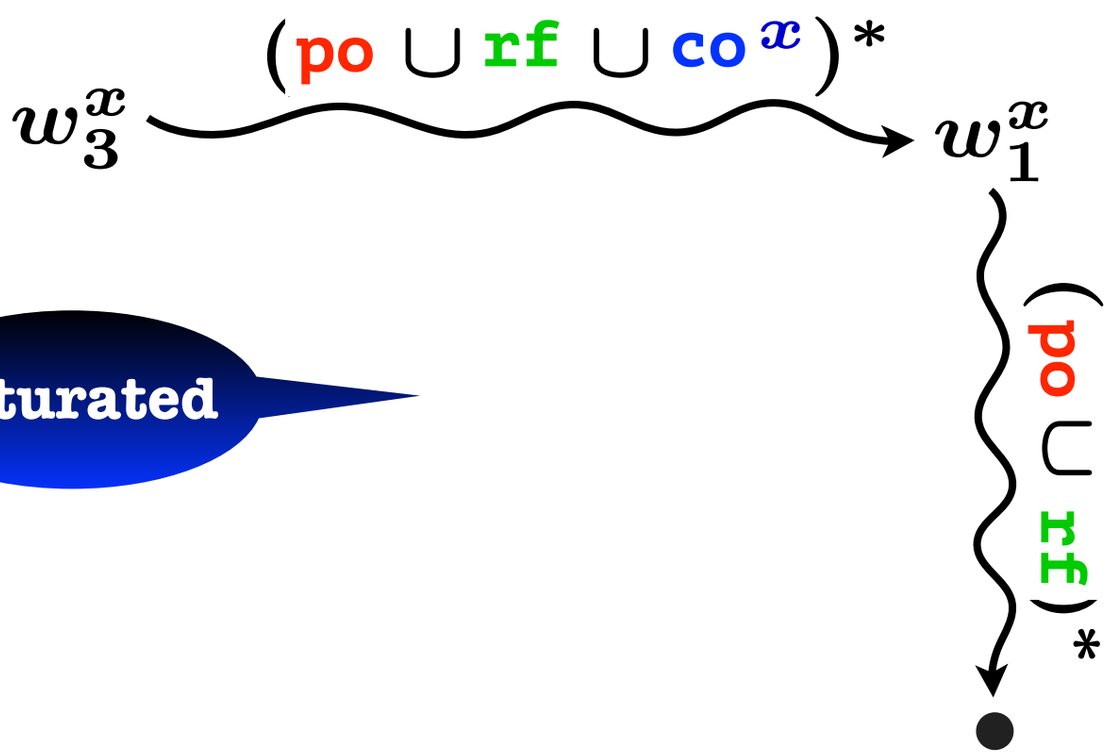

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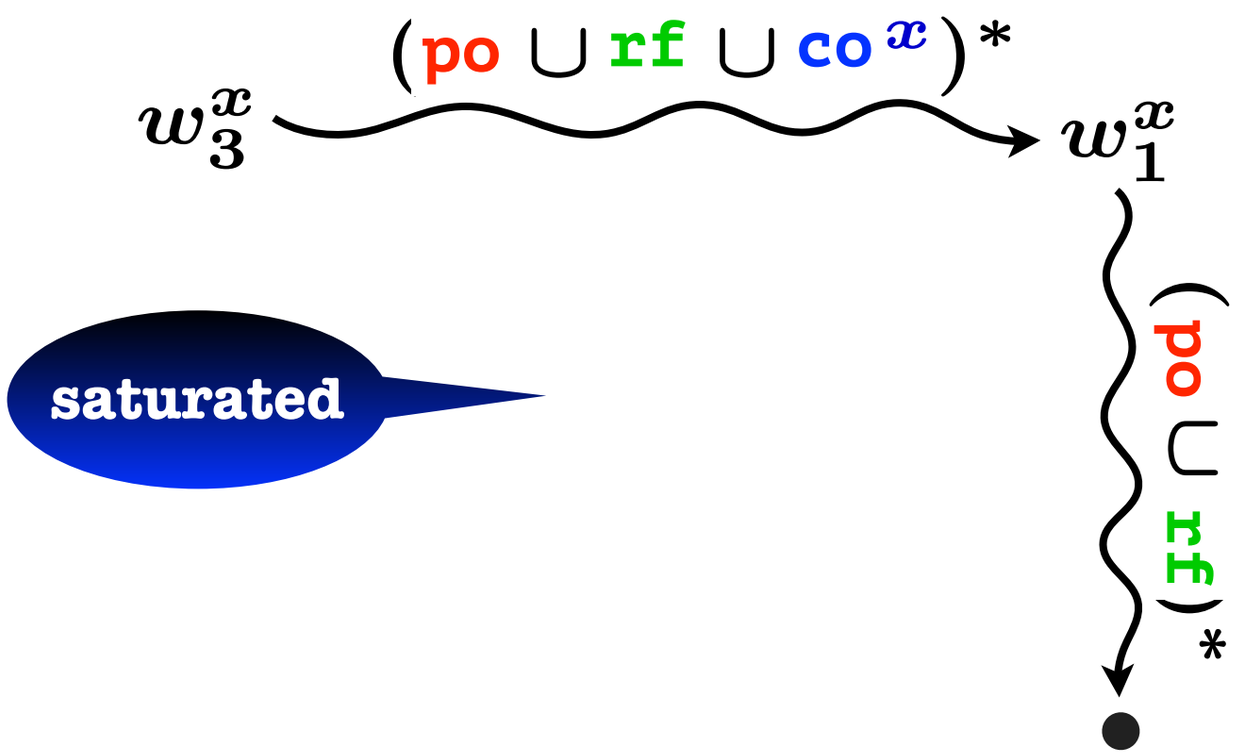

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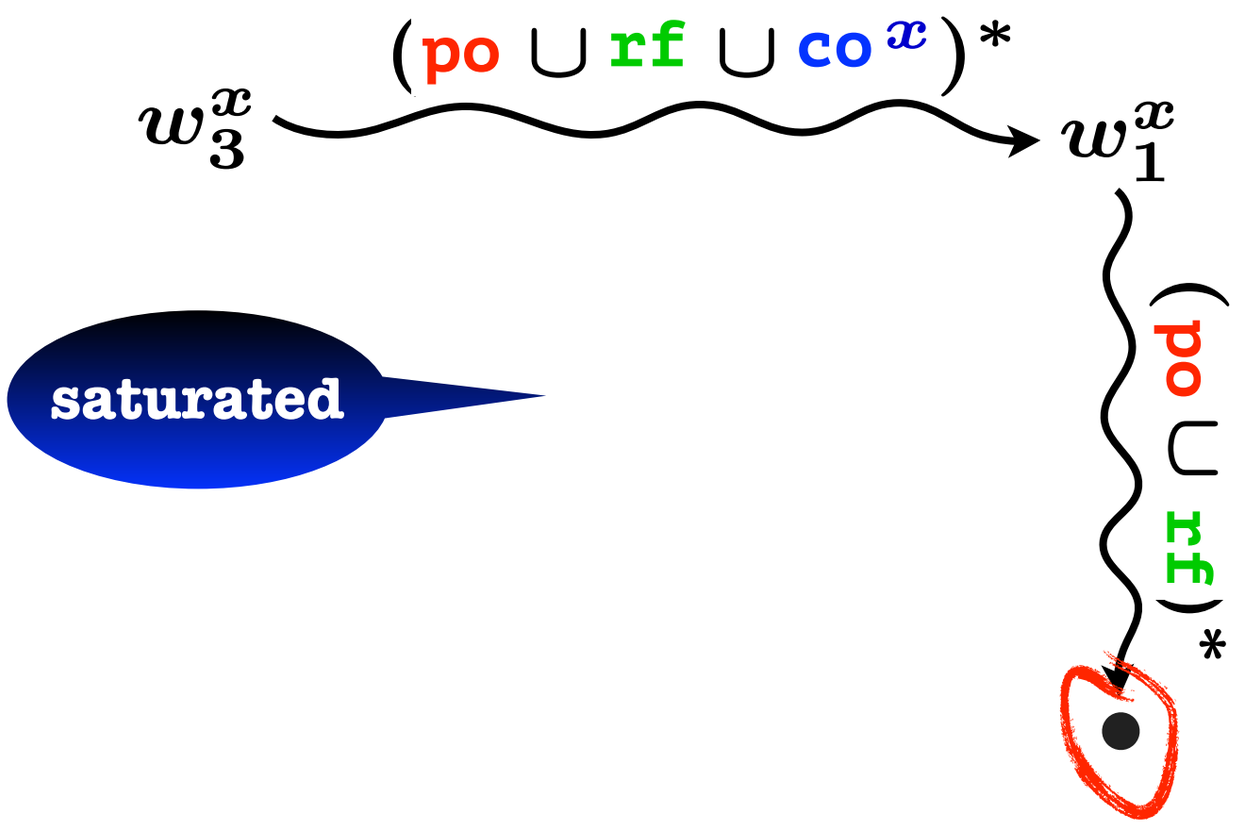

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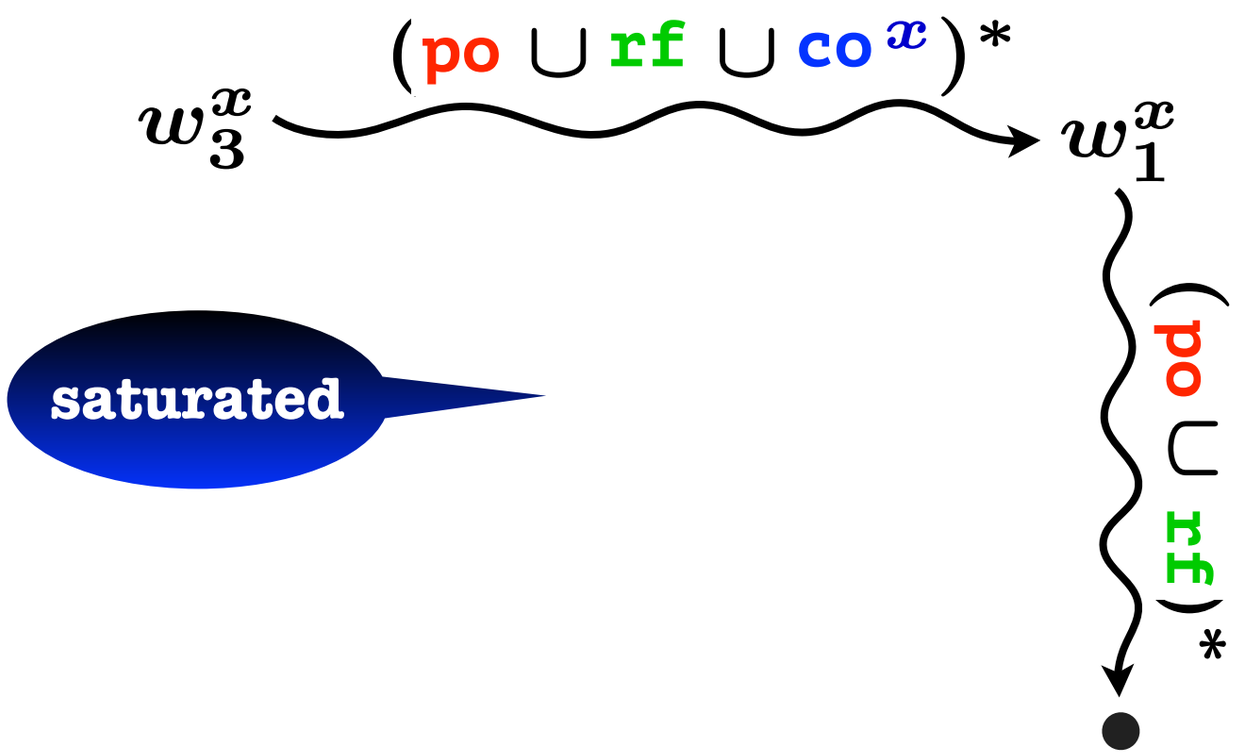

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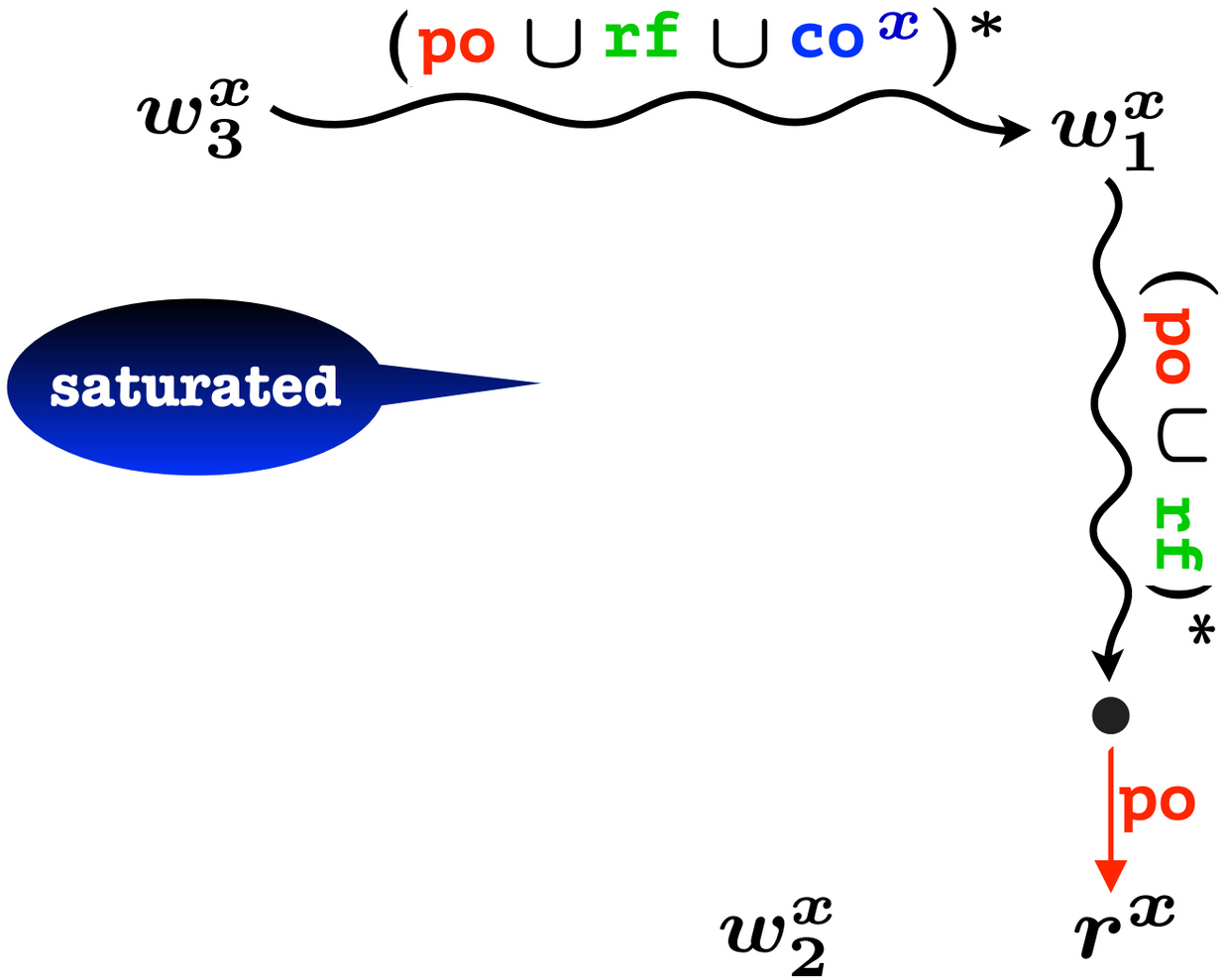

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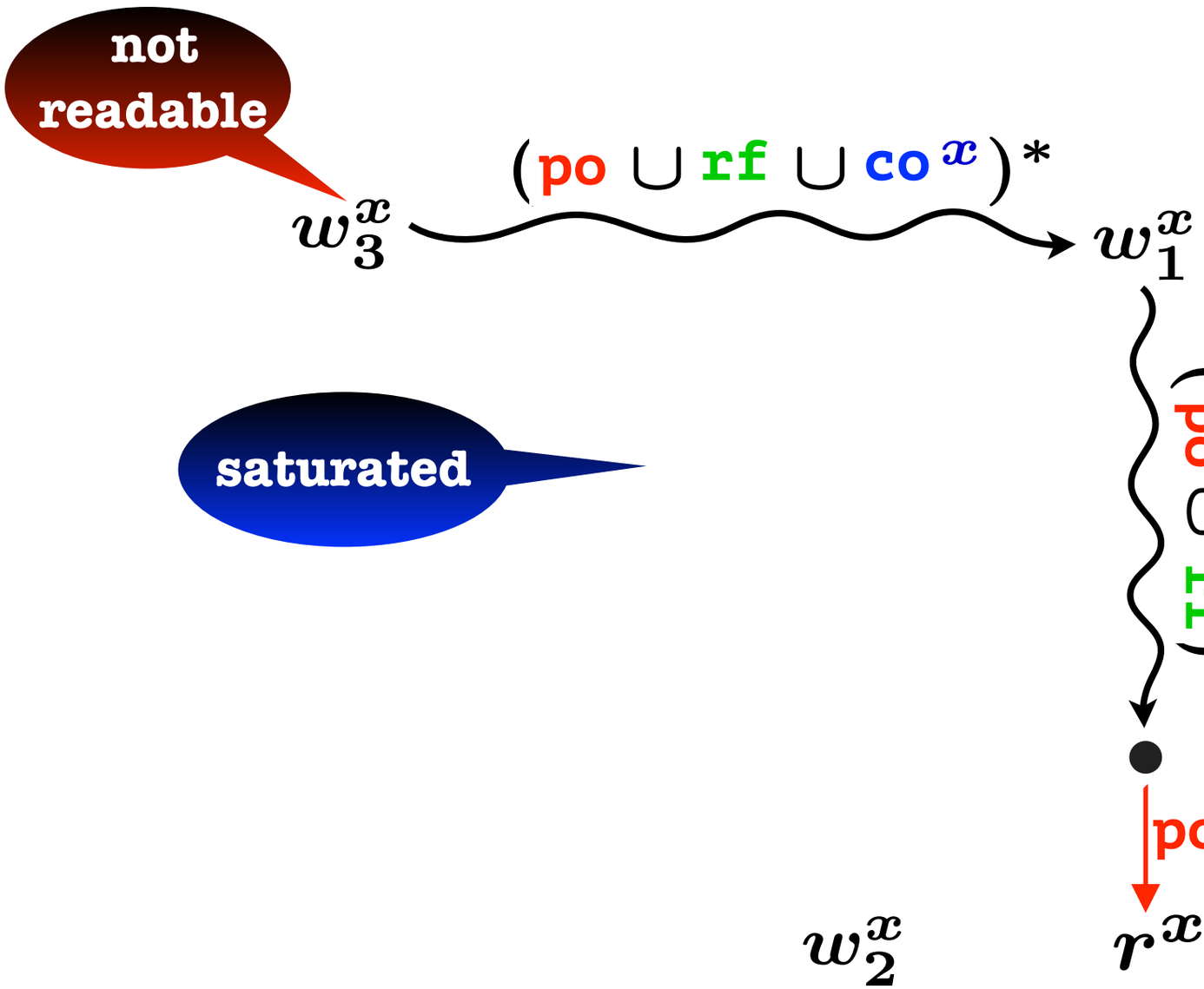

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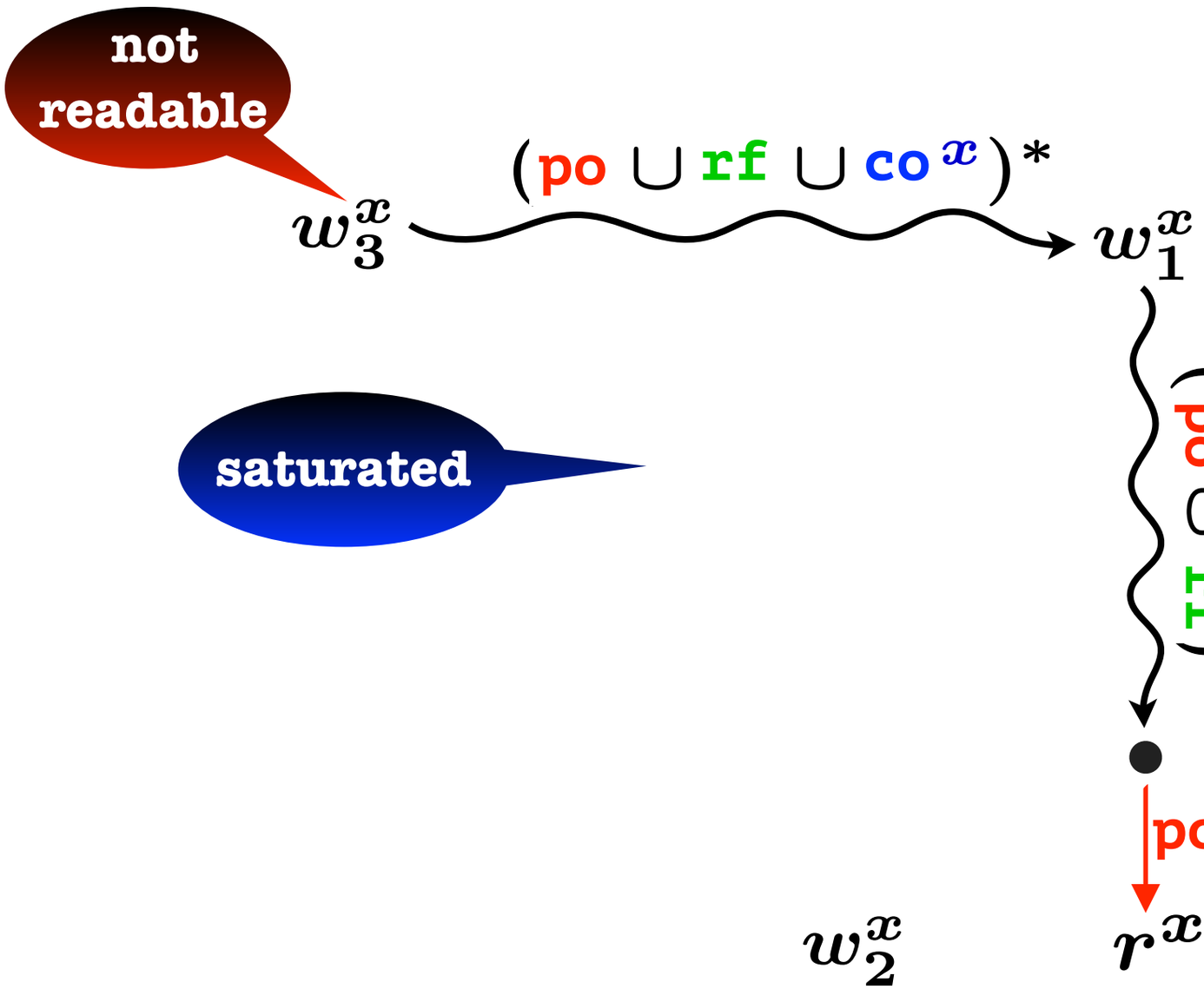

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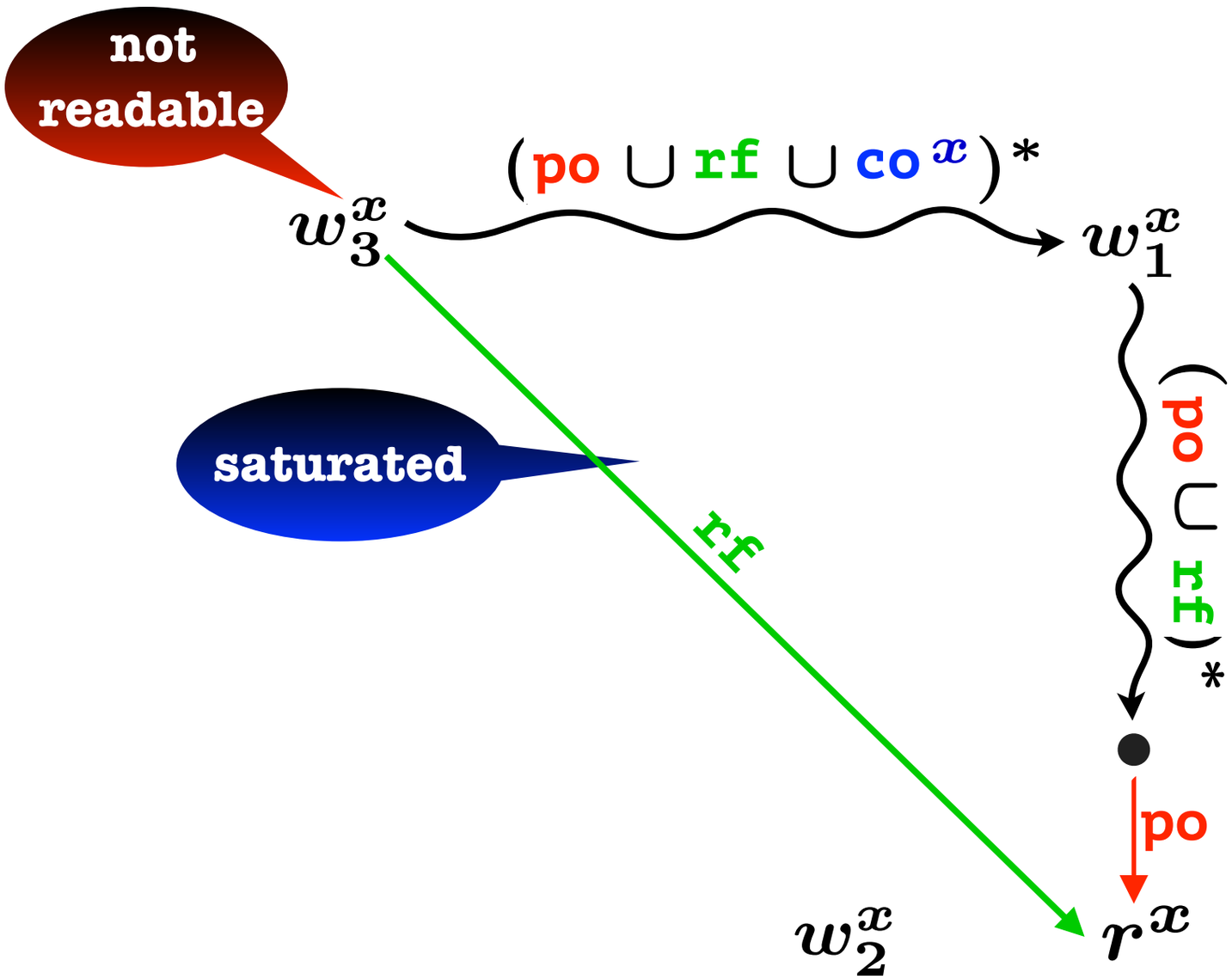

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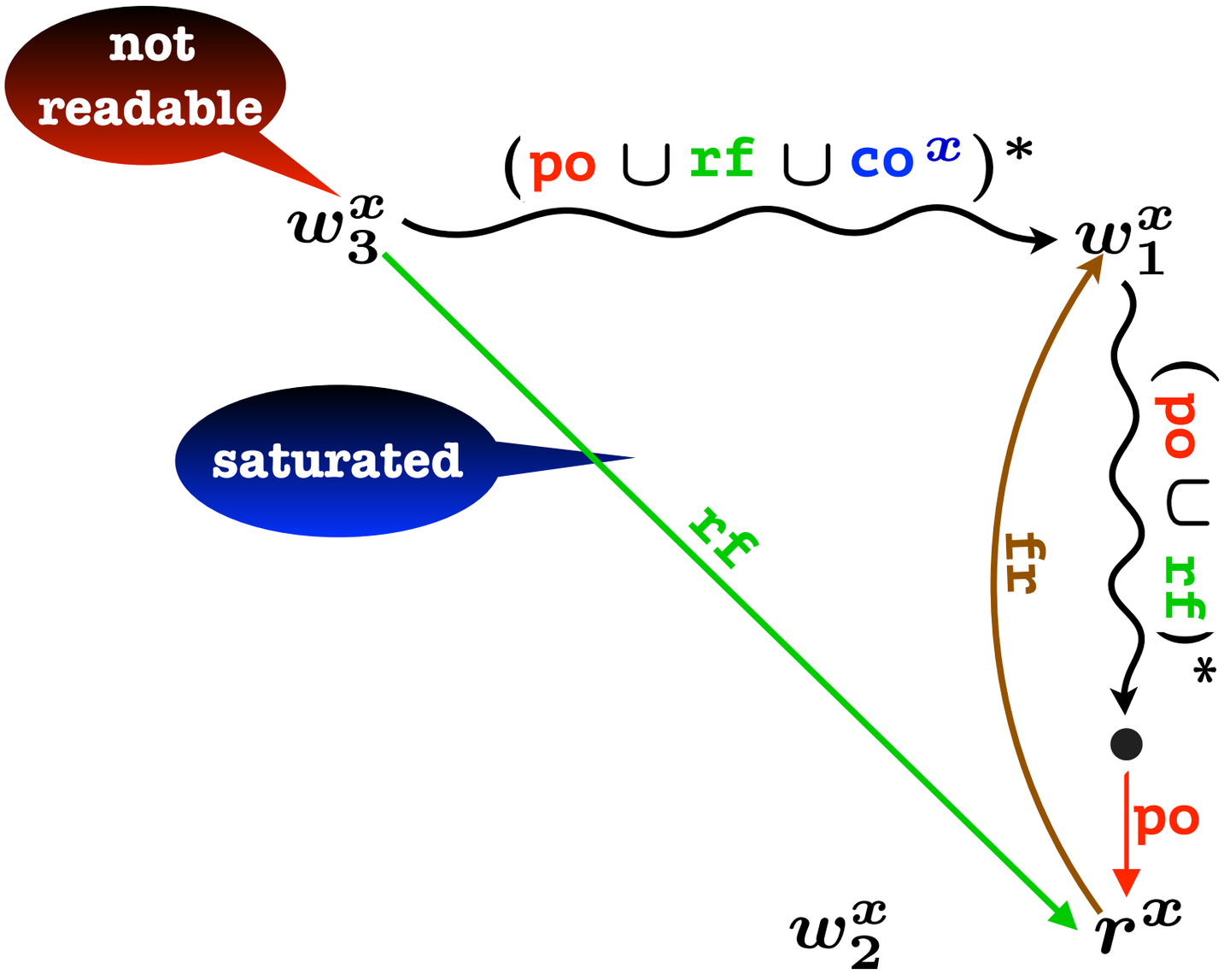

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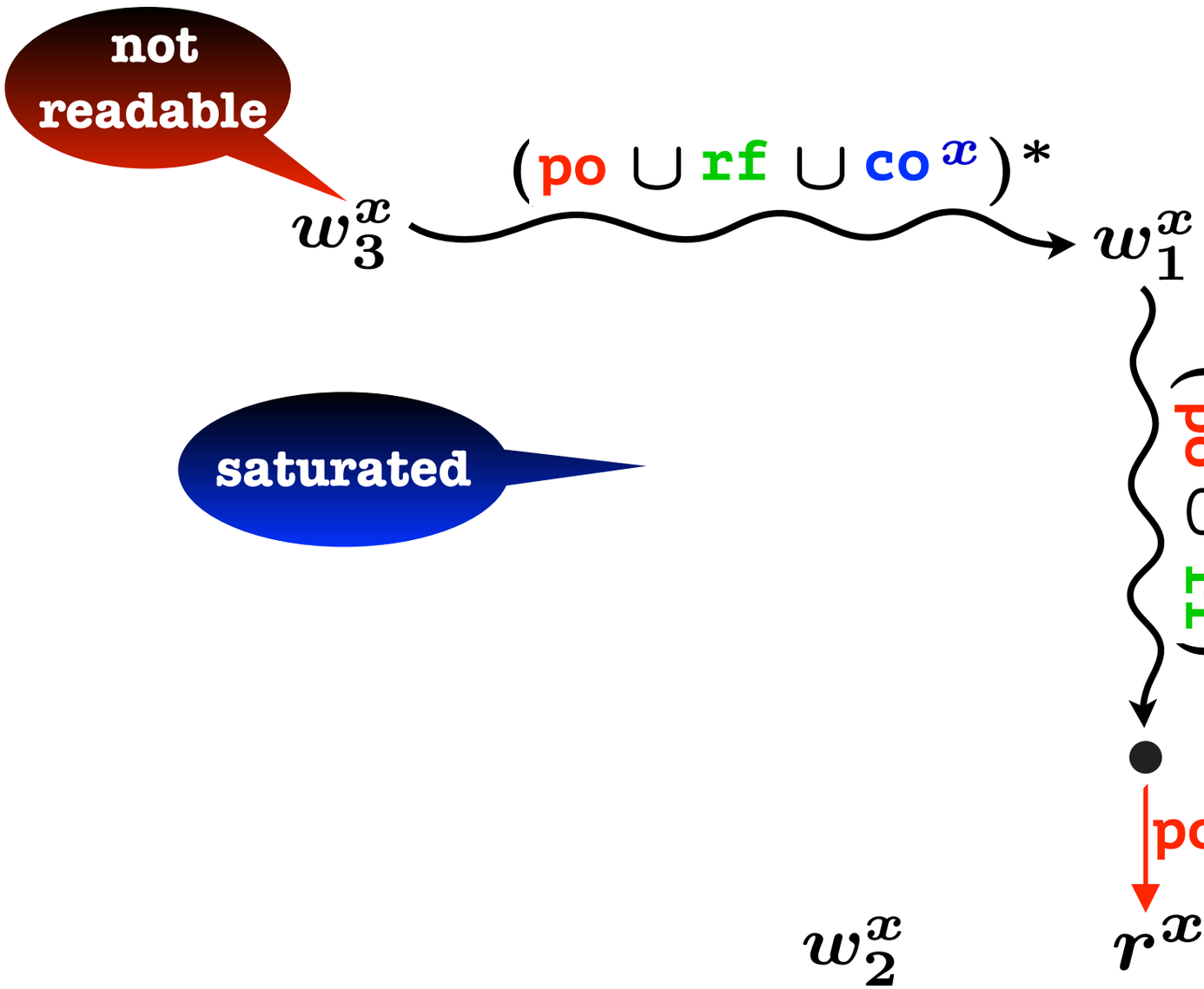

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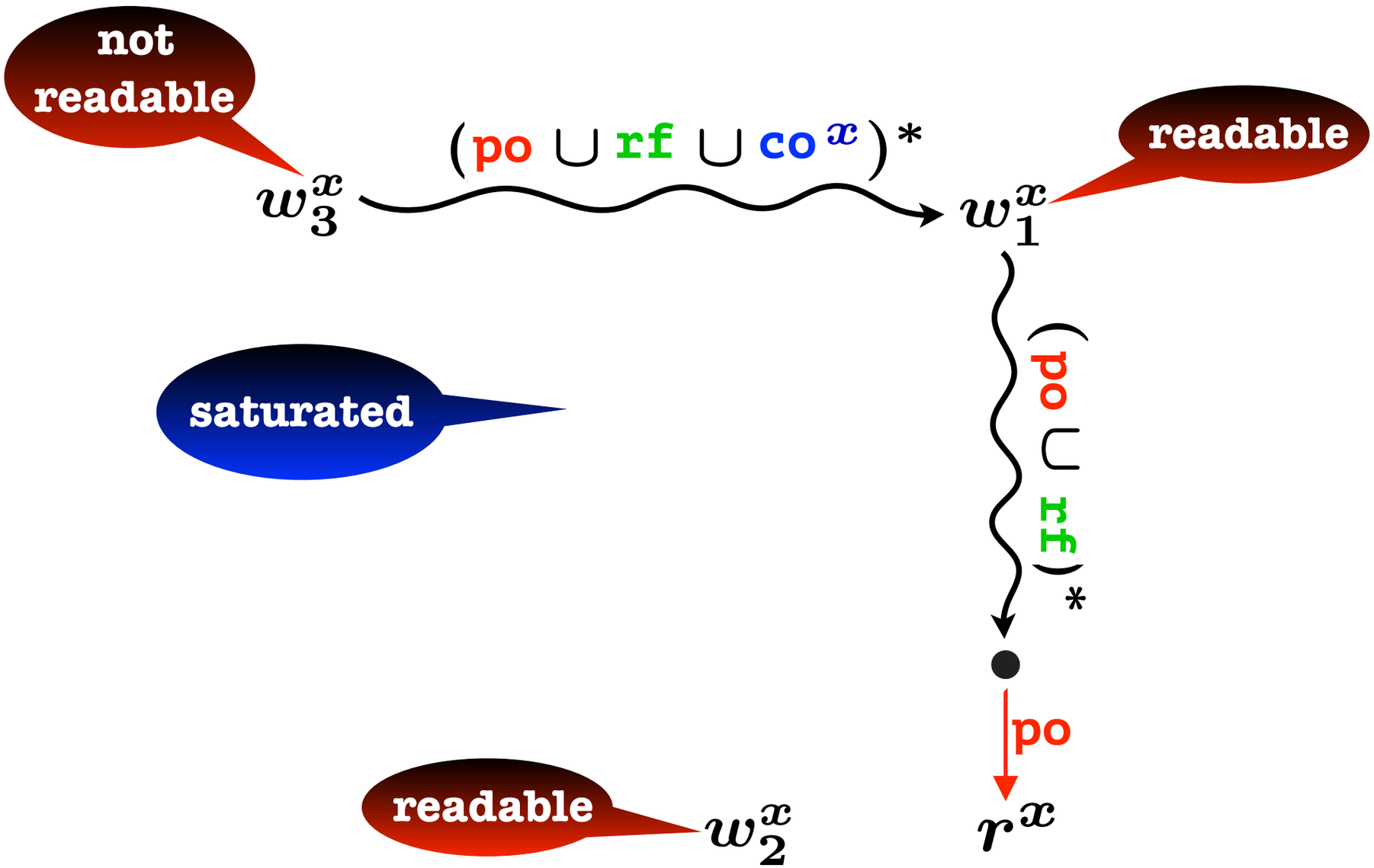

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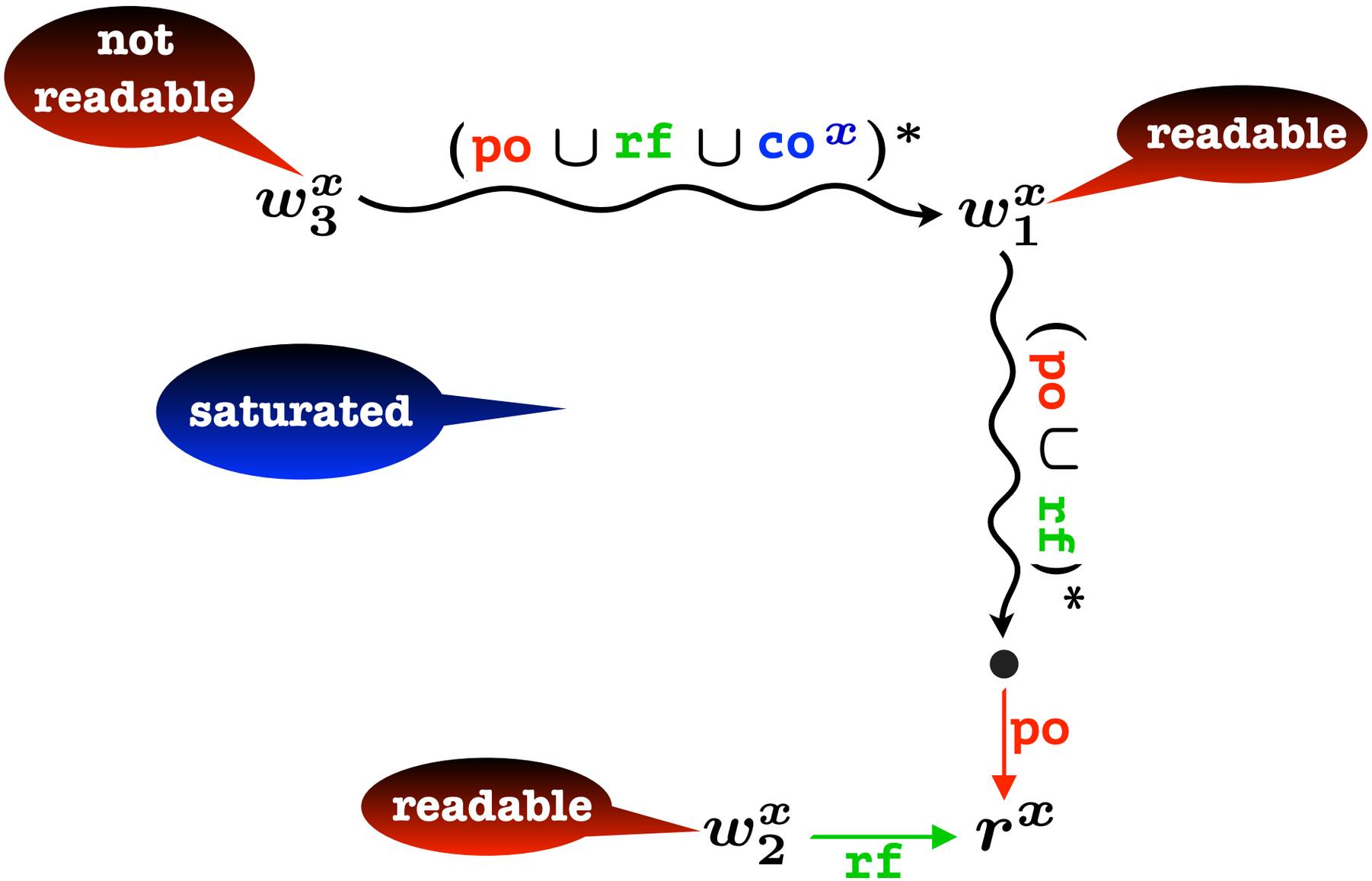

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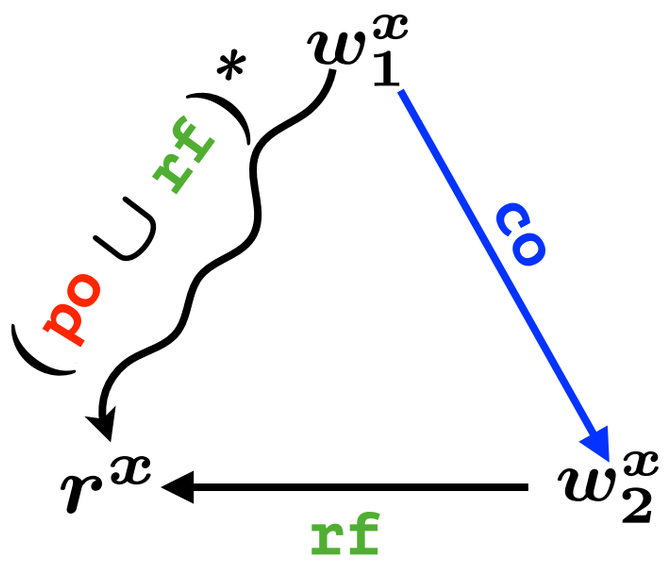

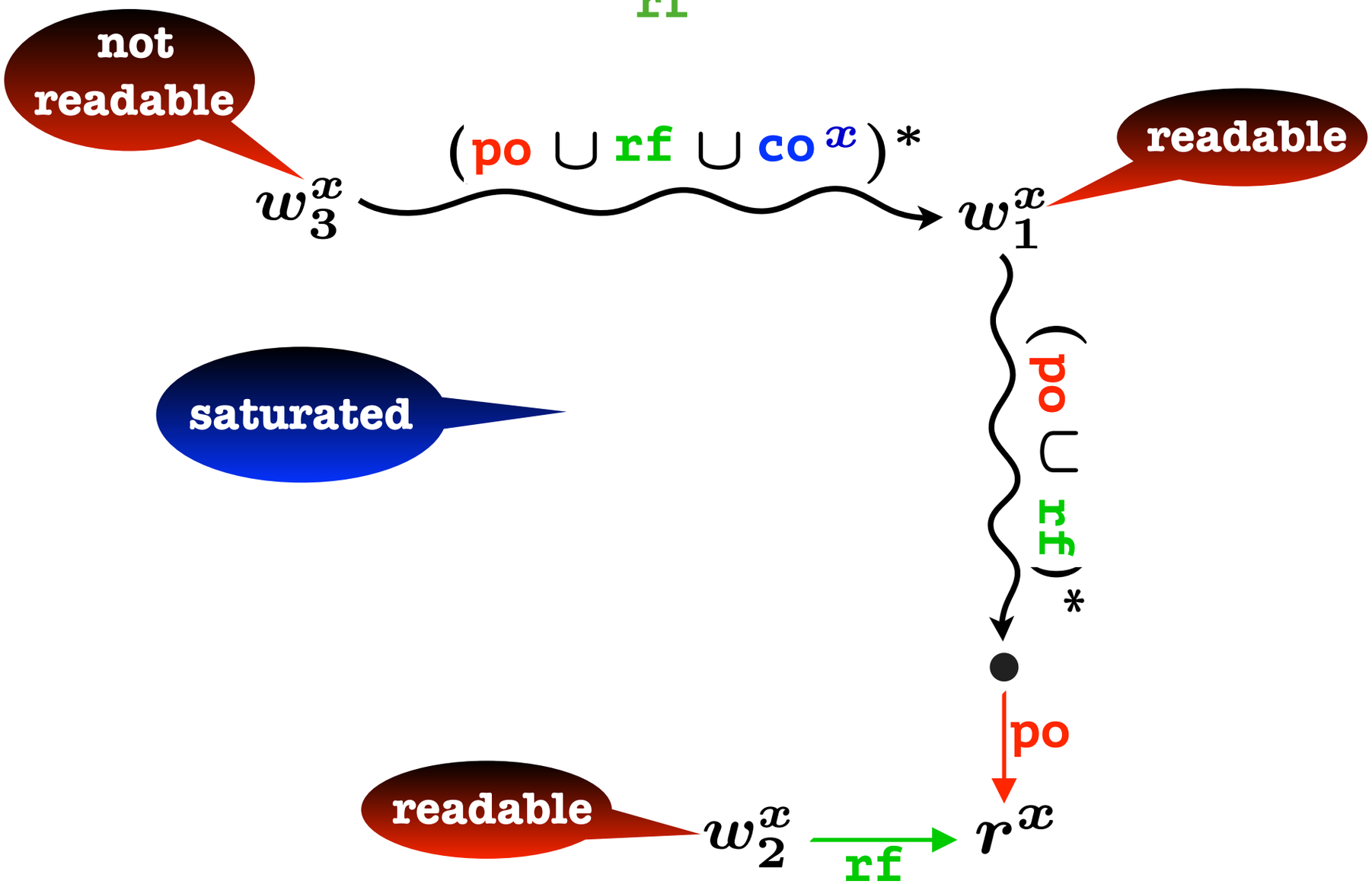

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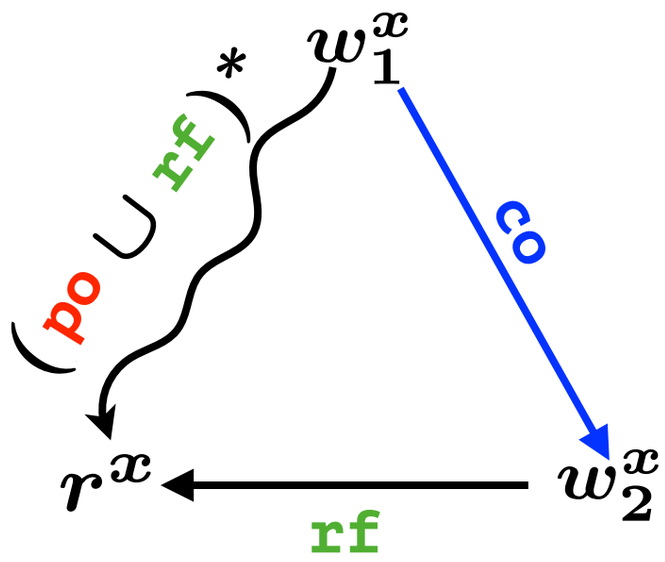

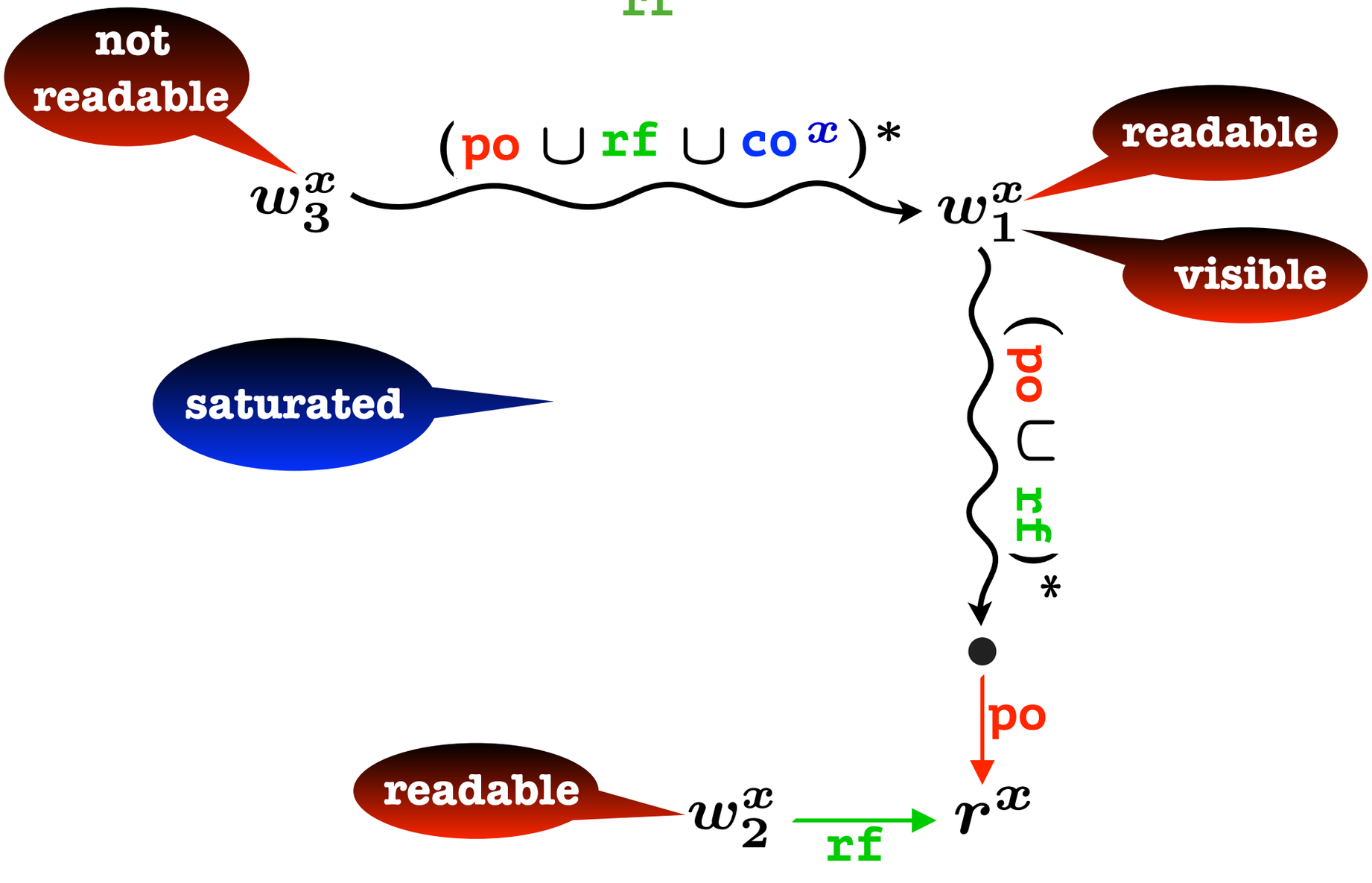

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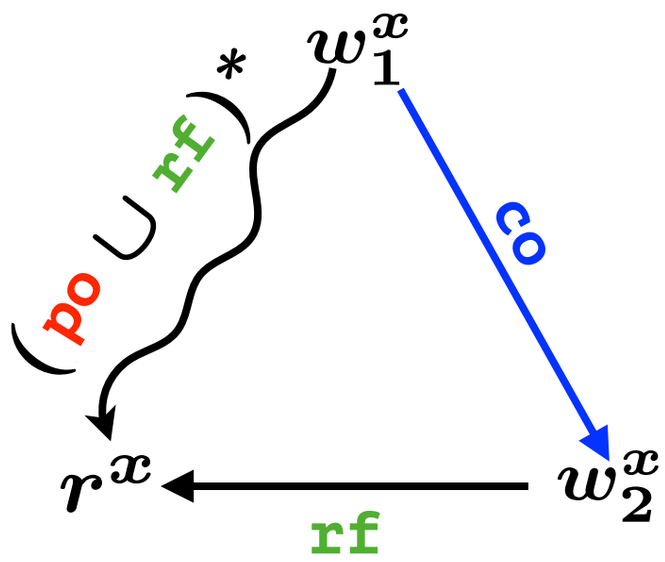

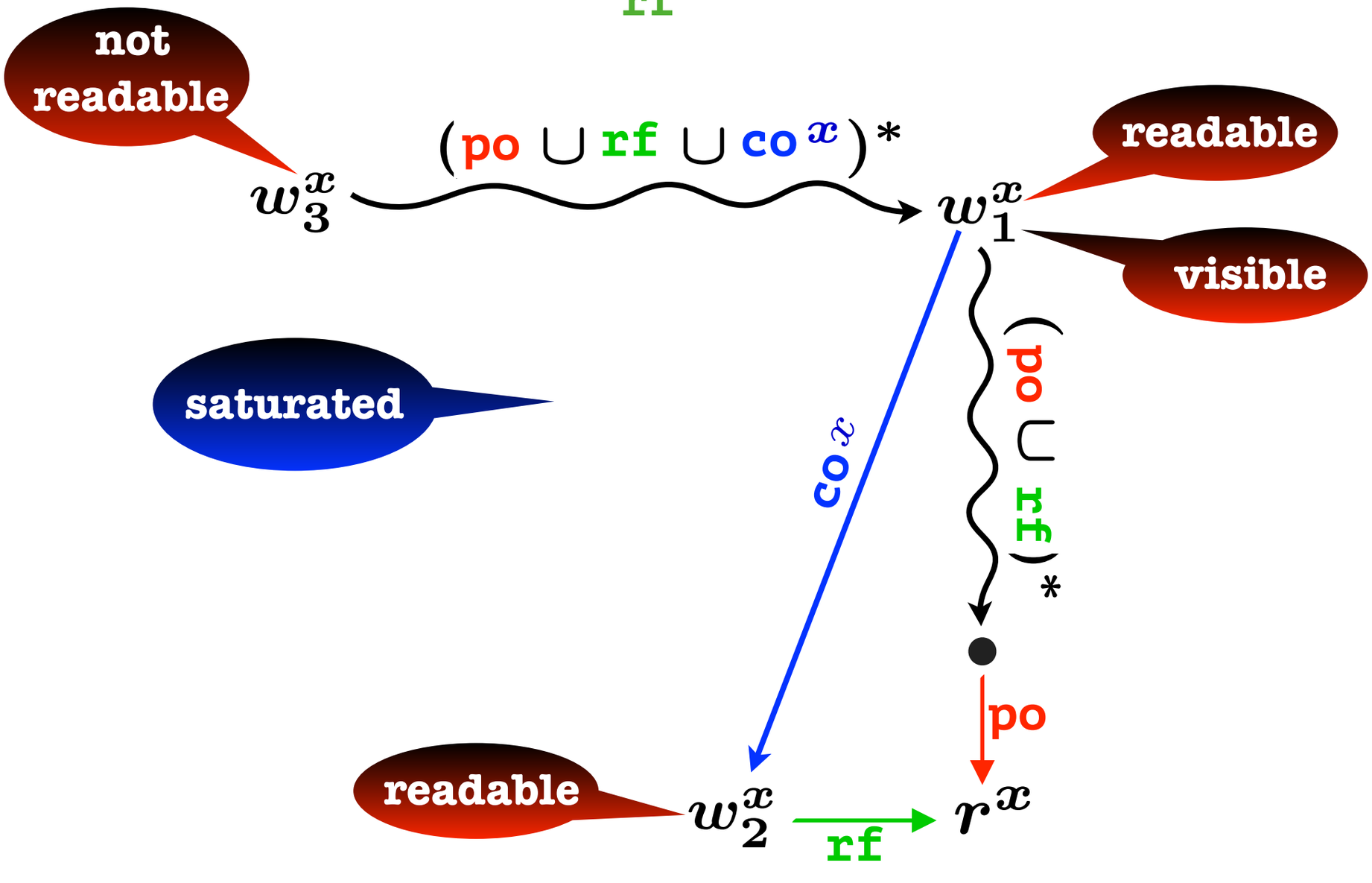

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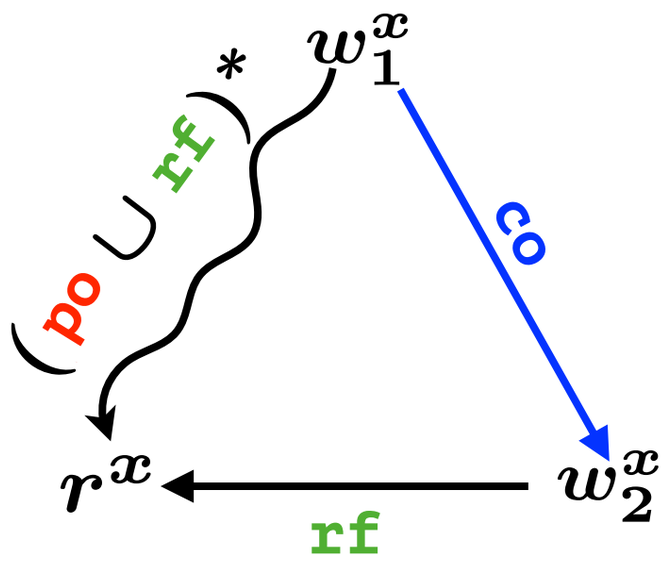

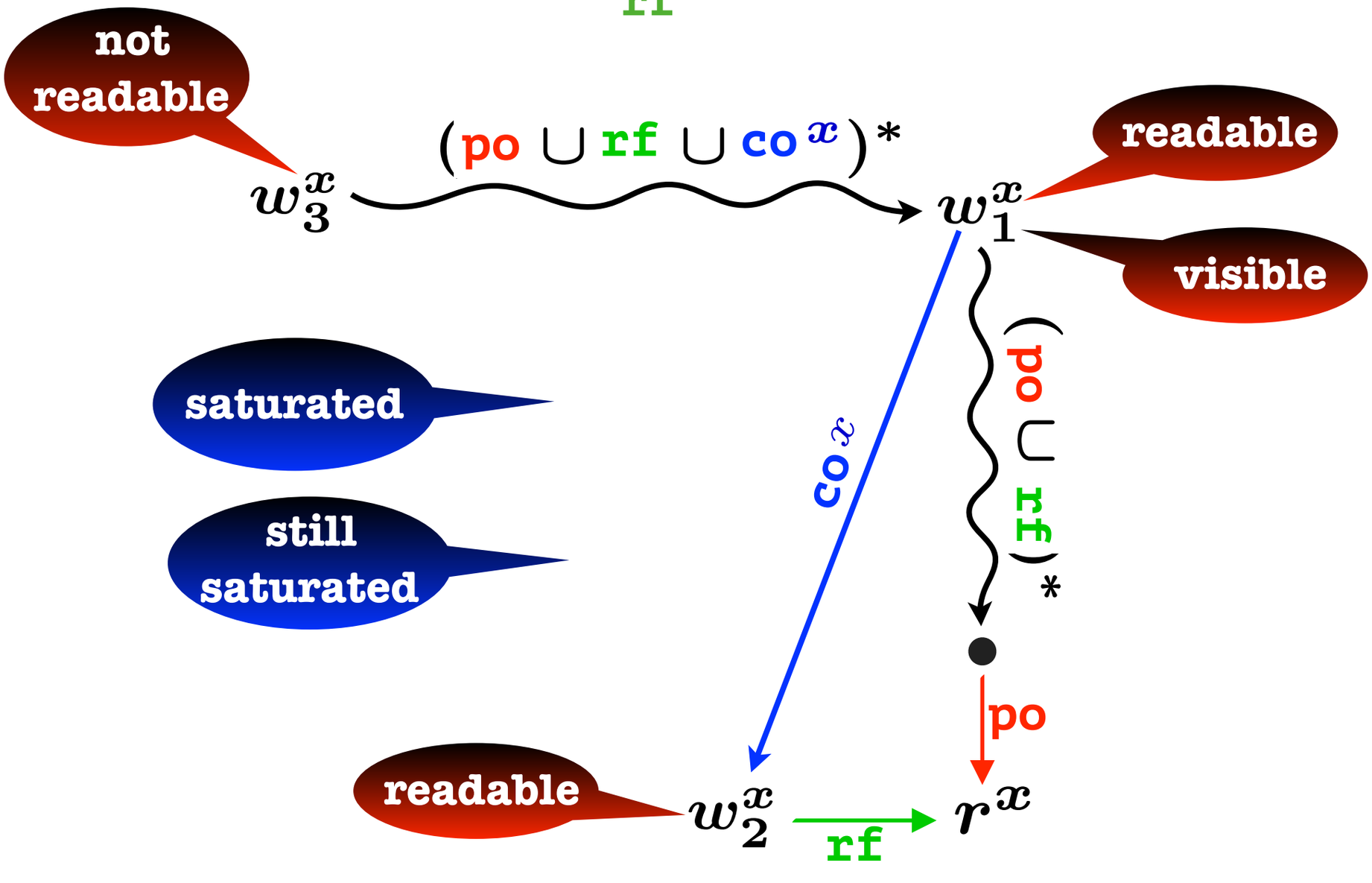

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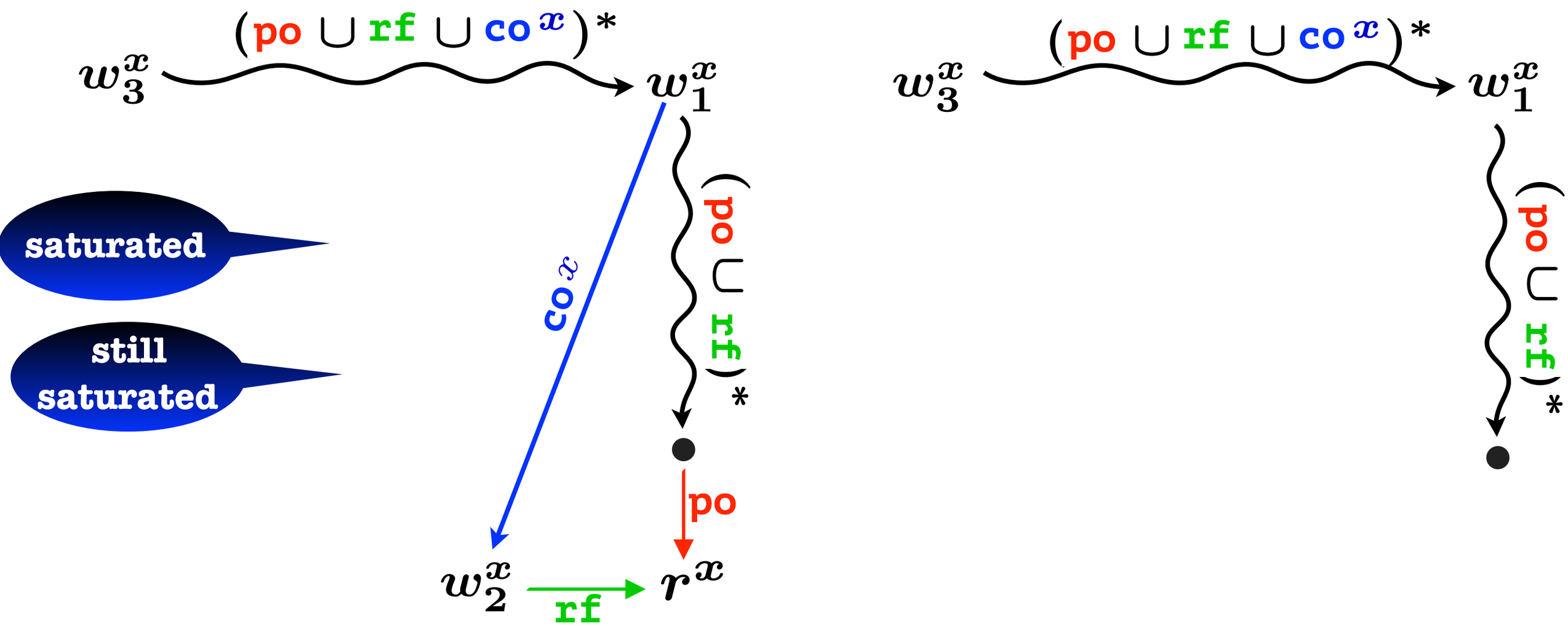

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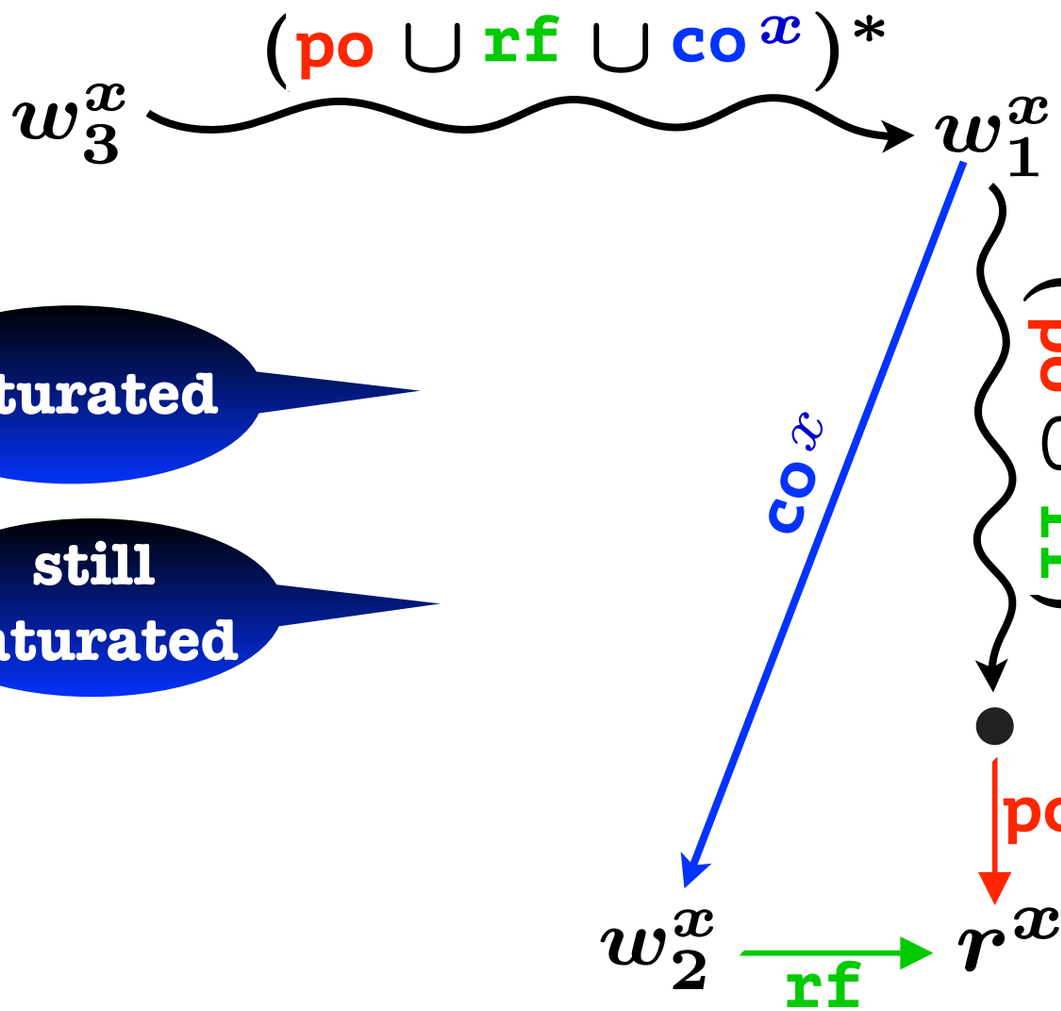
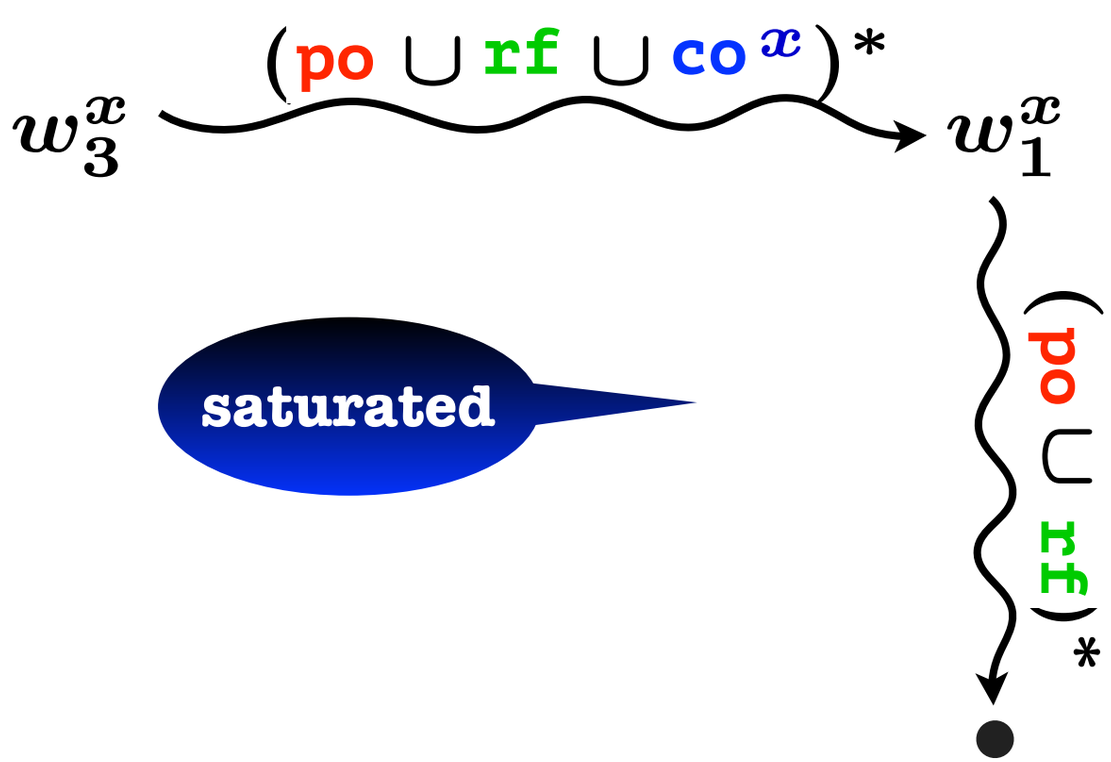
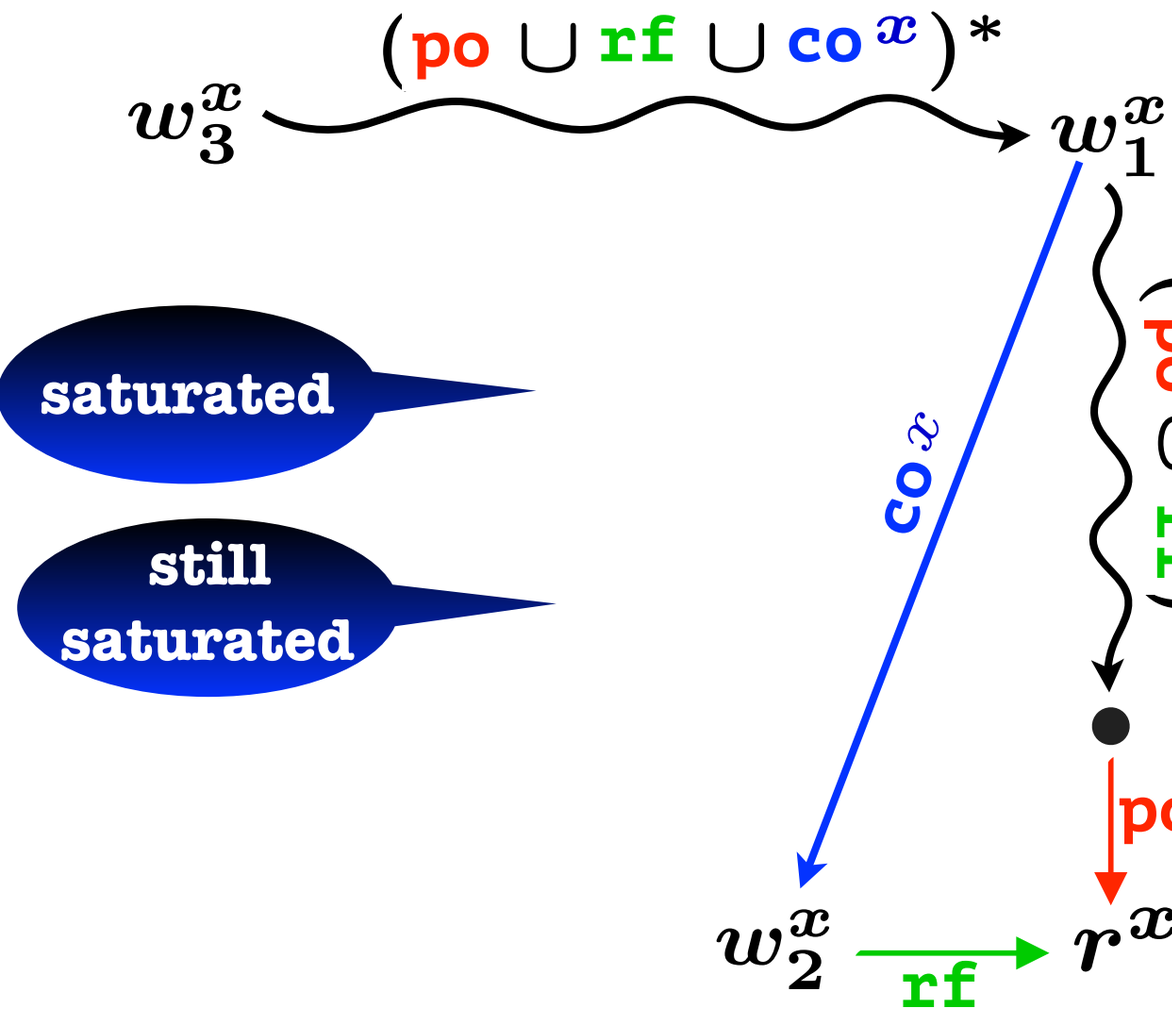
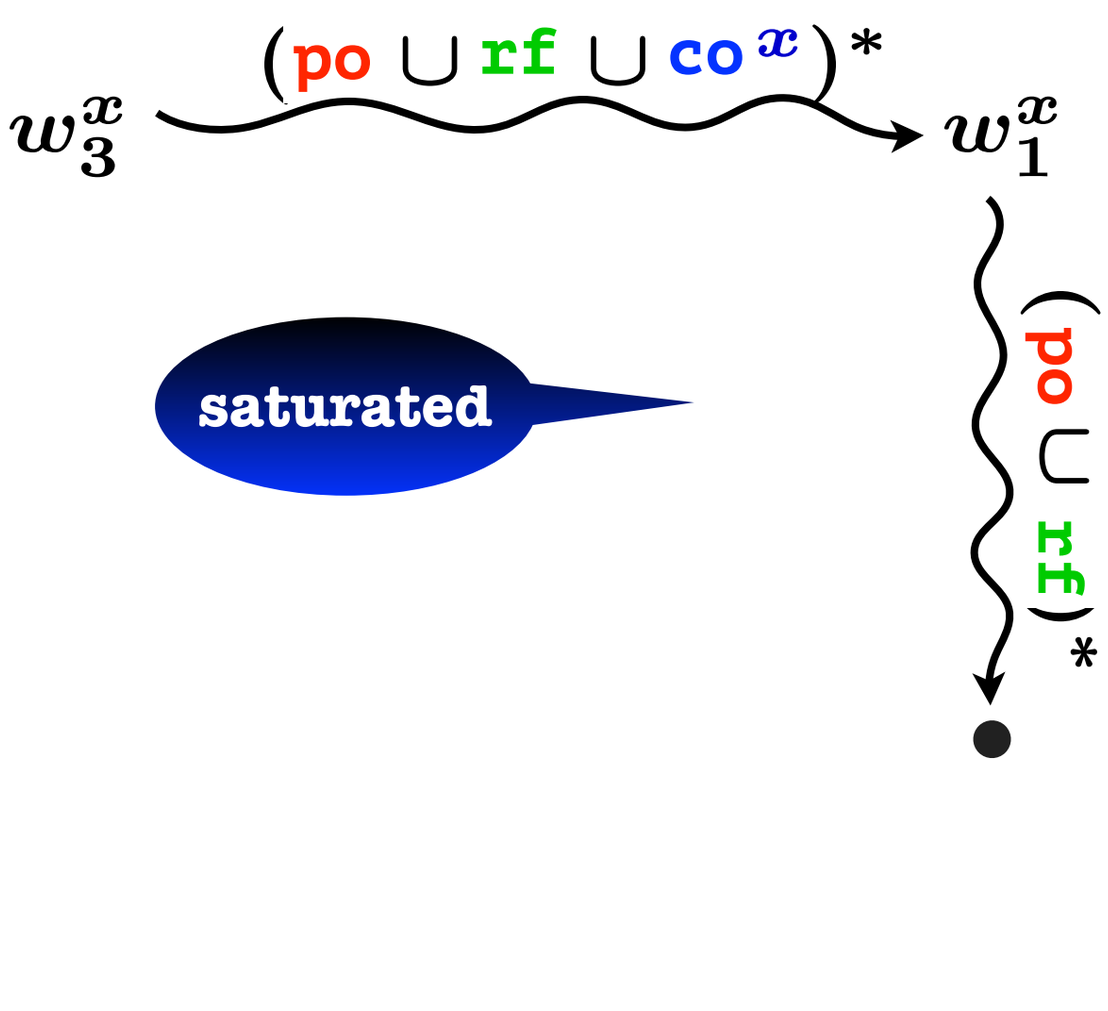


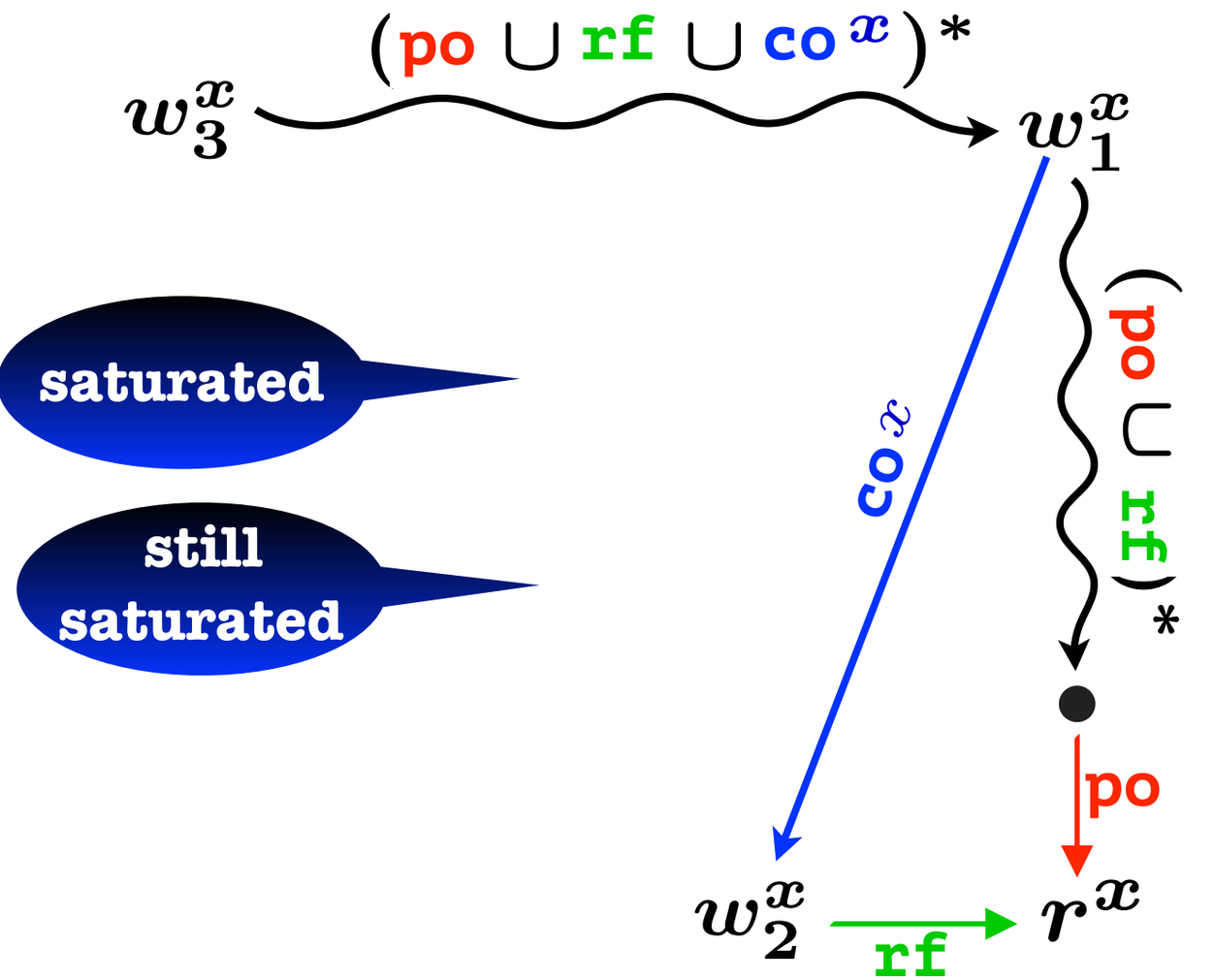


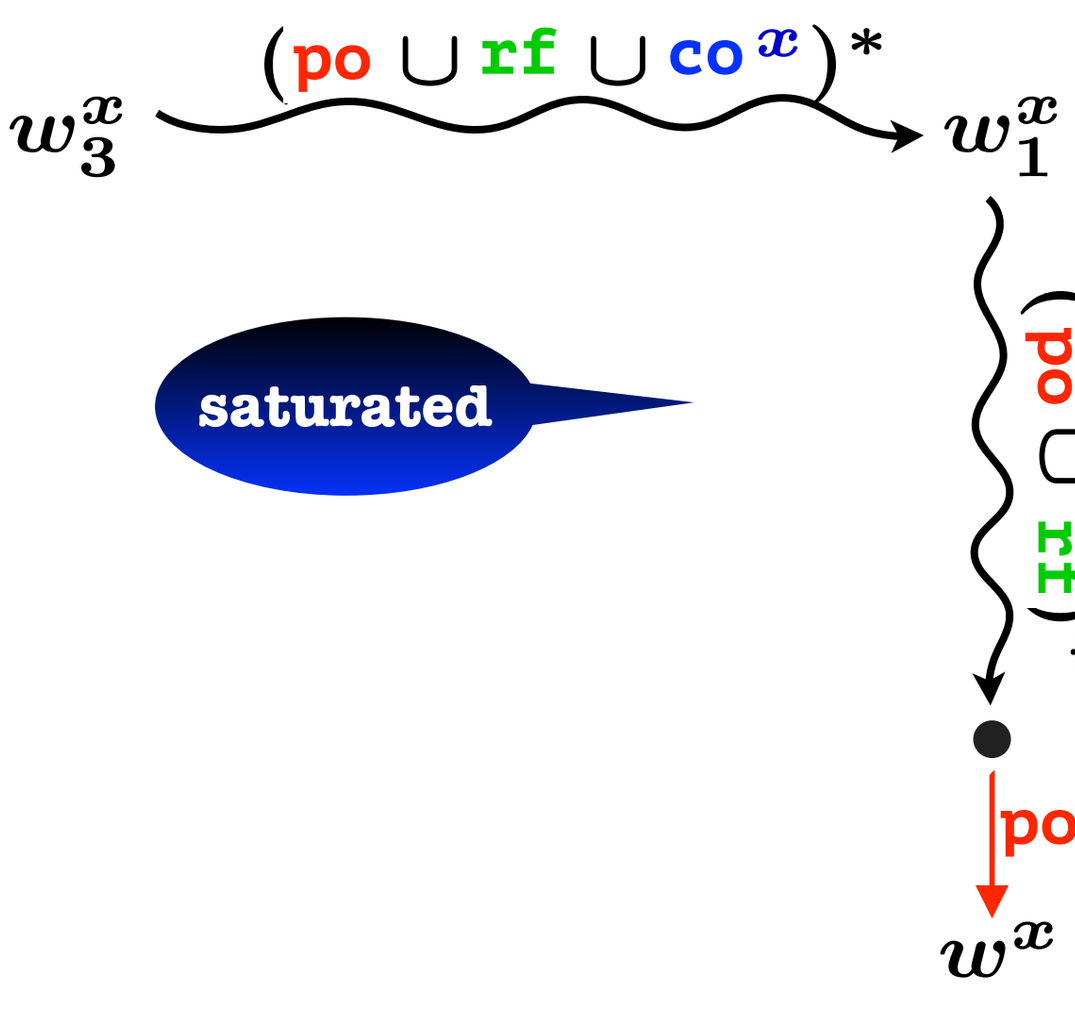
Adding Events to Traces



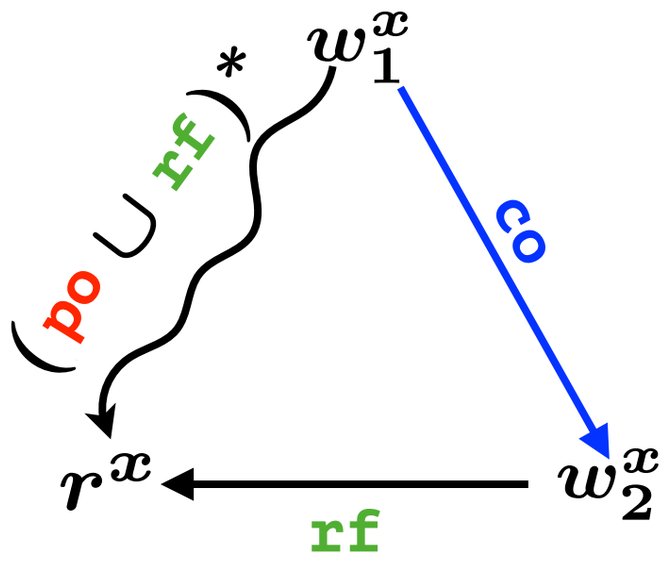


Adding Events to Traces



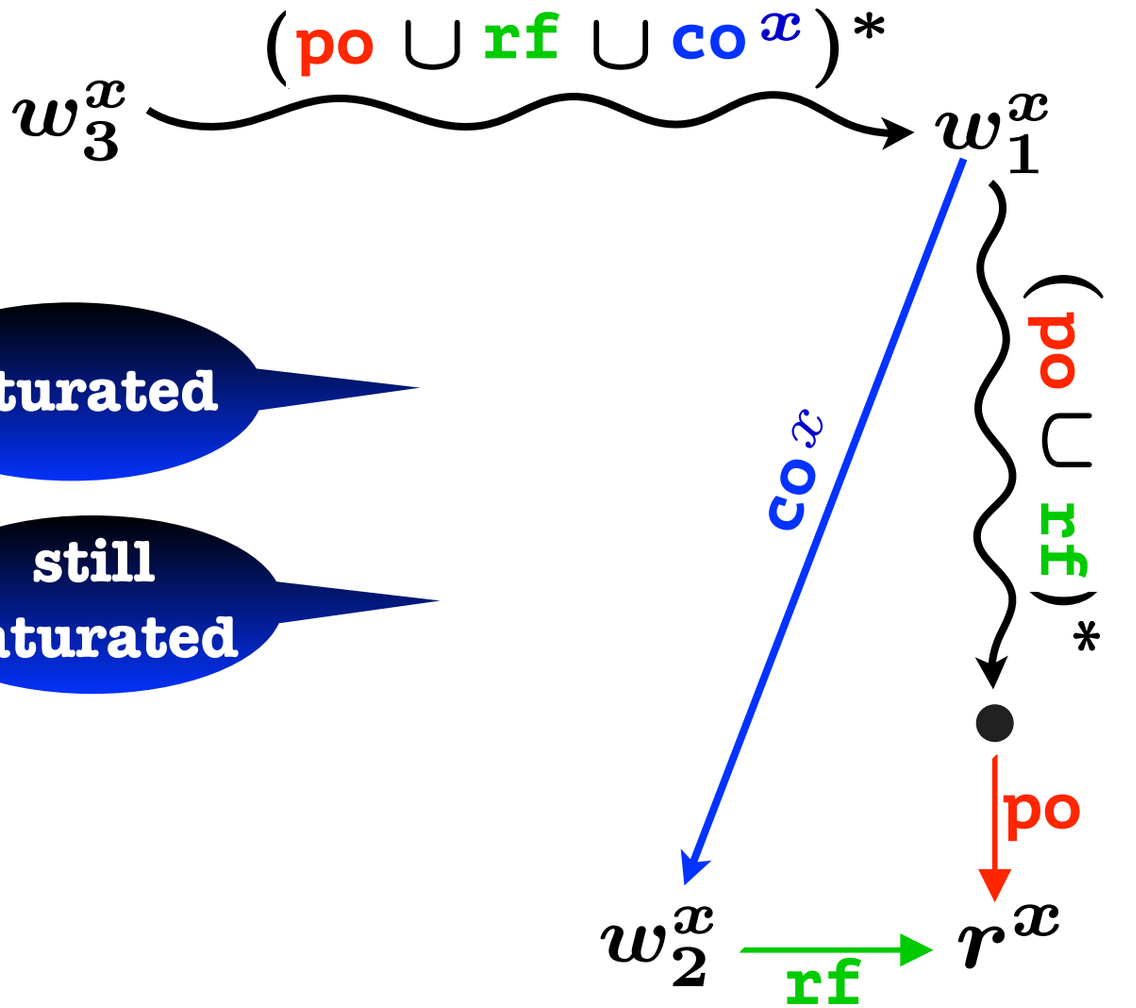


Adding Events to Traces

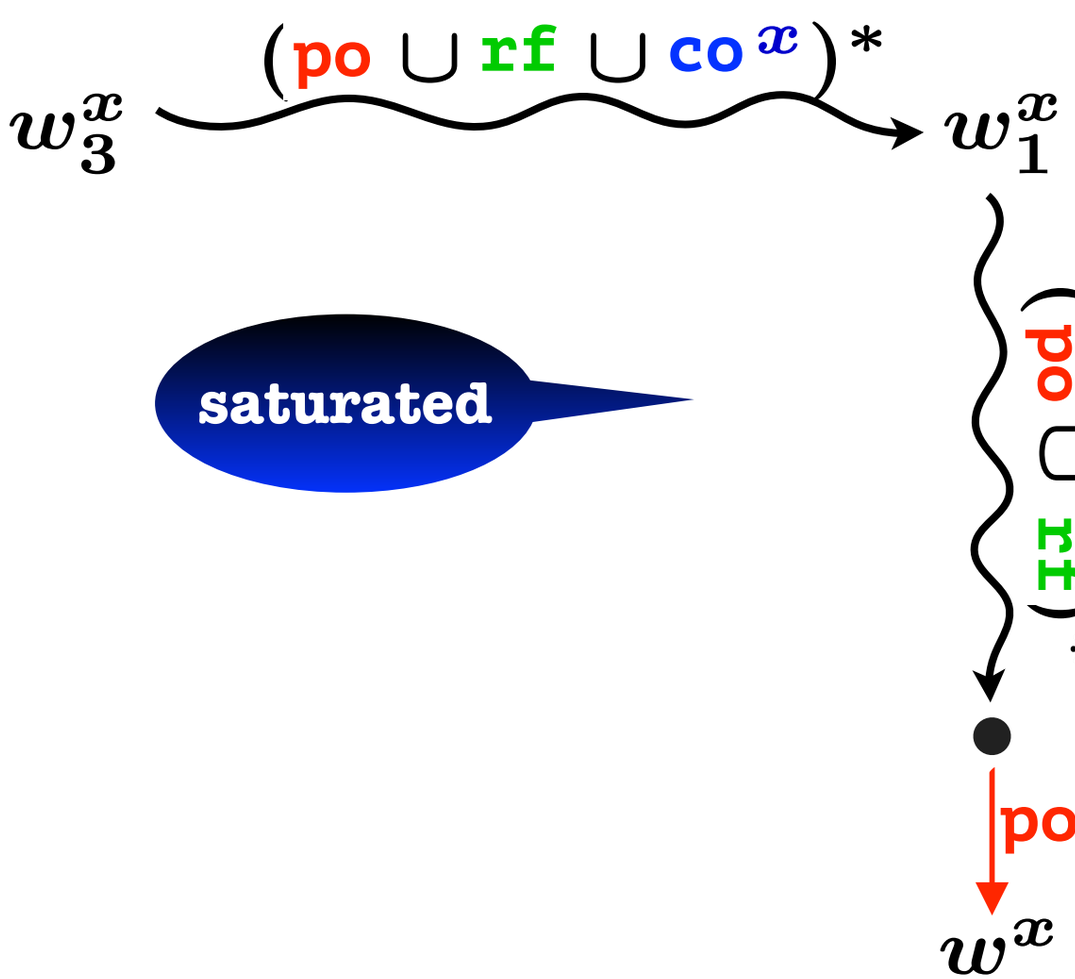


saturated

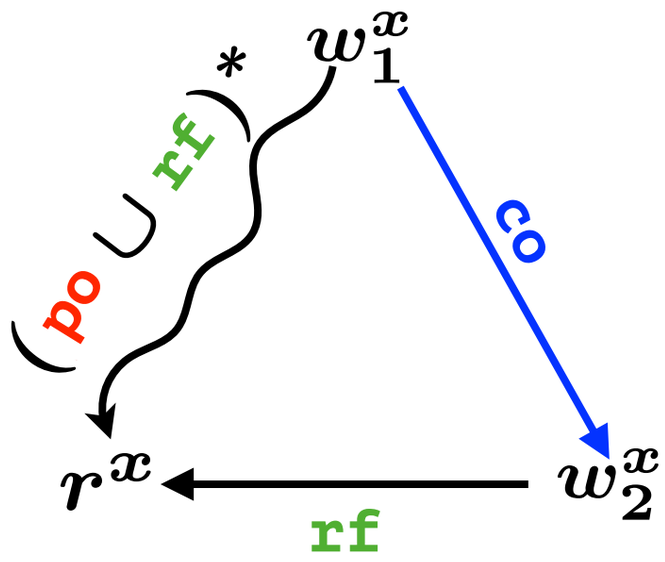
still saturated

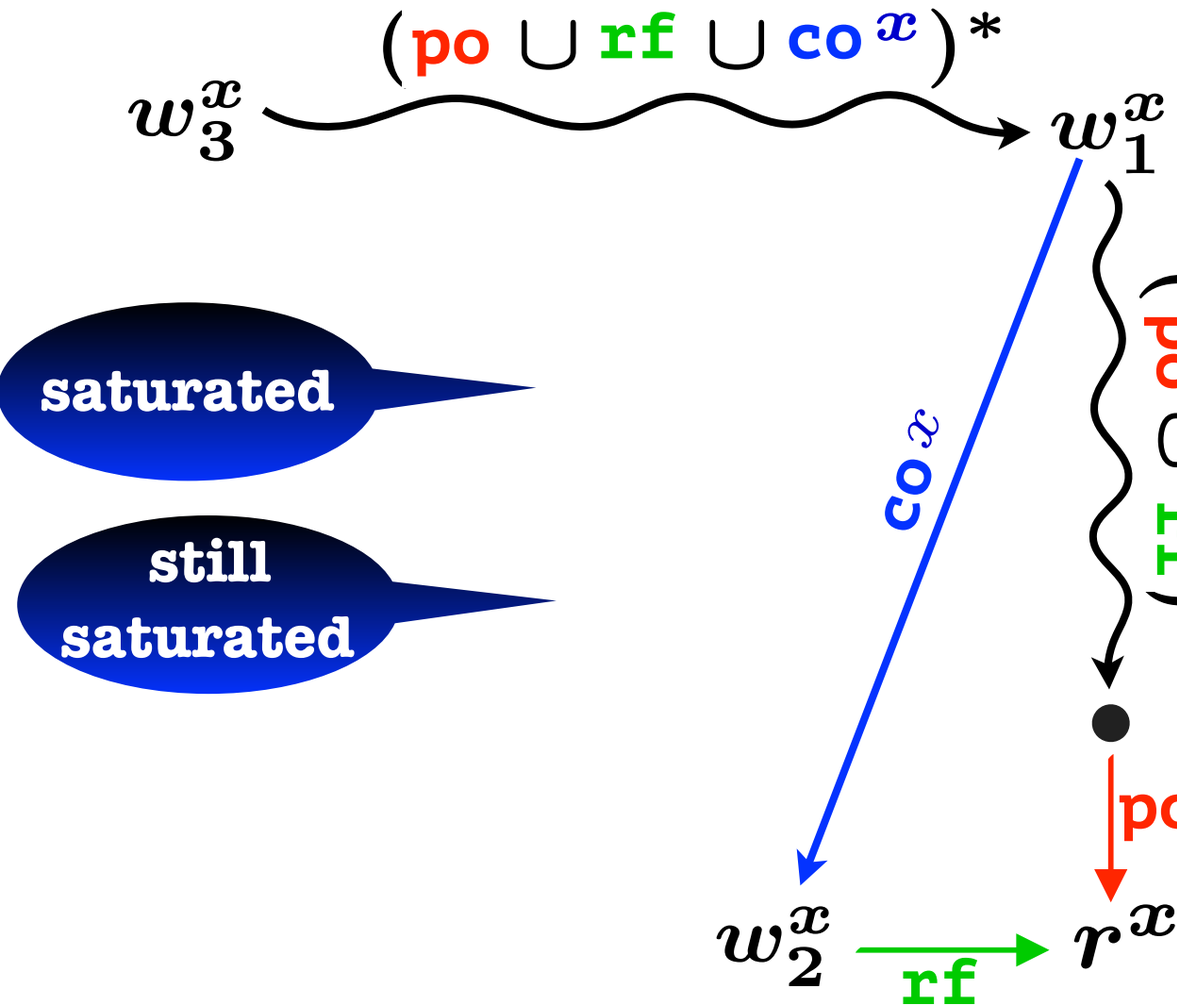


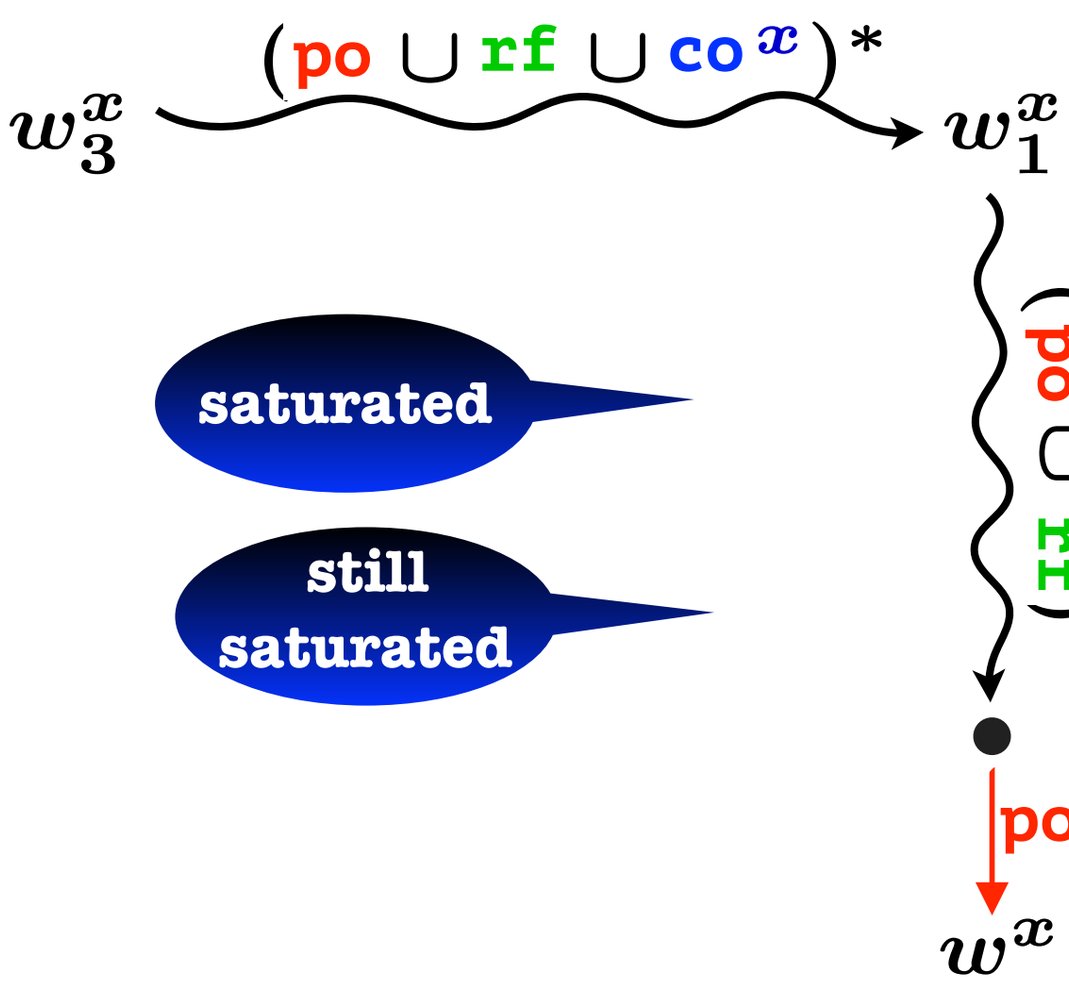
saturated



Adding Events to Traces







DPOR under the RA-semantics

- Build traces one after one
- For a given trace τ :
  - Build τ incrementally
  - extend τ by one event e at a time

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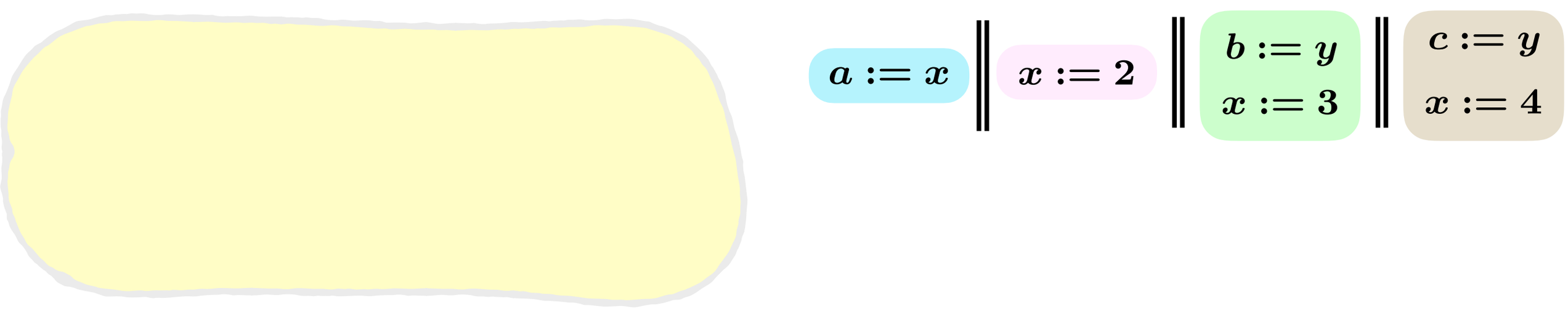

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

$x := 2$

$x := 2$

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

$x := 2$

$x := 2$

$a := x \parallel x := 2 \parallel \begin{matrix} b := y \\ x := 3 \end{matrix} \parallel \begin{matrix} c := y \\ x := 4 \end{matrix}$

$x := 2$

$x := 2$

$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

$x := 2$

$x := 2$

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

$x := 2$

$a := 0$ $a := 2$

$x := 2$

$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

$x := 2$

$a := 0$ $\quad$ $a := 2$

read-branching

$x := 2$

$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

schedule

$x := 2$

$a := 0$

$a := 2$

read-branching

$x := 2$

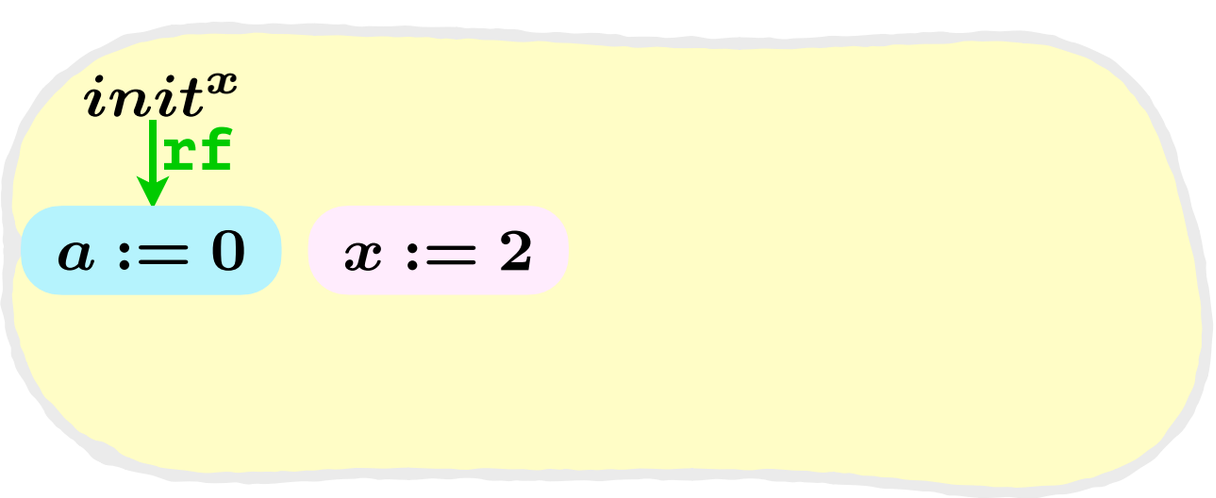
$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

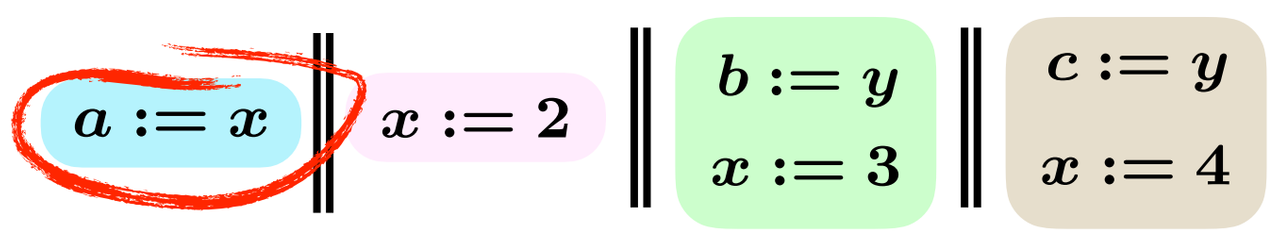
schedule

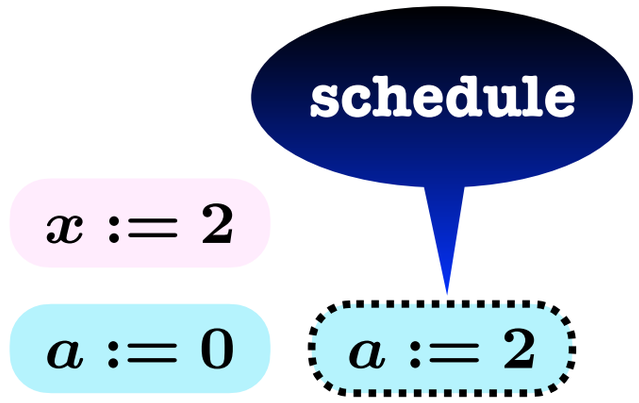
$x := 2$

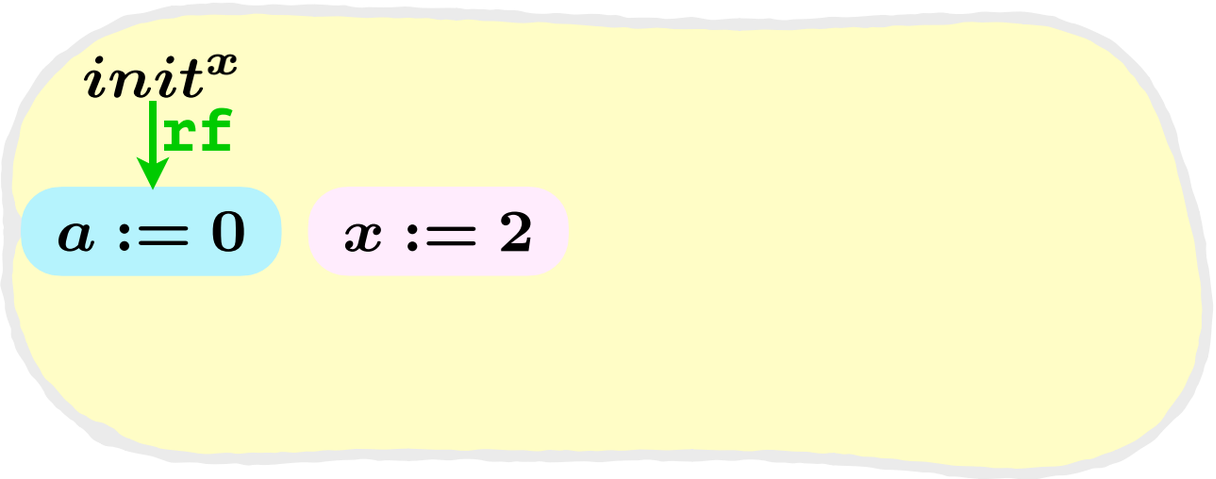
$a := 0$

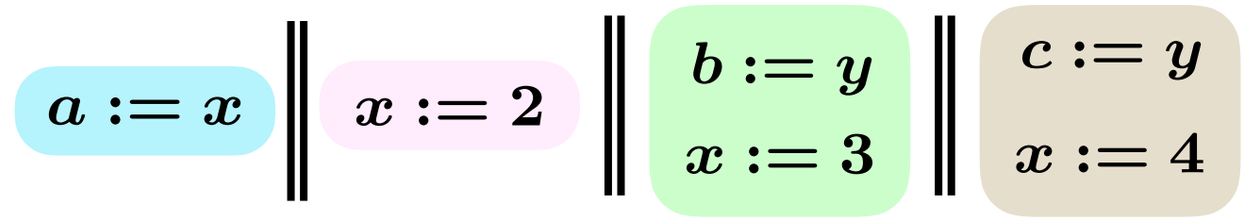
$a := 2$

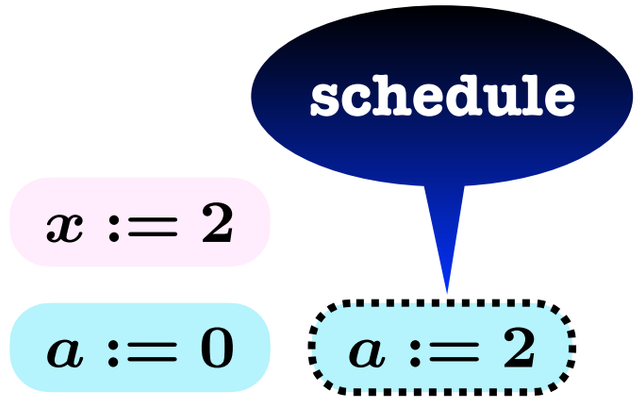

$init^x$
rf
$a := 0$
$x := 2$

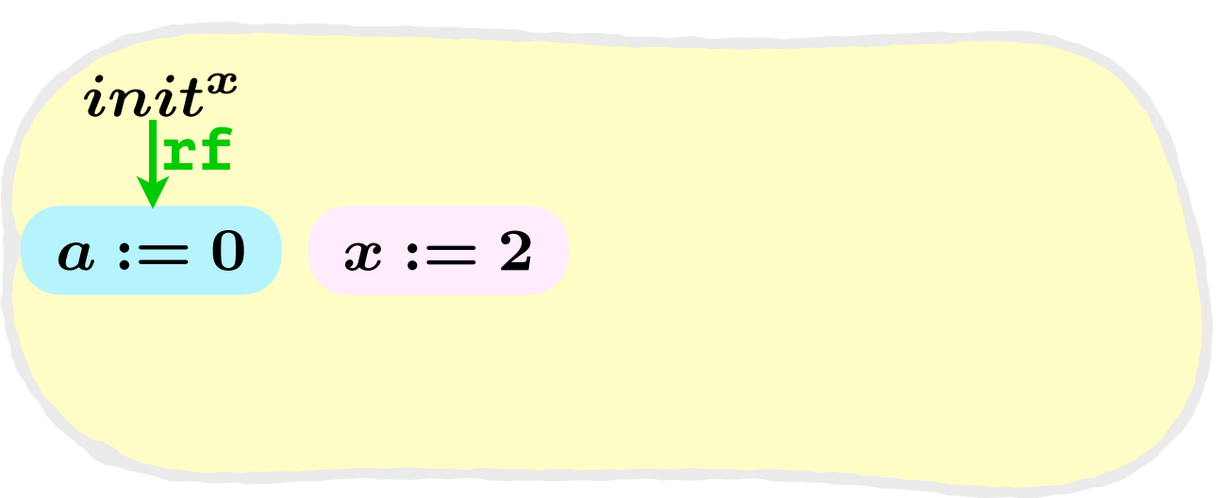

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

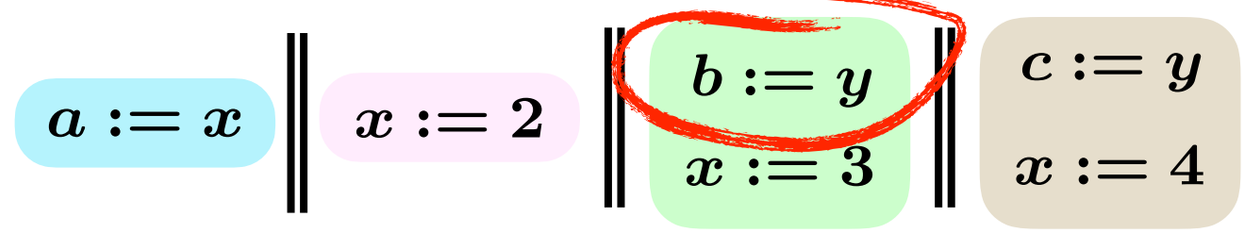

schedule
$x := 2$
$a := 0$
$a := 2$

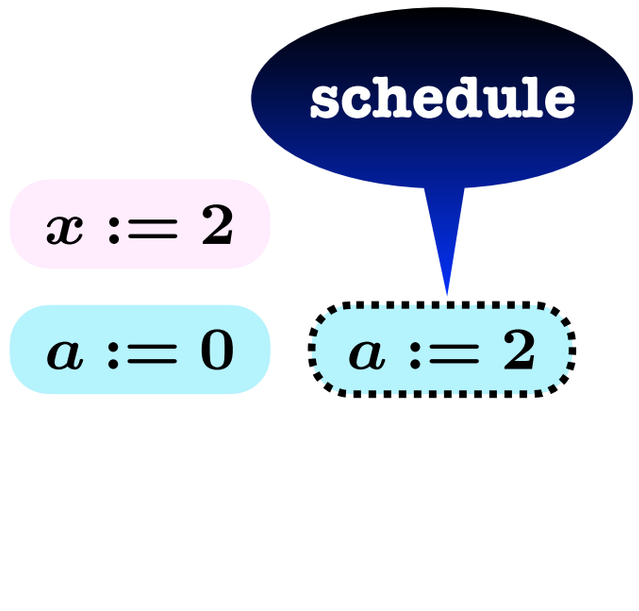

$init^x$
rf
$a := 0$
$x := 2$

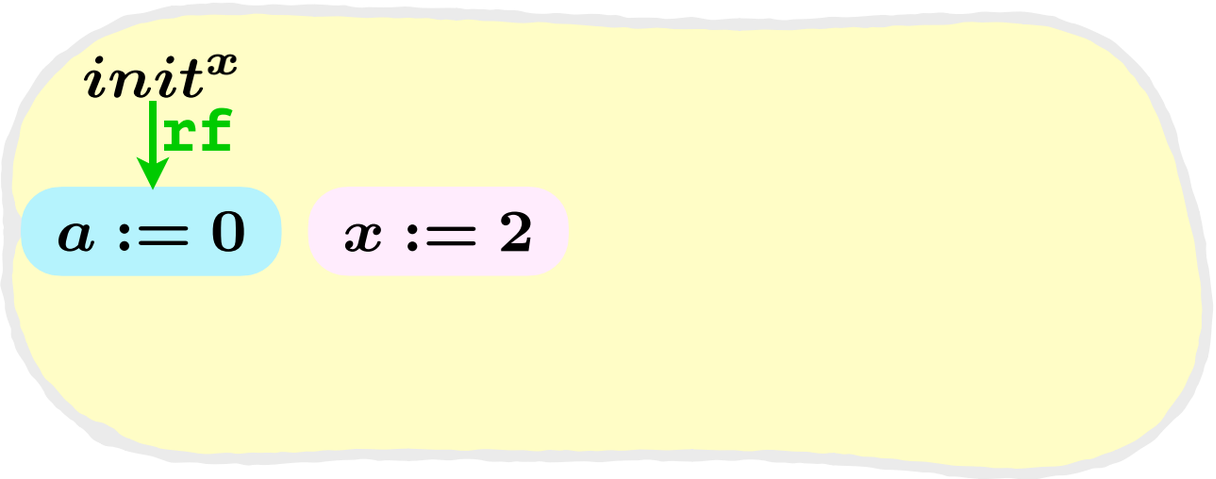

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

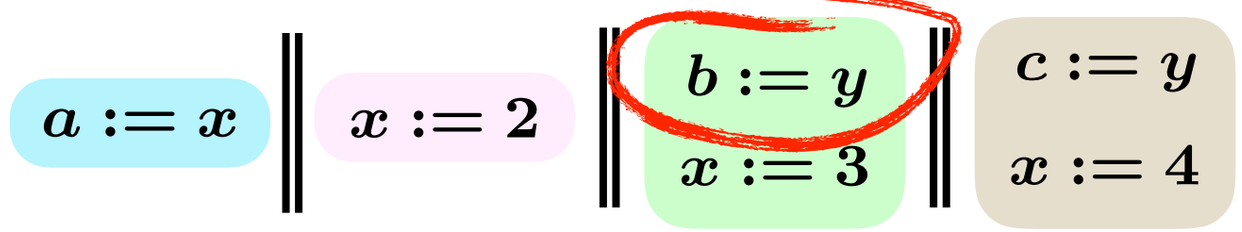

schedule
$x := 2$
$a := 0$
$a := 2$

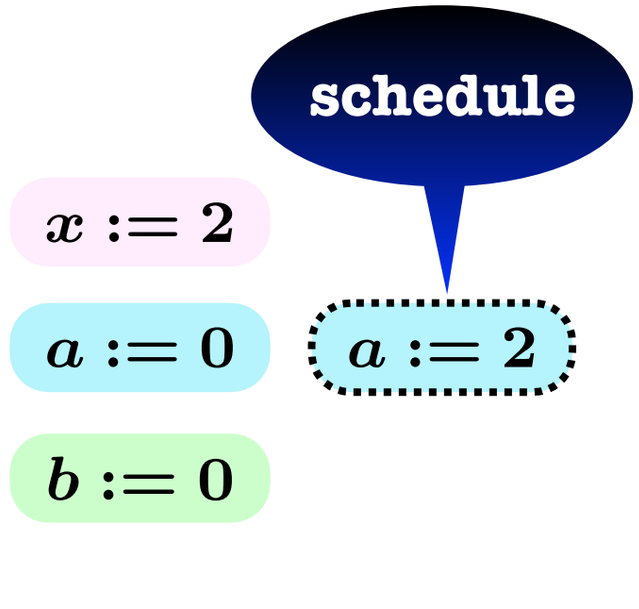

$init^x$
rf
$a := 0$
$x := 2$

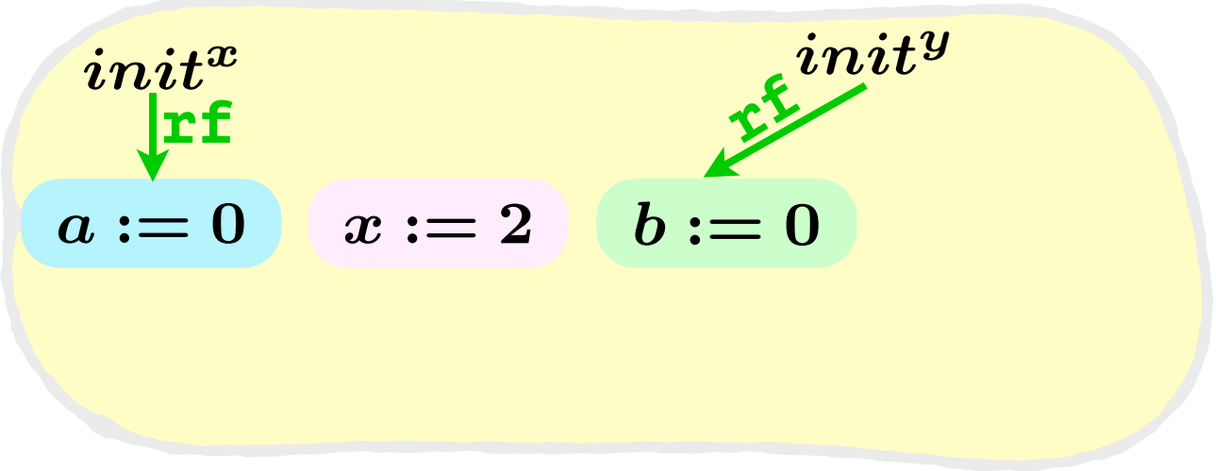

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

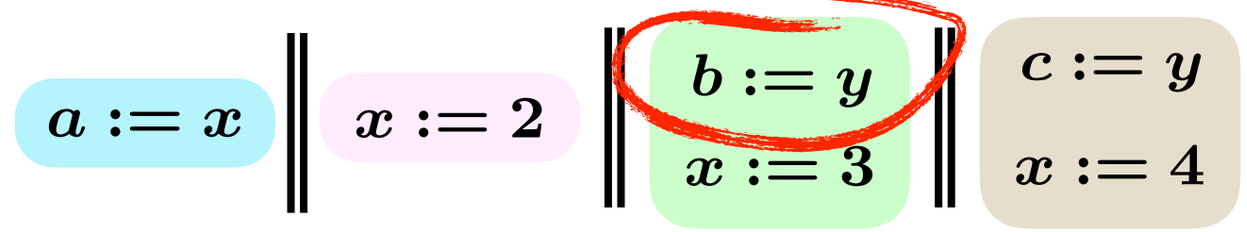

schedule
$x := 2$
$a := 0$
$a := 2$

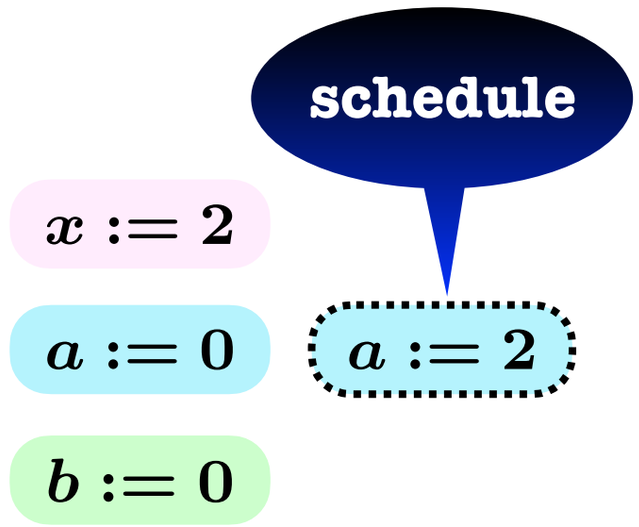

$init^x$
rf
$a := 0$
$x := 2$

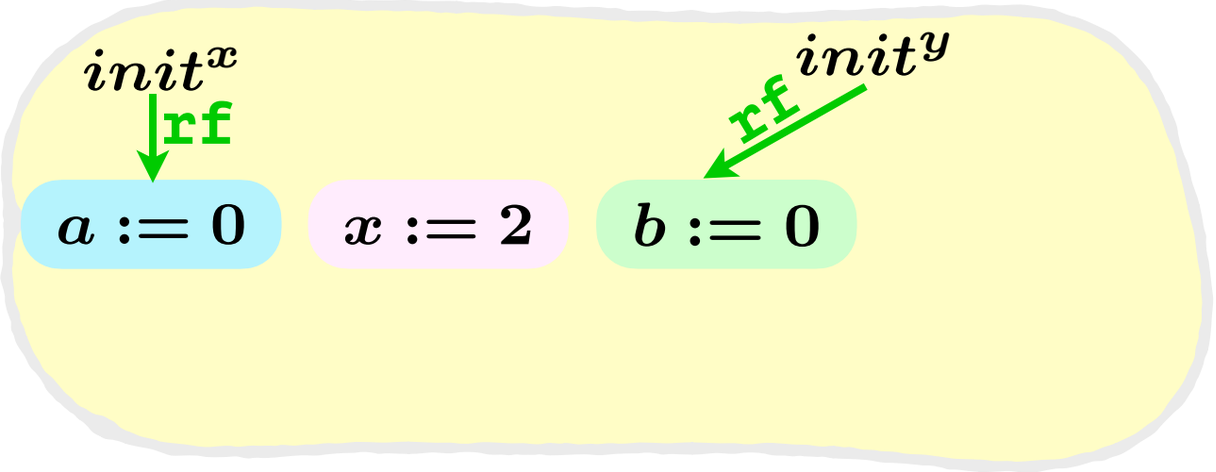

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$


schedule
$x := 2$
$a := 0$
$b := 0$
$a := 2$

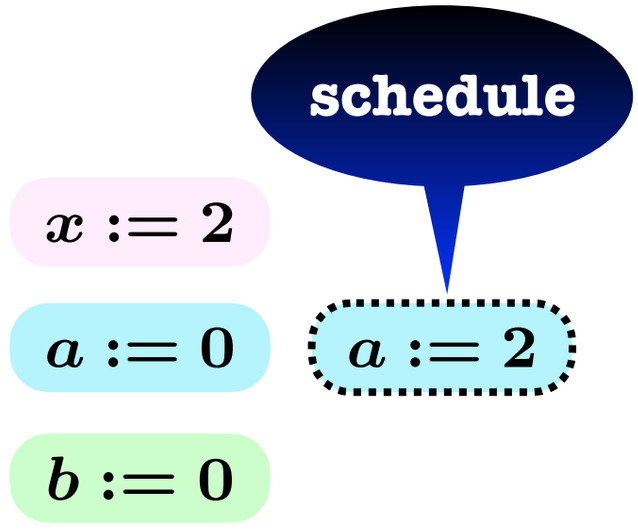

$init^x$
rf
$init^y$
rf
$a := 0$
$x := 2$
$b := 0$

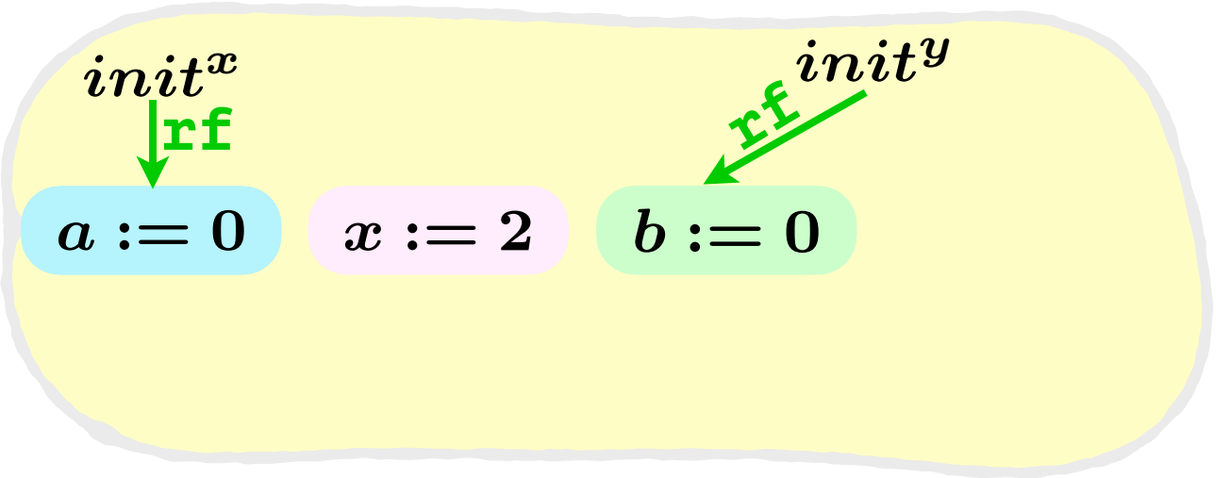

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

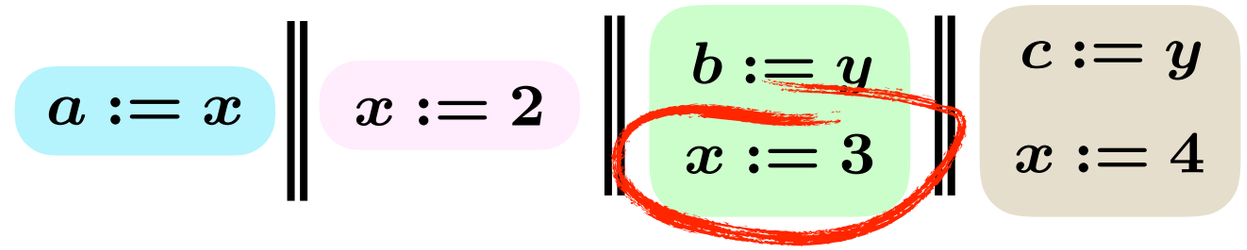

schedule
$x := 2$
$a := 0$
$b := 0$
$a := 2$

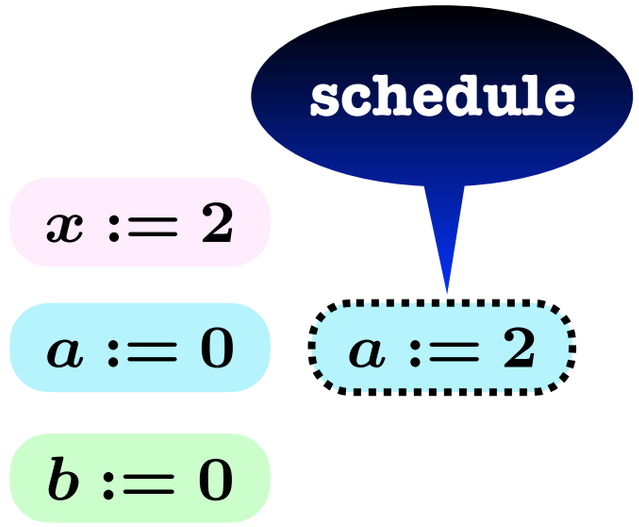


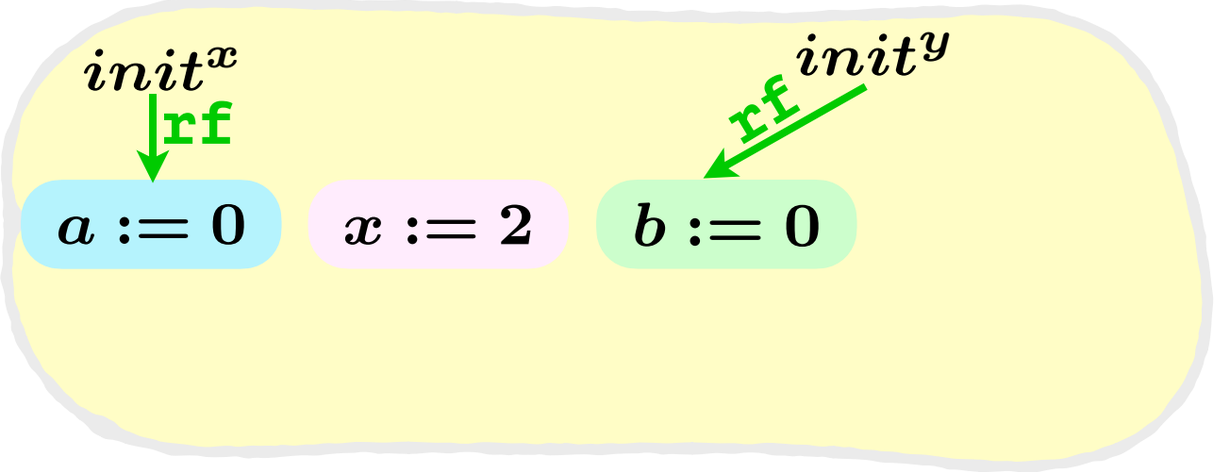
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

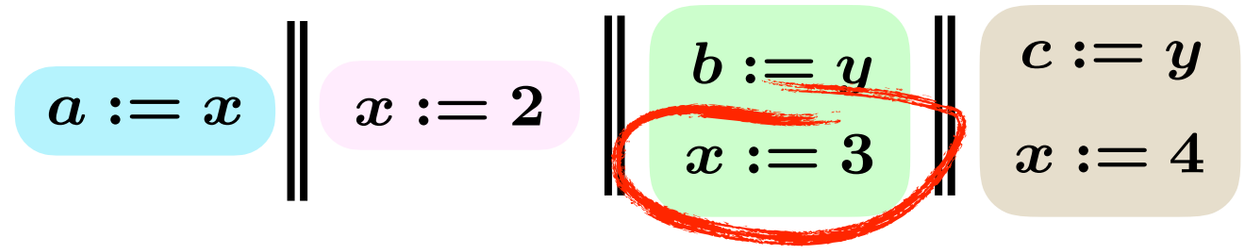


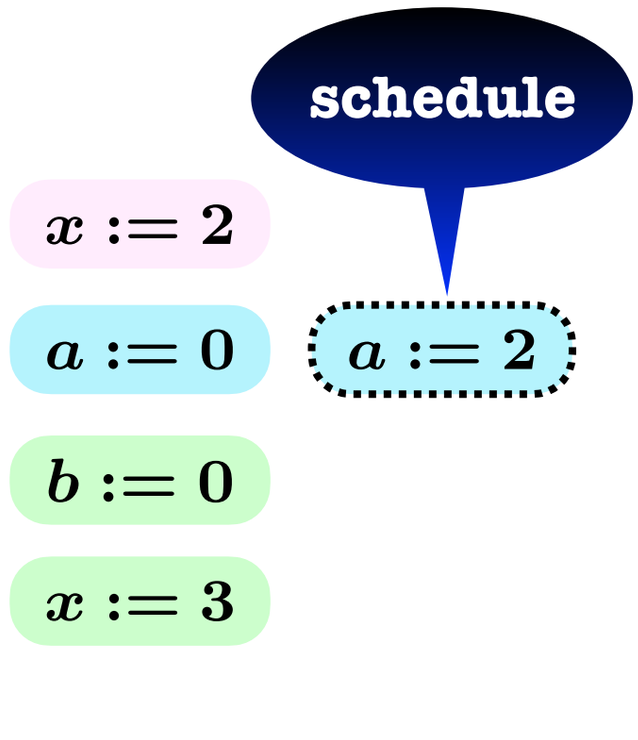

$init^x$
rf
$init^y$
rf
$a := 0$
$x := 2$
$b := 0$

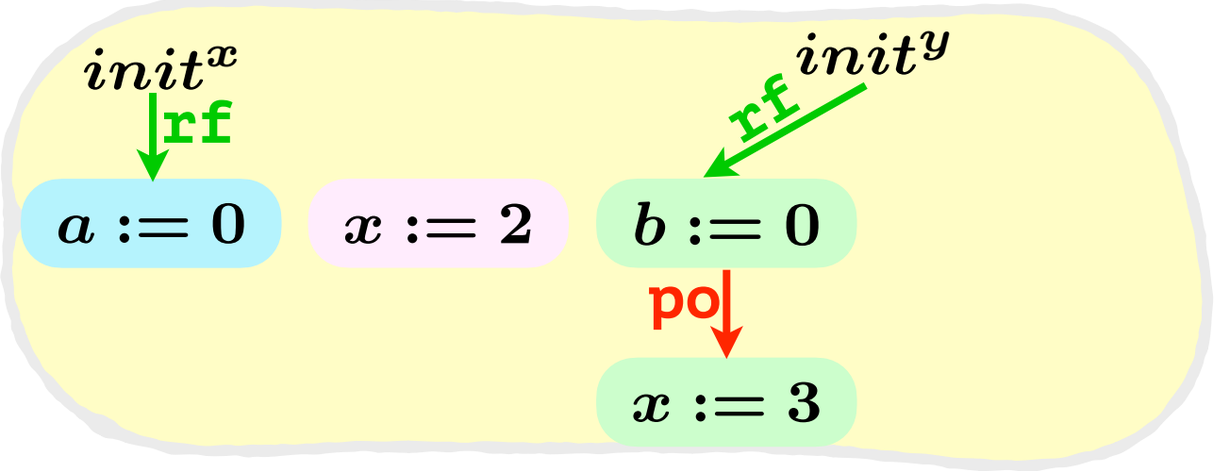

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

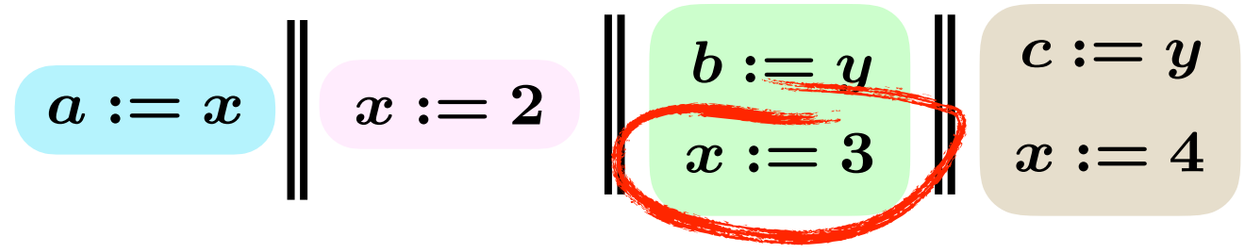

schedule
$x := 2$
$a := 0$
$b := 0$
$a := 2$


$init^x$
rf
$init^y$
rf
$a := 0$
$x := 2$
$b := 0$

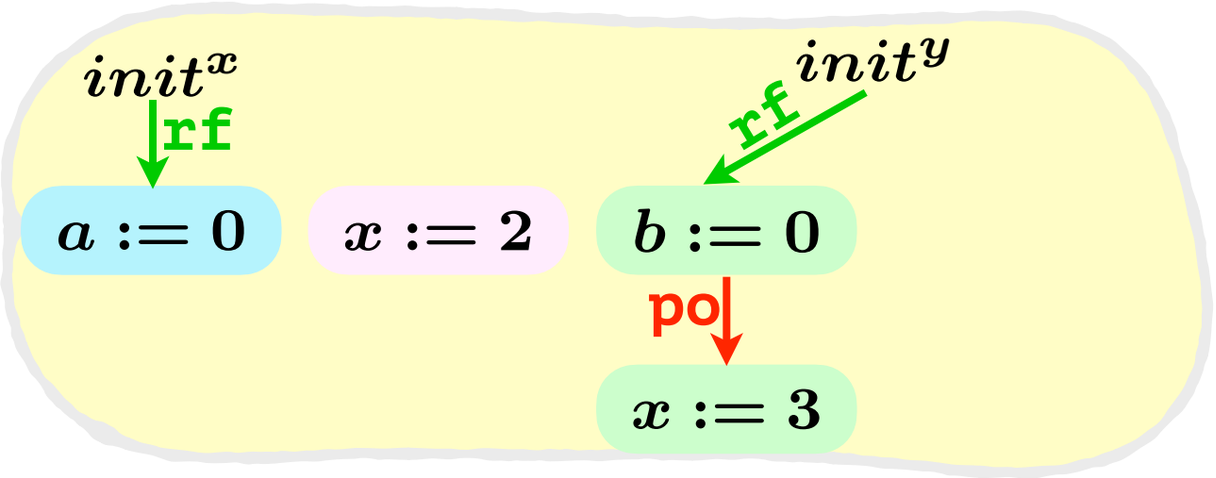

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

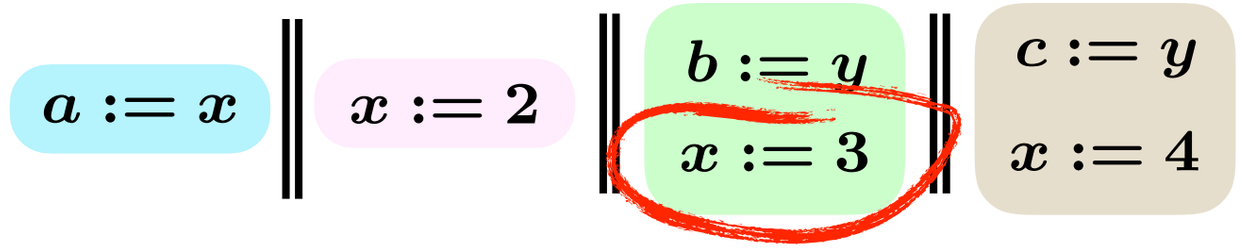

schedule
$x := 2$
$a := 0$
$b := 0$
$x := 3$
$a := 2$

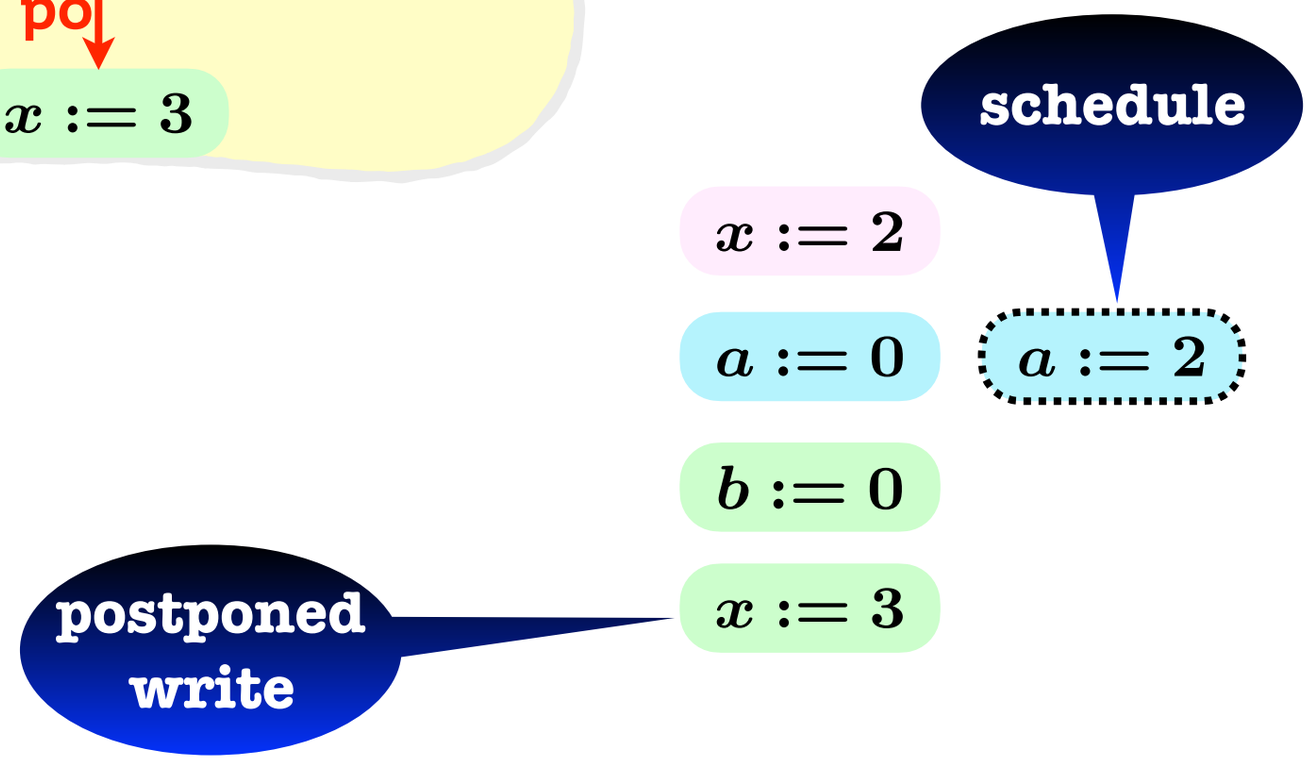

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
$b := 0$
po
$x := 3$

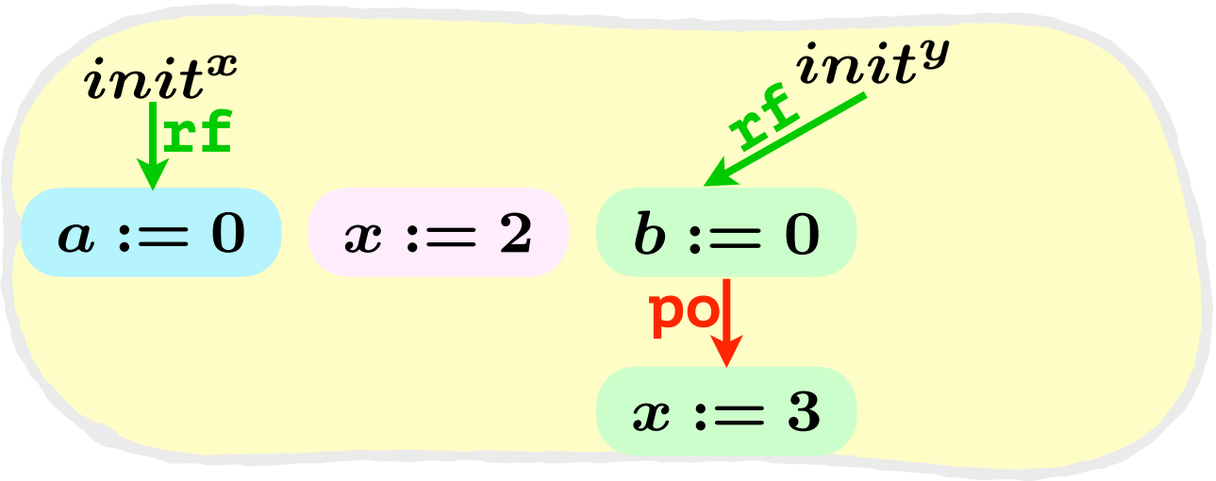

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

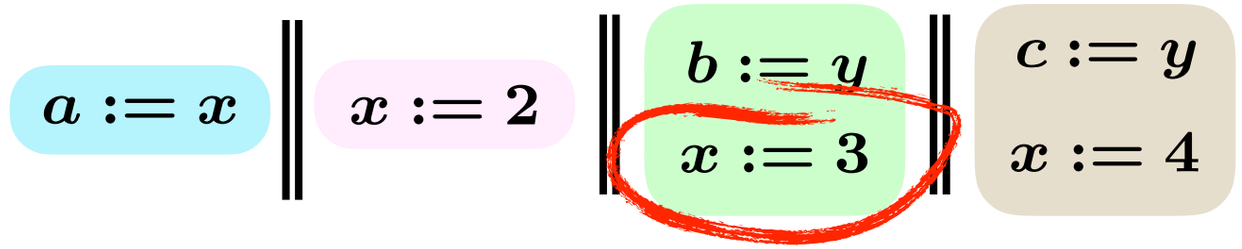
schedule
$x := 2$
$a := 0$
$b := 0$
$x := 3$
$a := 2$

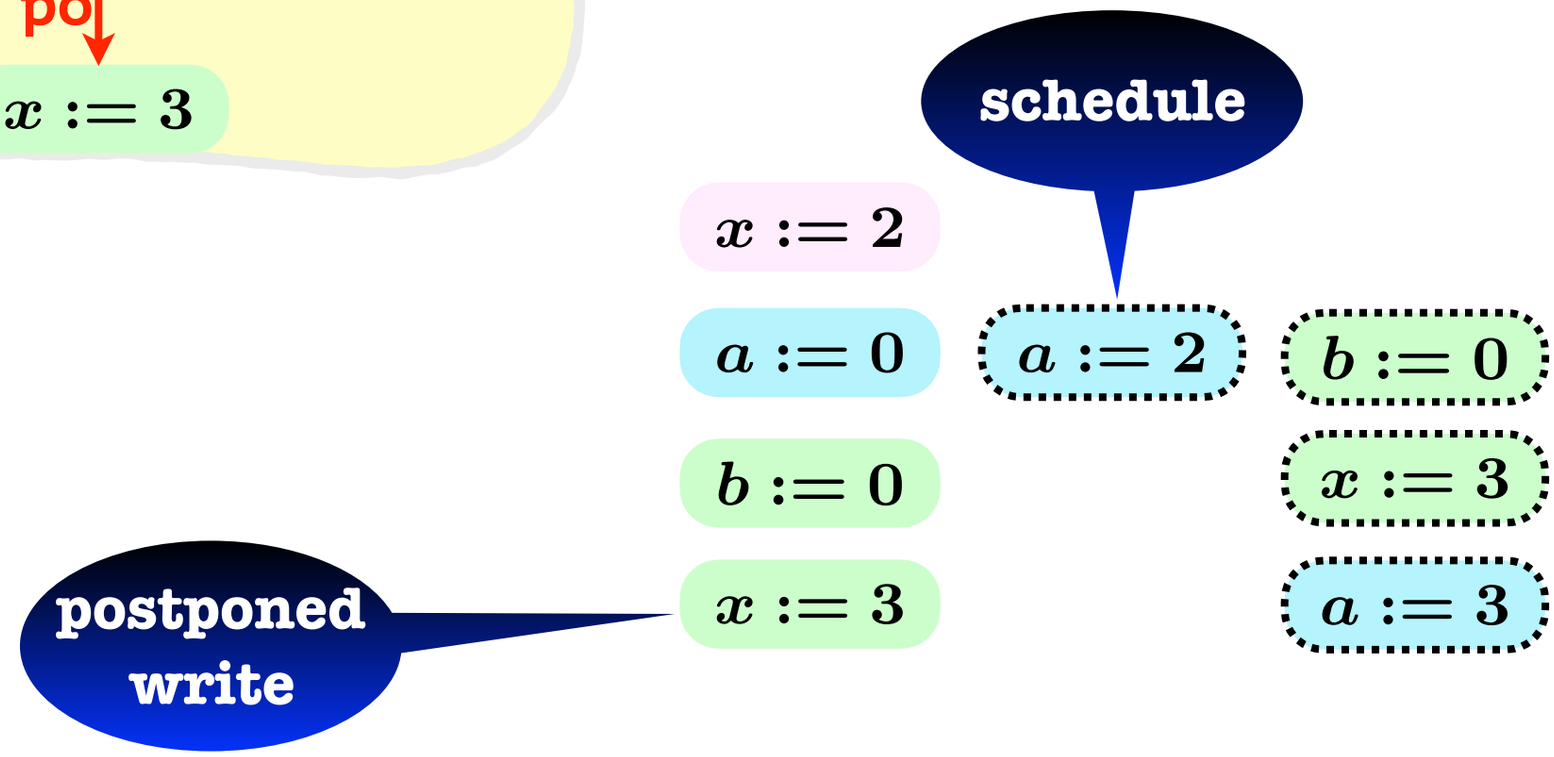

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
$b := 0$
po
$x := 3$

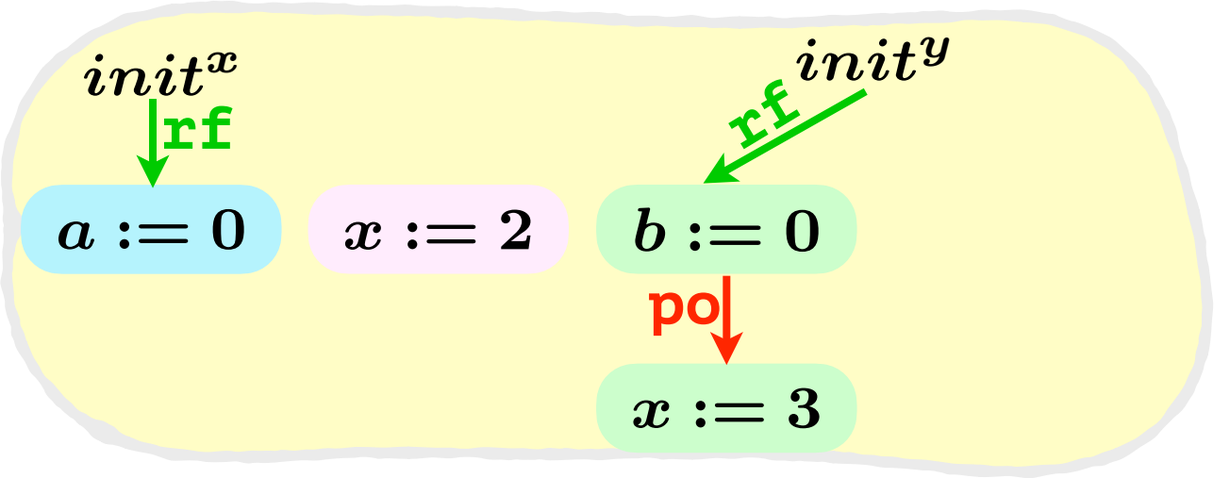

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

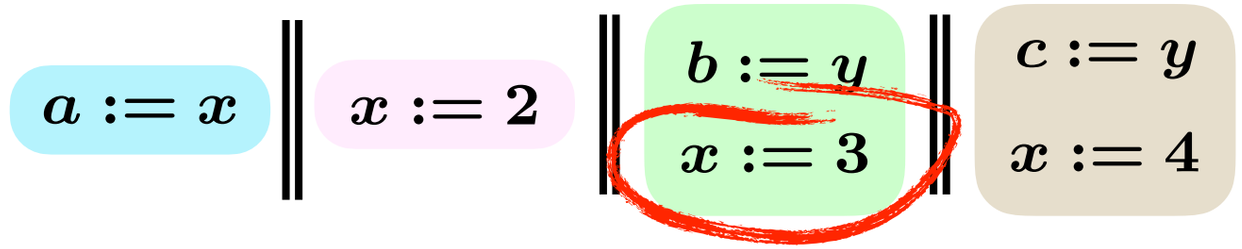

schedule
$x := 2$
$a := 0$
$a := 2$
$b := 0$
$x := 3$
postponed write


$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
$b := 0$
po
$x := 3$

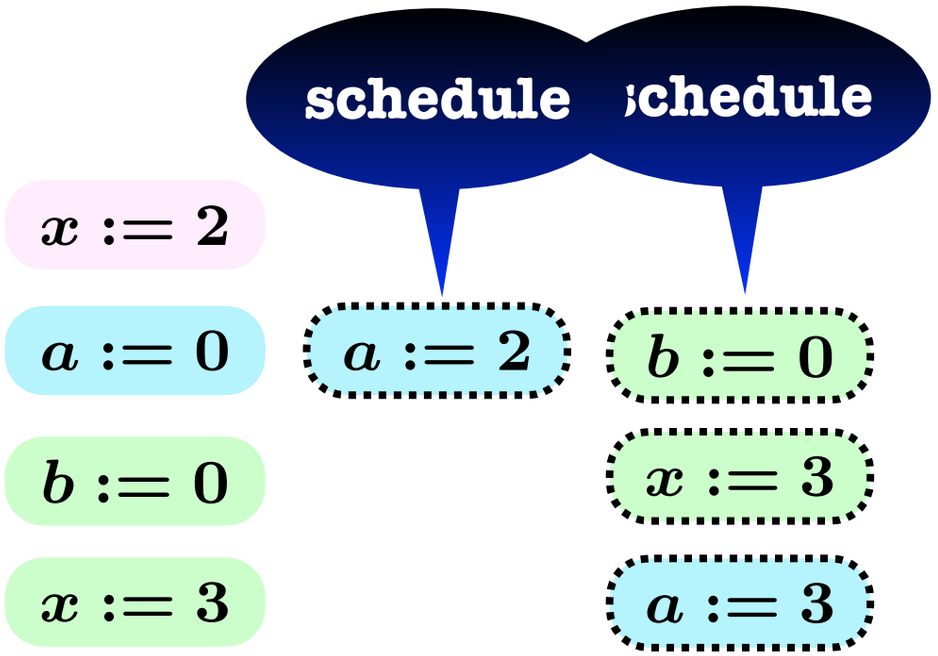

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

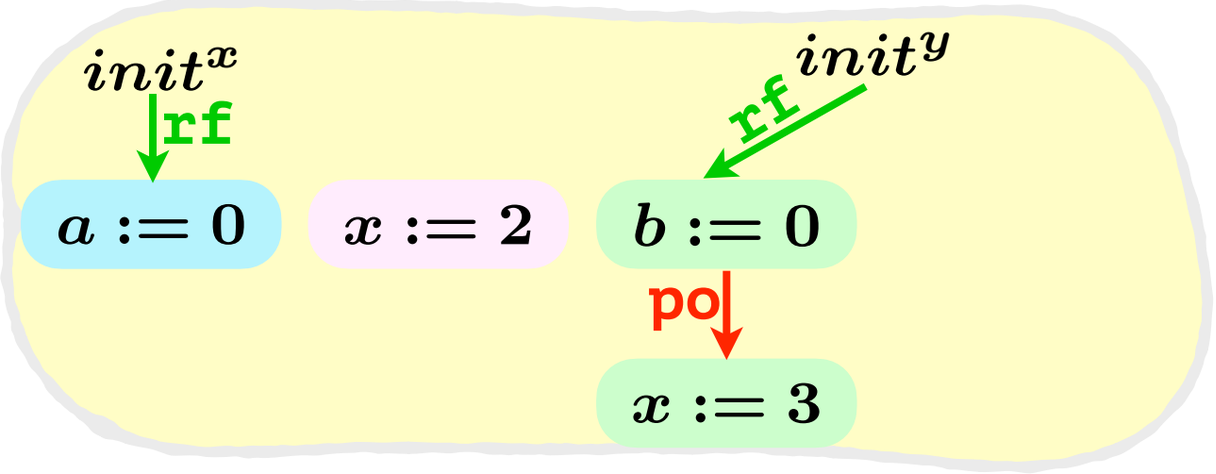

schedule
$x := 2$
$a := 0$
$b := 0$
$x := 3$
$a := 2$
$b := 0$
$x := 3$
$a := 3$
postponed write

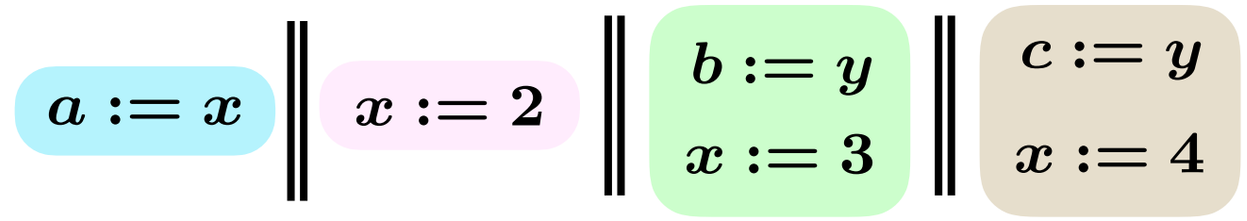

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
$b := 0$
po
$x := 3$

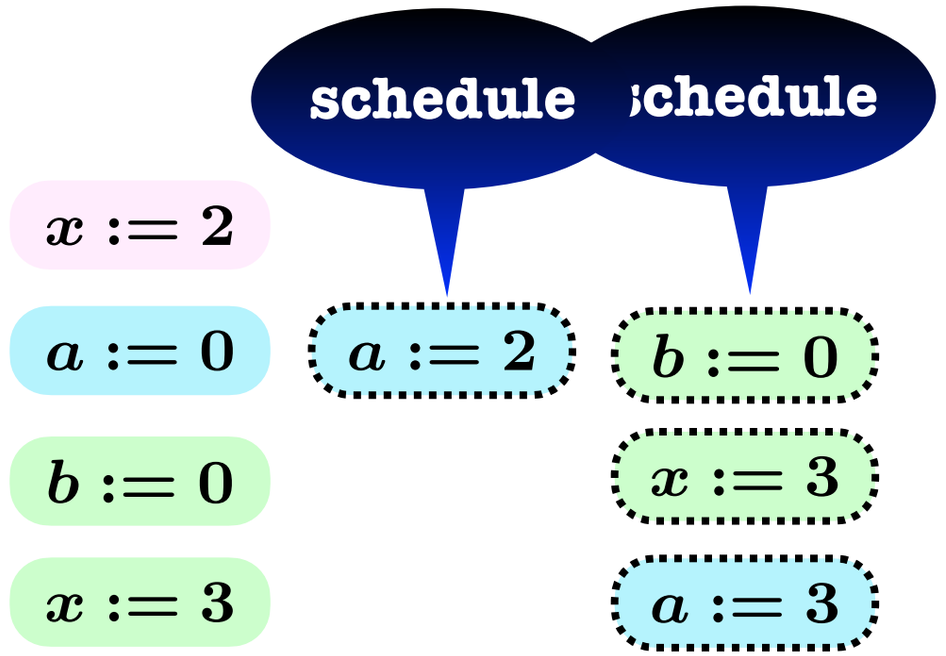

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

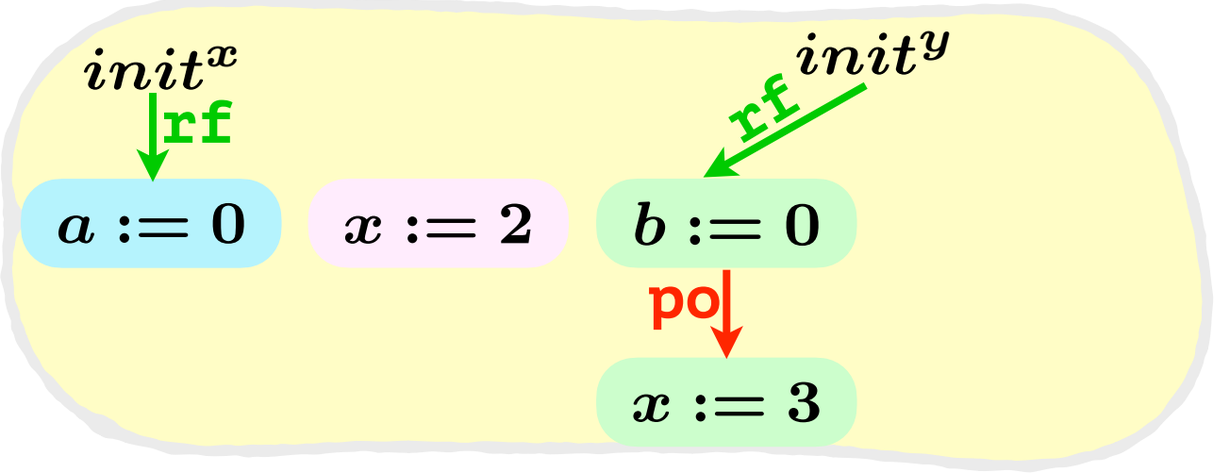
postponed write

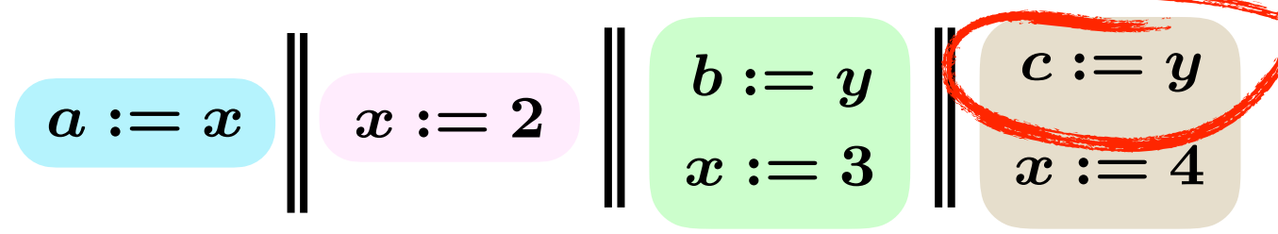

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$x := 3$
$a := 2$
$b := 0$
$x := 3$
$a := 3$

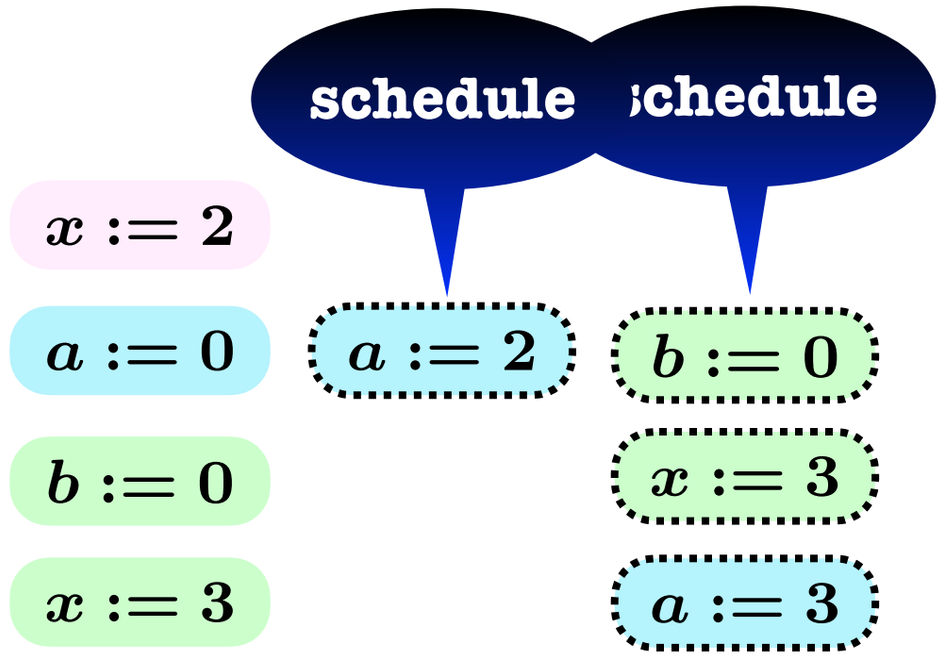

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
$b := 0$
po
$x := 3$

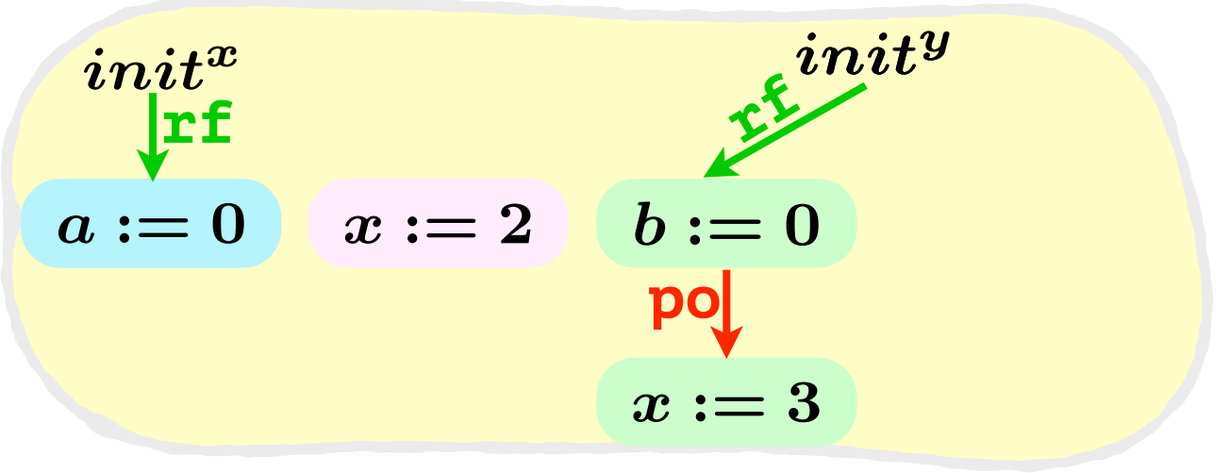

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$


schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$x := 3$
$a := 2$
$b := 0$
$x := 3$
$a := 3$

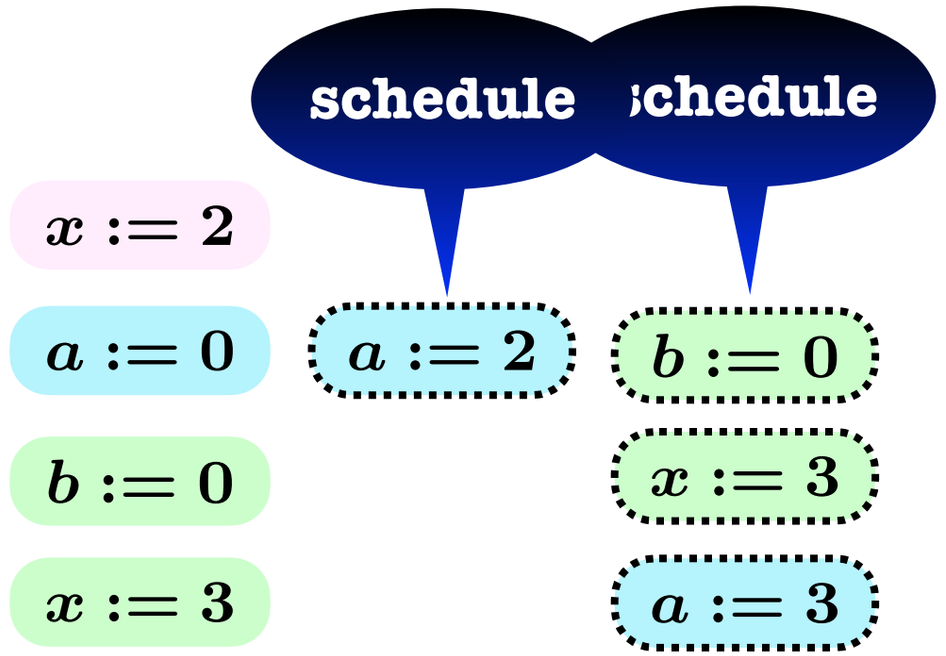

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
$b := 0$
po
$x := 3$


$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

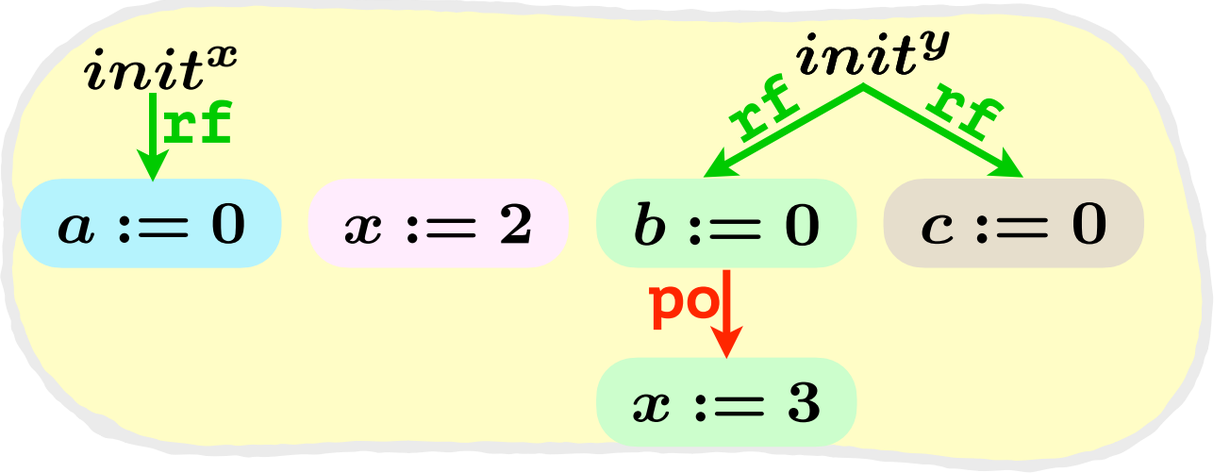

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$a := 2$
$b := 0$
$x := 3$
$x := 3$
$a := 3$

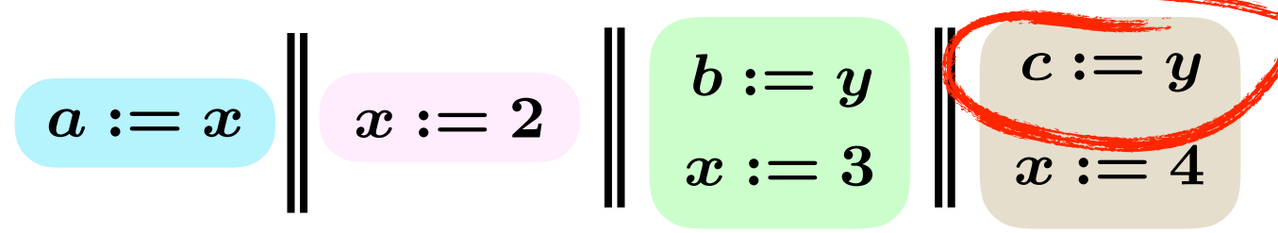


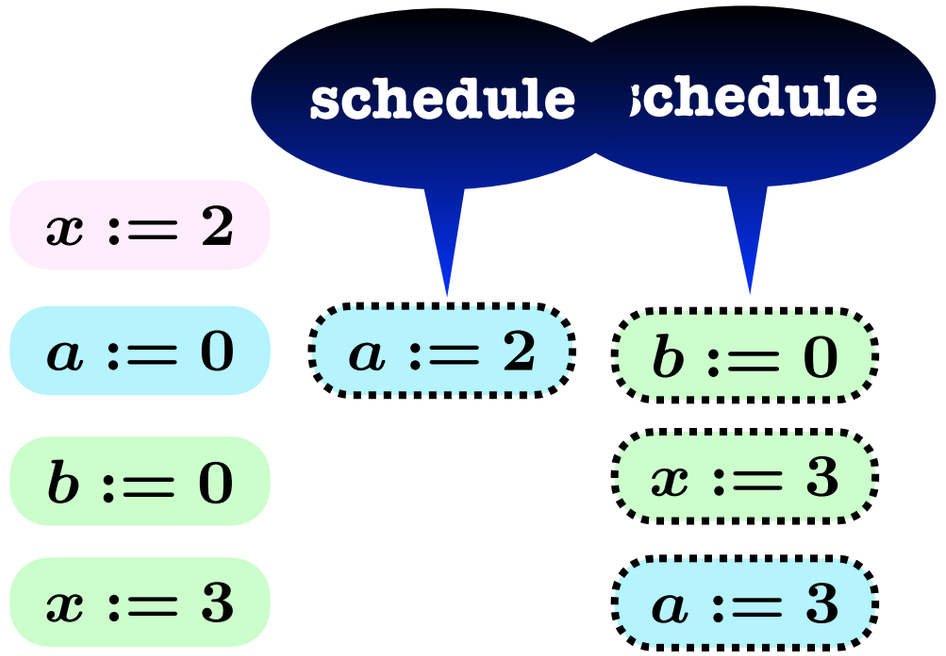
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$



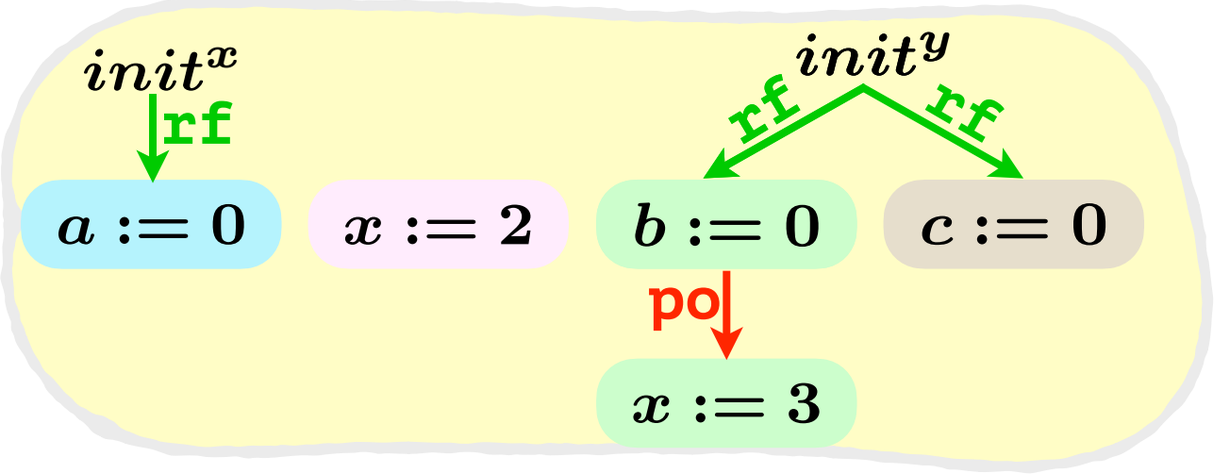
$c := 0$

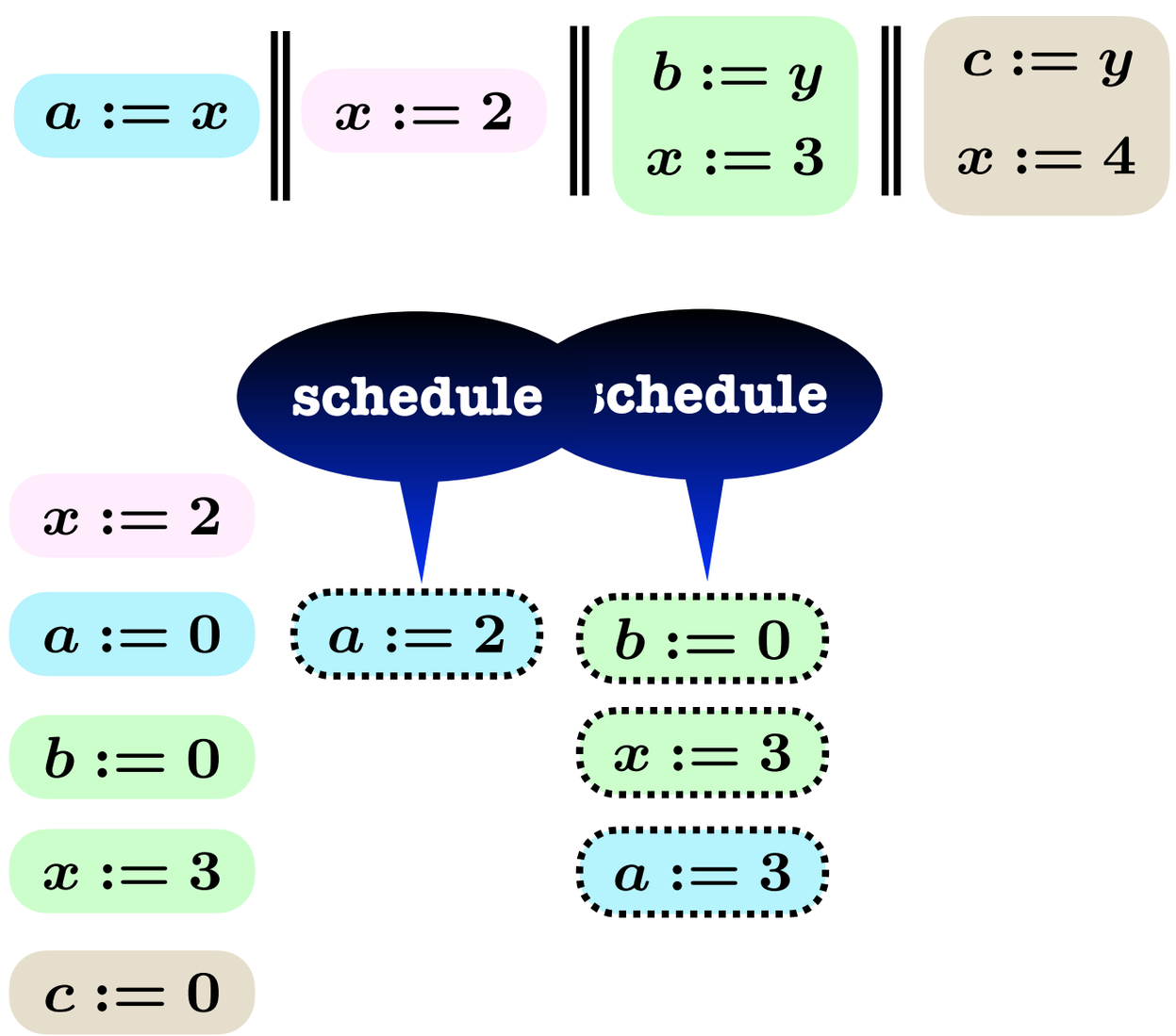

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
$x := 3$

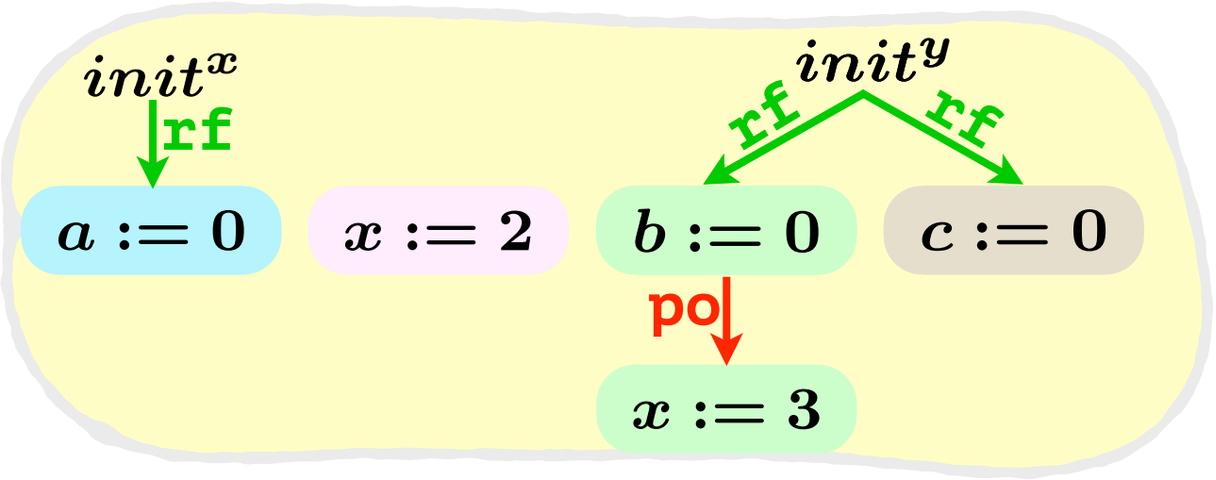

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

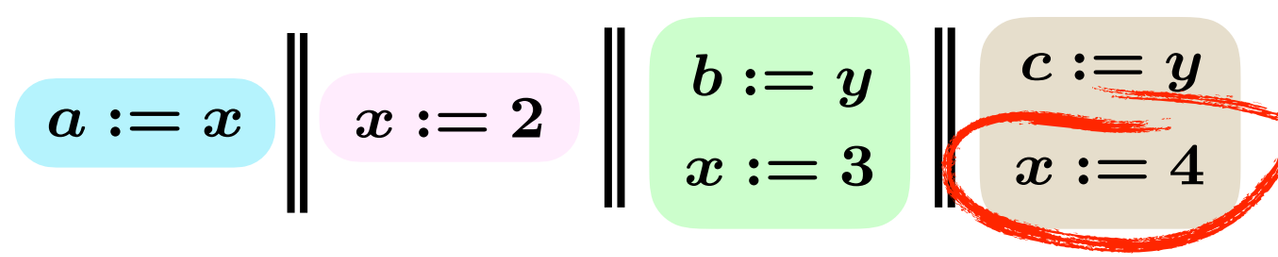

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$a := 2$
$b := 0$
$x := 3$
$x := 3$
$a := 3$

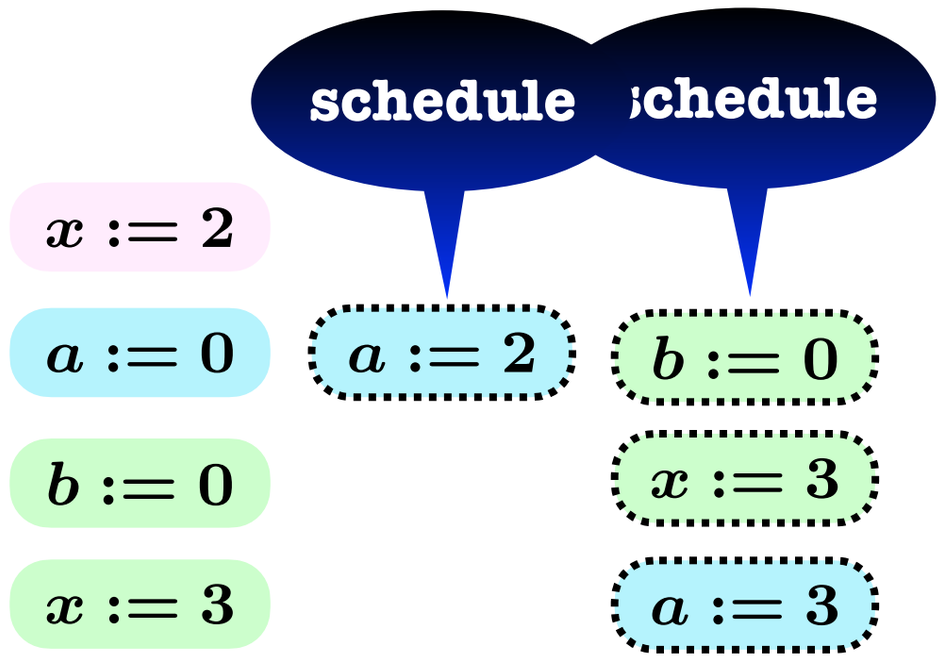
$c := 0$


$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
$x := 3$

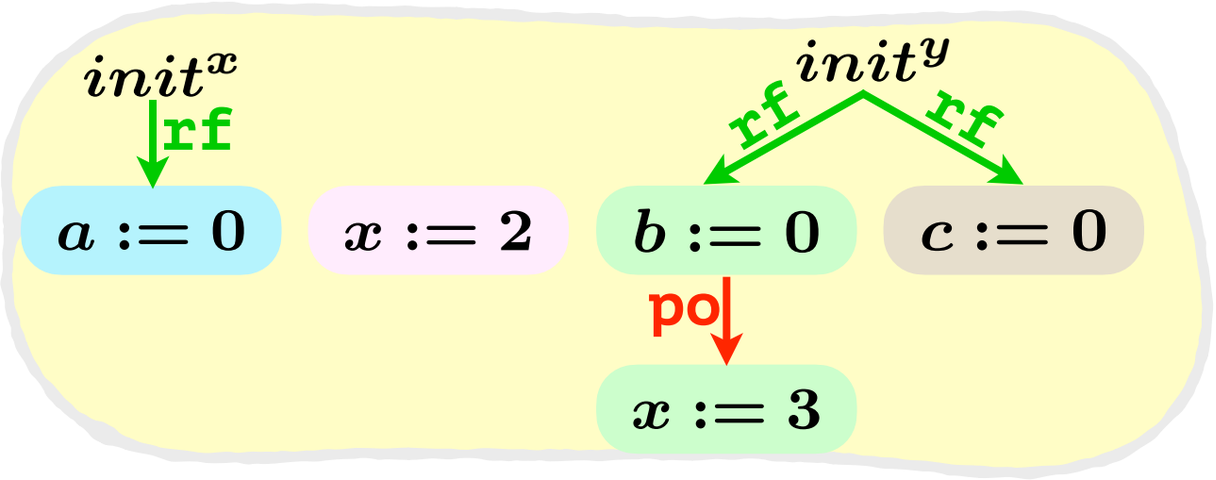

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$
schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$a := 2$
$b := 0$
$x := 3$
$x := 3$
$a := 3$
$c := 0$


$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
$x := 3$


$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$


schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$a := 2$
$b := 0$
$x := 3$
$x := 3$
$a := 3$

$c := 0$



$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

schedule schedule

$x := 2$

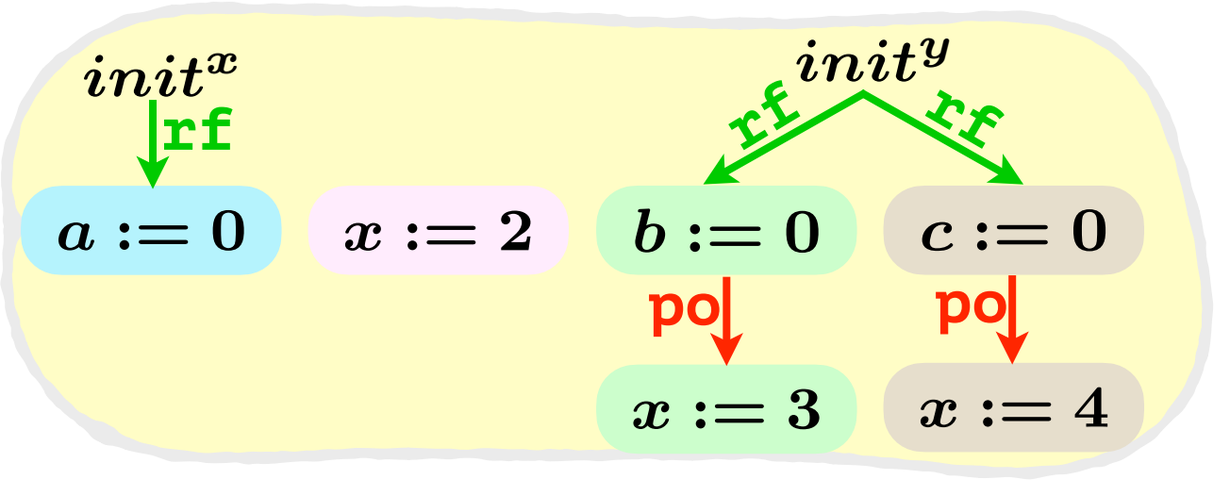
$a := 0$ $a := 2$ $b := 0$

$b := 0$ $x := 3$

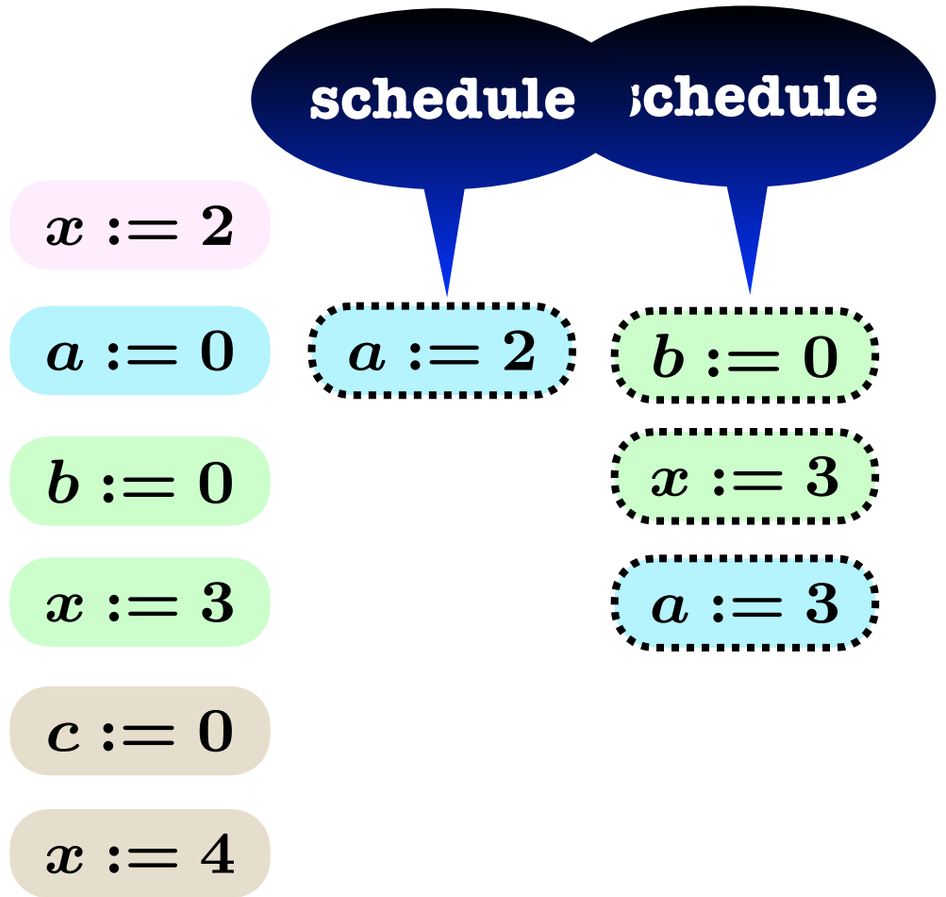
$x := 3$ $a := 3$

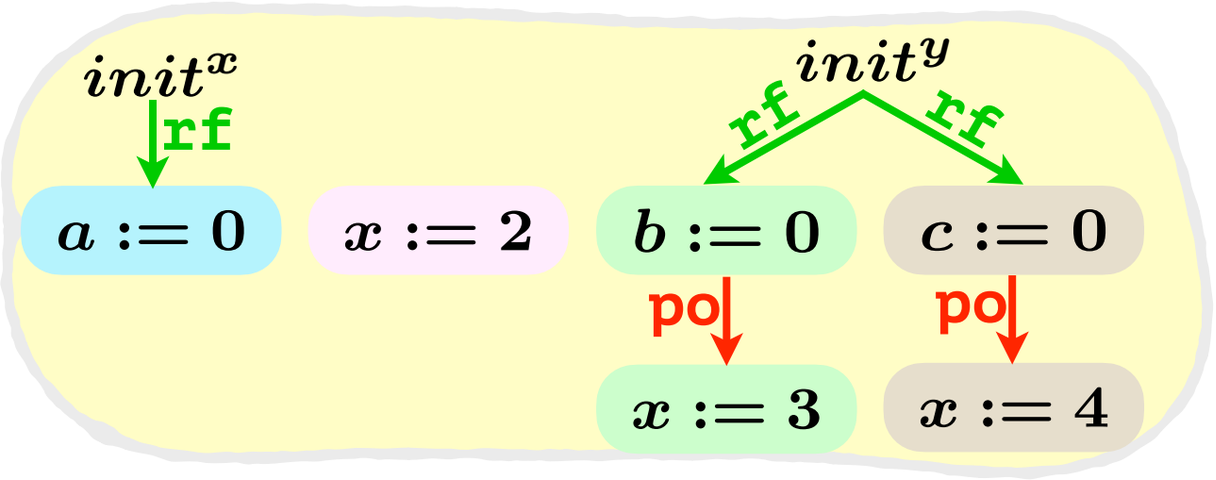
$c := 0$

$x := 4$

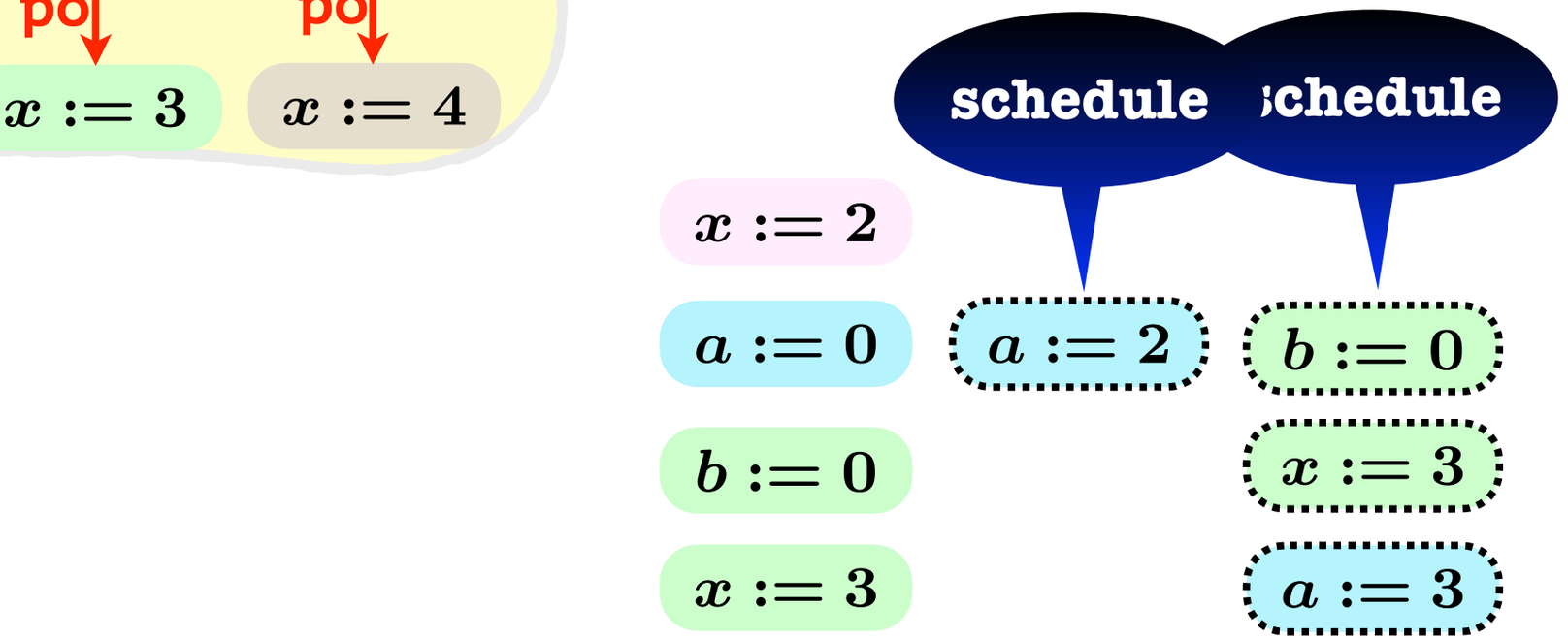


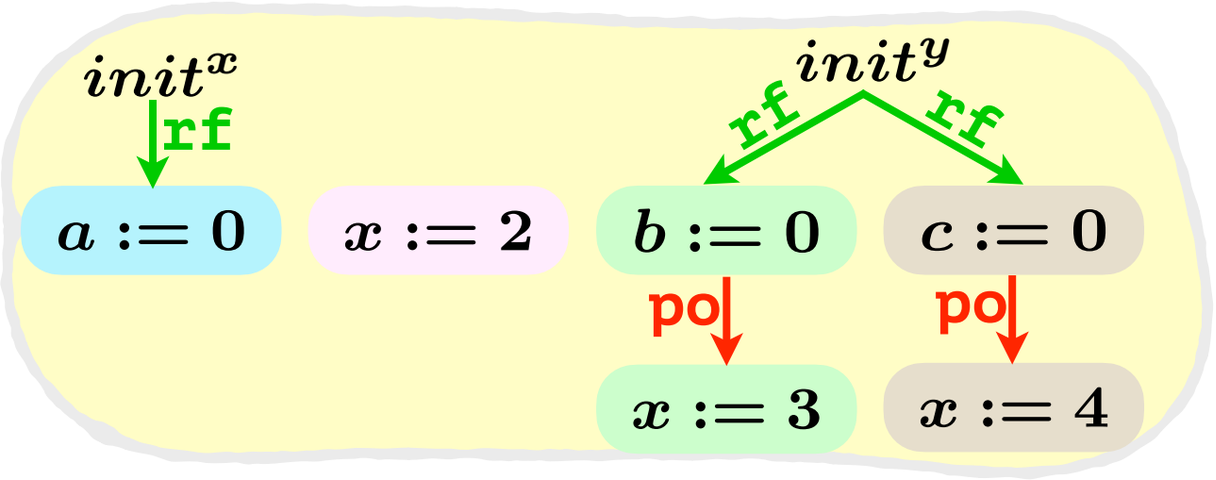
$$a := x \;\|\; x := 2 \;\|\; b := y;\ x := 3 \;\|\; c := y;\ x := 4$$





$$a := x \;\|\; x := 2 \;\|\; b := y \; x := 3 \;\|\; c := y \; x := 4$$





$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

schedule schedule

$x := 2$

$a := 0$ $a := 2$ $b := 0$ $c := 0$

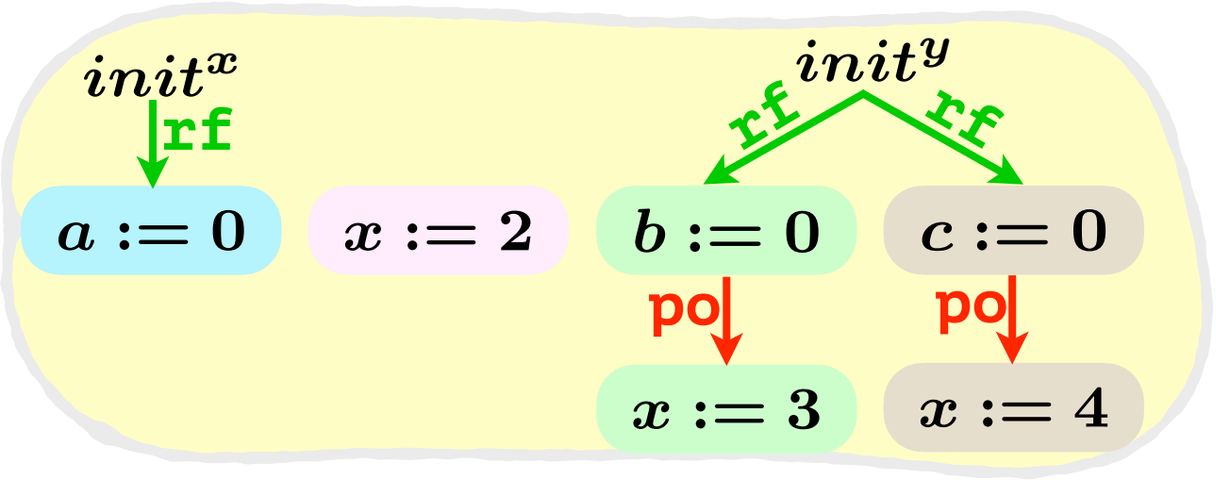
$b := 0$ $x := 3$ $x := 4$

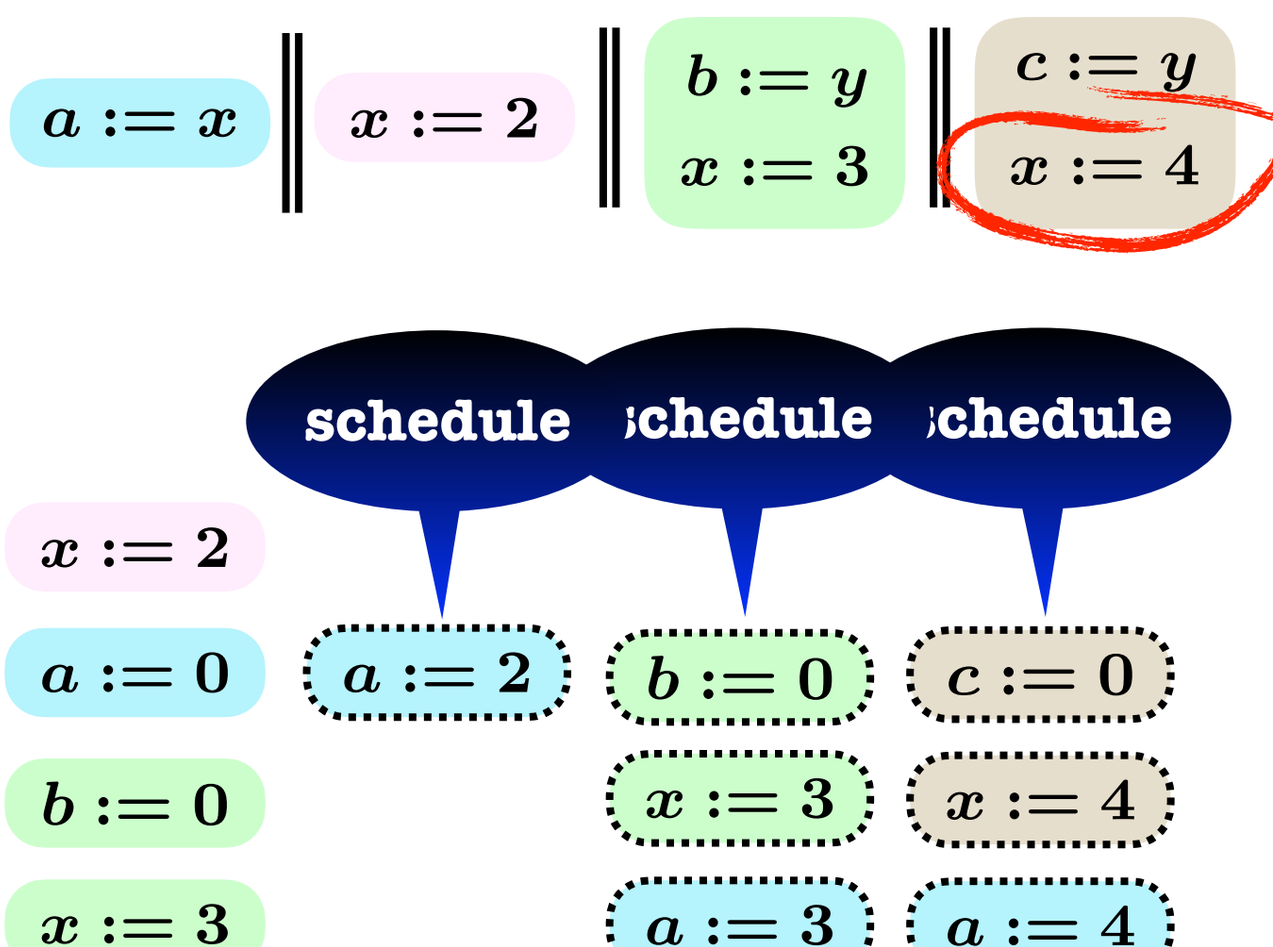
$x := 3$ $a := 3$ $a := 4$

$c := 0$

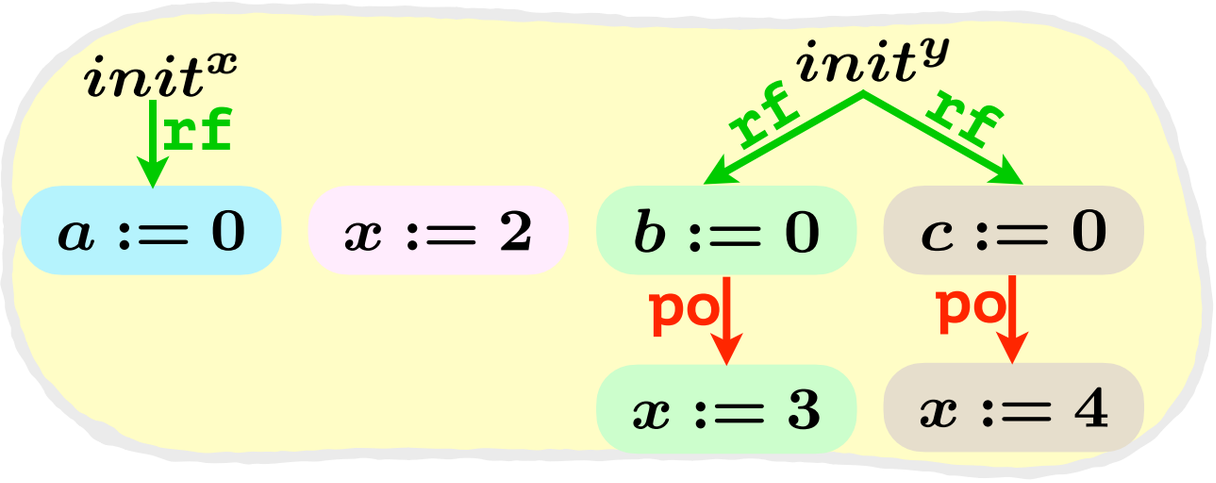
postponed write

$x := 4$


$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$


$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$
schedule
schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$x := 3$
$a := 2$
$b := 0$
$x := 3$
$a := 3$
$c := 0$
$x := 4$
$a := 4$

$c := 0$
postponed write
$x := 4$



$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

schedule schedule schedule

$x := 2$ $a := 2$ $b := 0$ $c := 0$

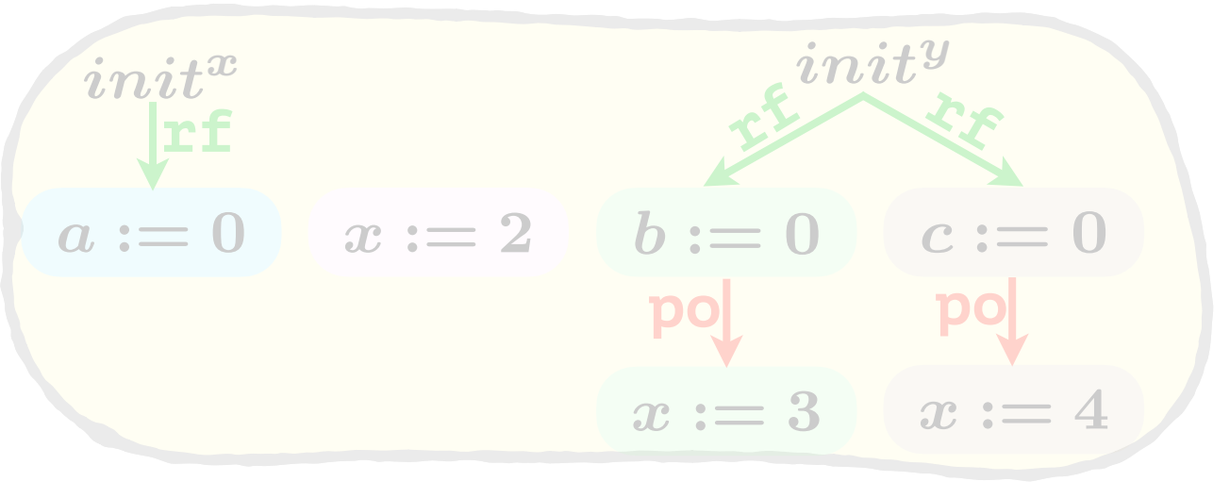
$a := 0$ $x := 3$ $x := 4$

$b := 0$ $a := 3$ $a := 4$

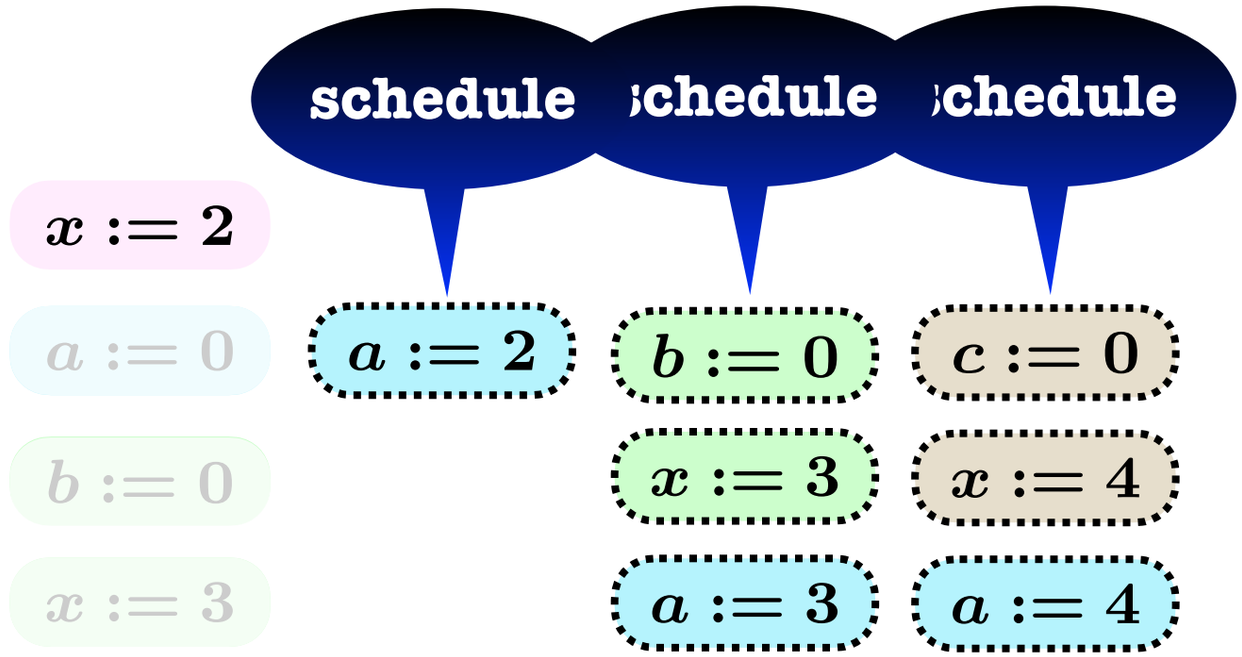
$x := 3$

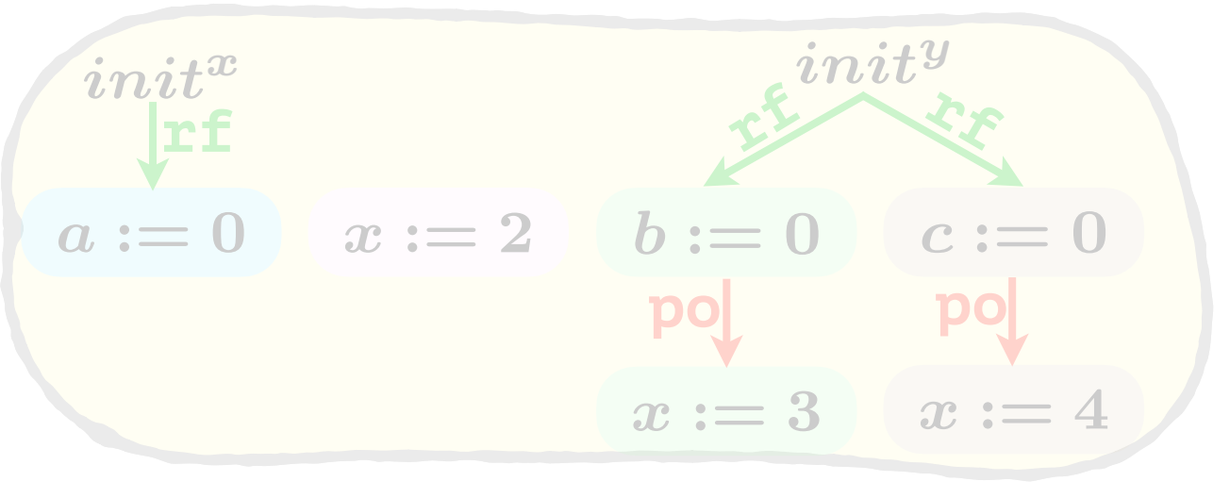
$c := 0$

$x := 4$

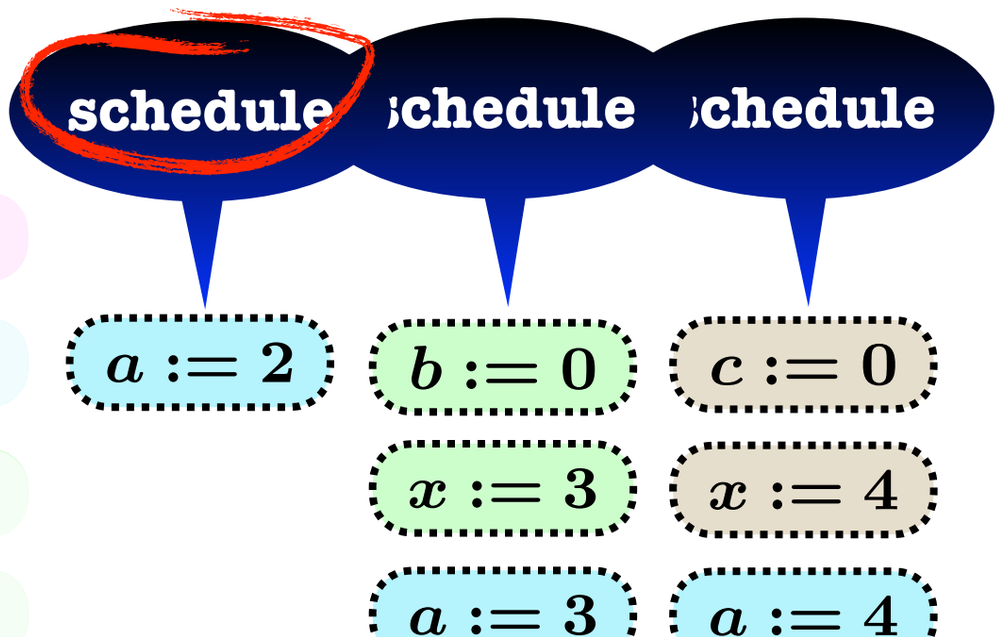


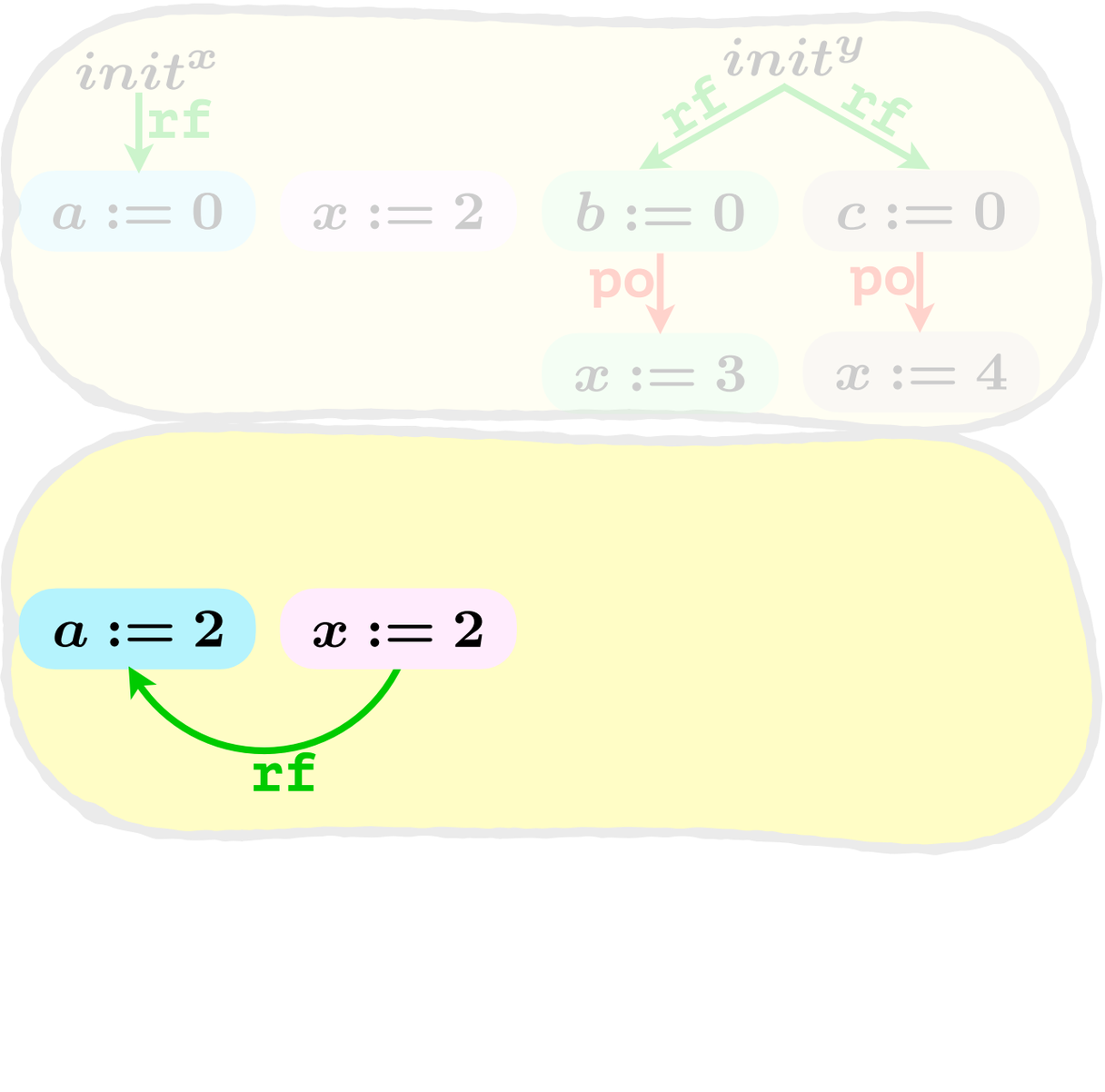
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$





$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$





$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

$x := 2$

$a := 0$

$b := 0$

$x := 3$

$c := 0$

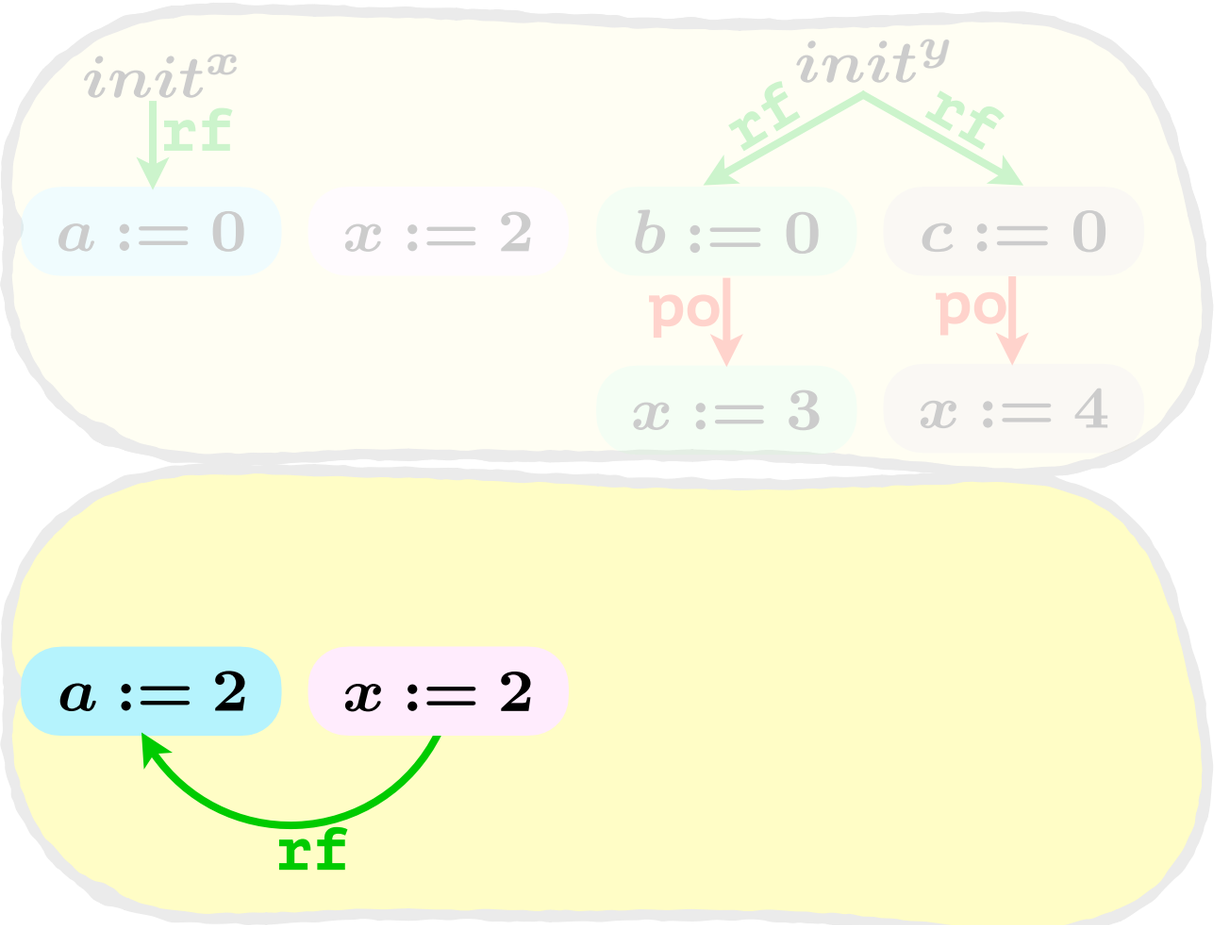
$x := 4$

schedule schedule

$a := 2$ $b := 0$ $c := 0$

$x := 3$ $x := 4$

$a := 3$ $a := 4$



$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

schedule schedule

$x := 2$

$a := 0$

$b := 0$

$x := 3$

$a := 2$

$b := 0$

$x := 3$

$a := 3$

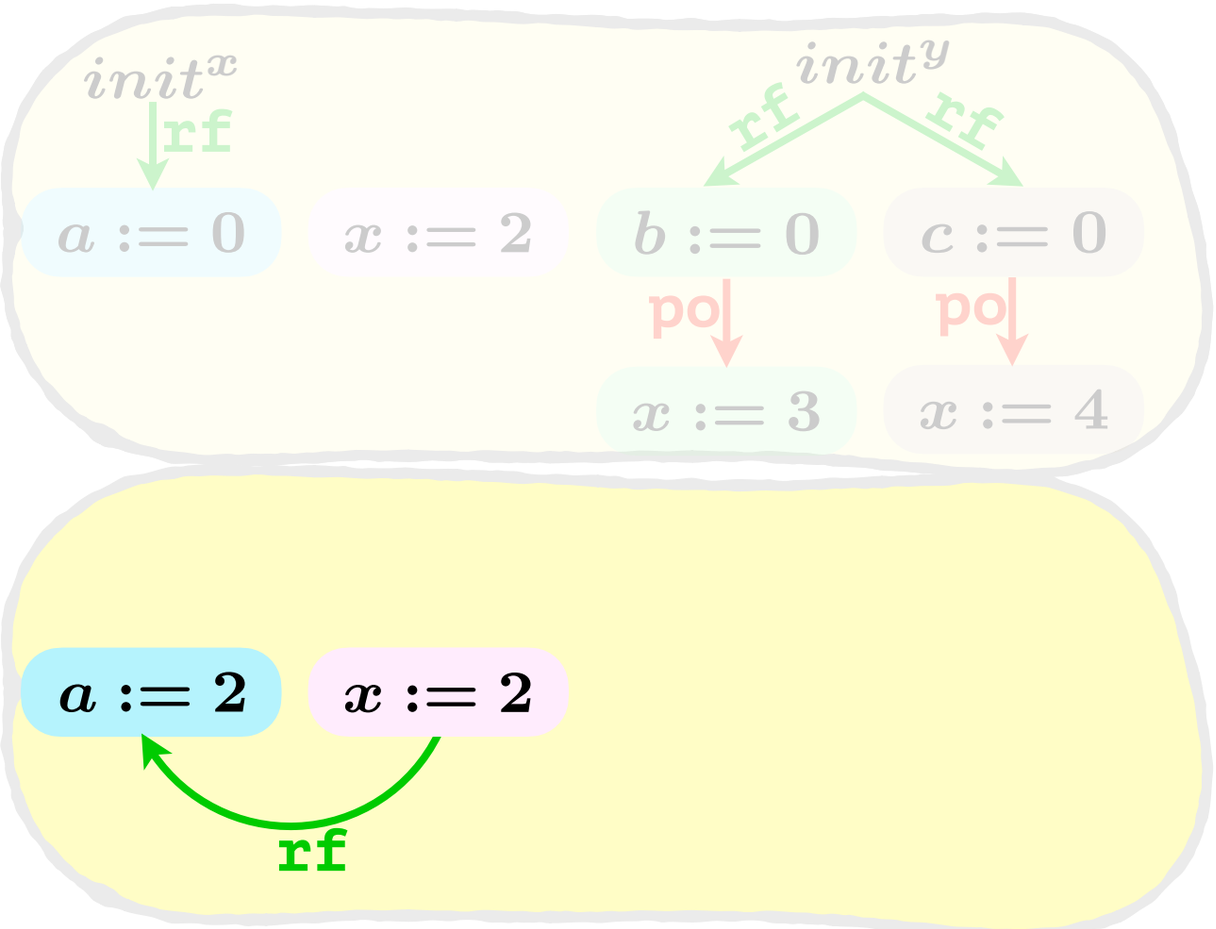
$c := 0$

$x := 4$

$a := 4$

$c := 0$

$x := 4$



$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

schedule schedule

$x := 2$

$a := 0$

$b := 0$

$x := 3$

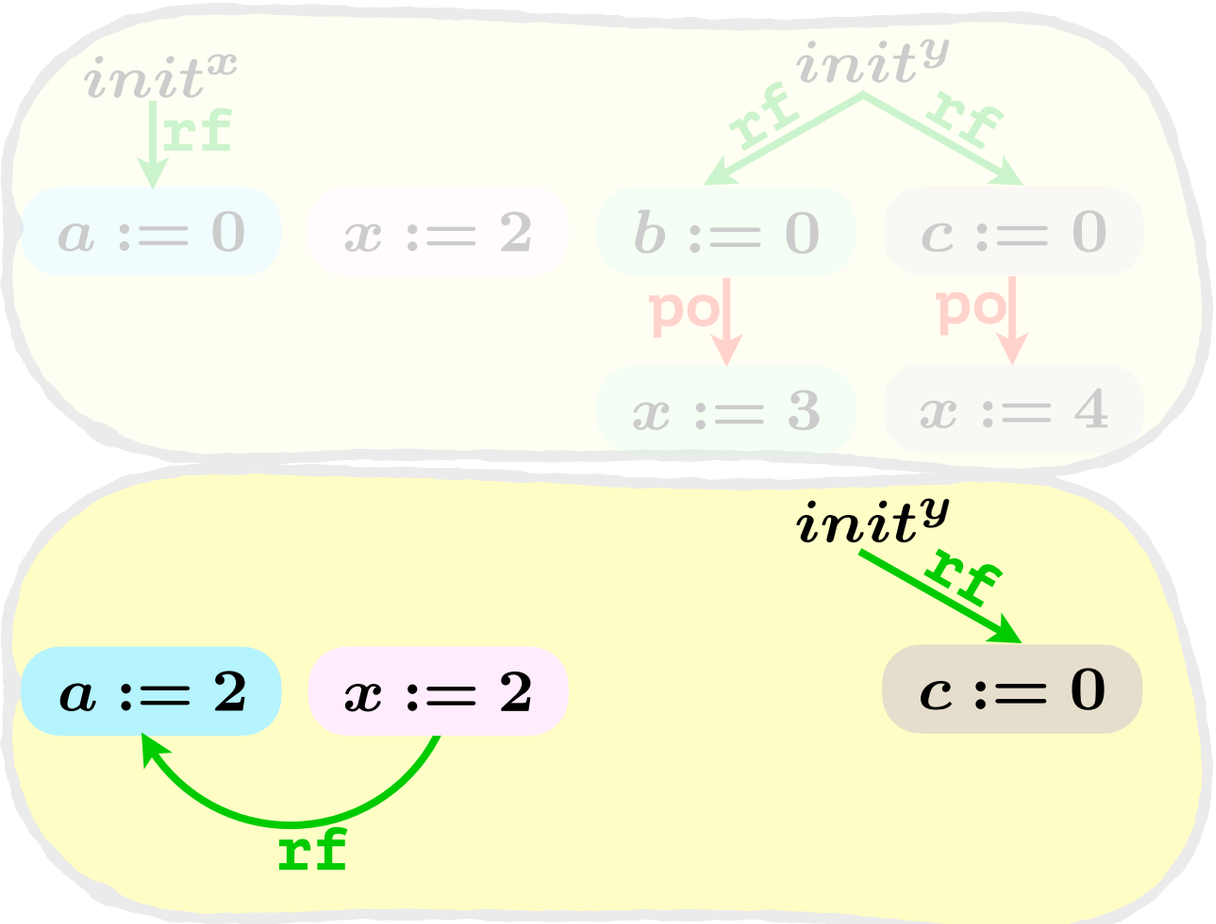
$c := 0$

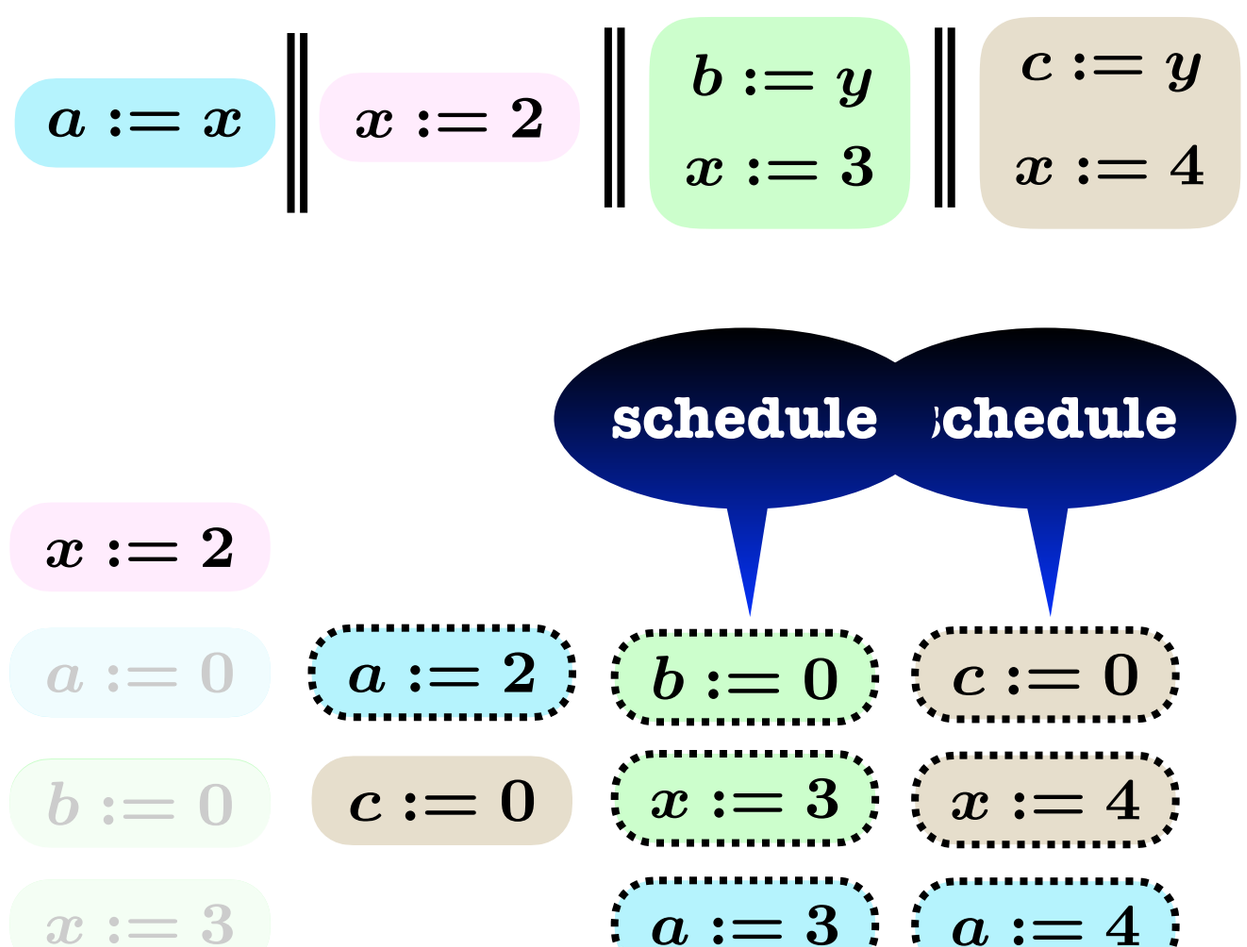
$x := 4$

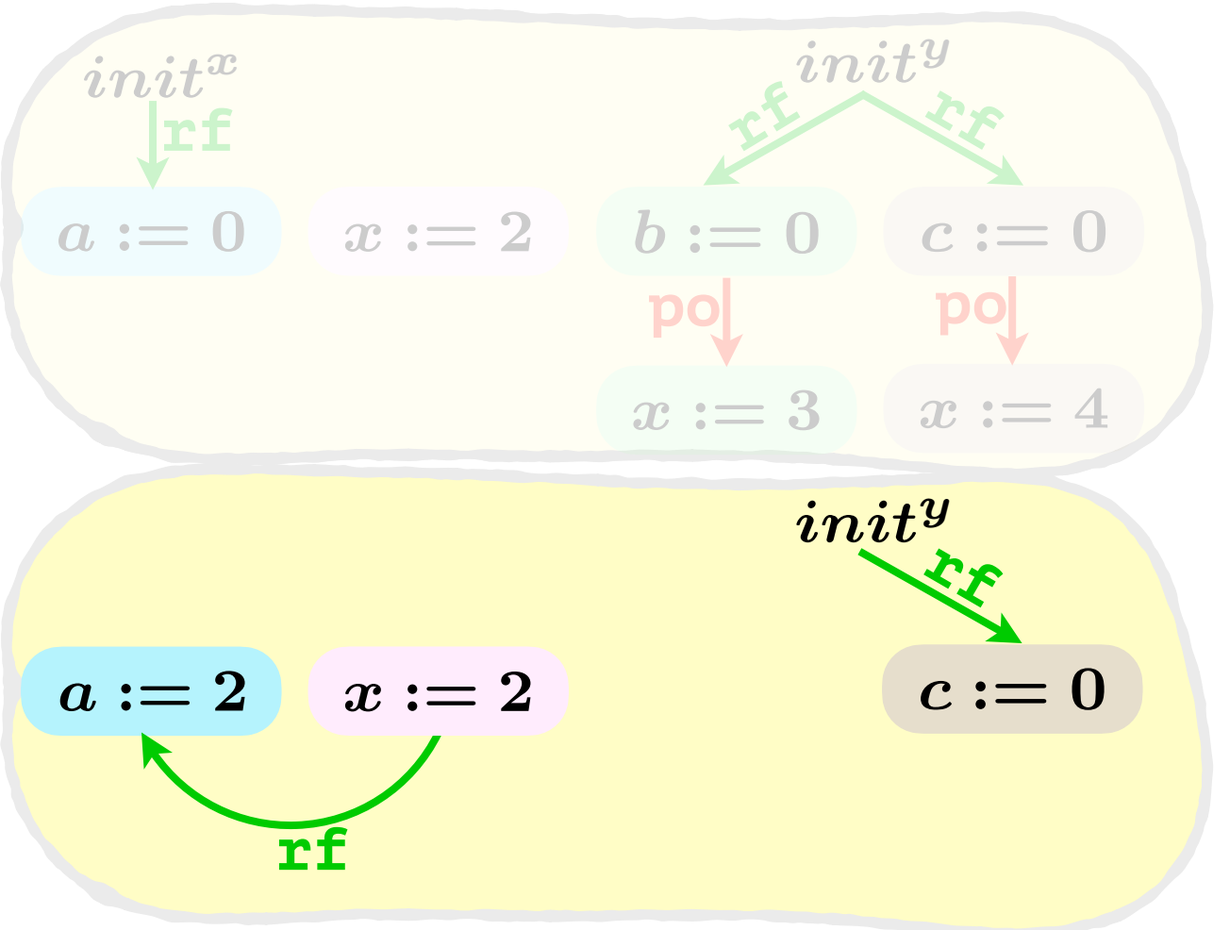
$a := 2$ $b := 0$ $c := 0$

$c := 0$ $x := 3$ $x := 4$

$a := 3$ $a := 4$


$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
$a := 2$
$x := 2$
$c := 0$
rf


$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$
schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$x := 4$



$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

schedule schedule

$x := 2$

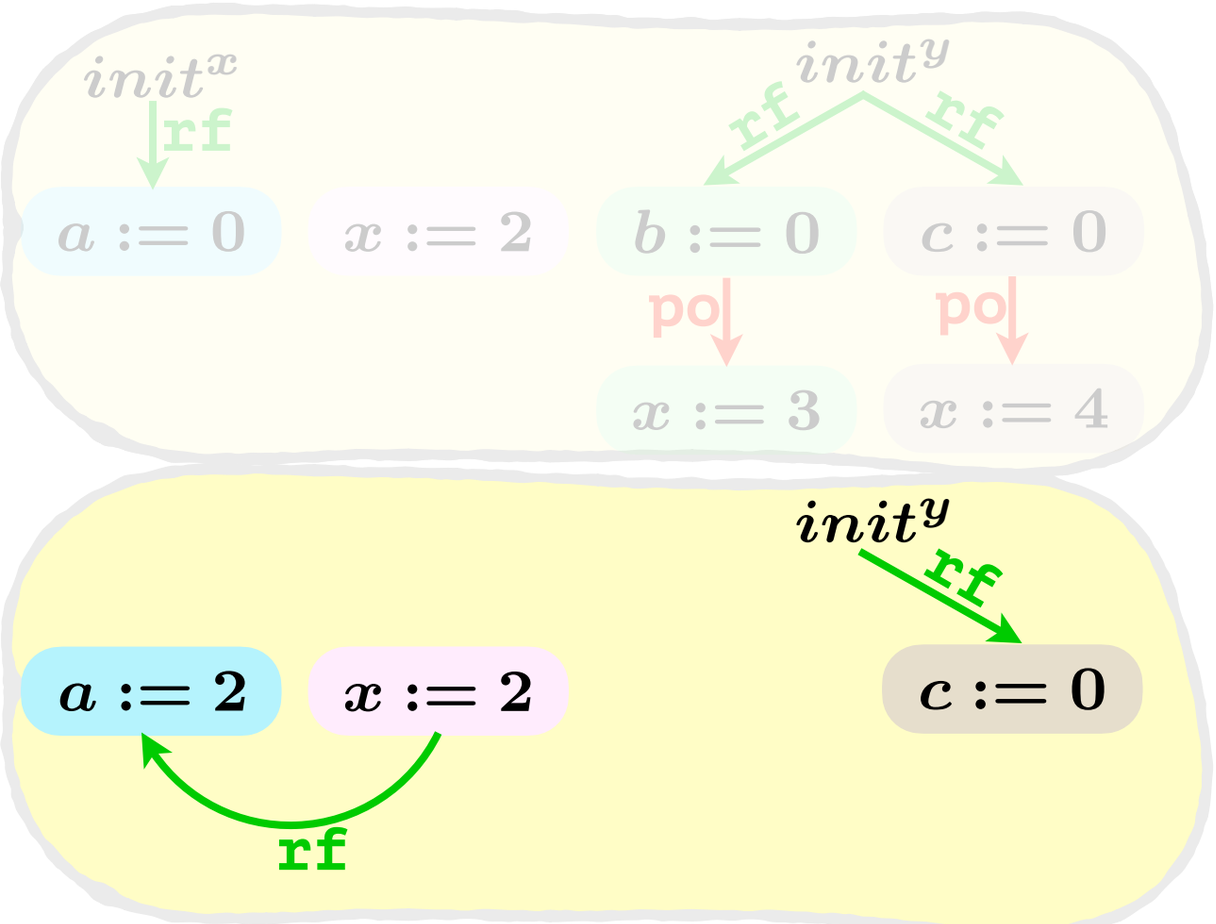
$a := 0$ $a := 2$ $b := 0$ $c := 0$

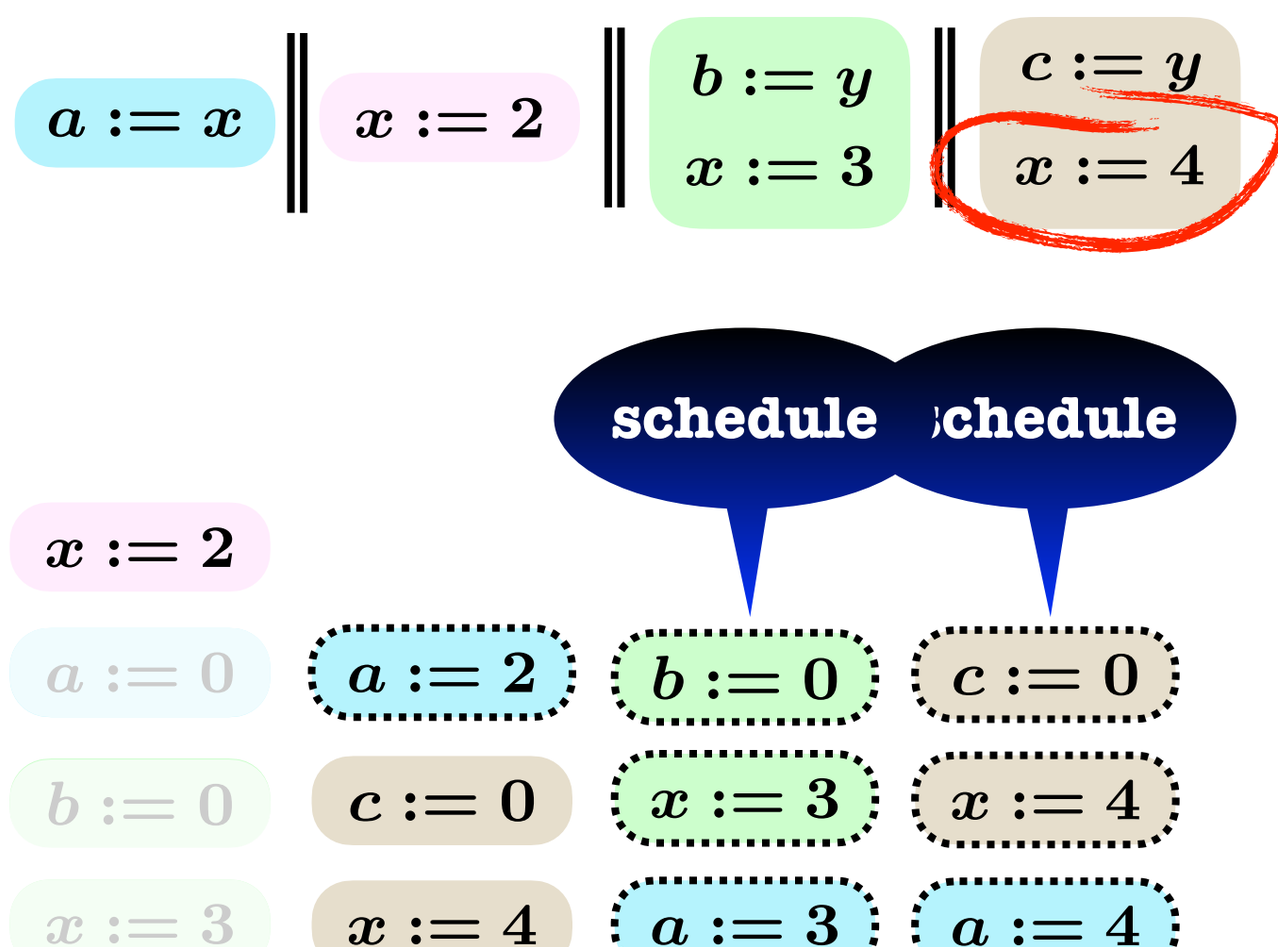
$b := 0$ $c := 0$ $x := 3$ $x := 4$

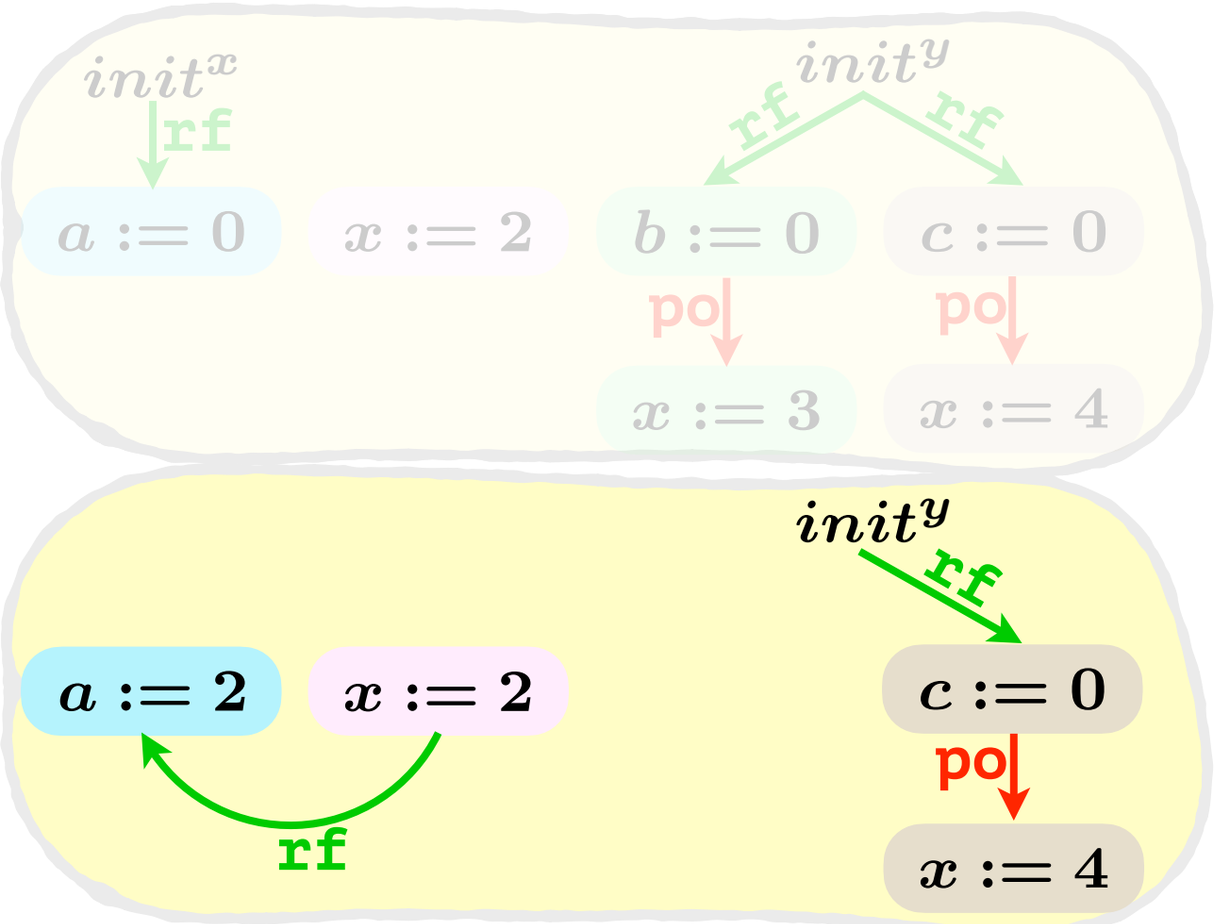
$x := 3$ $a := 3$ $a := 4$

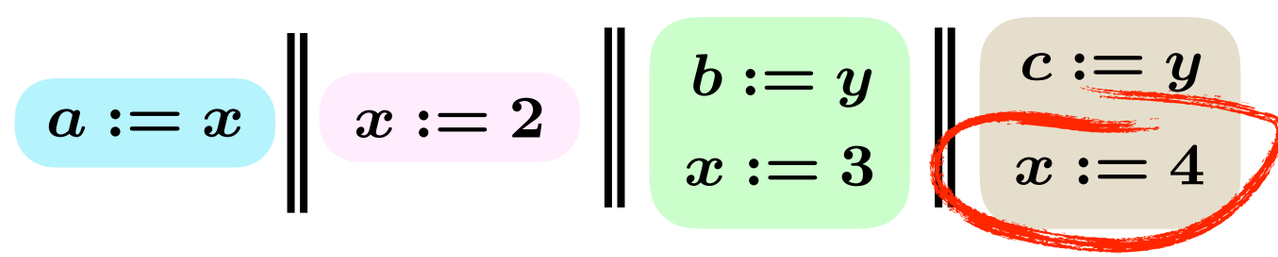
$c := 0$

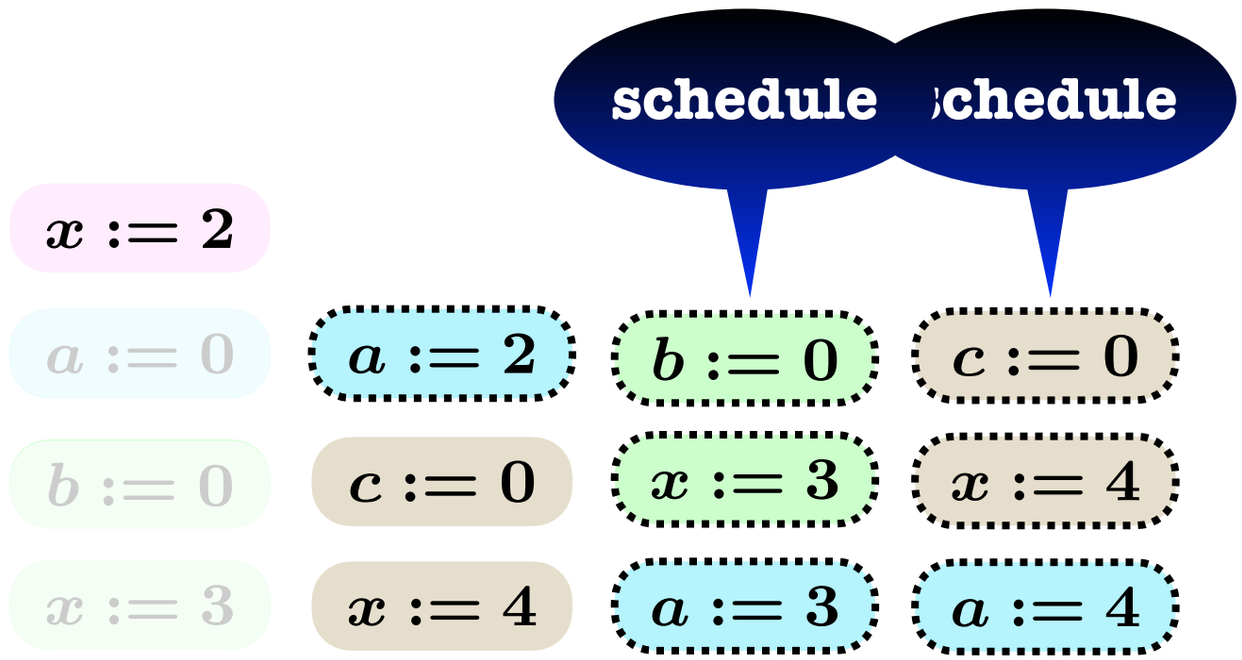
$x := 4$

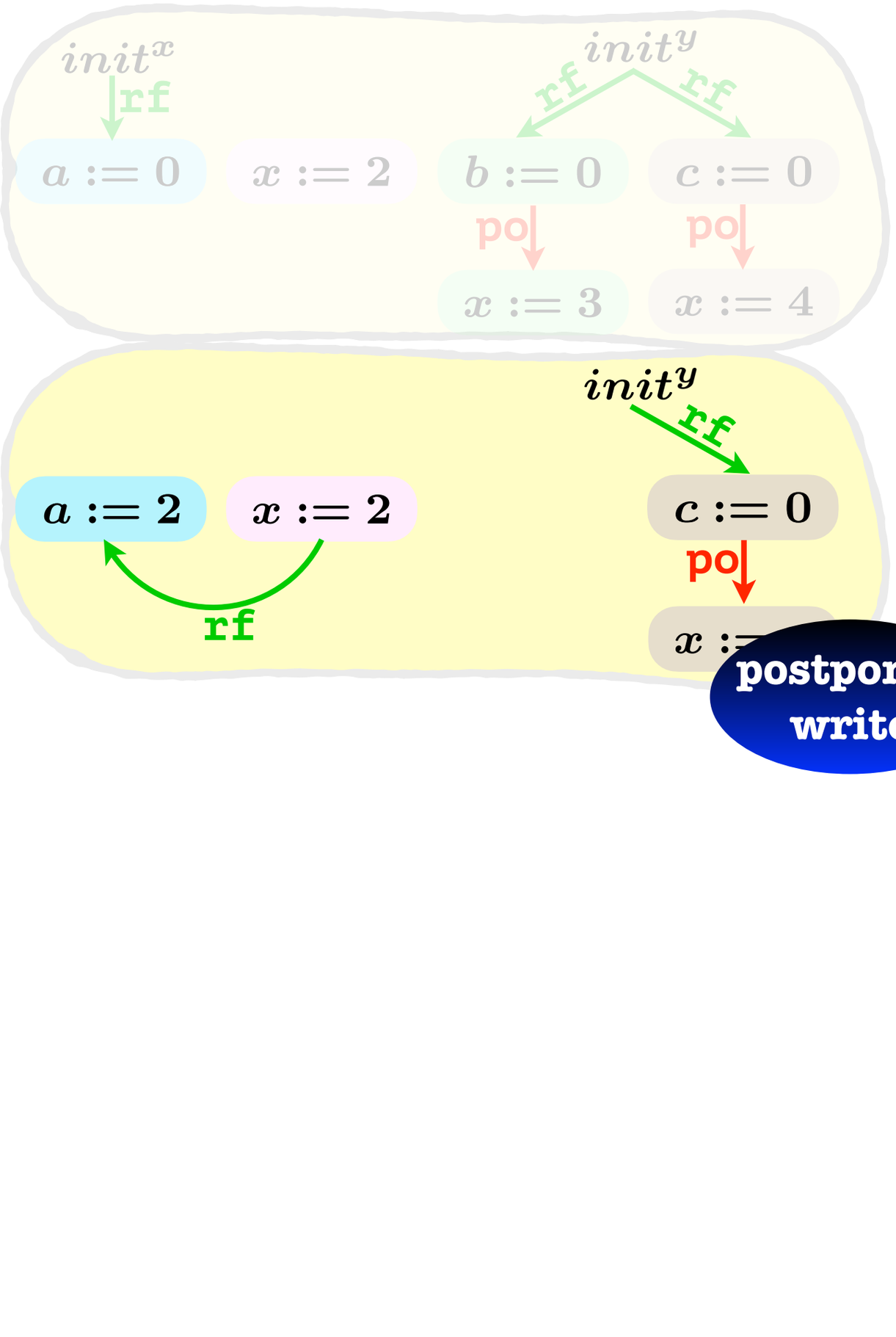

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
$a := 2$
$x := 2$
$c := 0$
rf


$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$
schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$

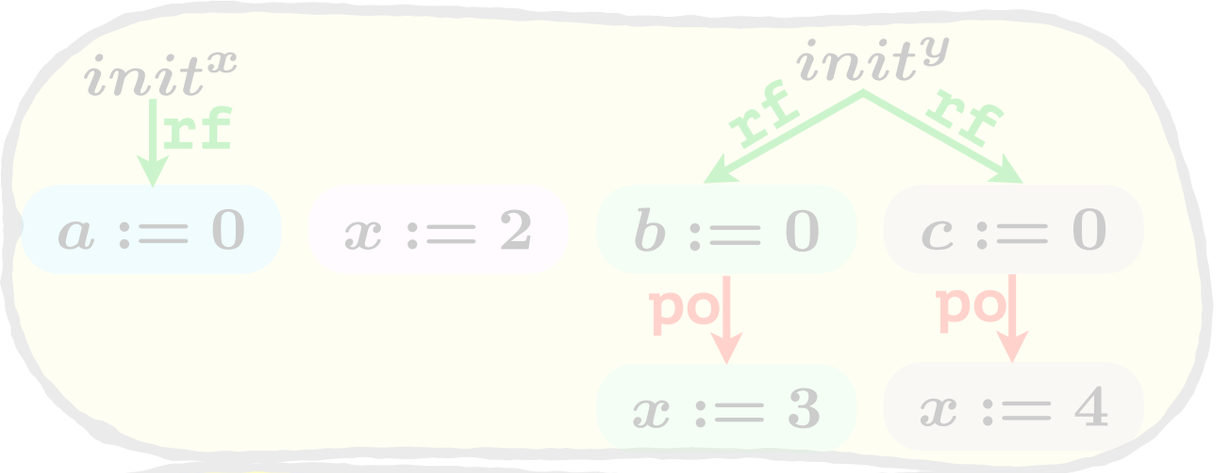

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
$a := 2$
$x := 2$
$c := 0$
po
rf
$x := 4$

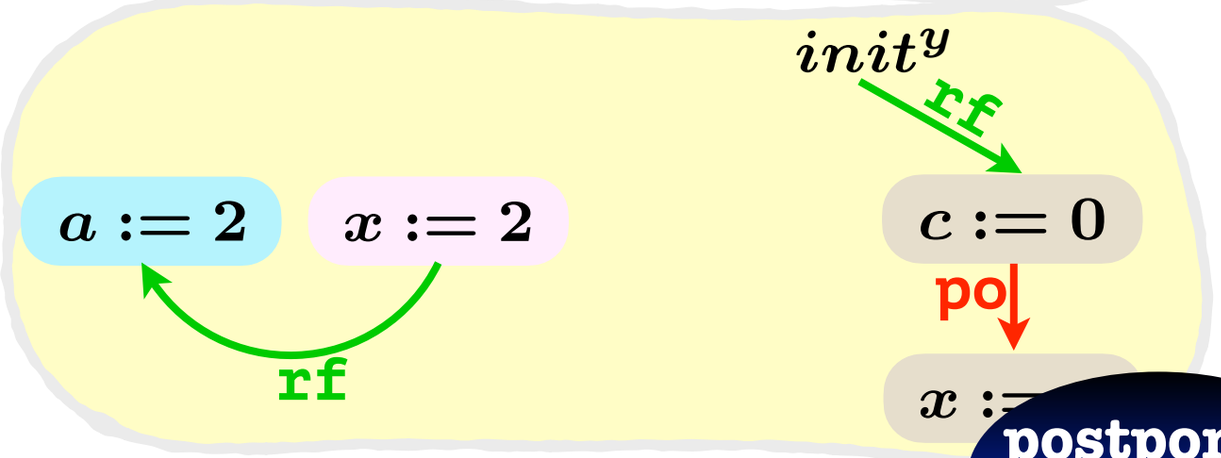

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

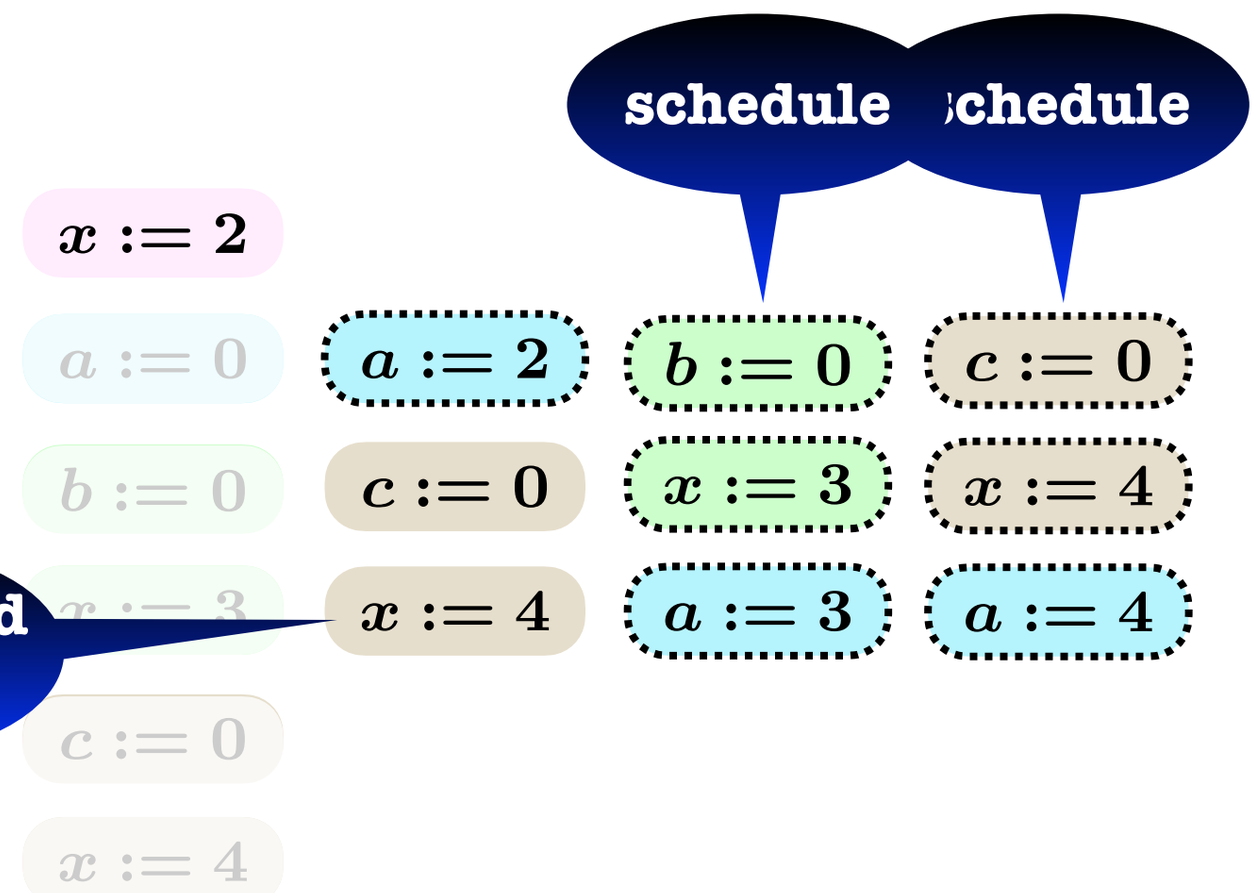

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$


$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
$a := 2$
$x := 2$
$c := 0$
po
rf
$x :=$
postponed write
$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$
schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$x := 4$

$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$



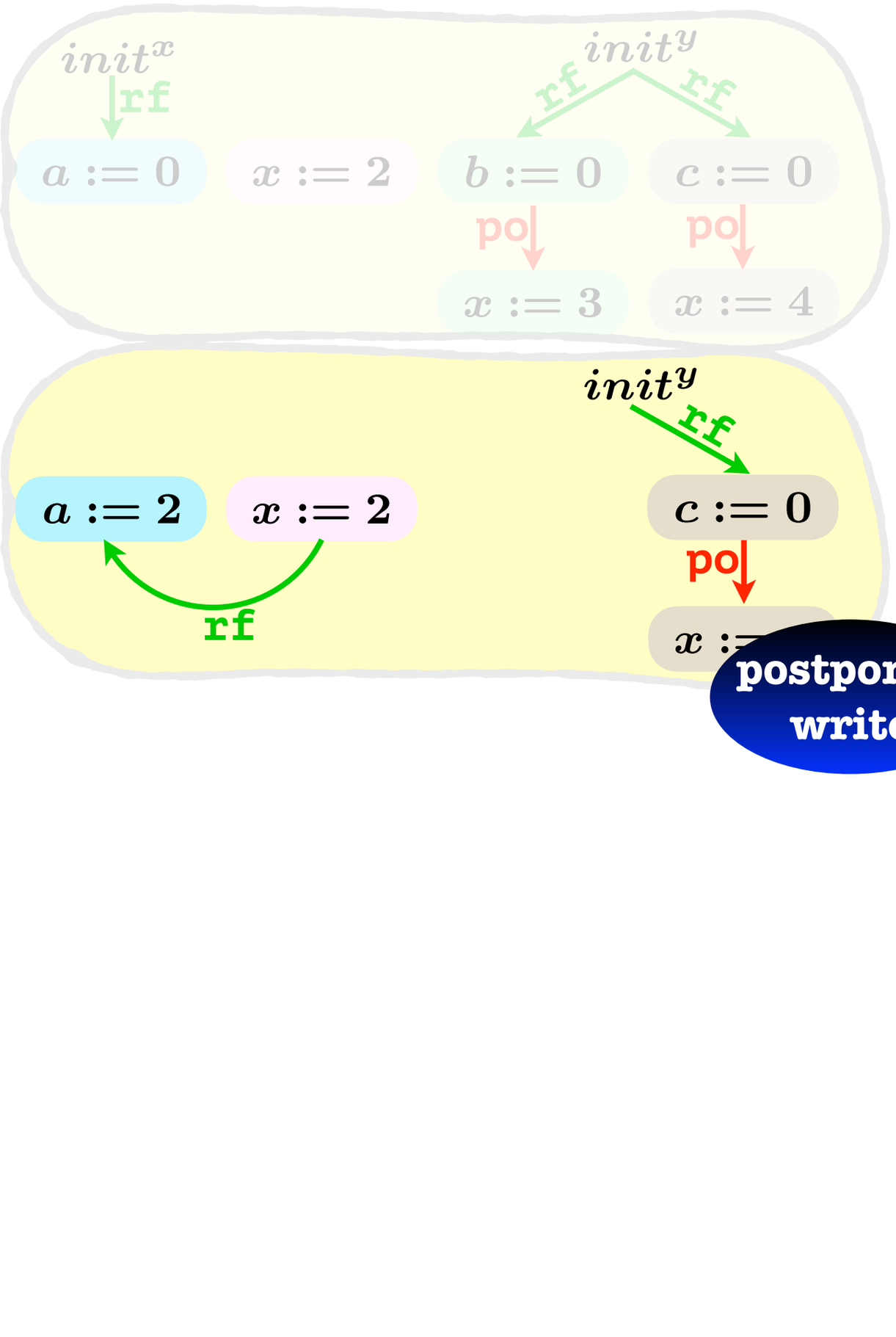


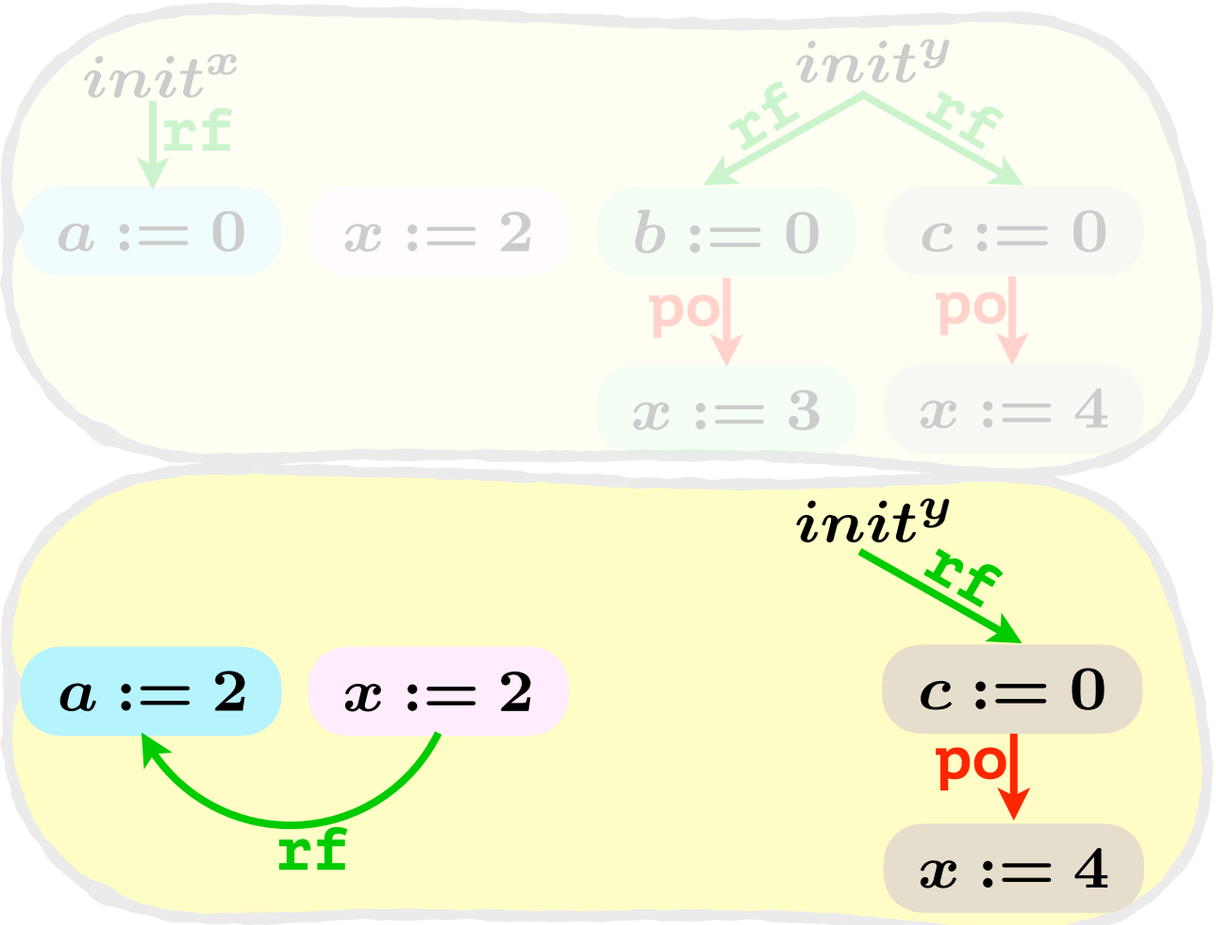


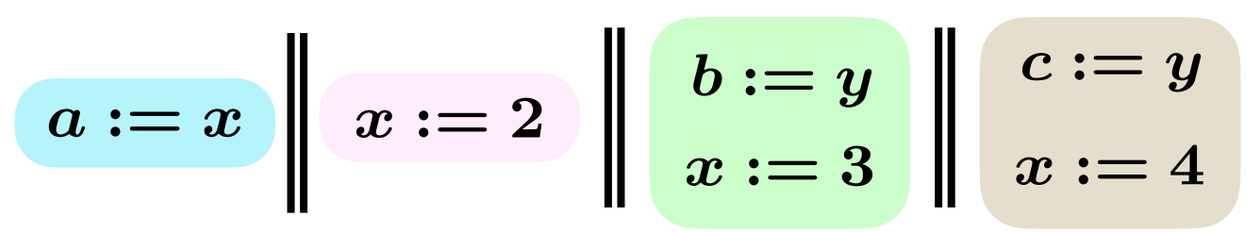
$c := 0$

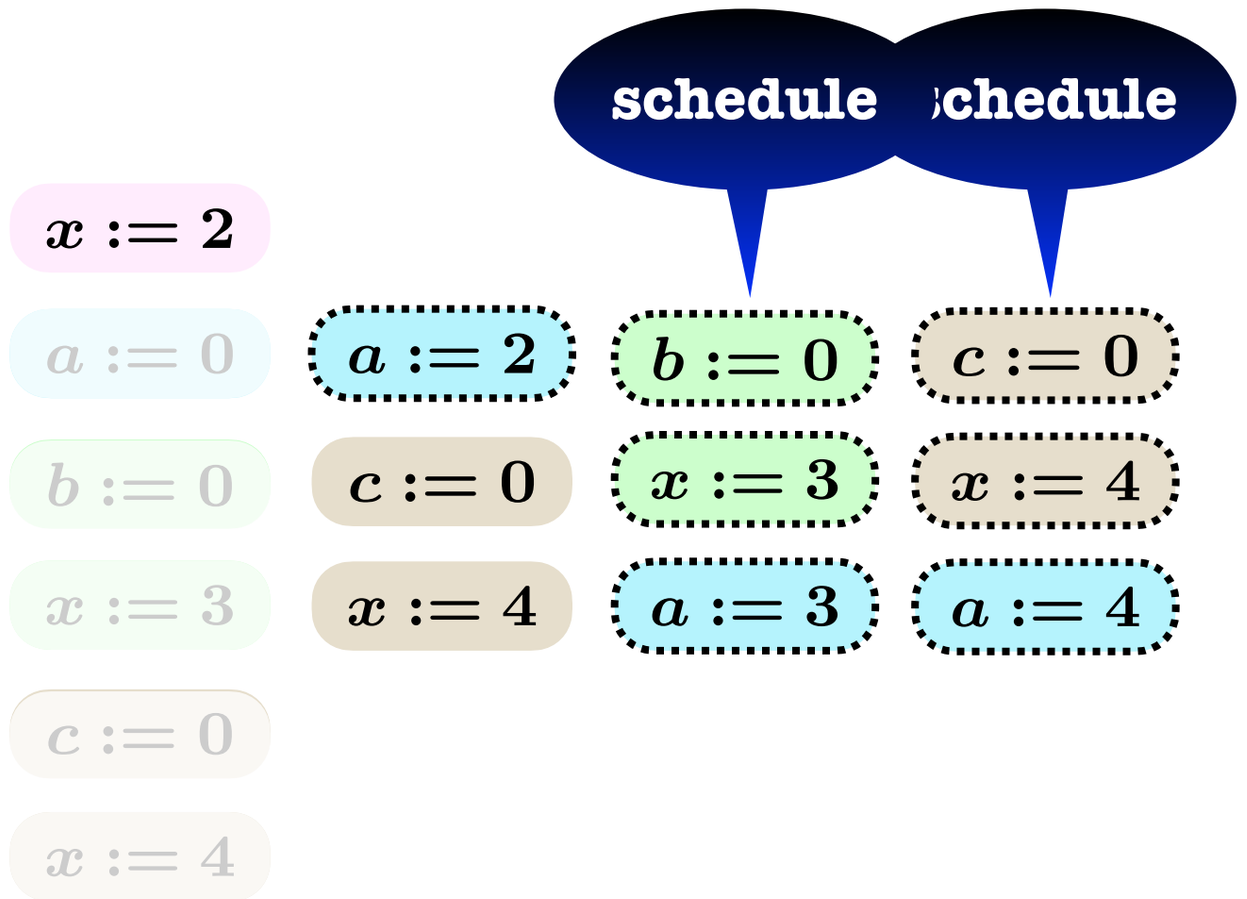
$x := 4$

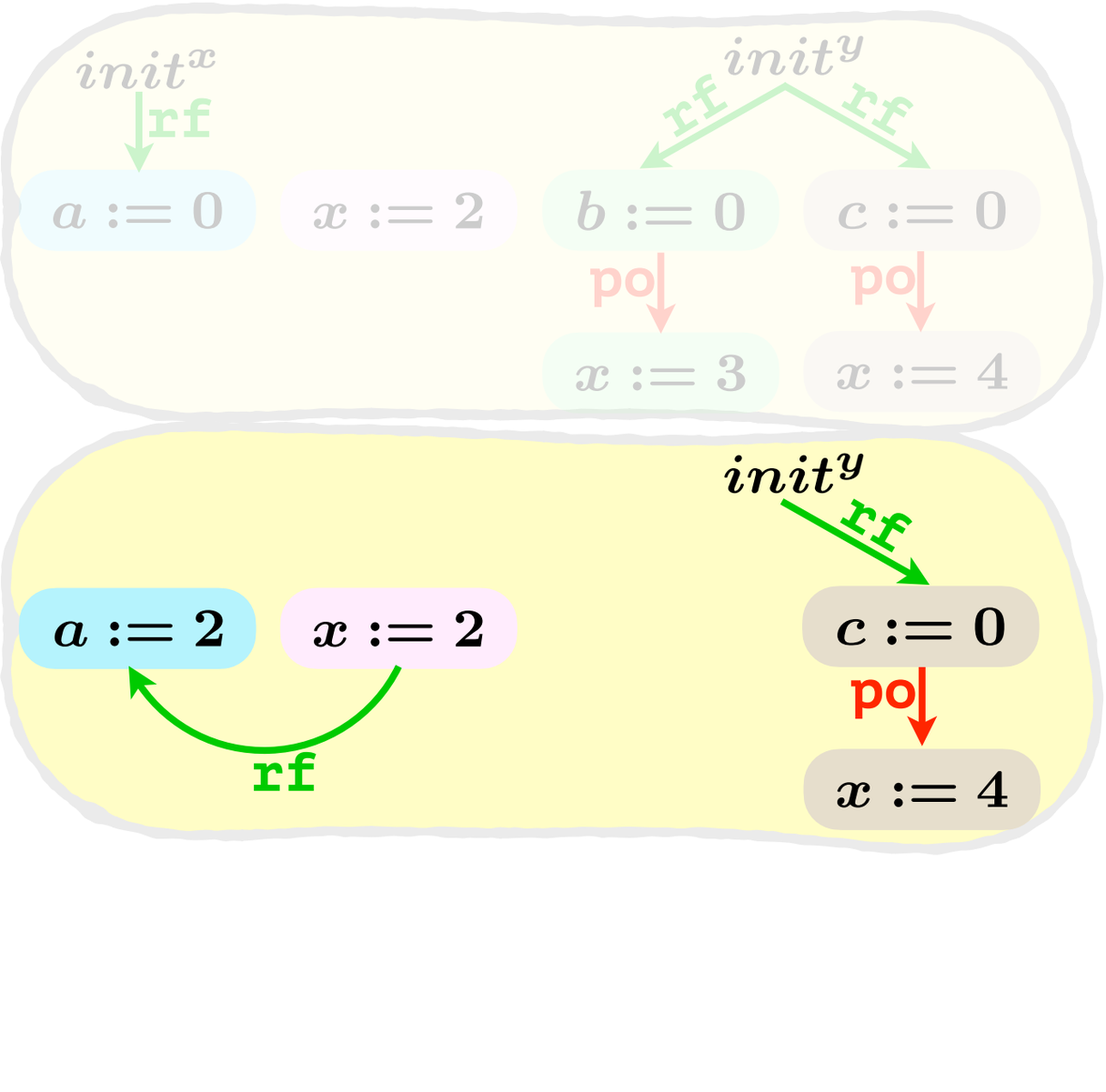
$a := 4$


$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
$a := 2$
$x := 2$
$c := 0$
po
rf
$x :=$
postponed write
$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$
schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$x := 4$

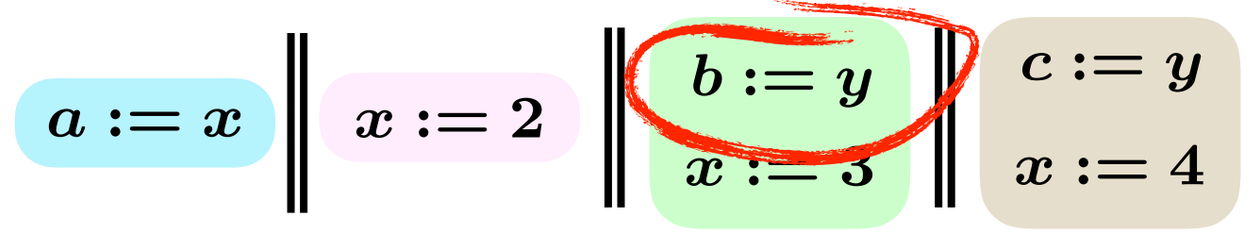

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
$a := 2$
$x := 2$
$c := 0$
po
rf
$x := 4$

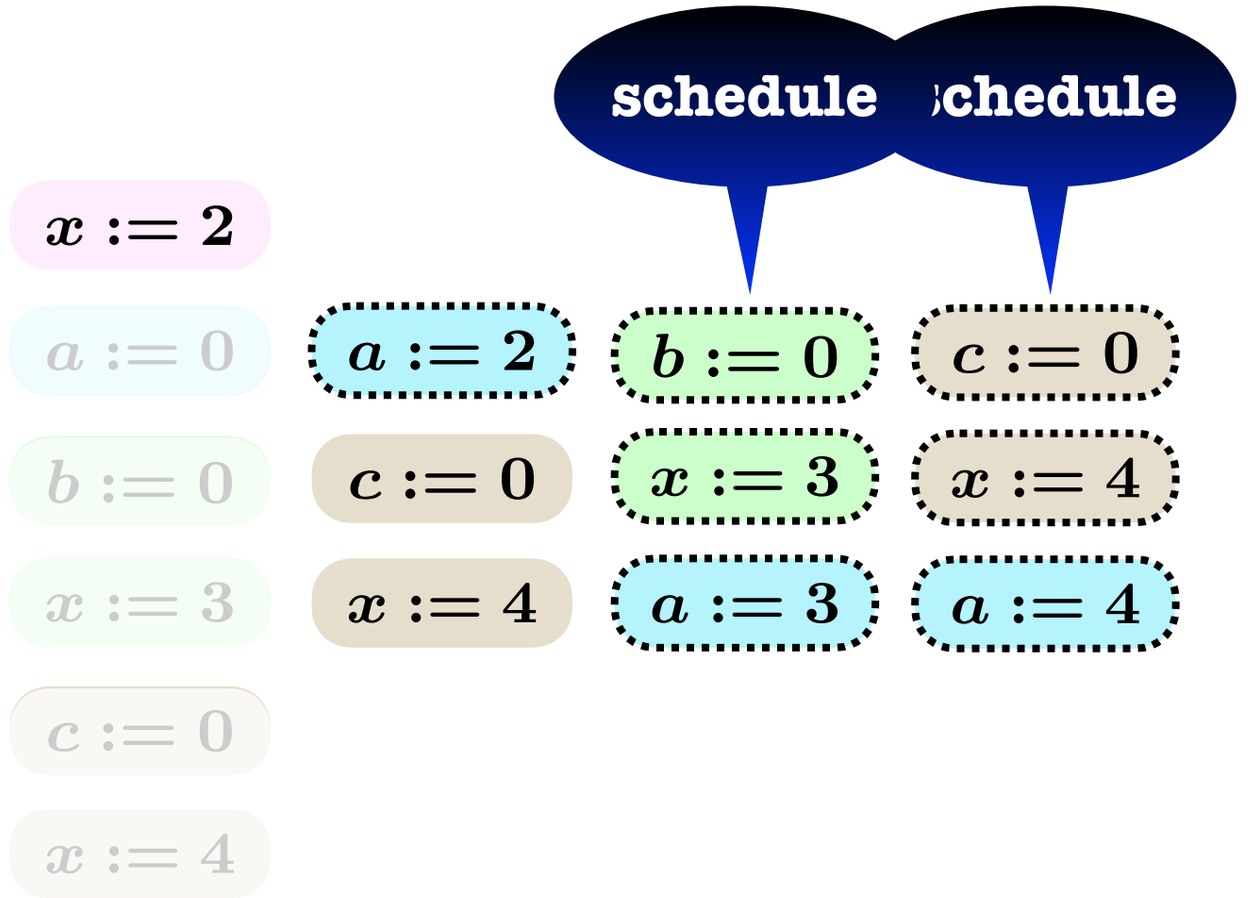

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$


schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$x := 4$

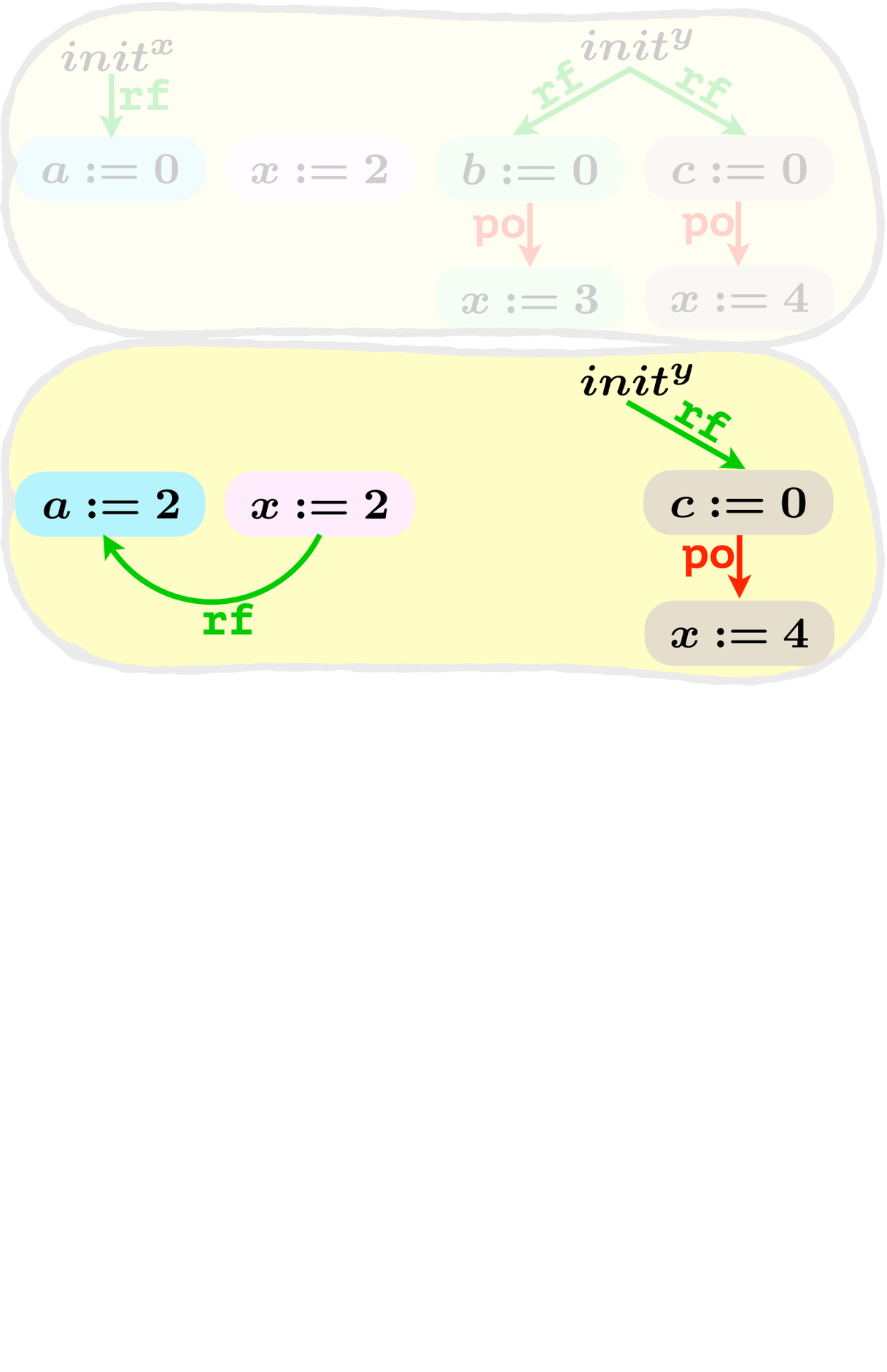

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

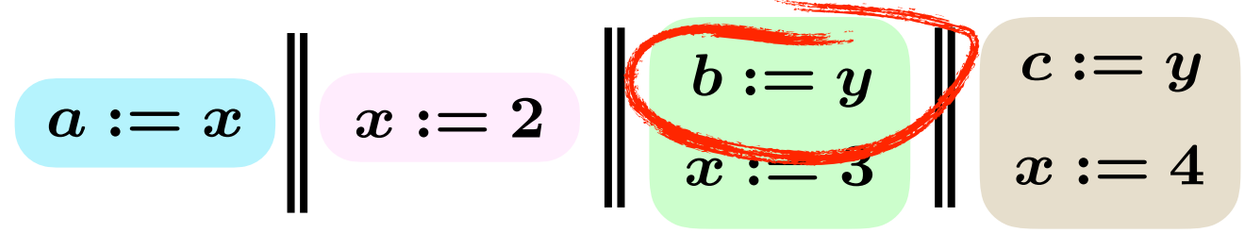
$init^y$
rf
$a := 2$
$x := 2$
$c := 0$
po
rf
$x := 4$

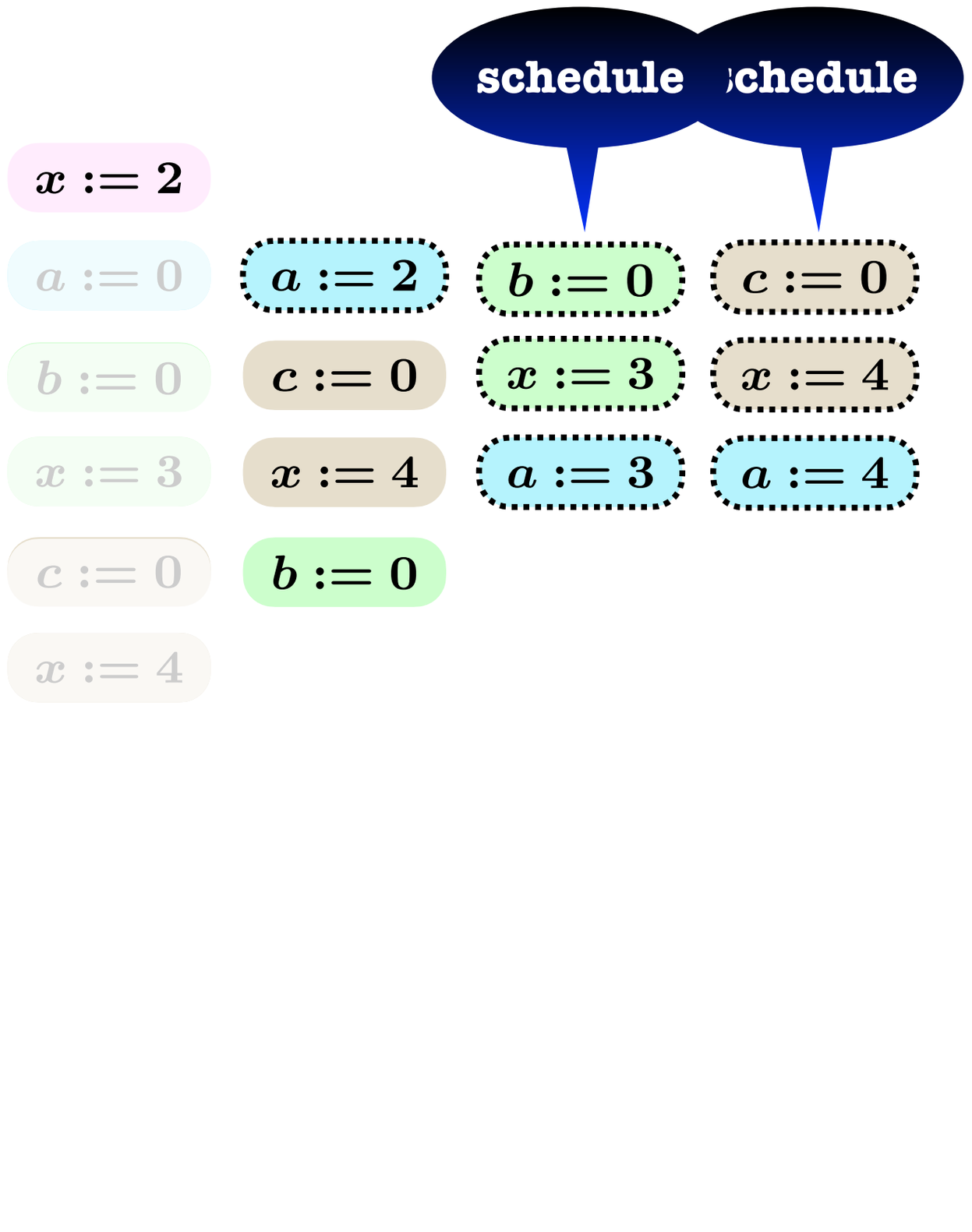

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

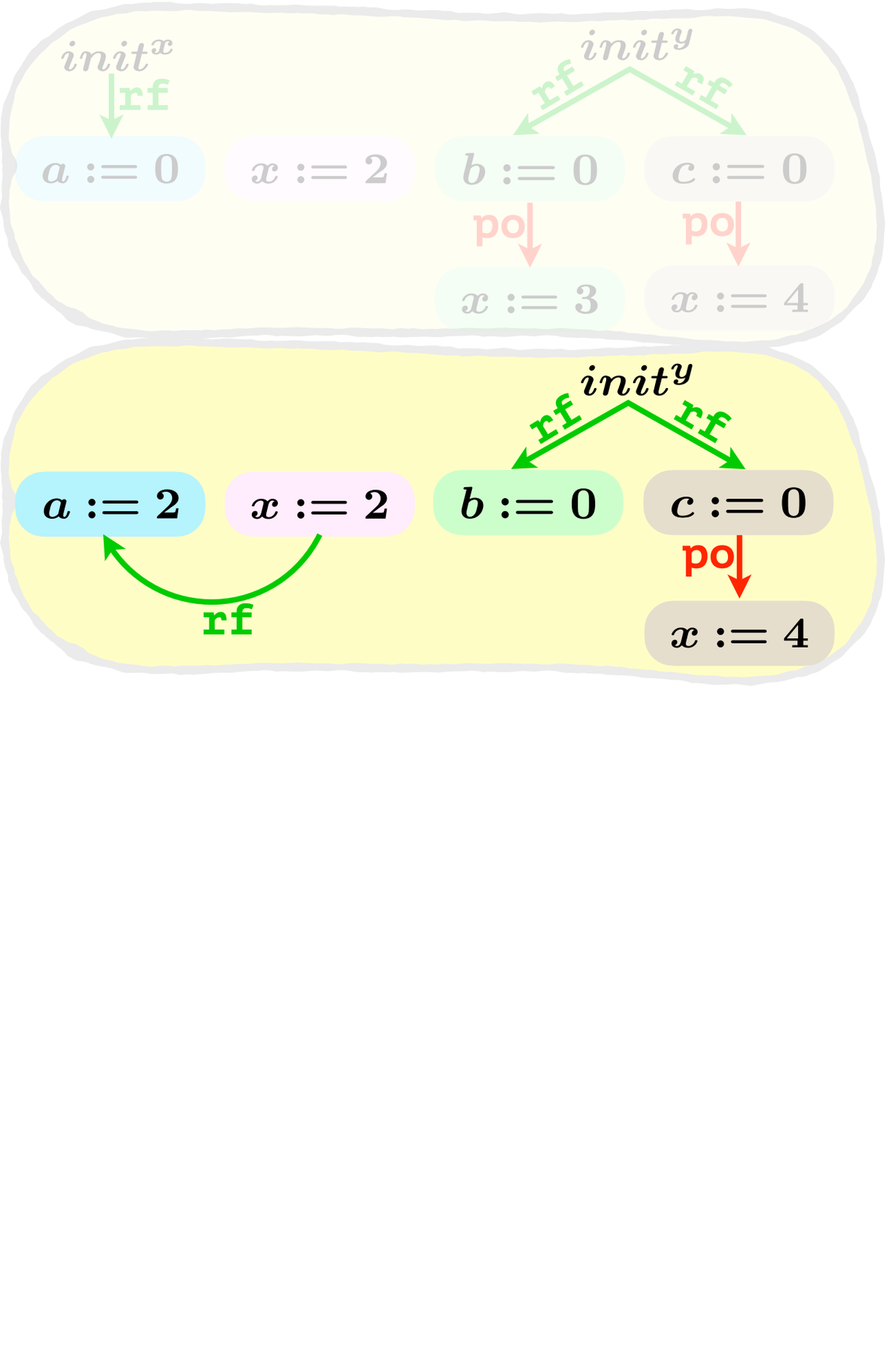

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$

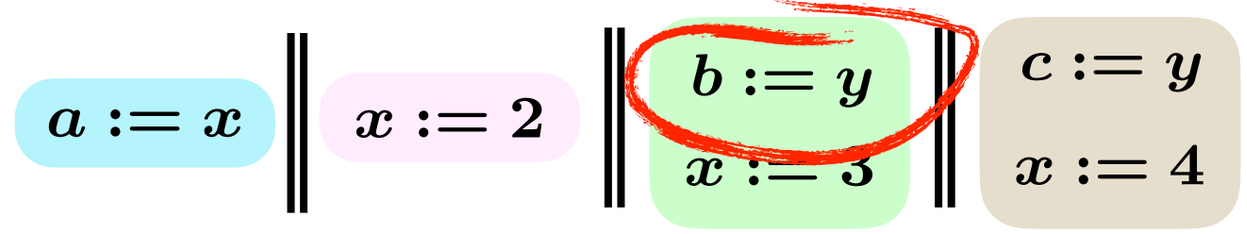
$c := 0$
$x := 4$

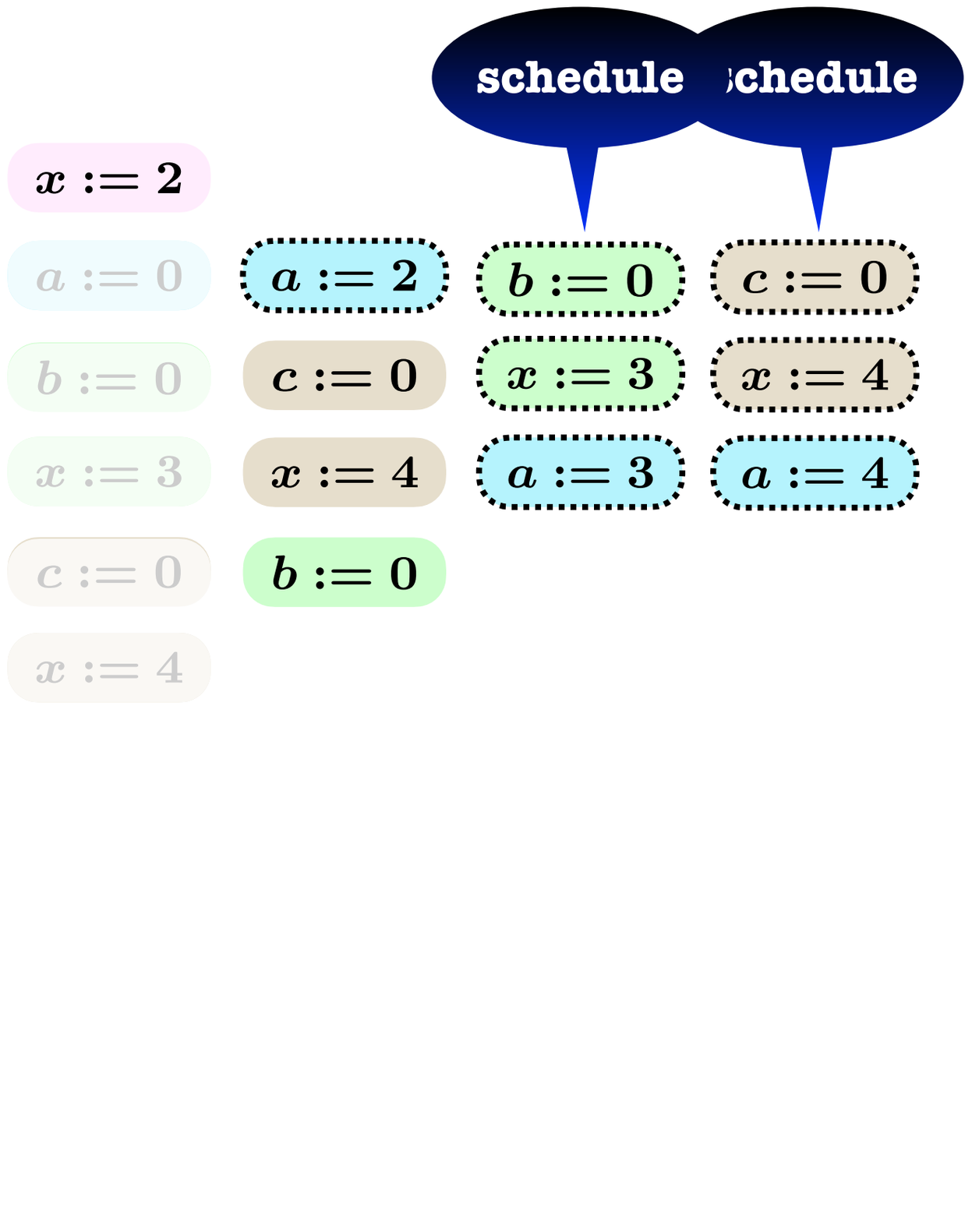

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
$a := 2$
$x := 2$
$c := 0$
po
rf
$x := 4$

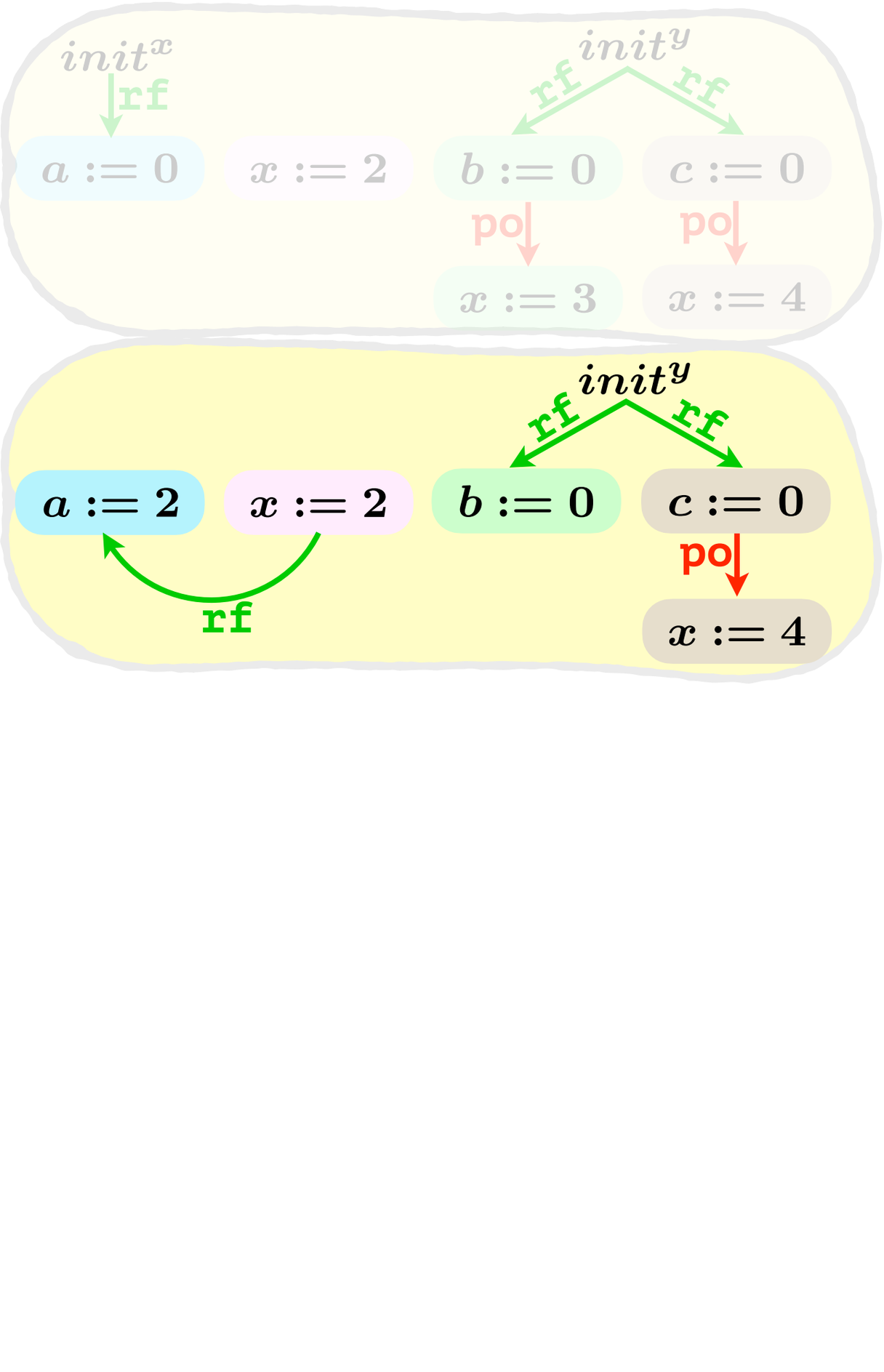

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

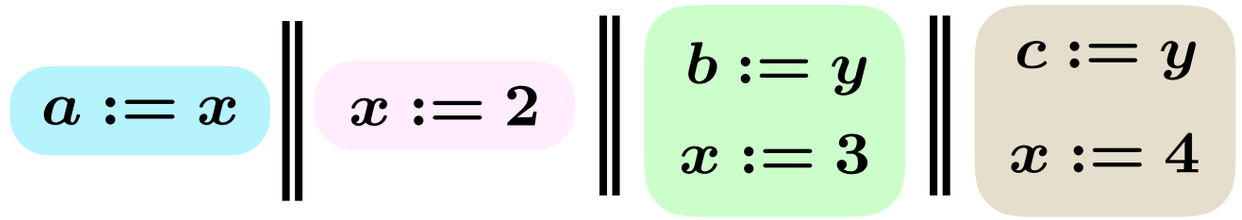

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$

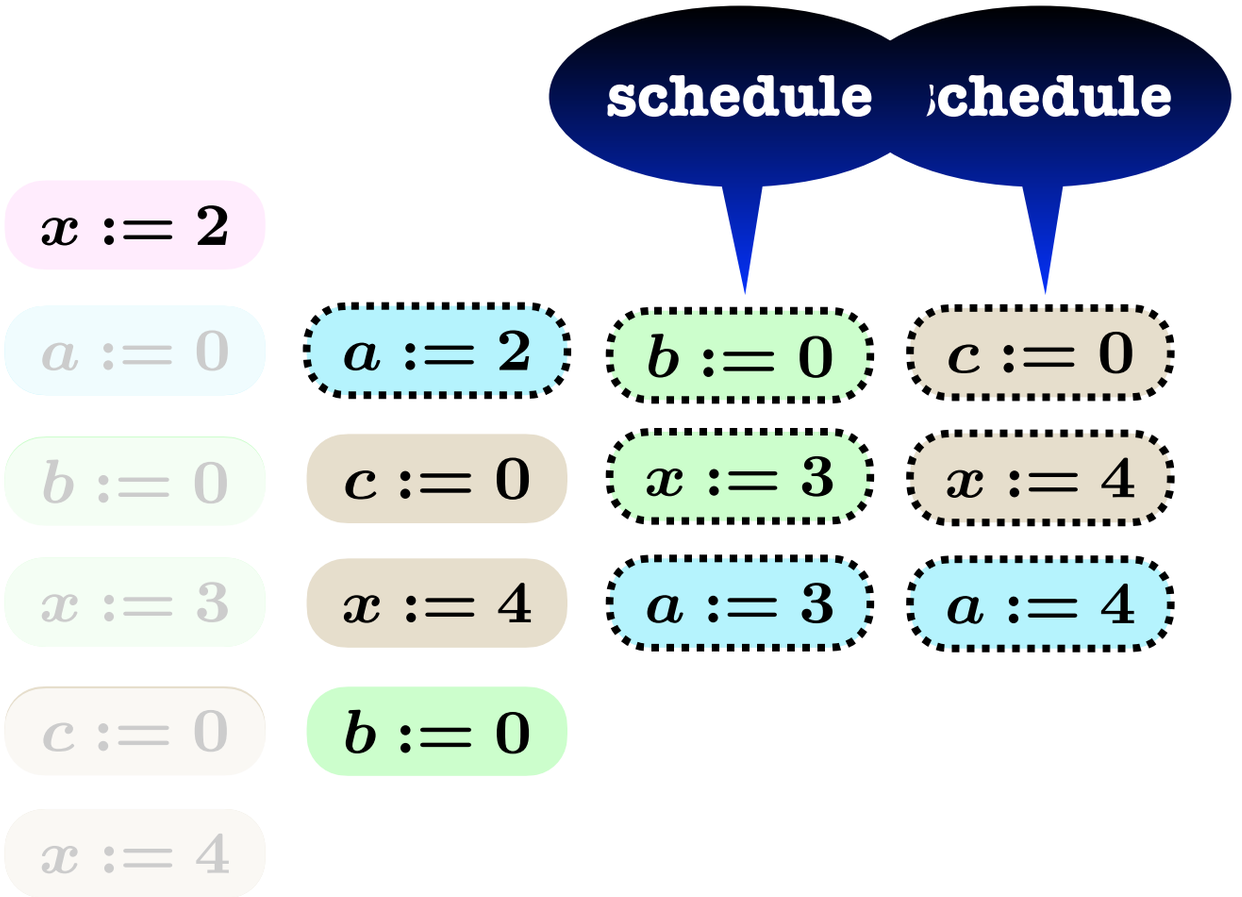

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 4$

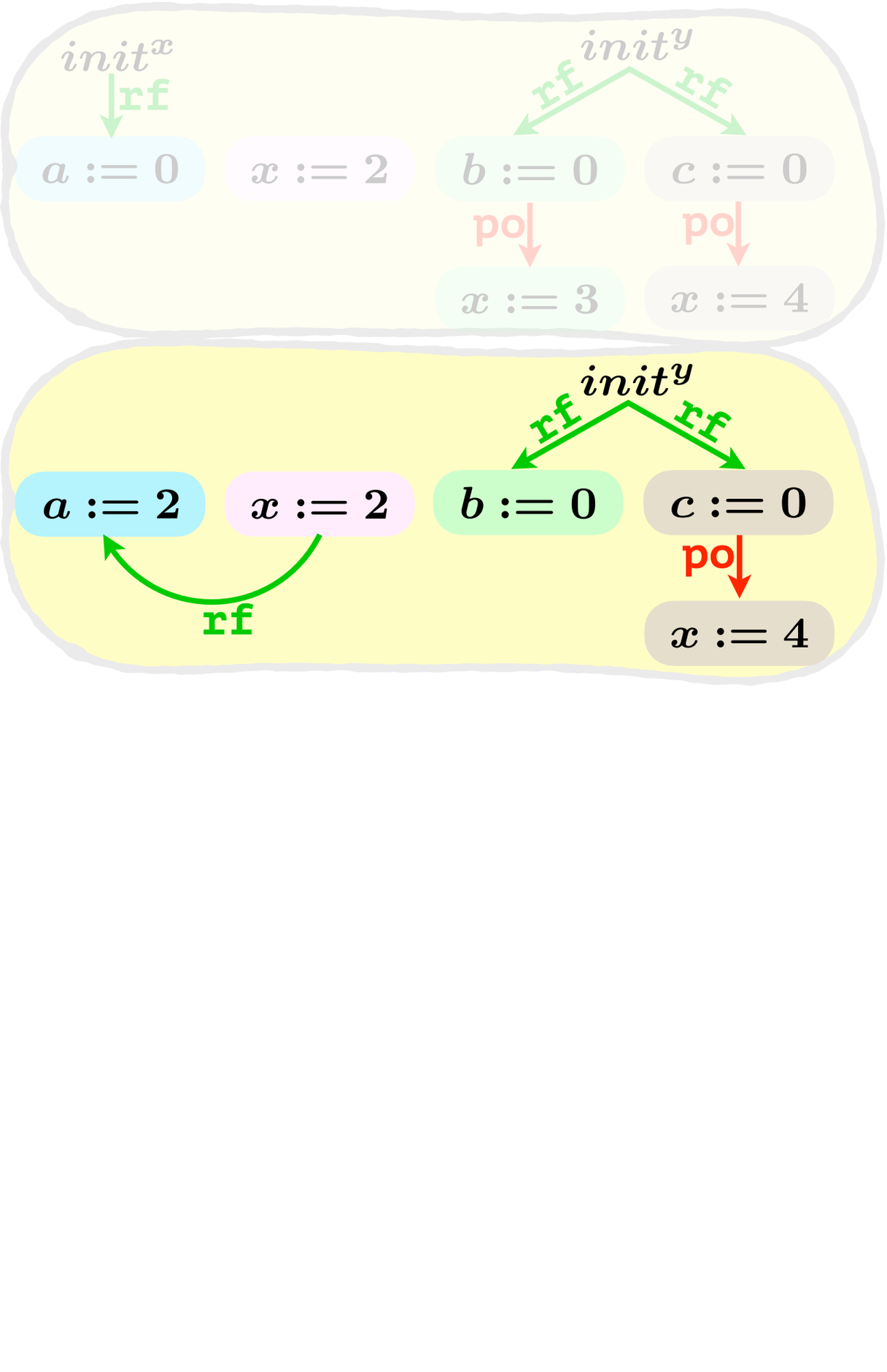

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

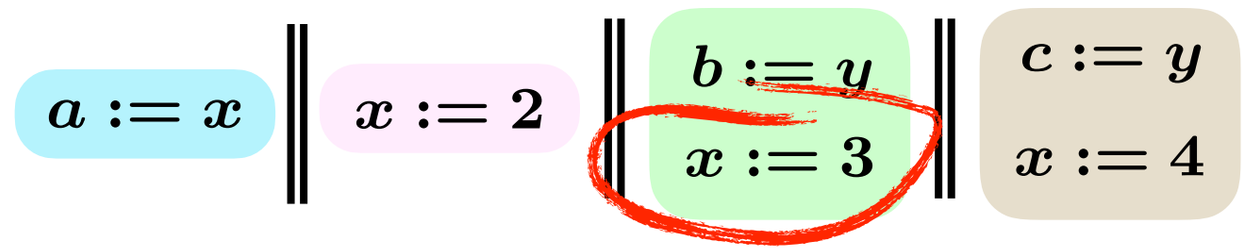

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$

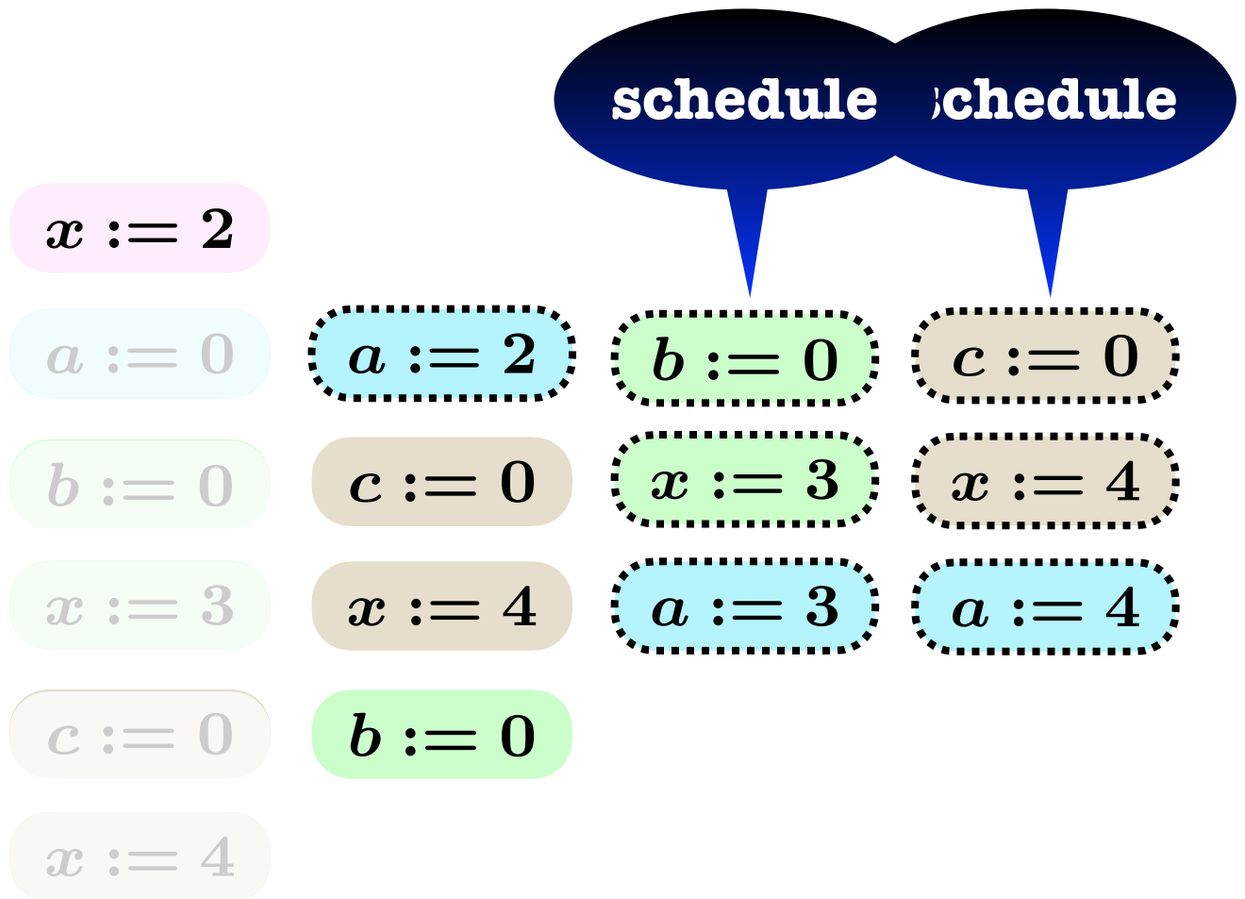

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 4$

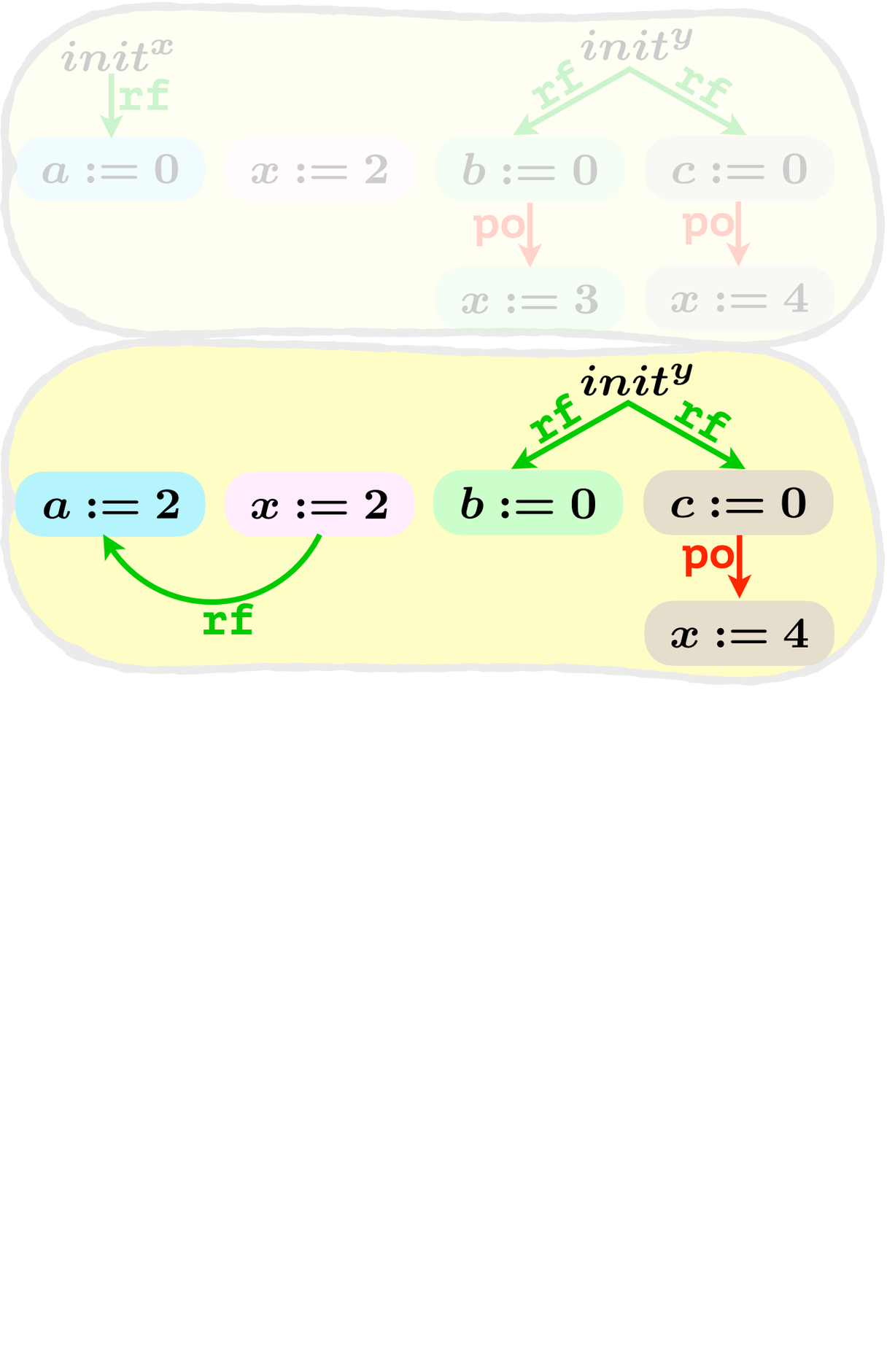

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

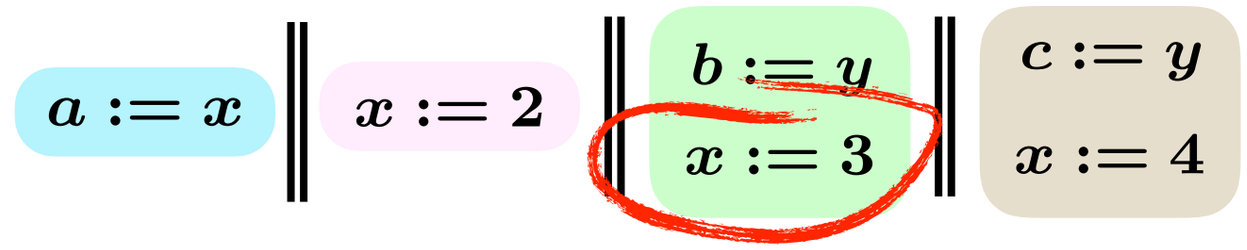

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$

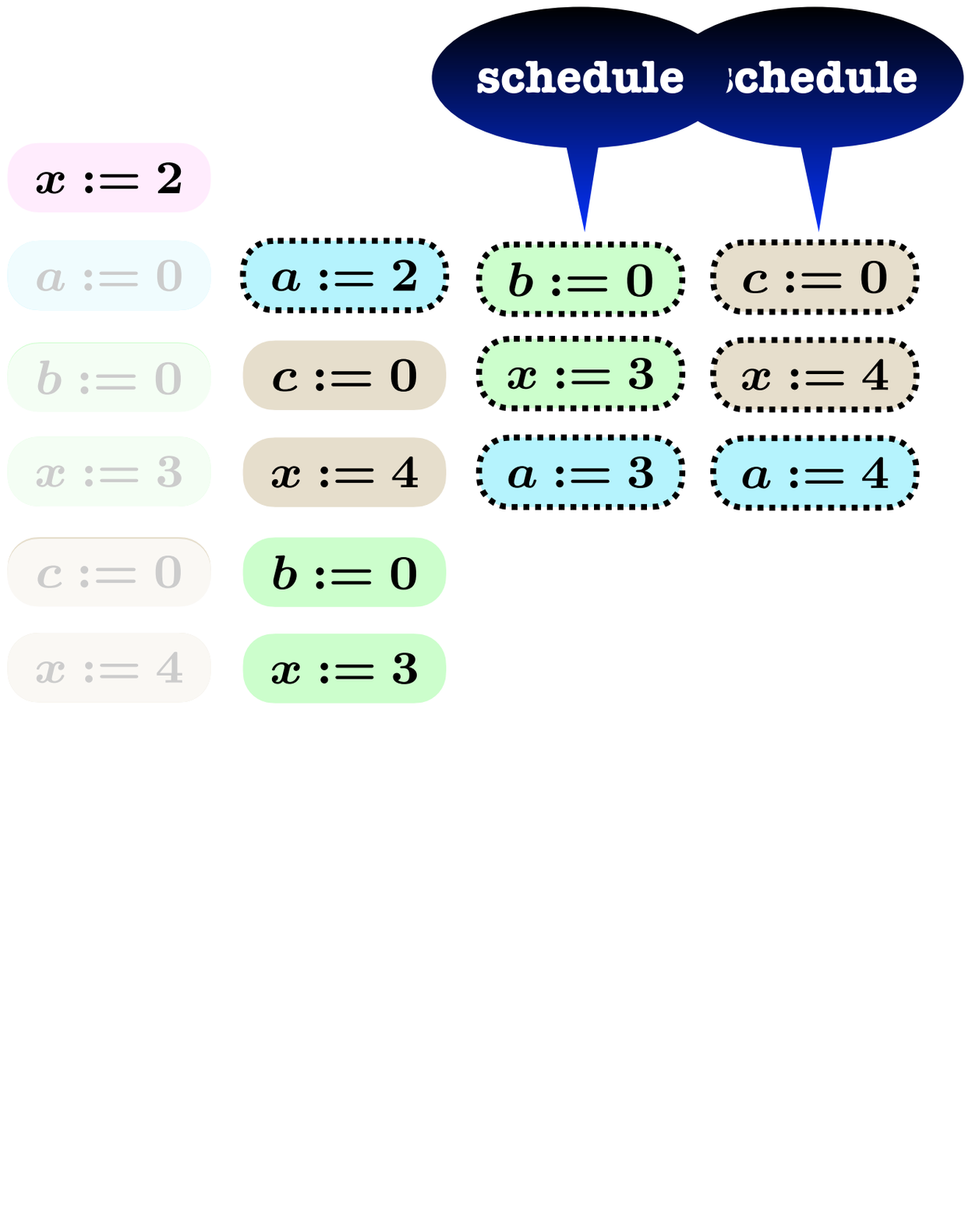

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 4$

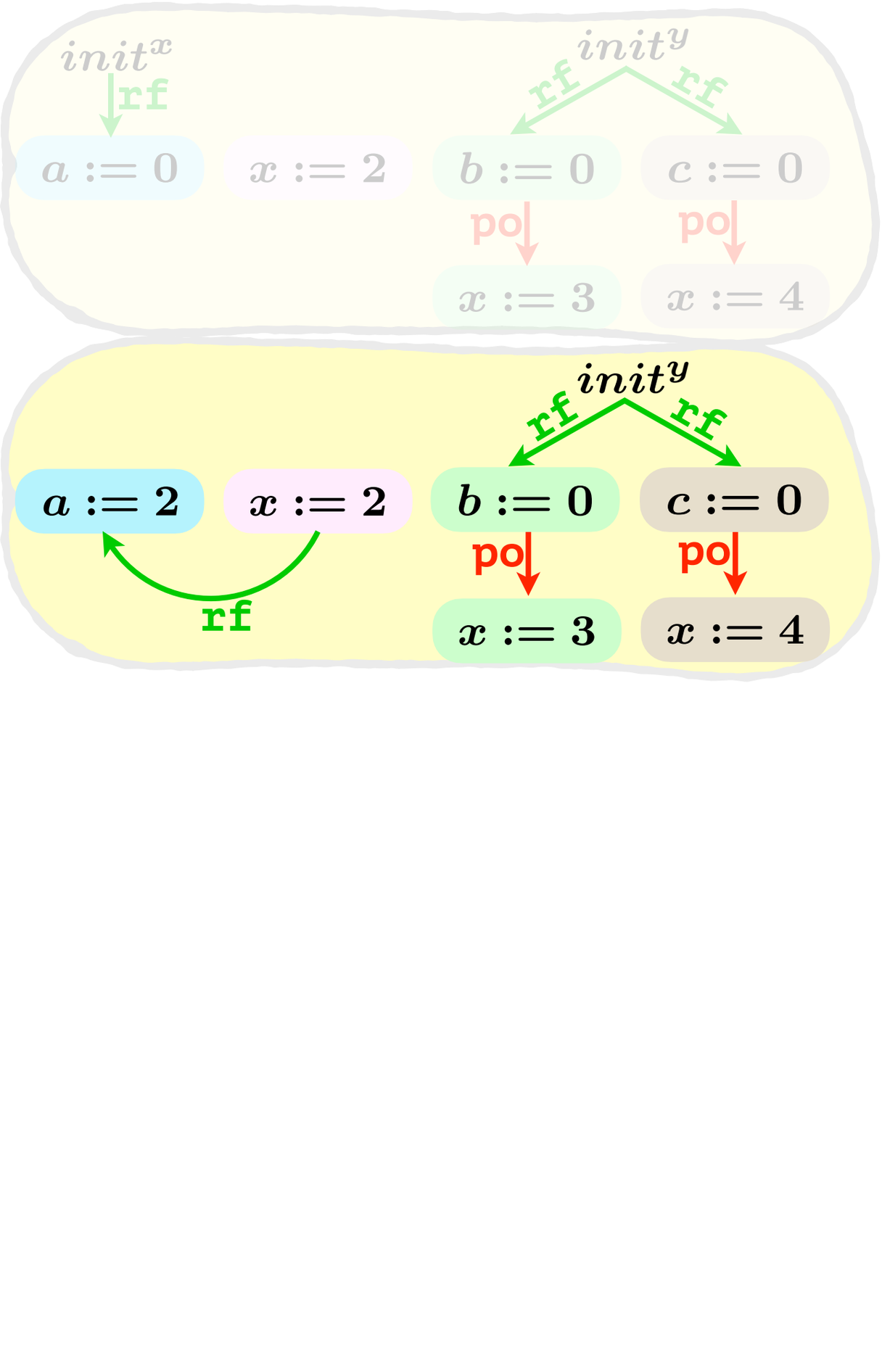

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

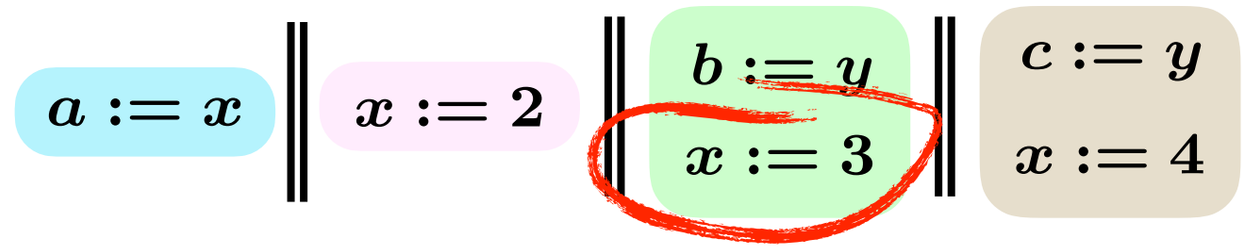

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$

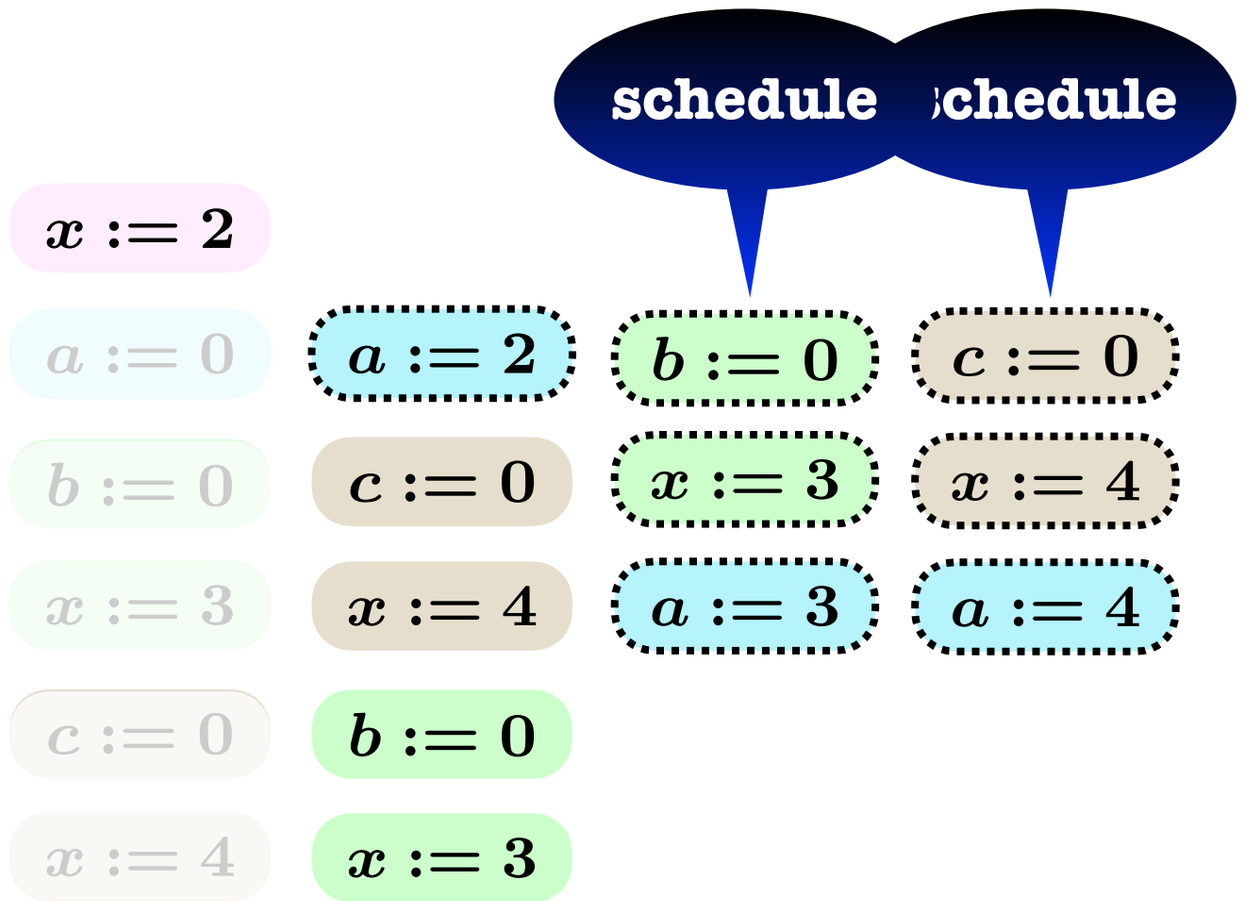

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 4$

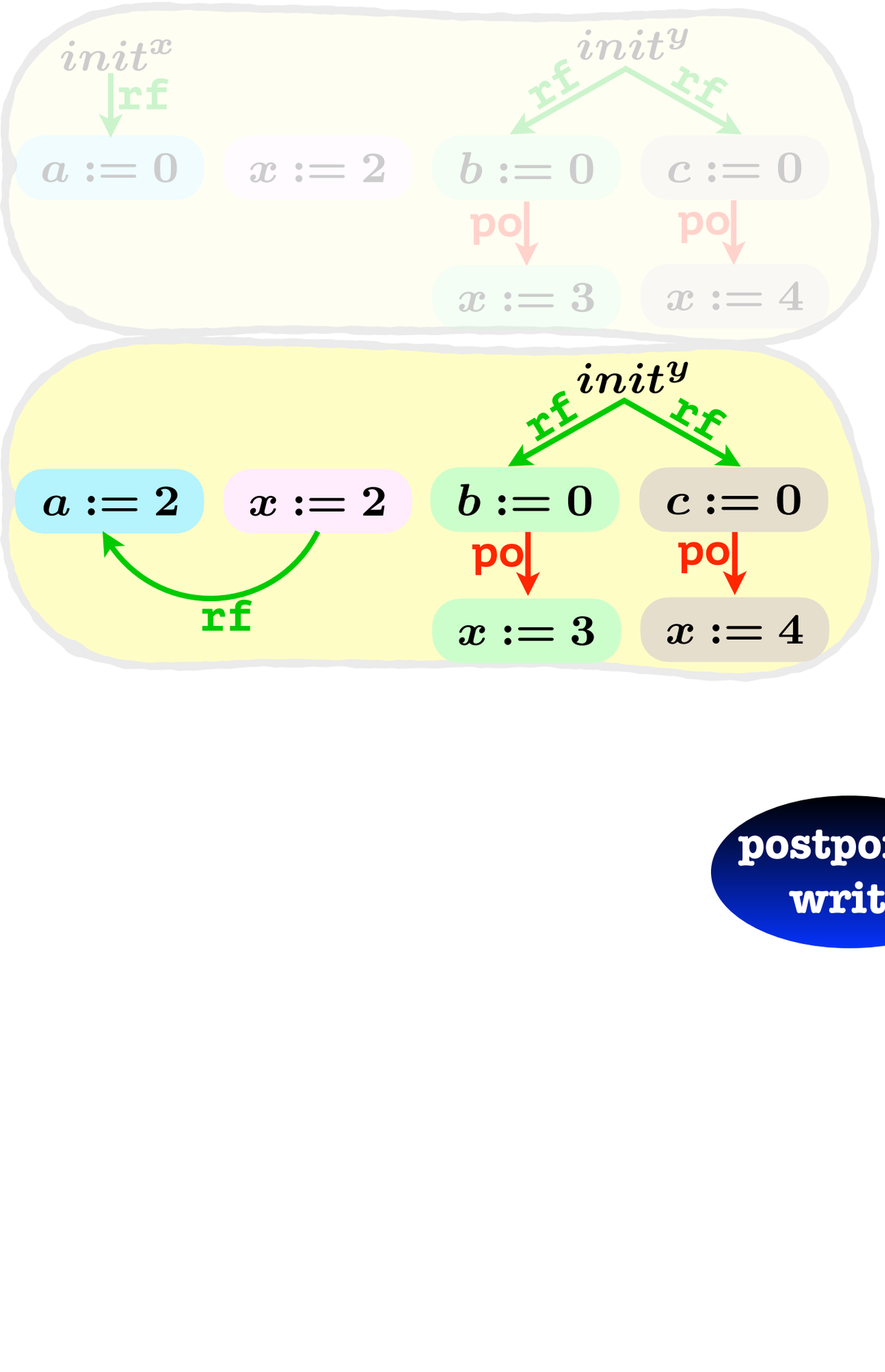

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

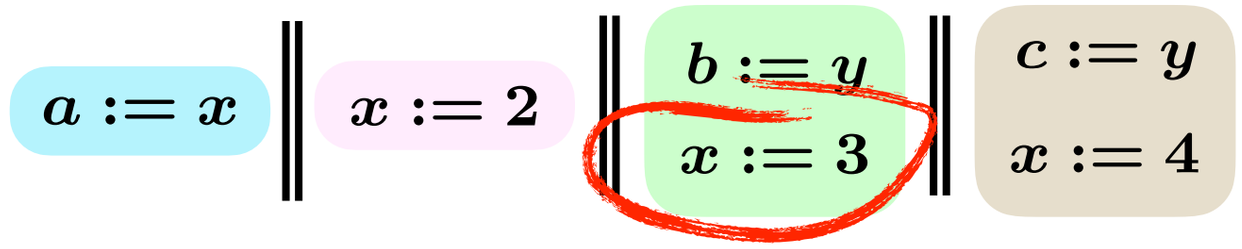

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$
$x := 3$

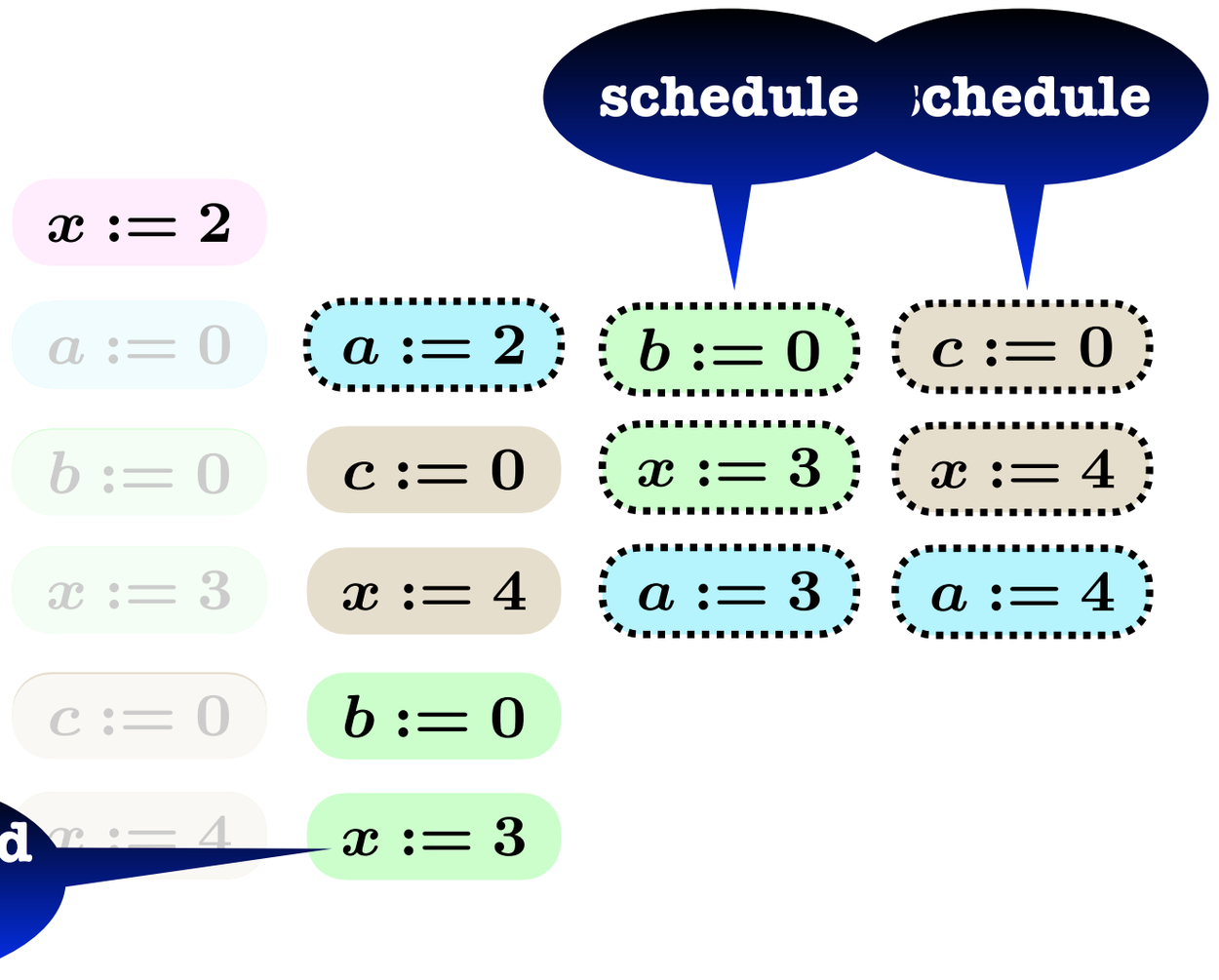

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

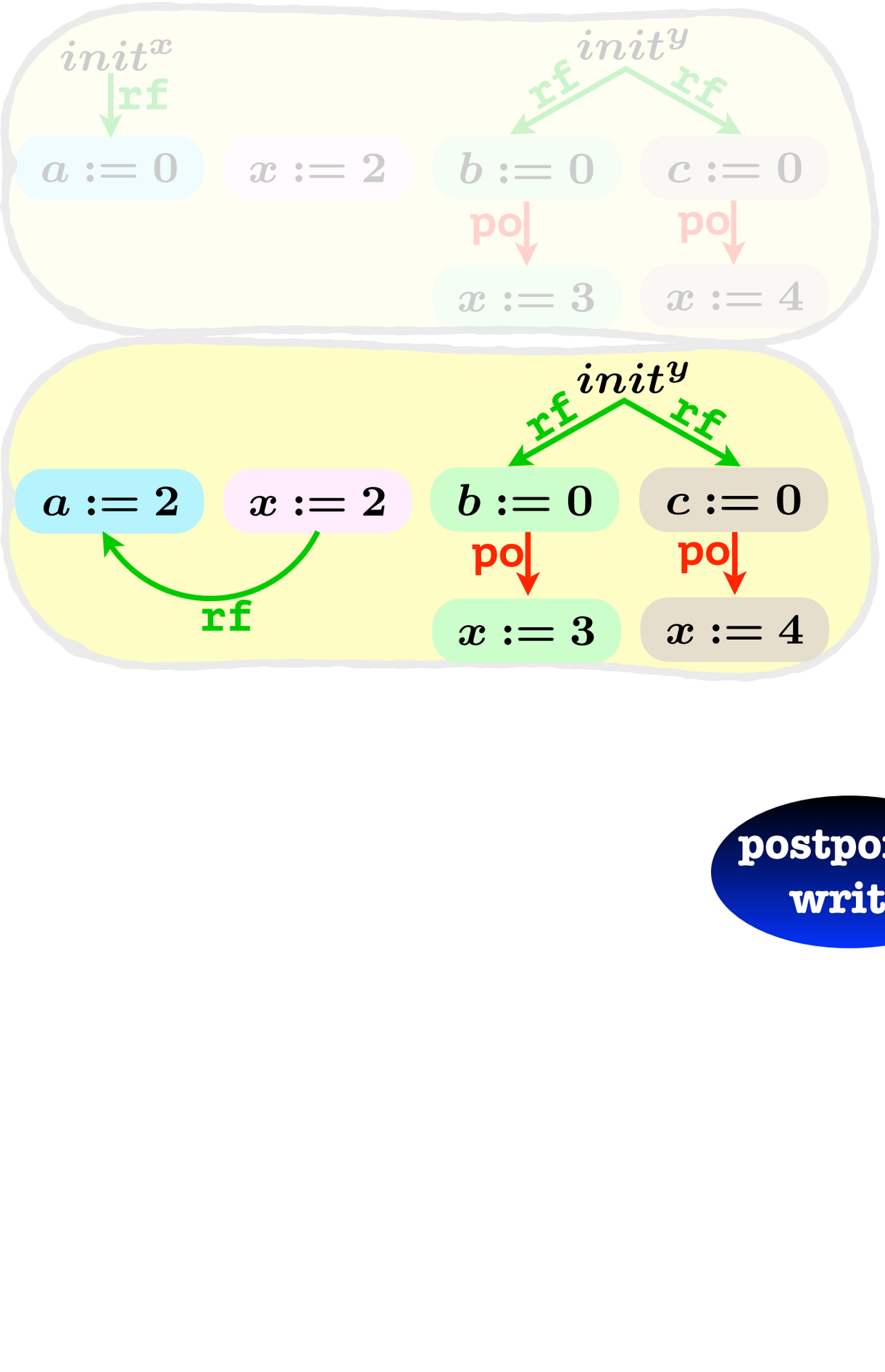

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

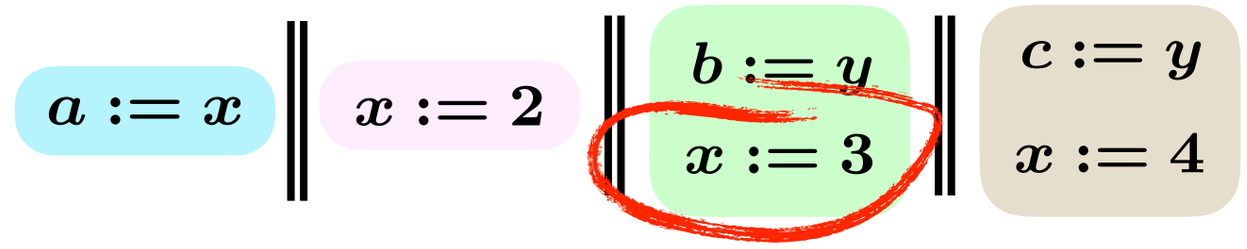

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$
$x := 3$

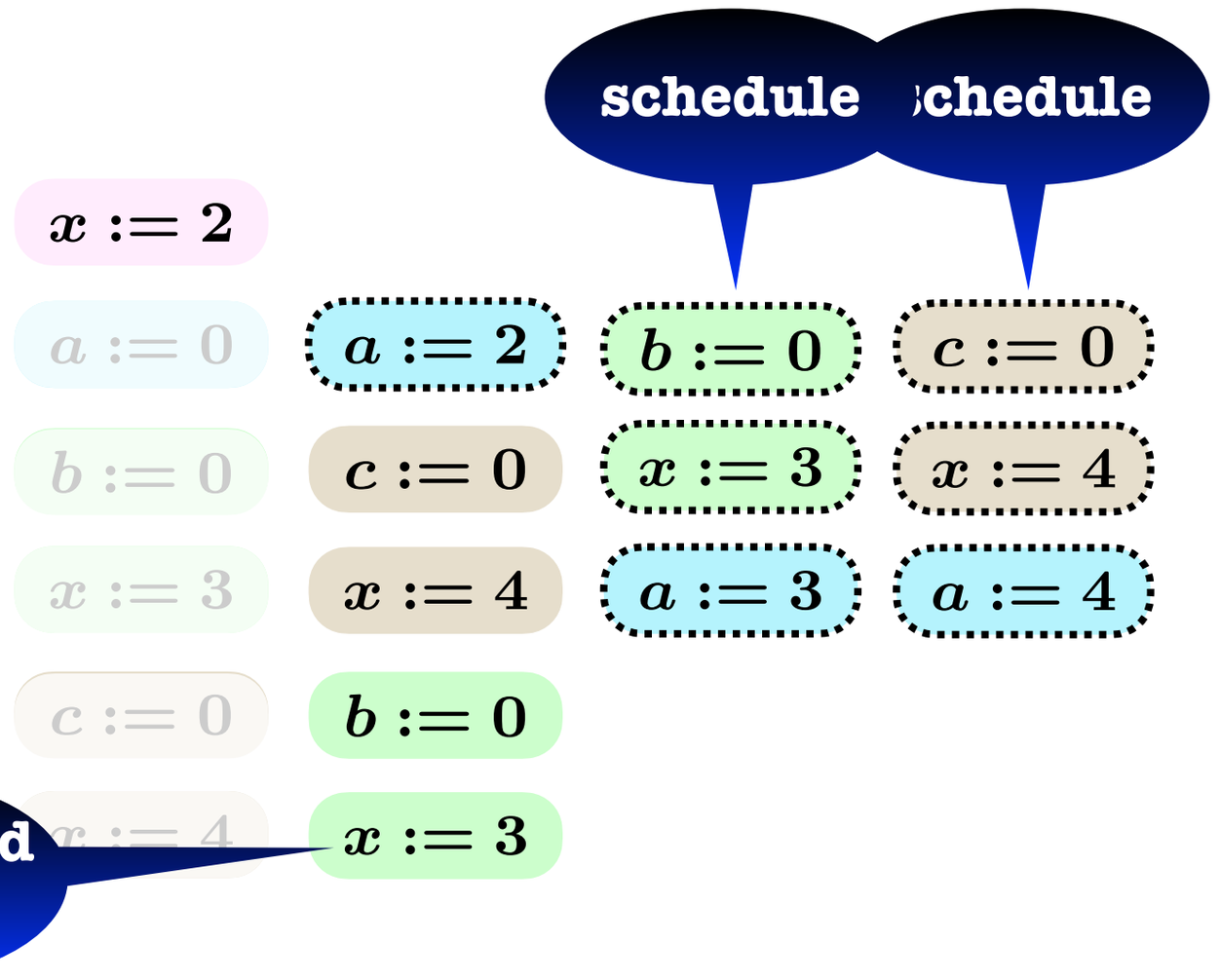

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

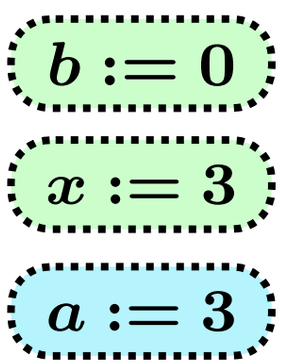

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

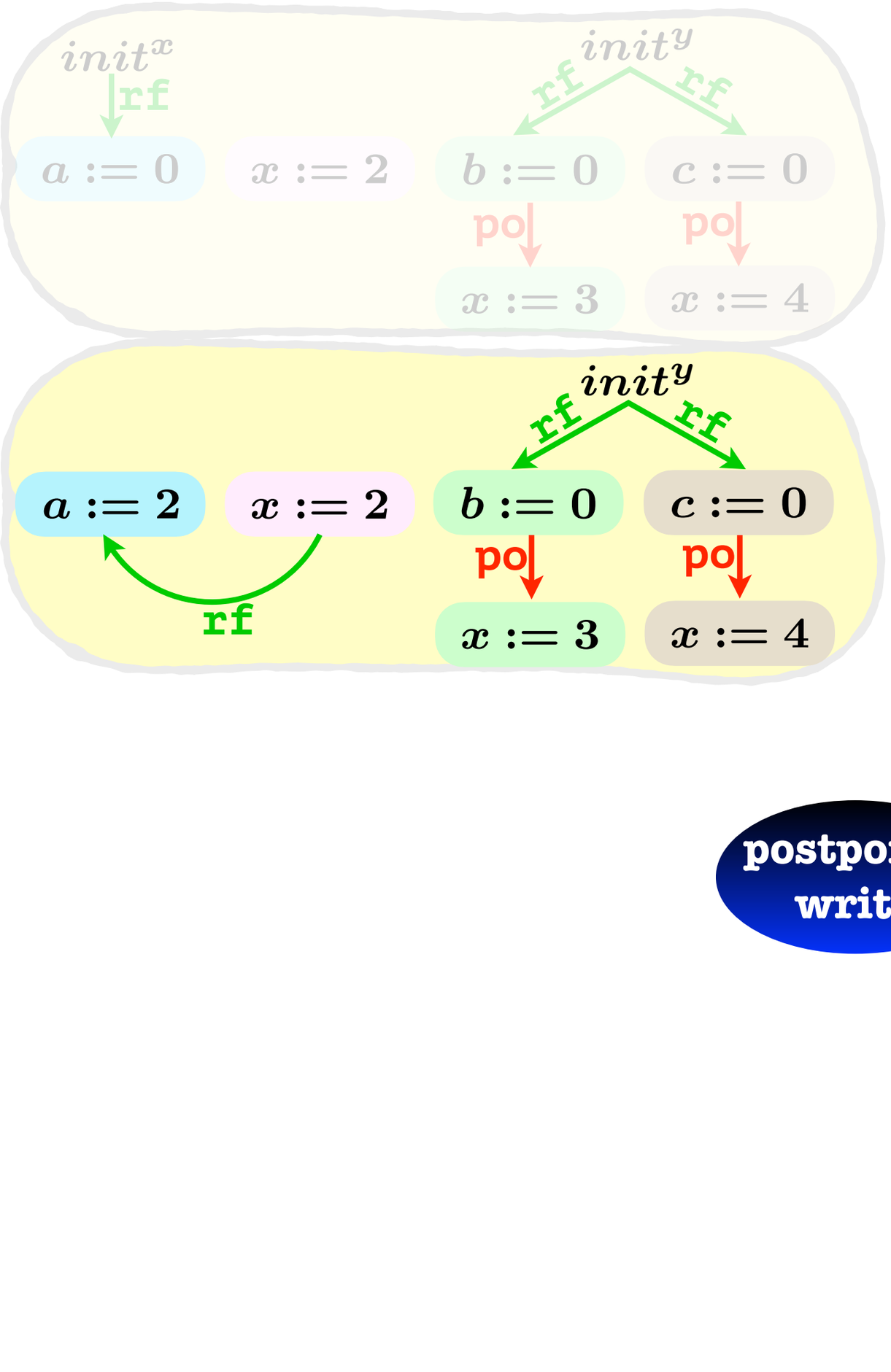

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
postponed write

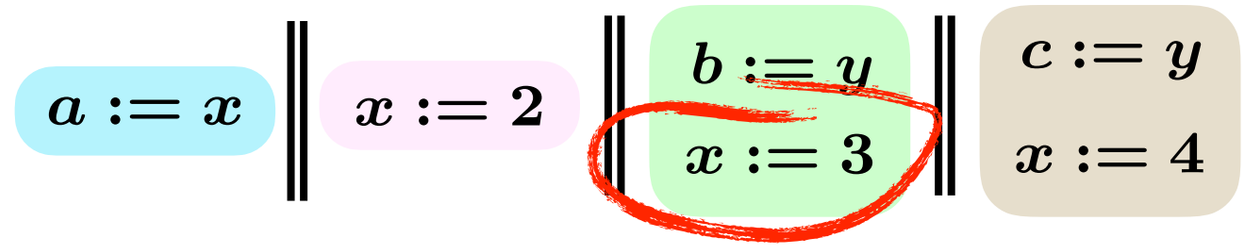

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

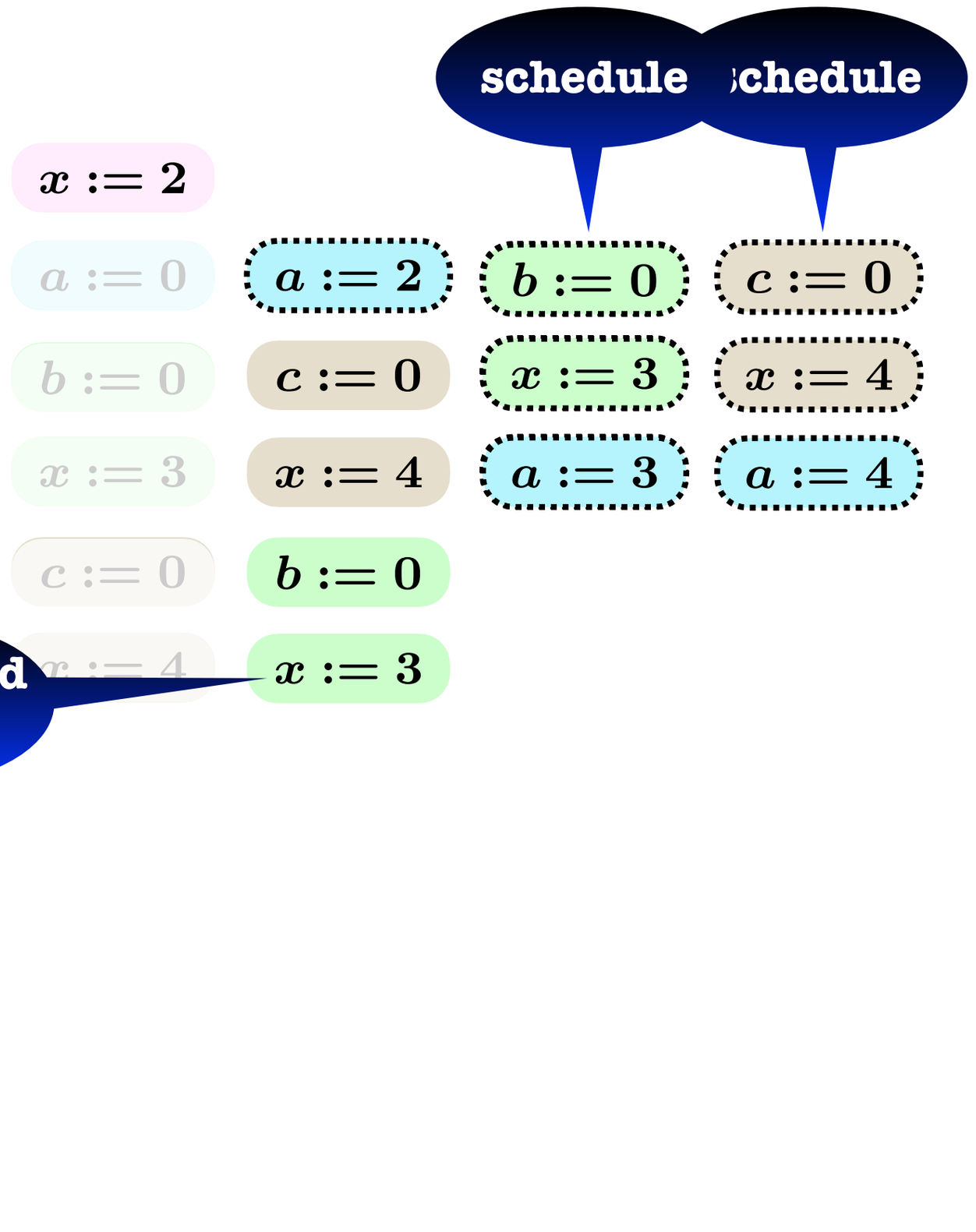

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

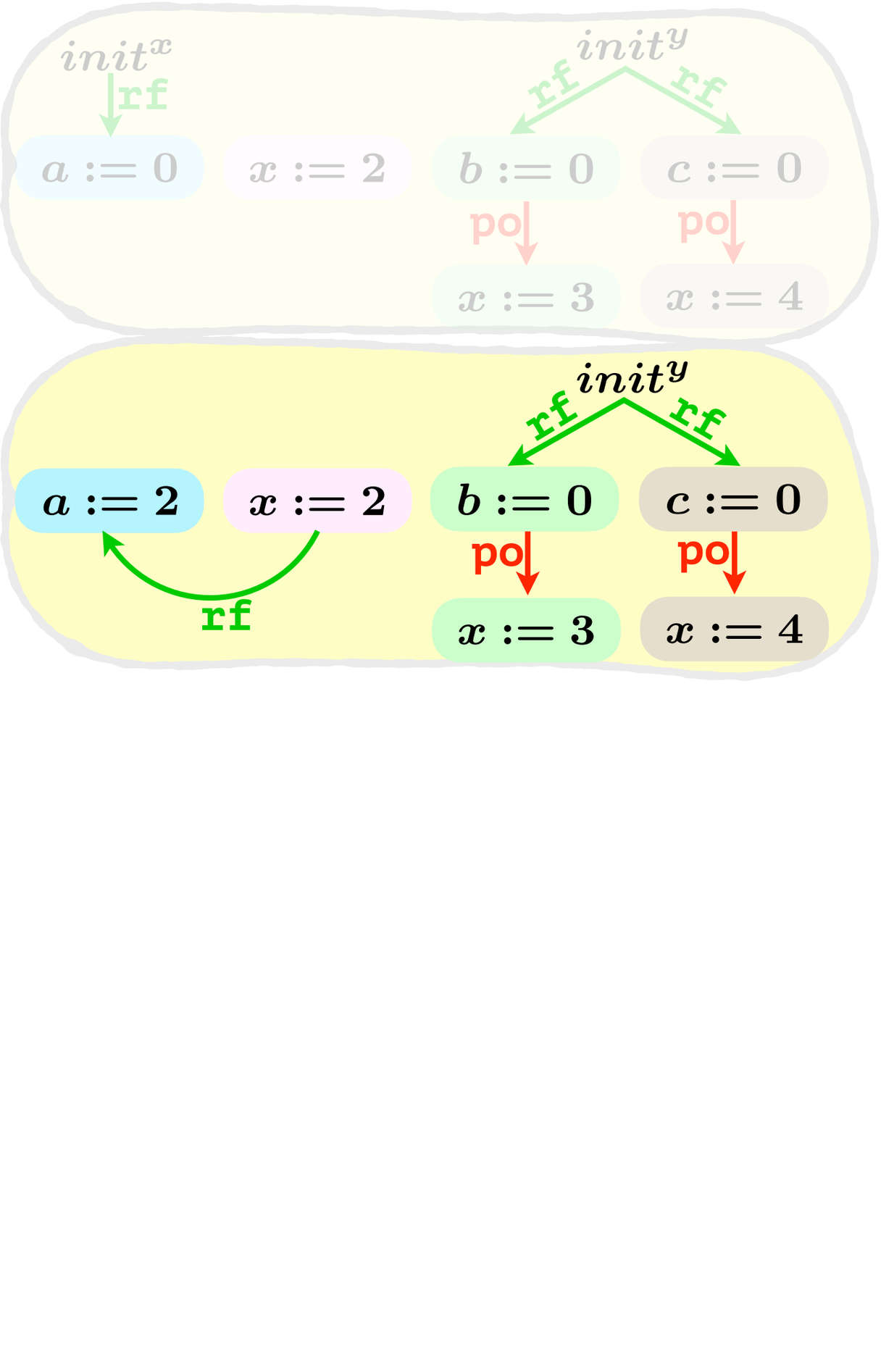

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
postponed write
$x := 3$


$b := 0$
$x := 3$
$a := 3$

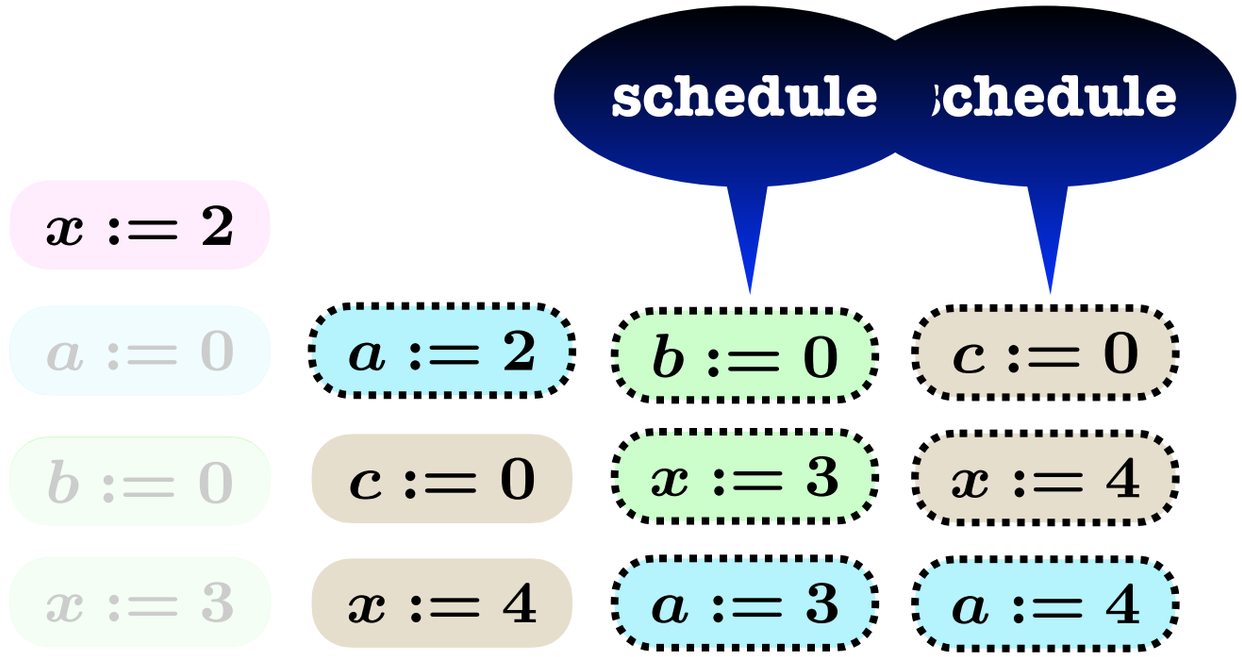

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

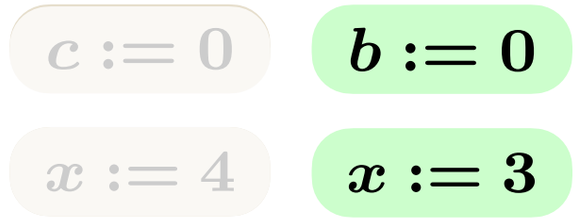

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

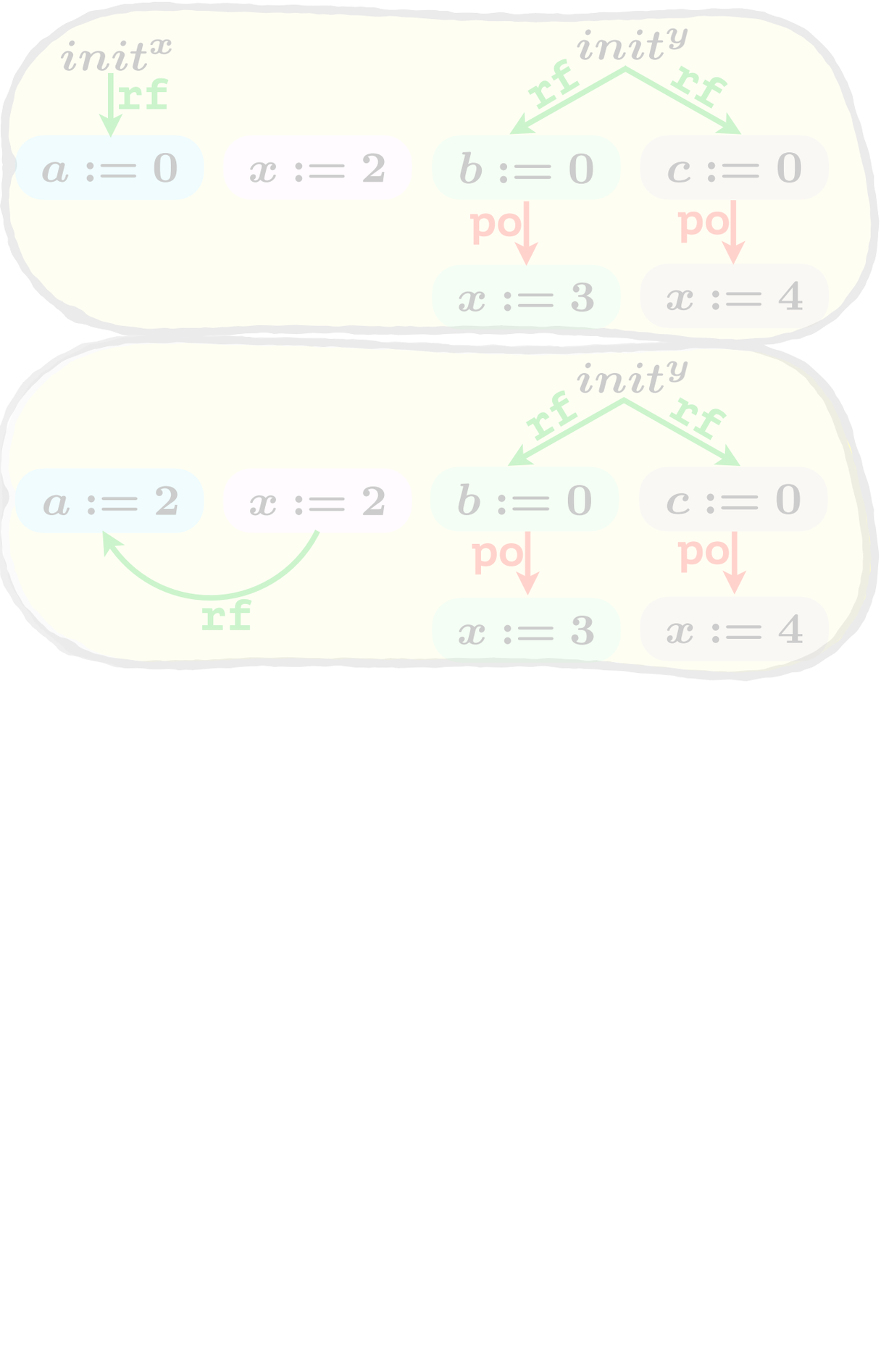

schedule
schedule
$x := 2$
$a := 0$
$b := 0$
$c := 0$
$a := 2$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
postponed write



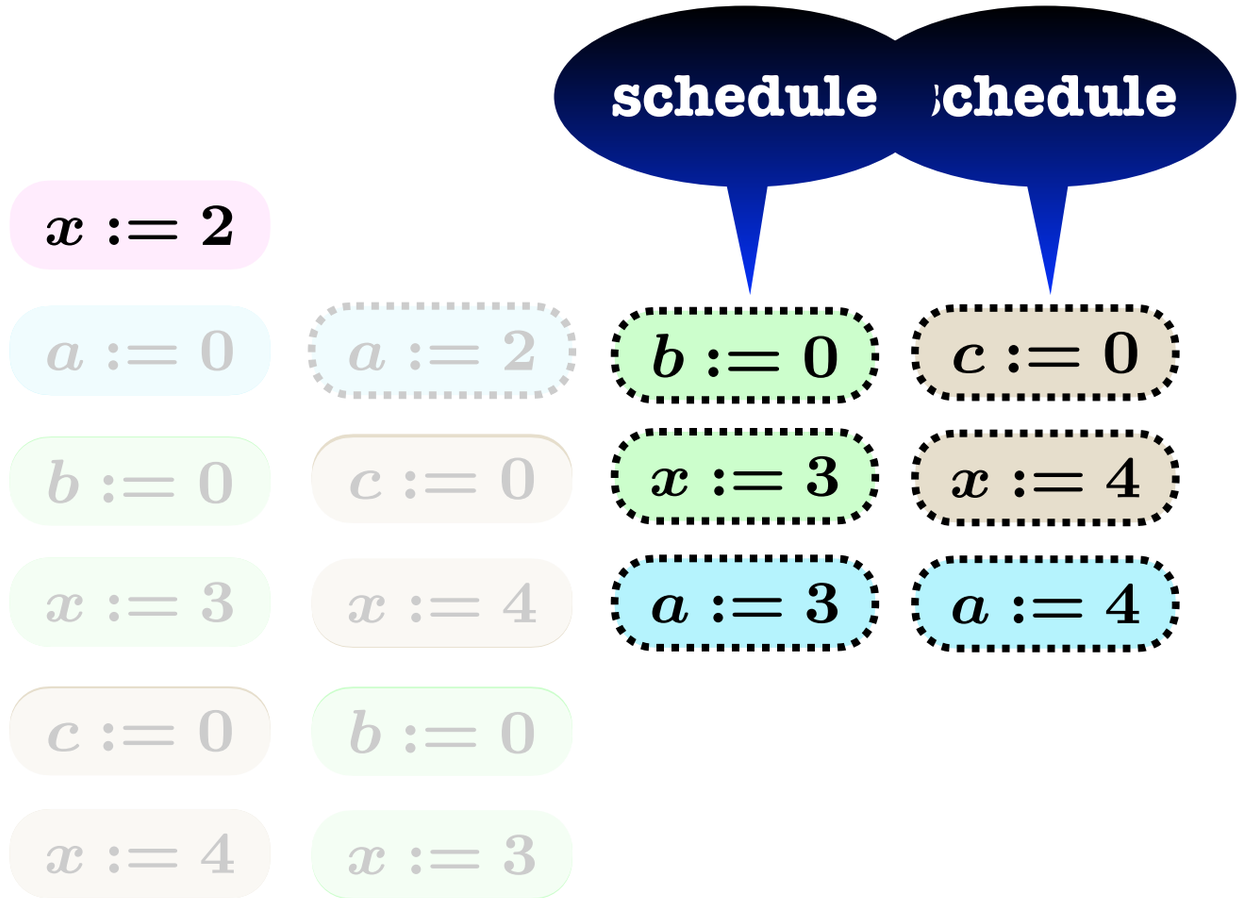
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

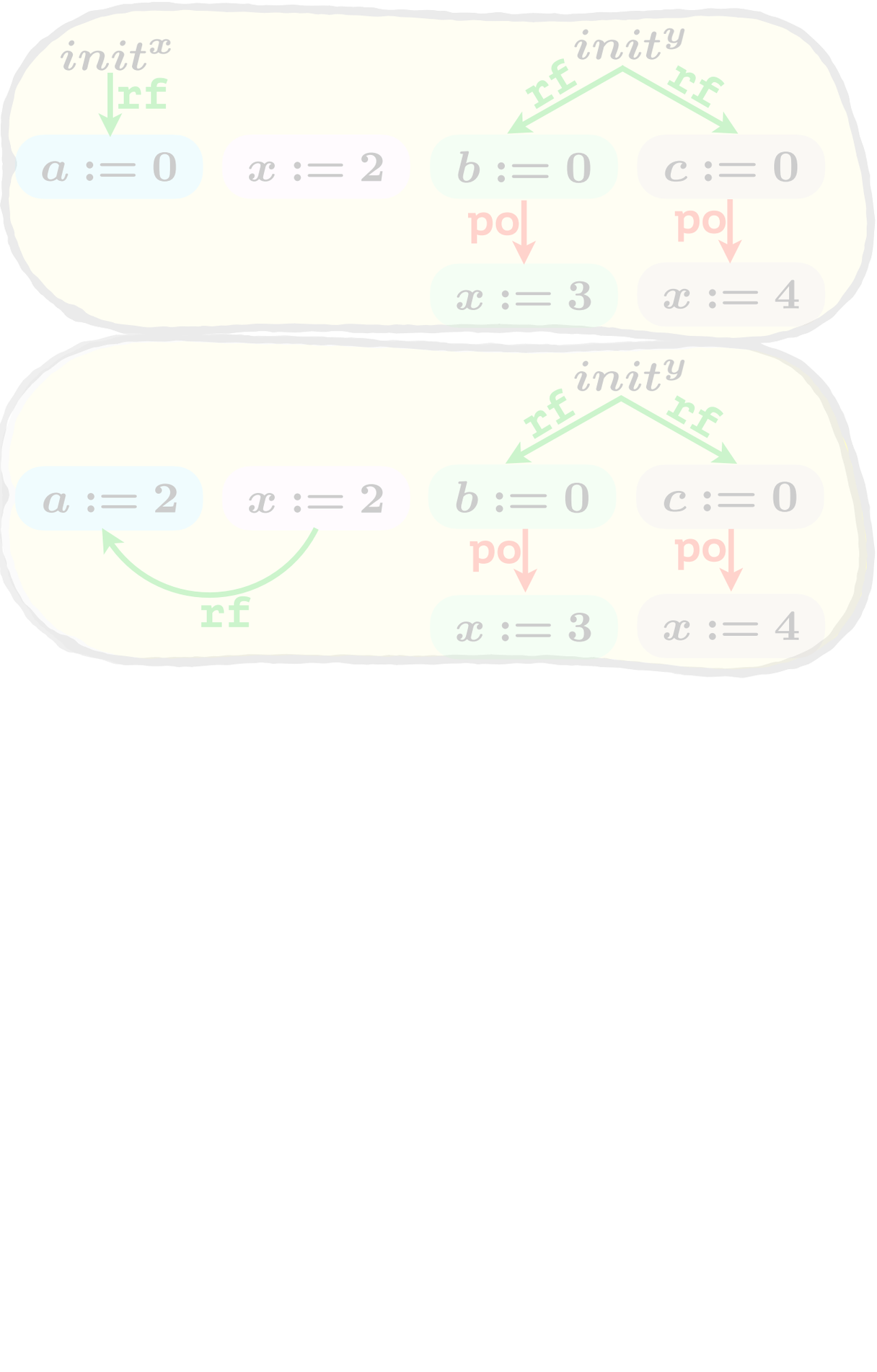




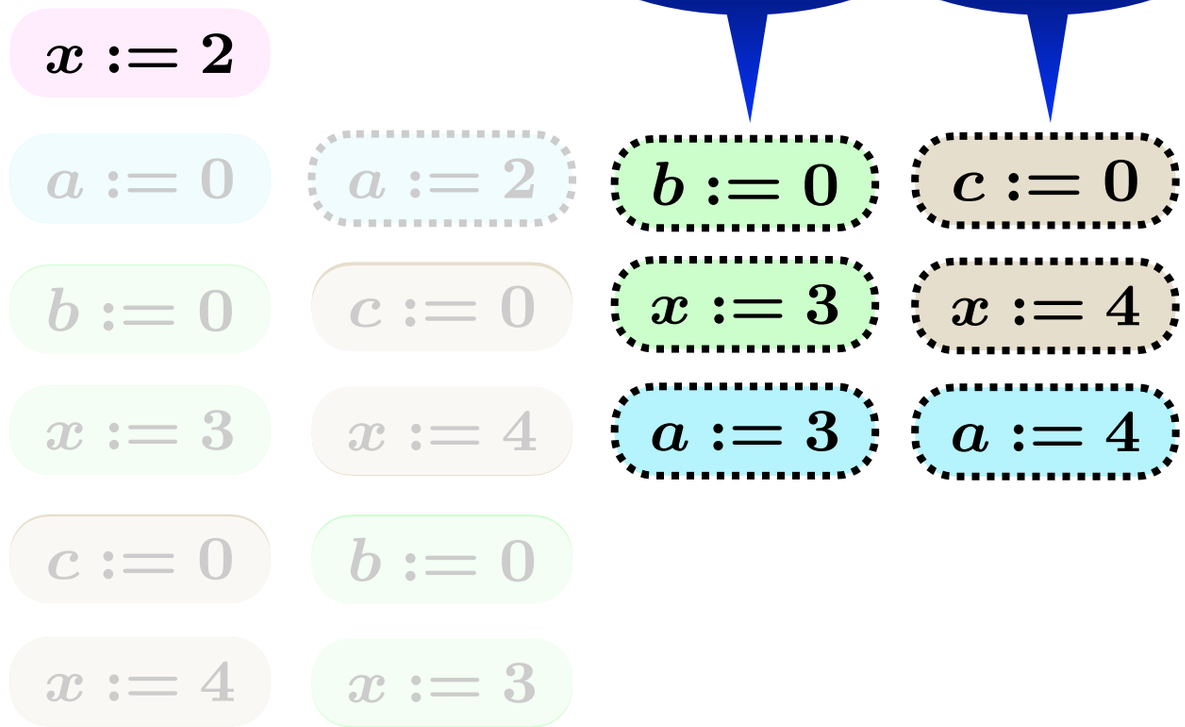


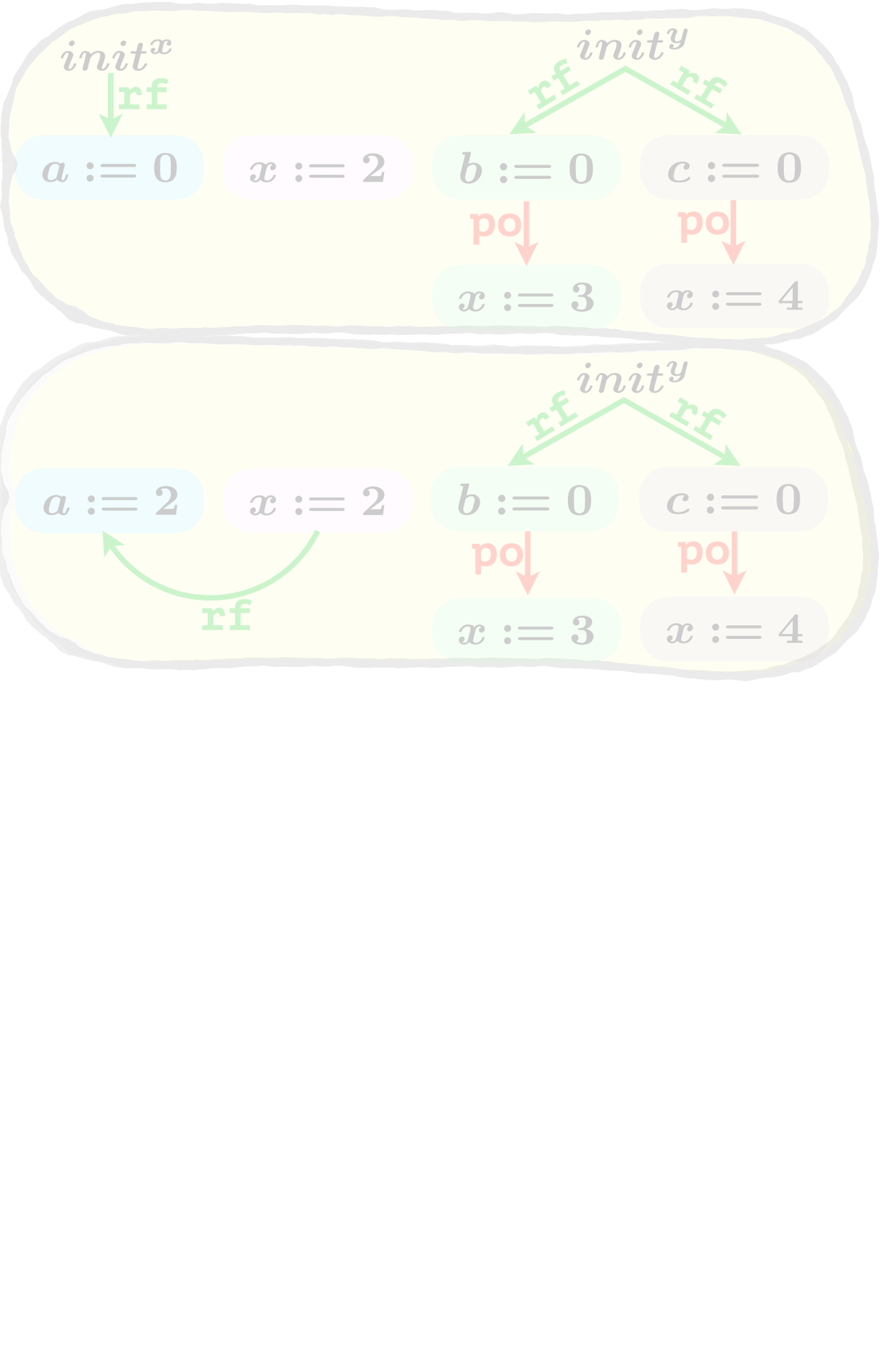
$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$





$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$



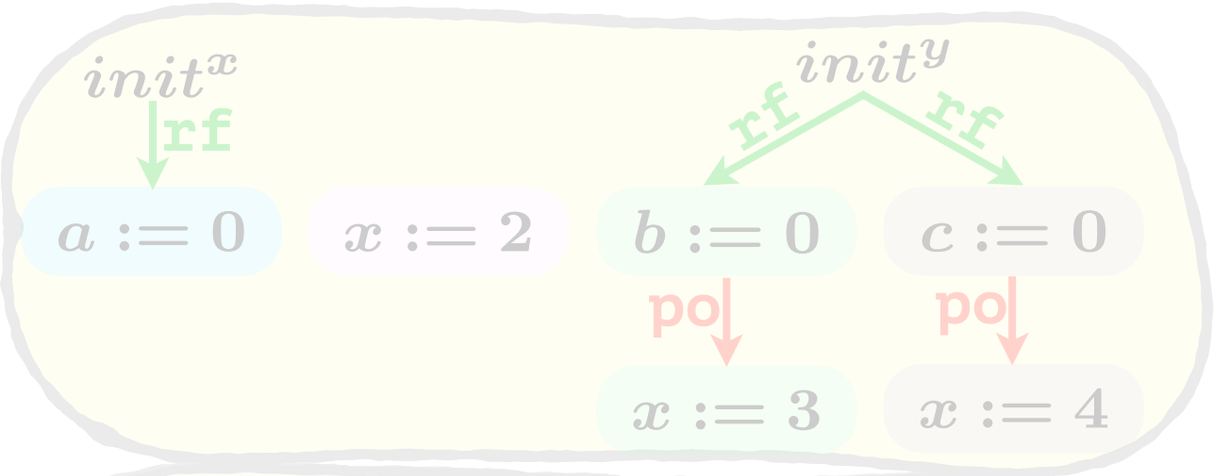


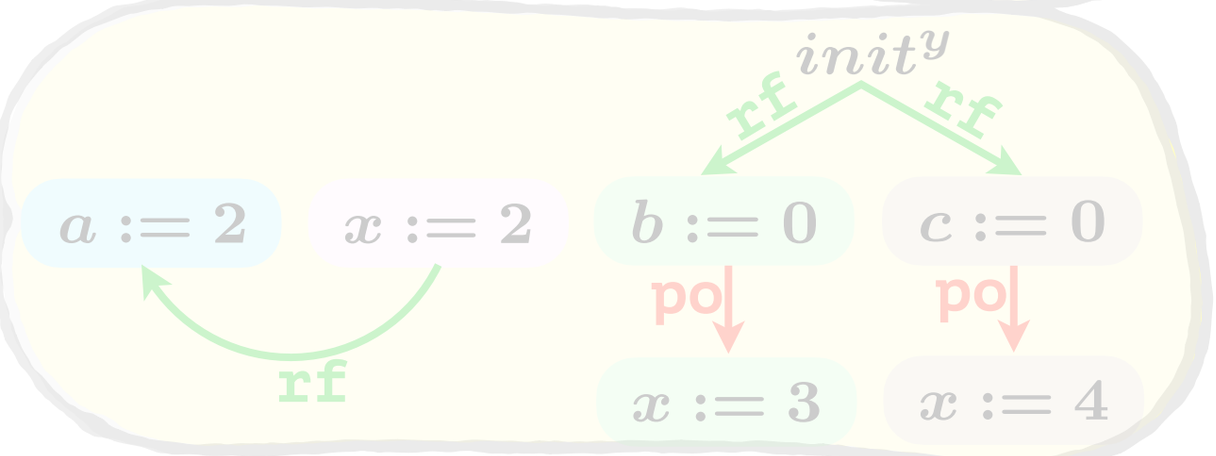
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

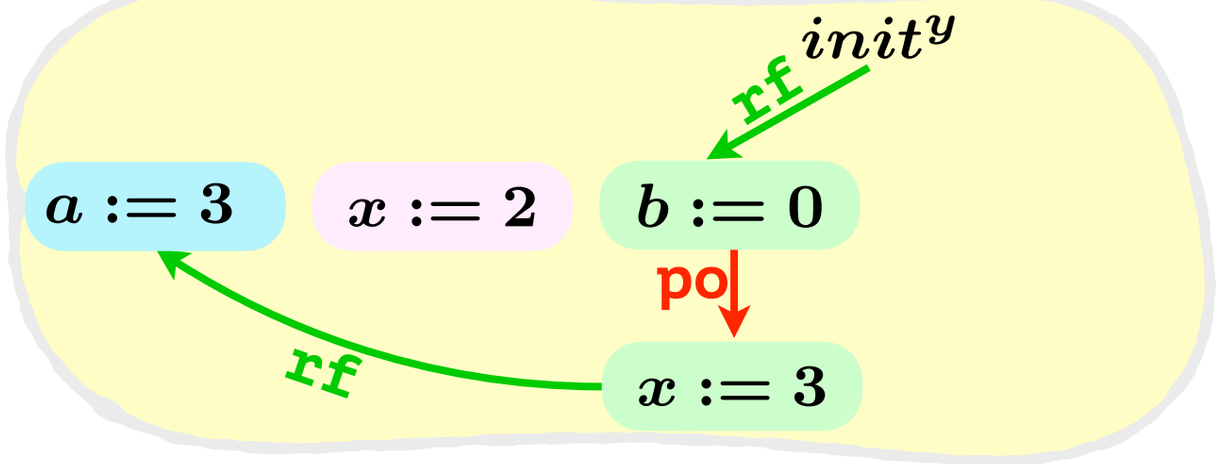
schedule

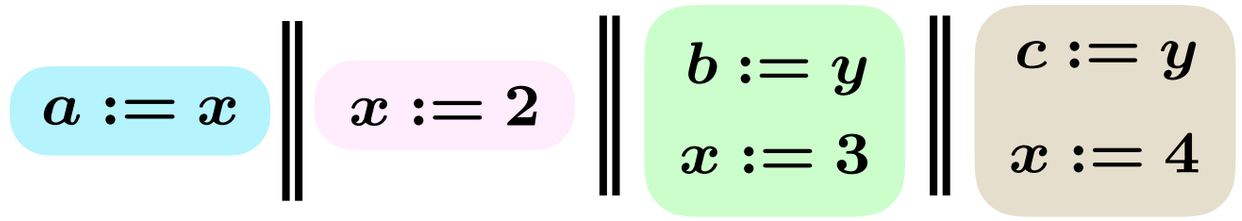
$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$		
$x := 4$	$x := 3$		

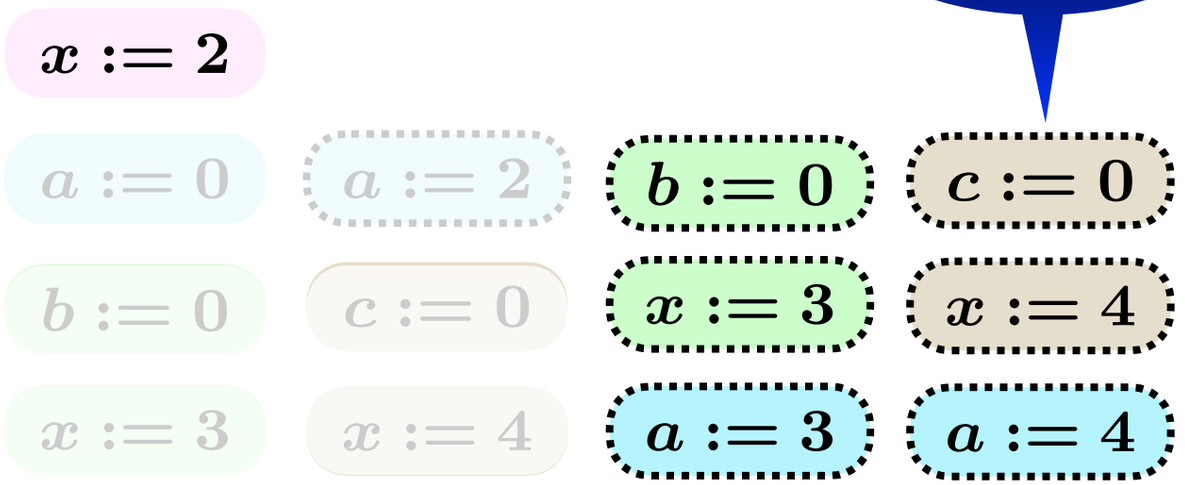

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

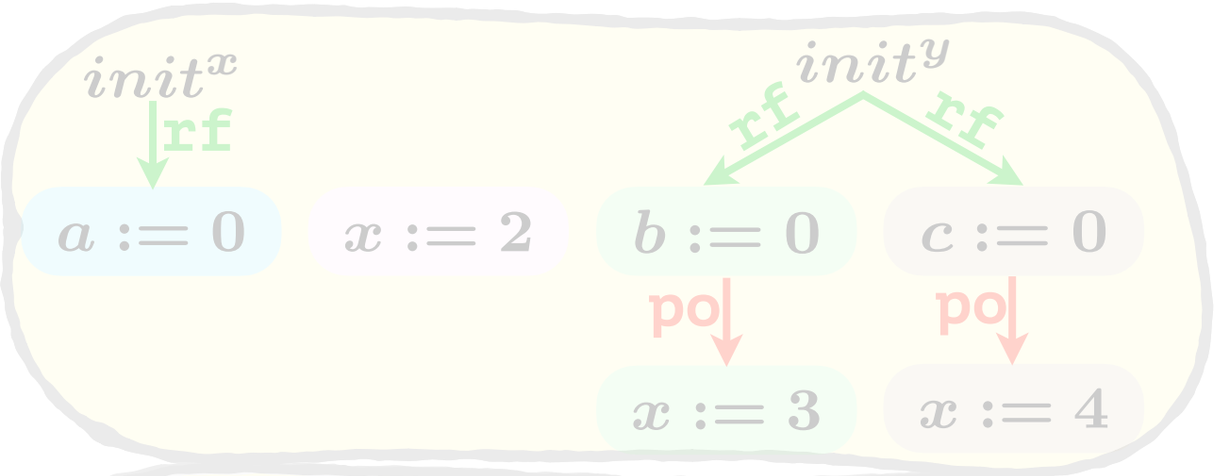

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

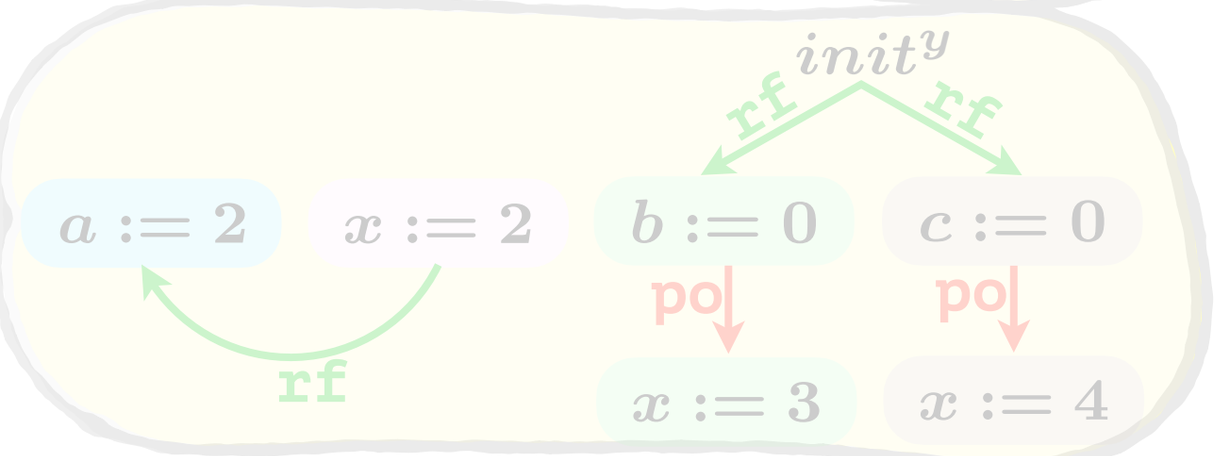

$init^y$
rf
$a := 3$
$x := 2$
$b := 0$
po
rf
$x := 3$

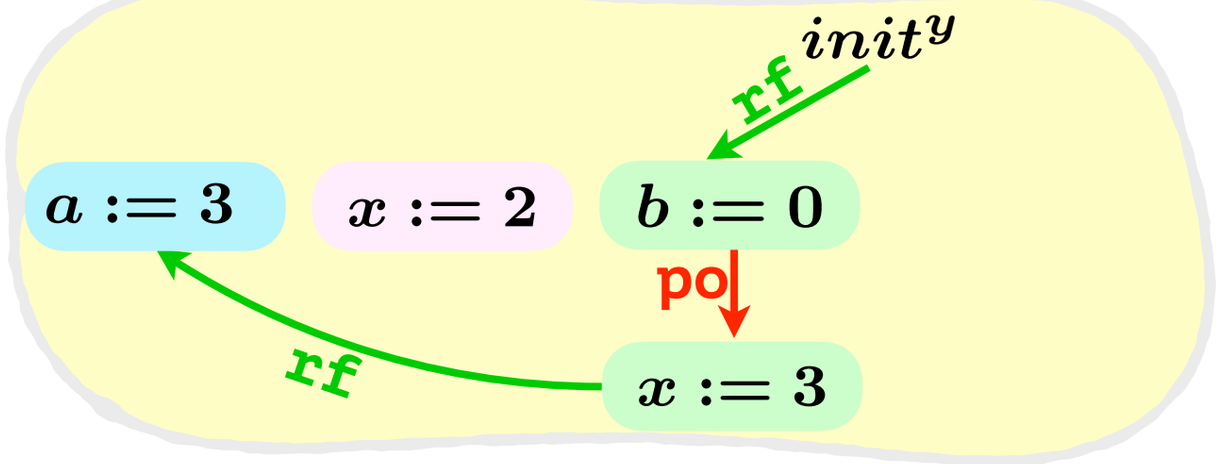

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

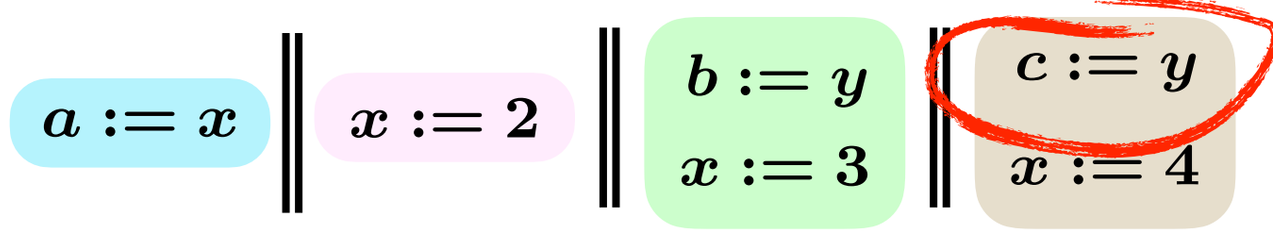
schedule


$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$
$x := 3$





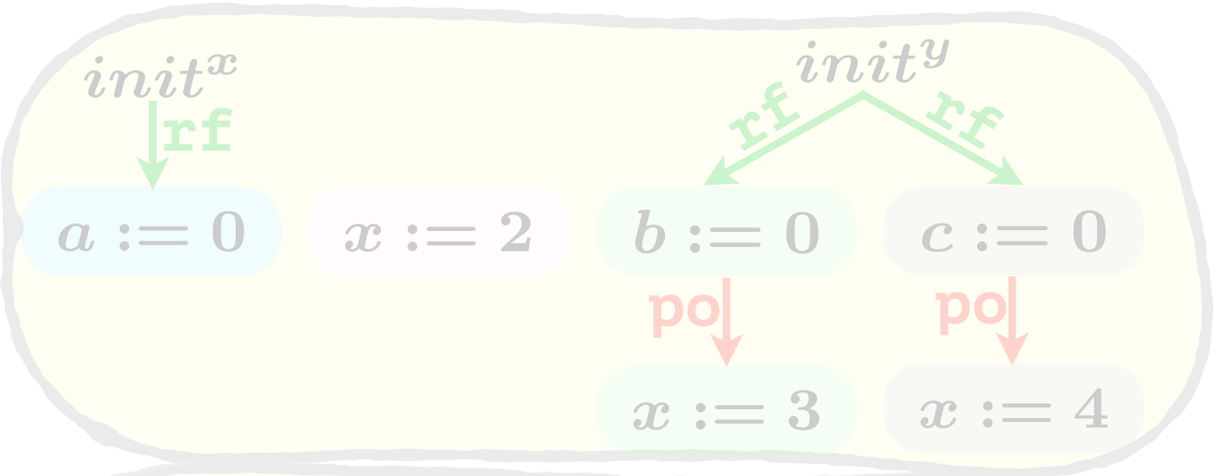


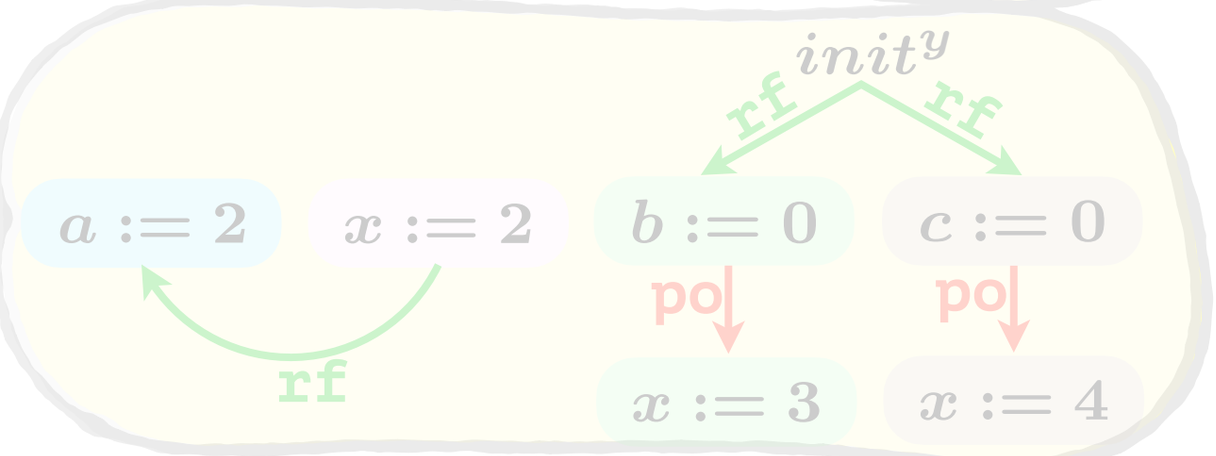


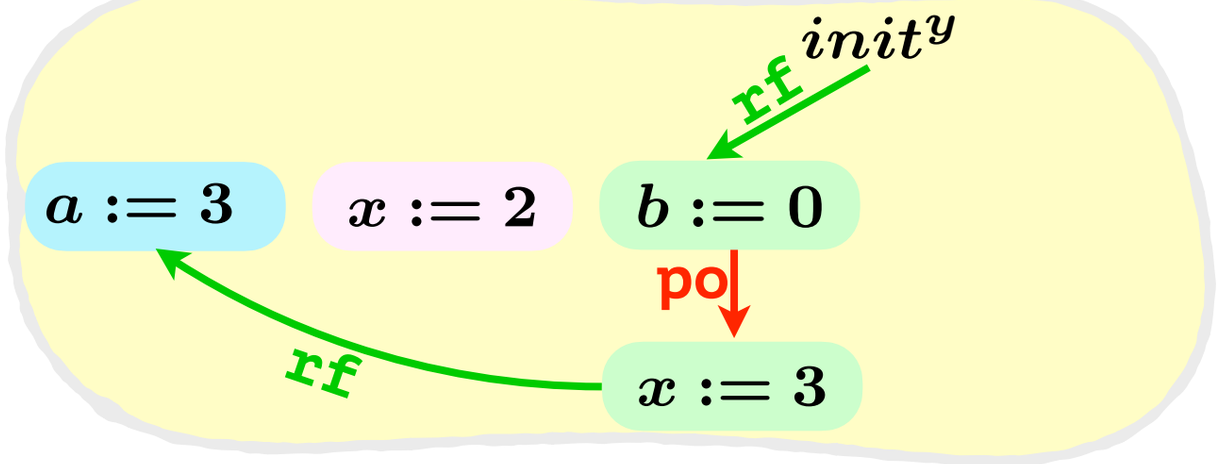
schedule

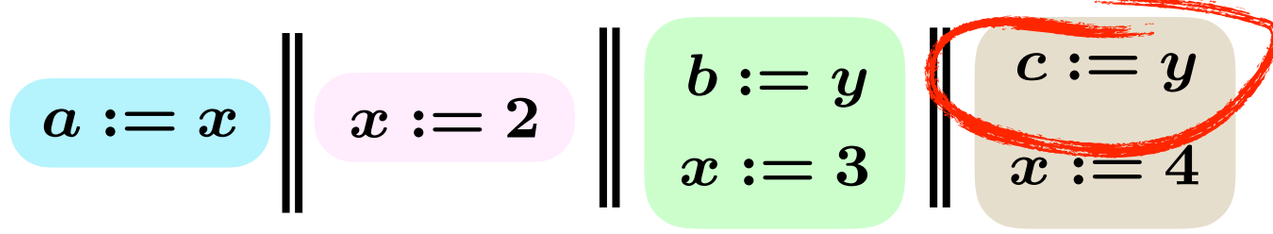
$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$x := 4$	$x := 3$





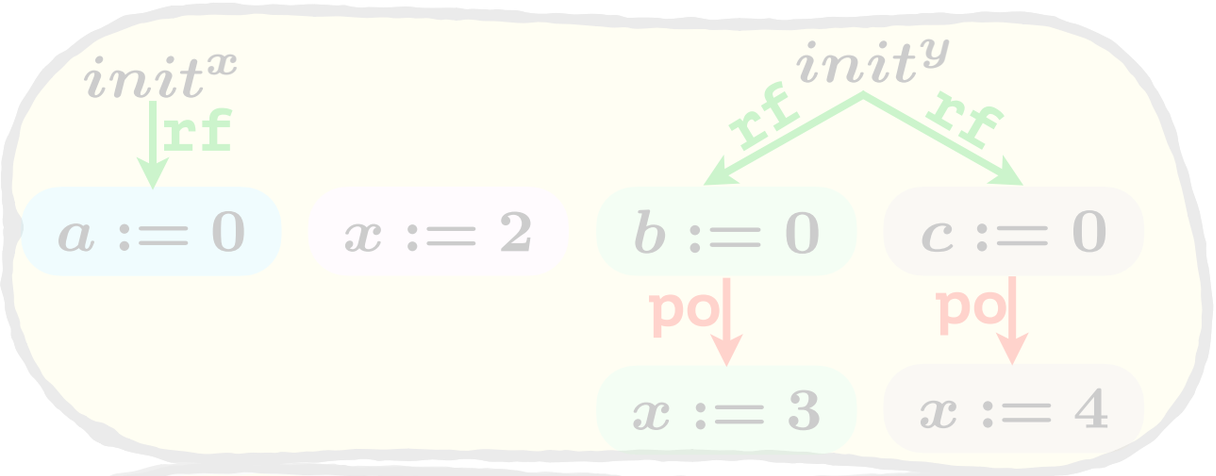


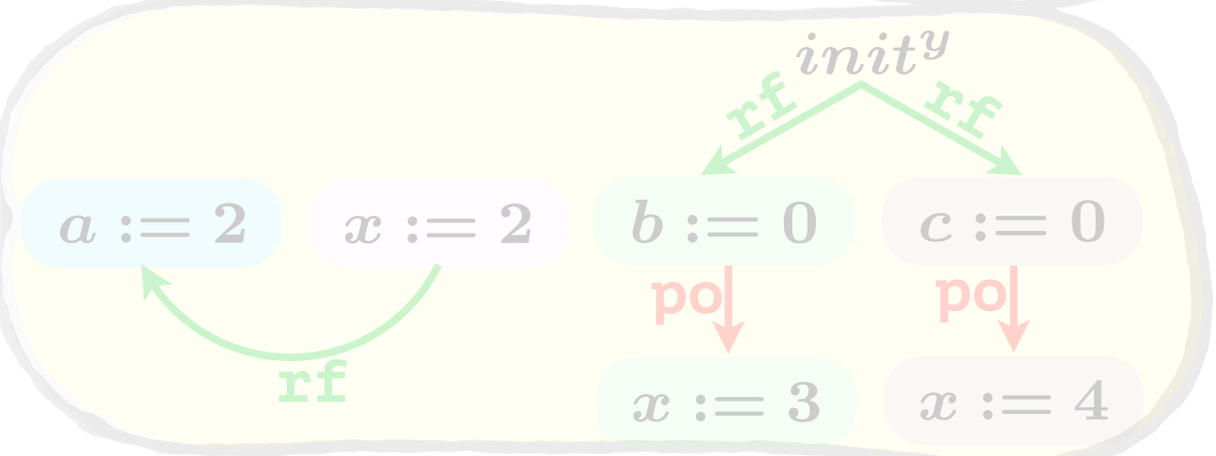


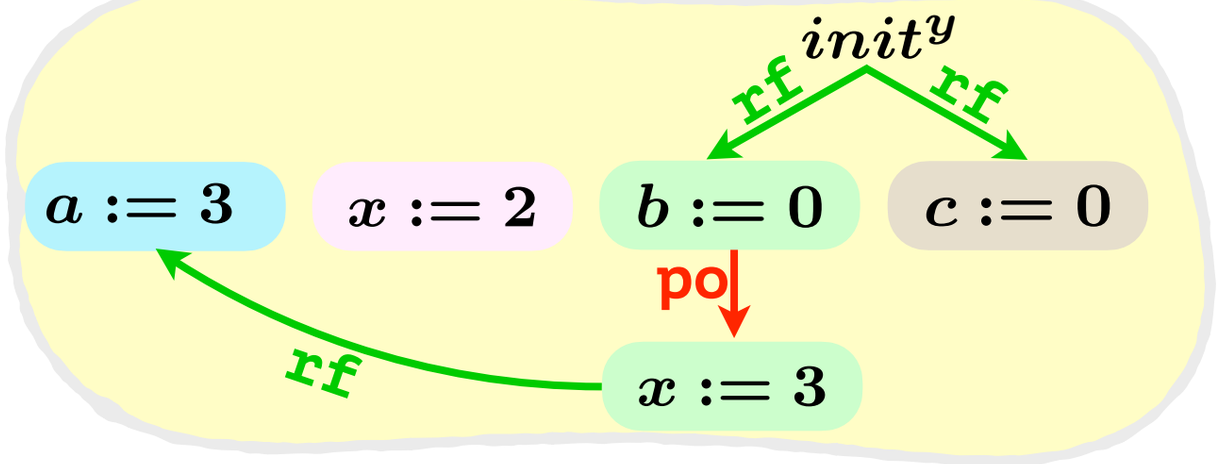
schedule

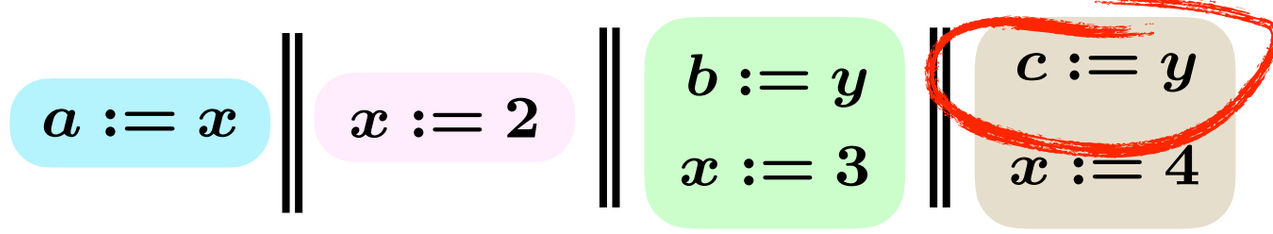
$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$		





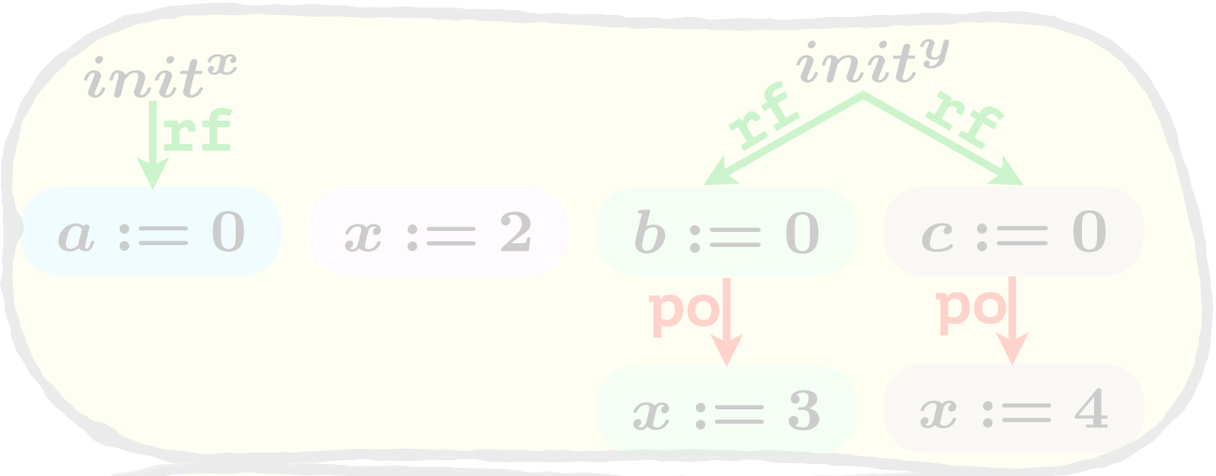


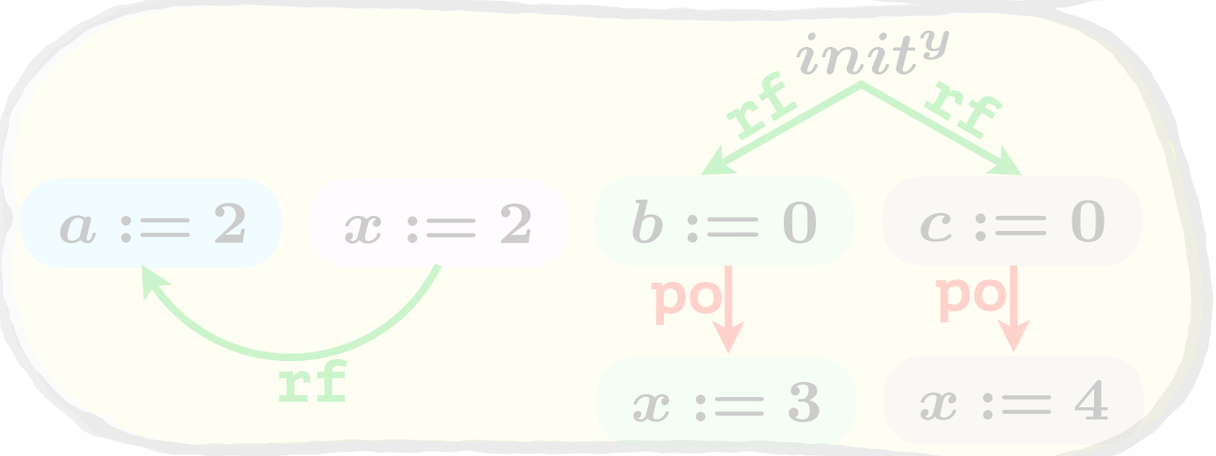


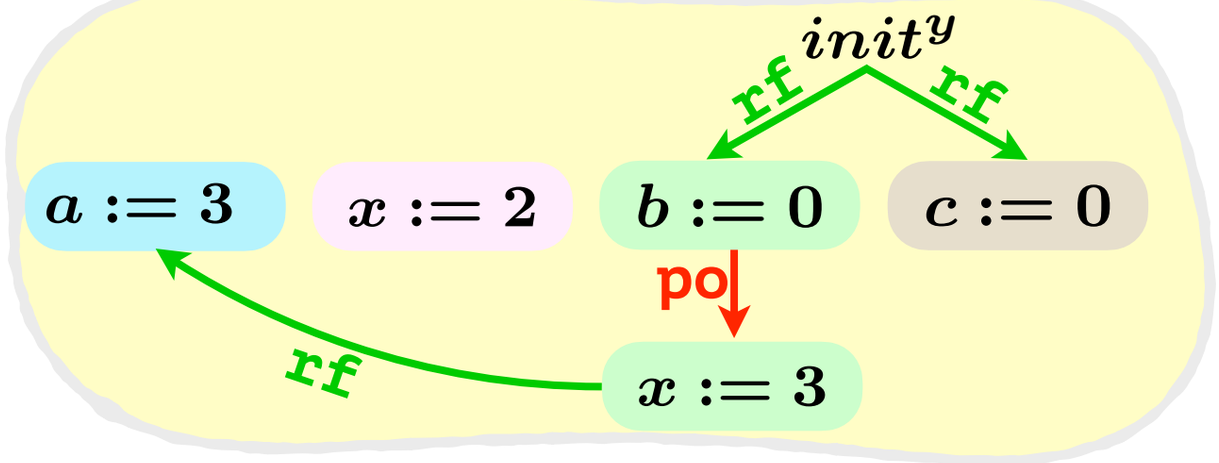
schedule

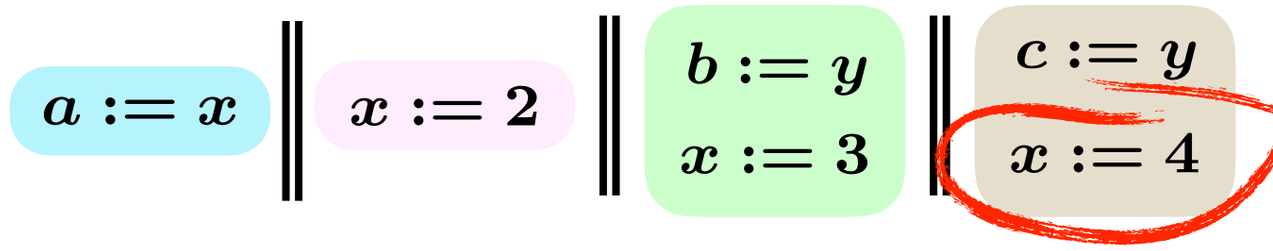
$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$		

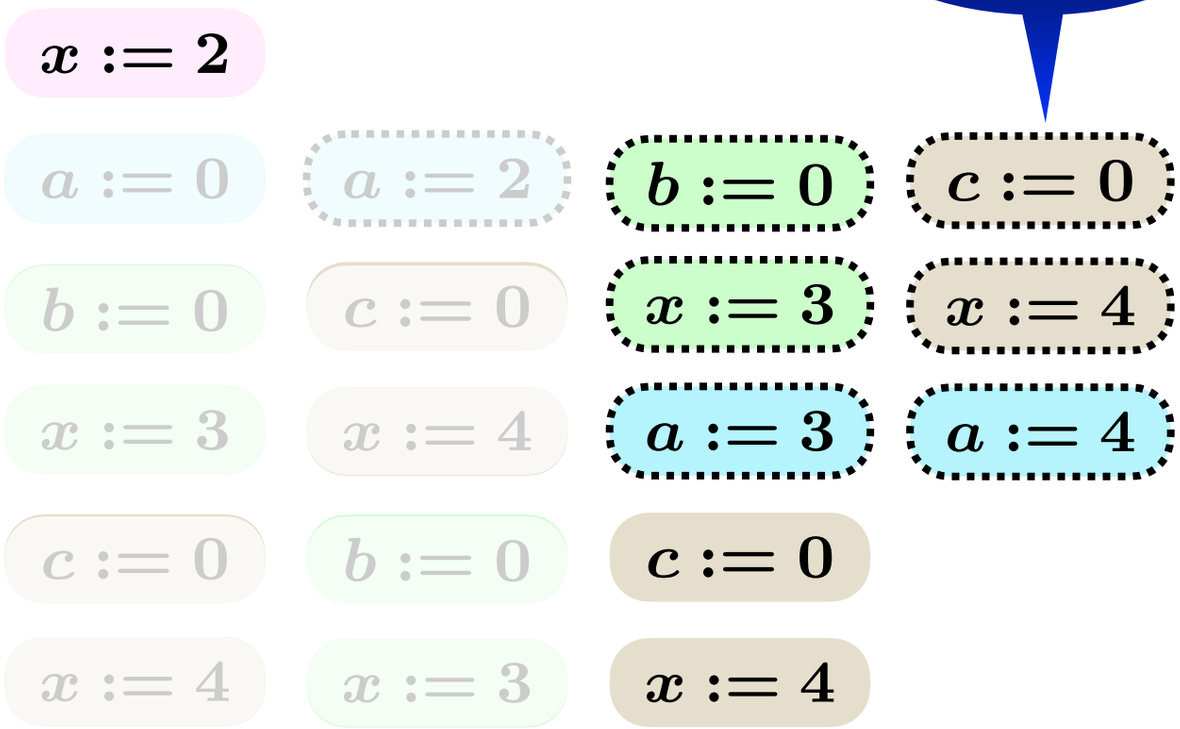

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

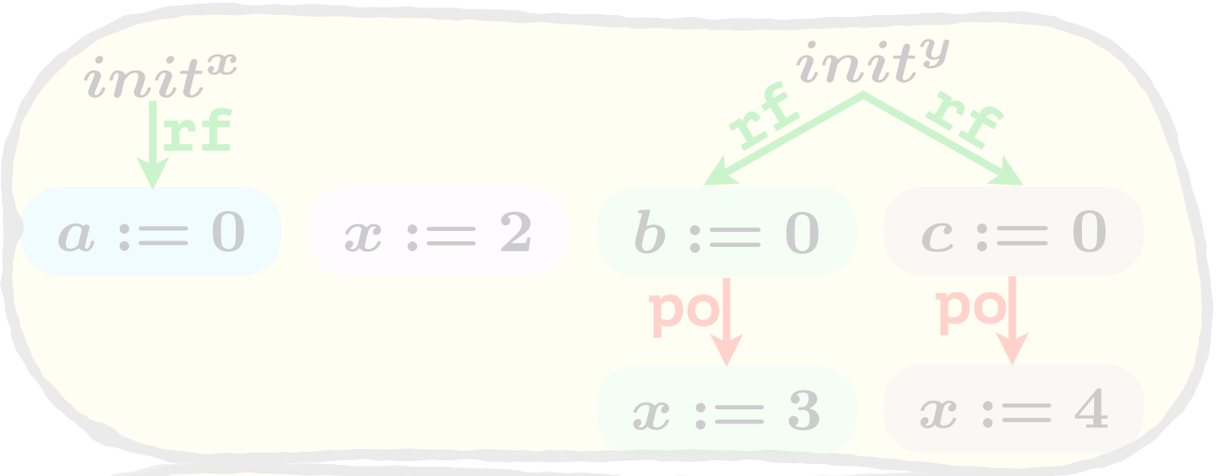

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

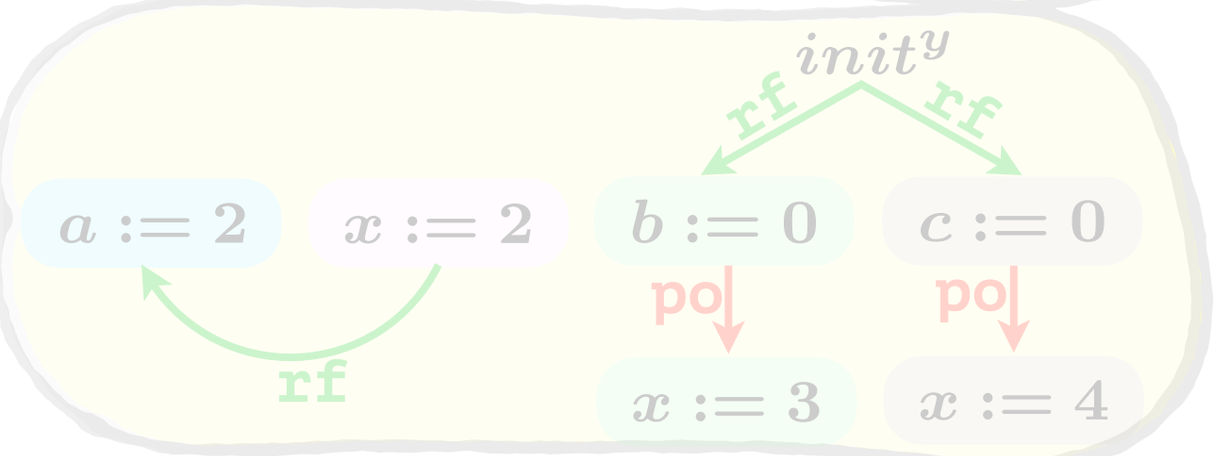

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 3$

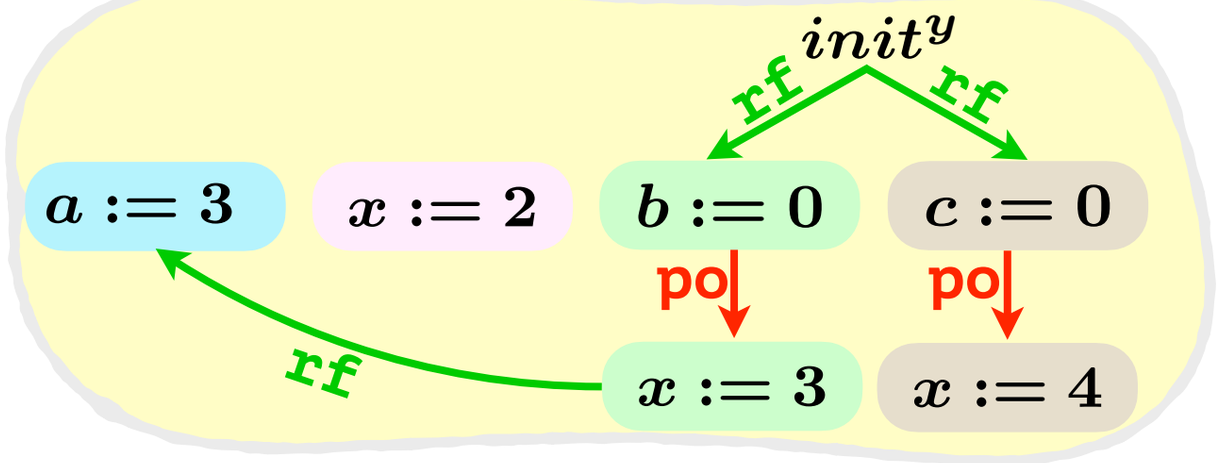

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

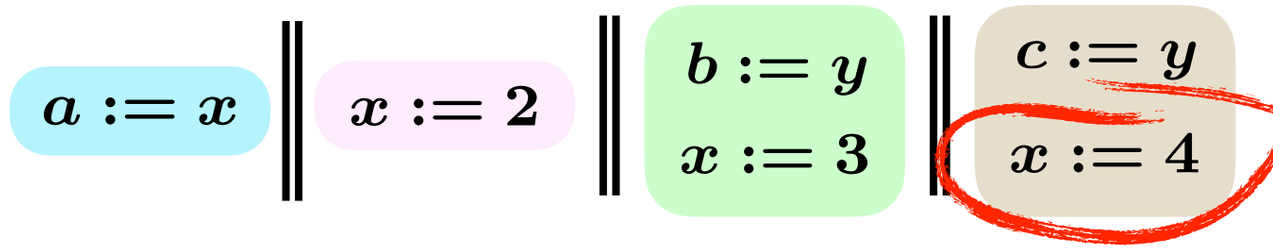
schedule


$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 3$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$x := 4$
$x := 3$
$x := 4$





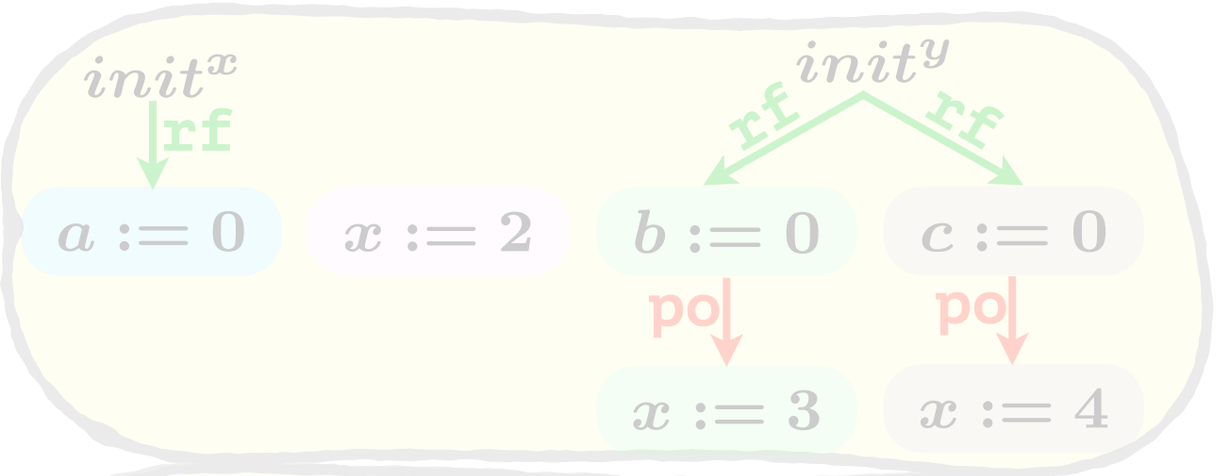


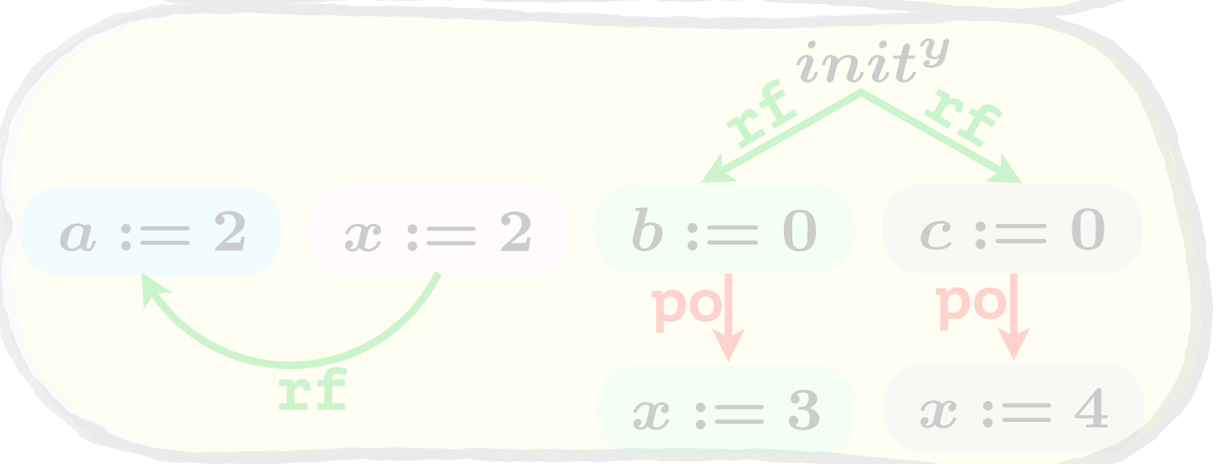


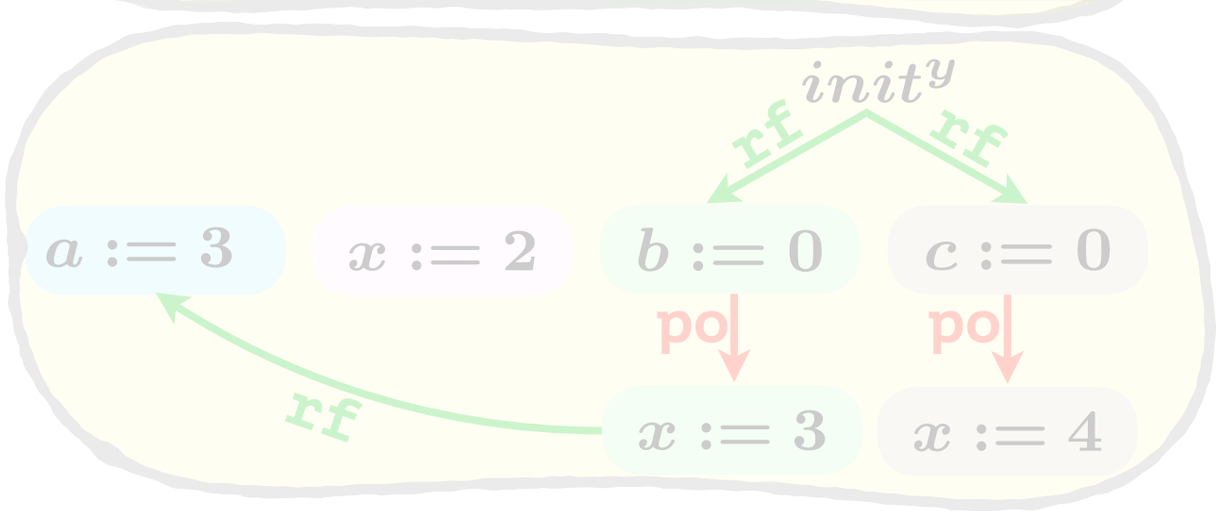
schedule

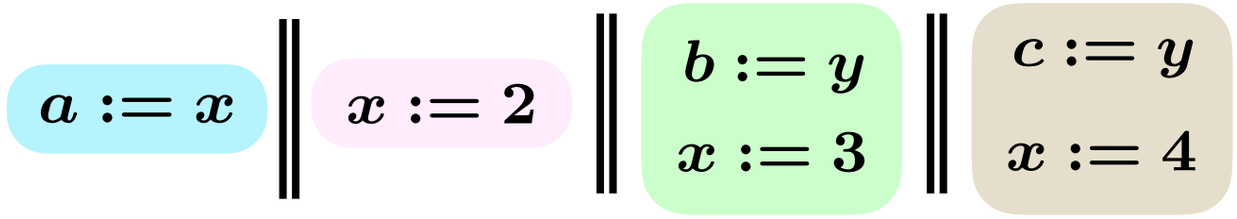
$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$	$x := 4$	





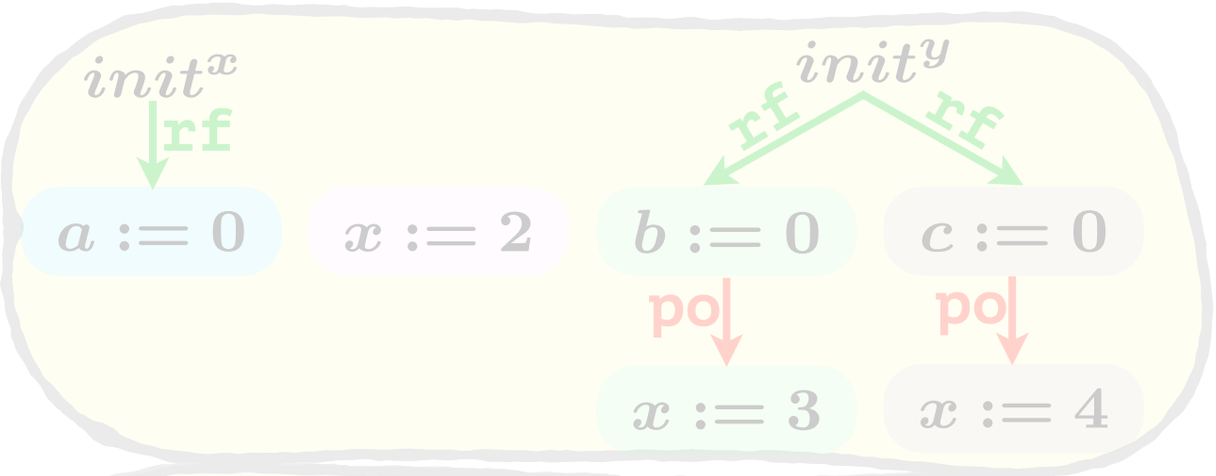


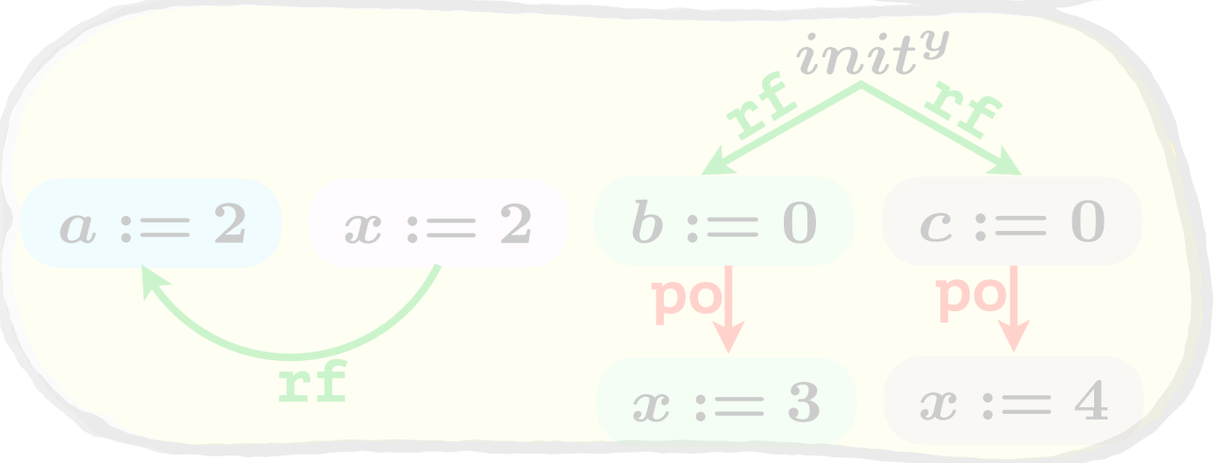


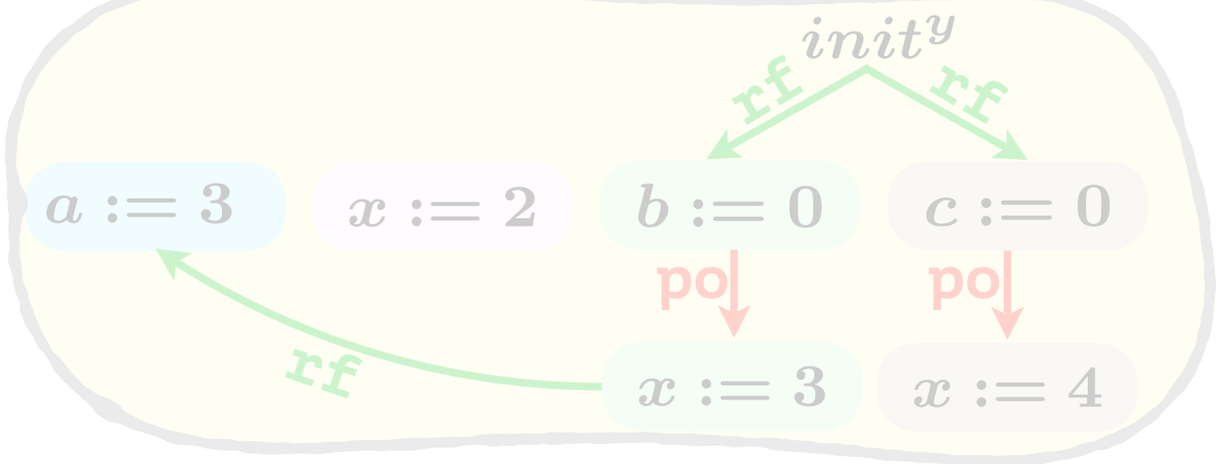
schedule

$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$	$x := 4$	





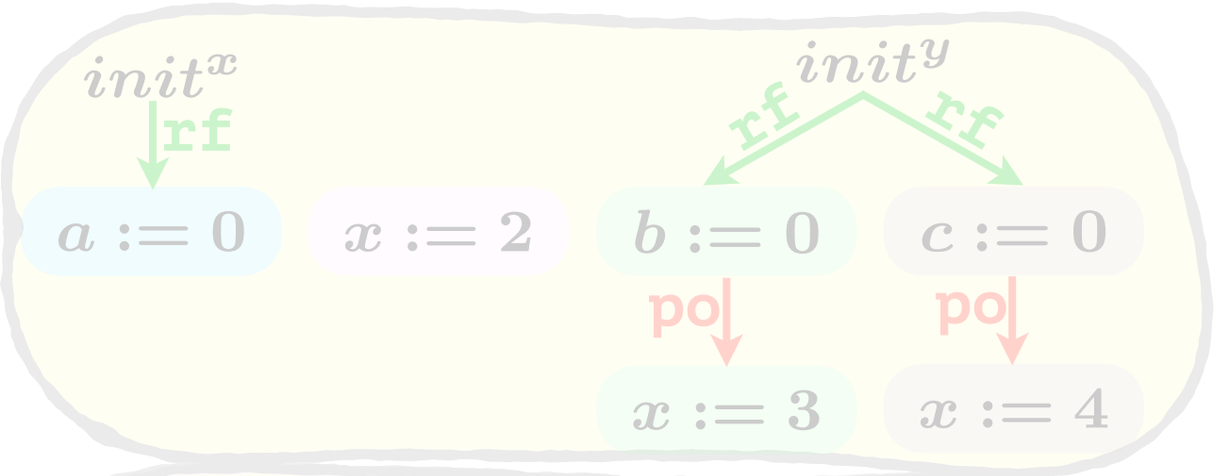


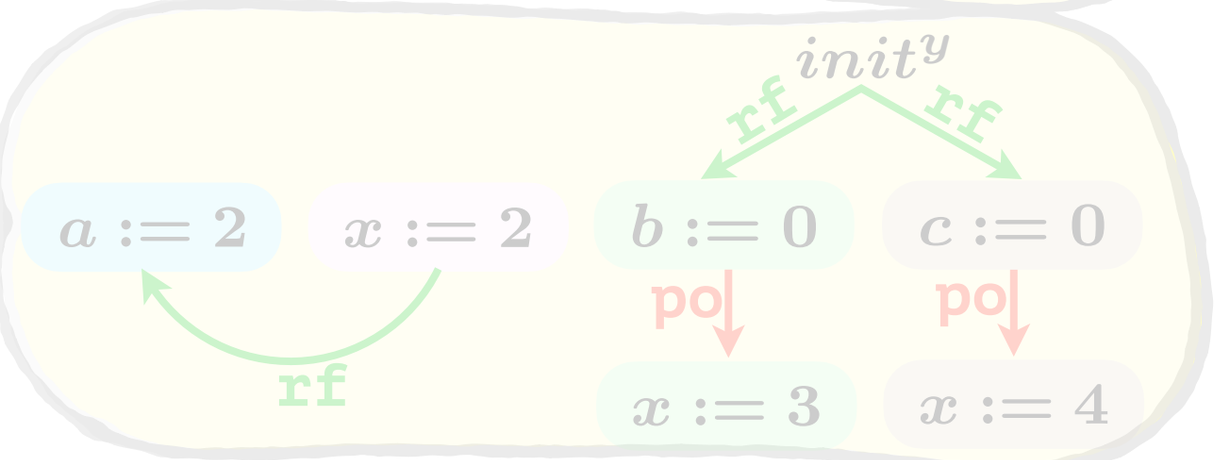
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

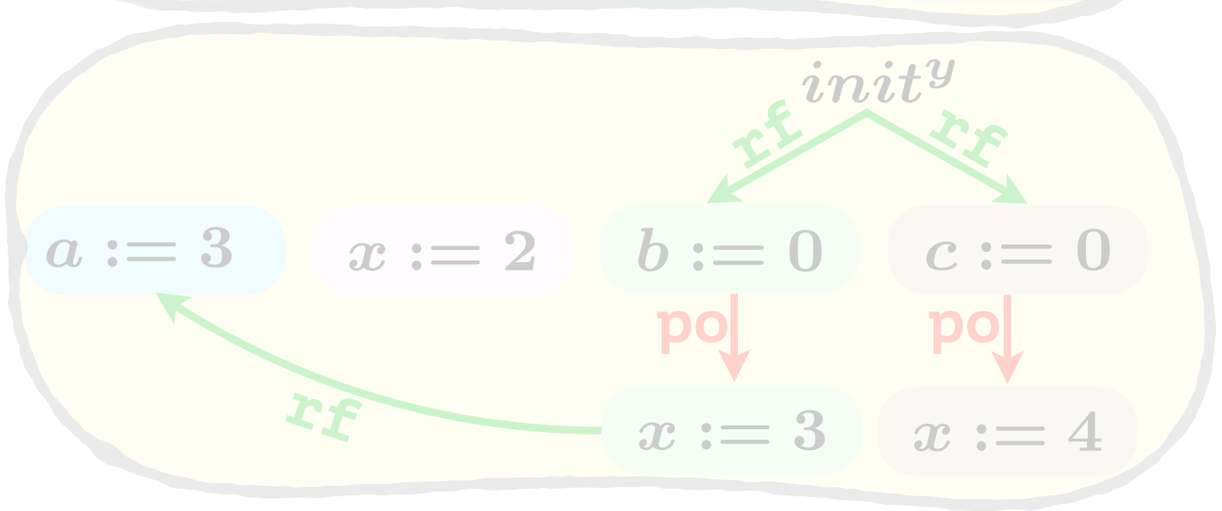
schedule

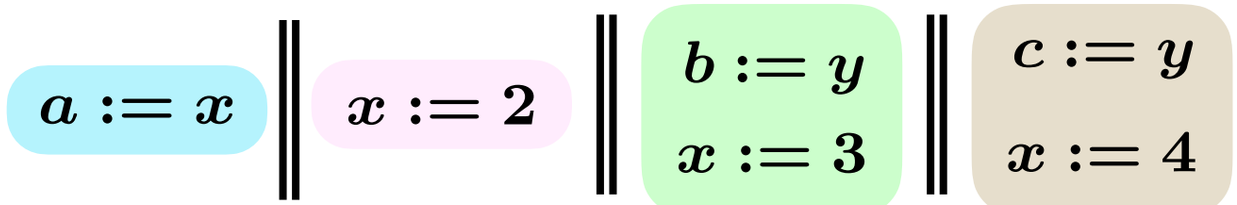
$x := 2$

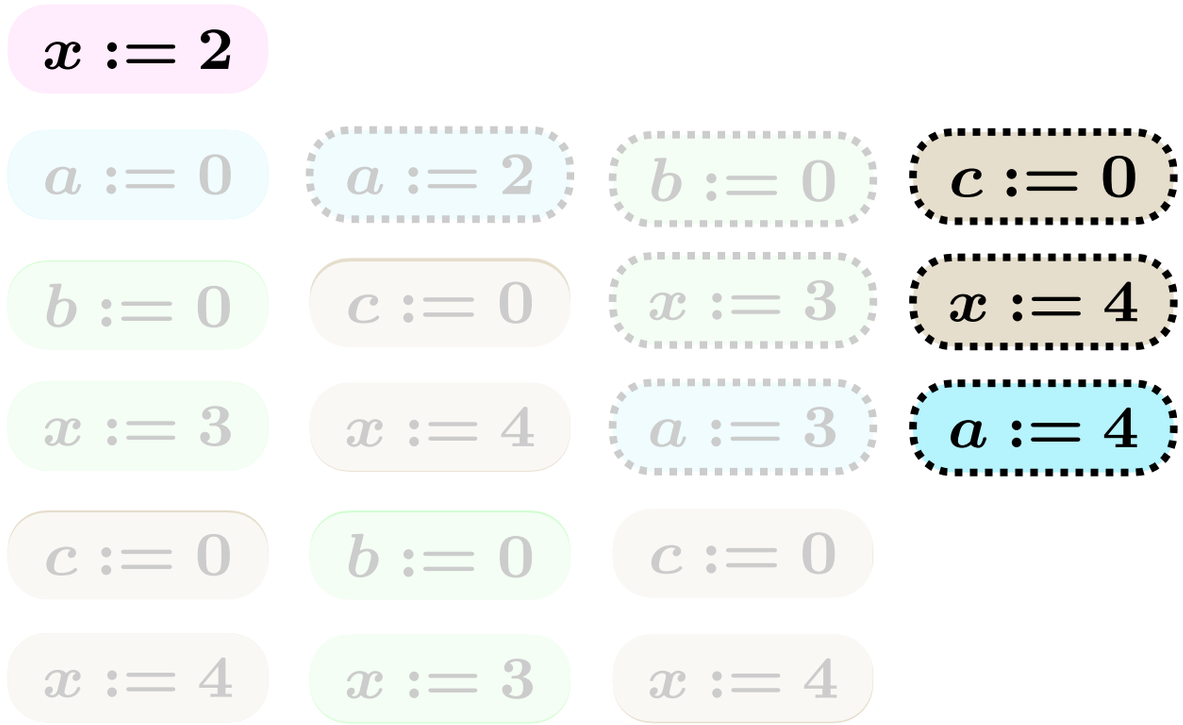
$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$	$x := 4$	

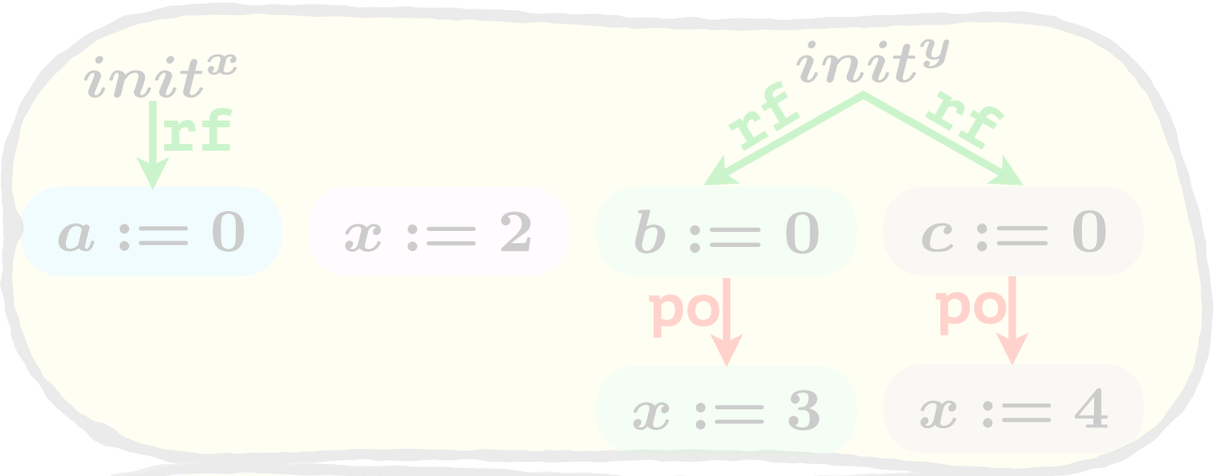

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

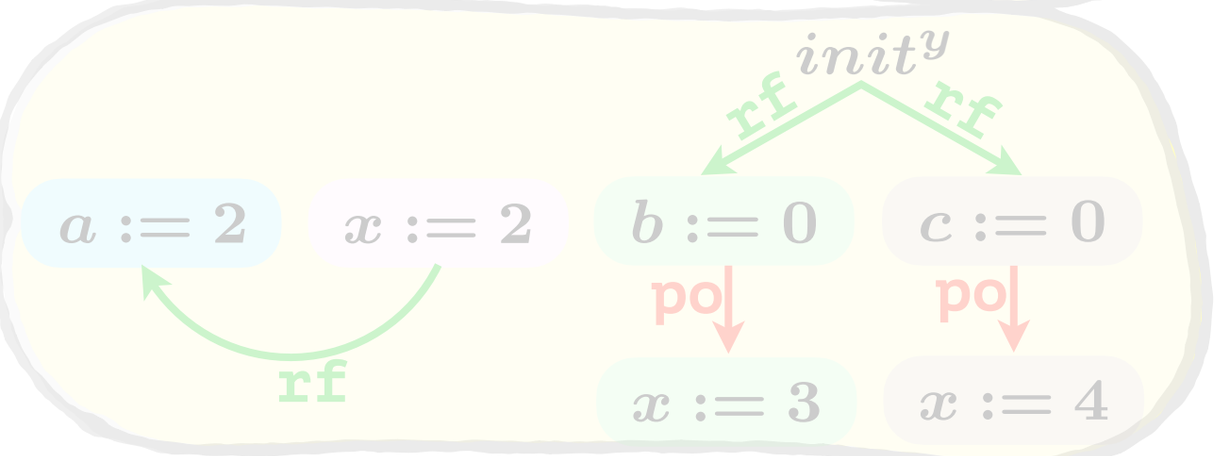

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

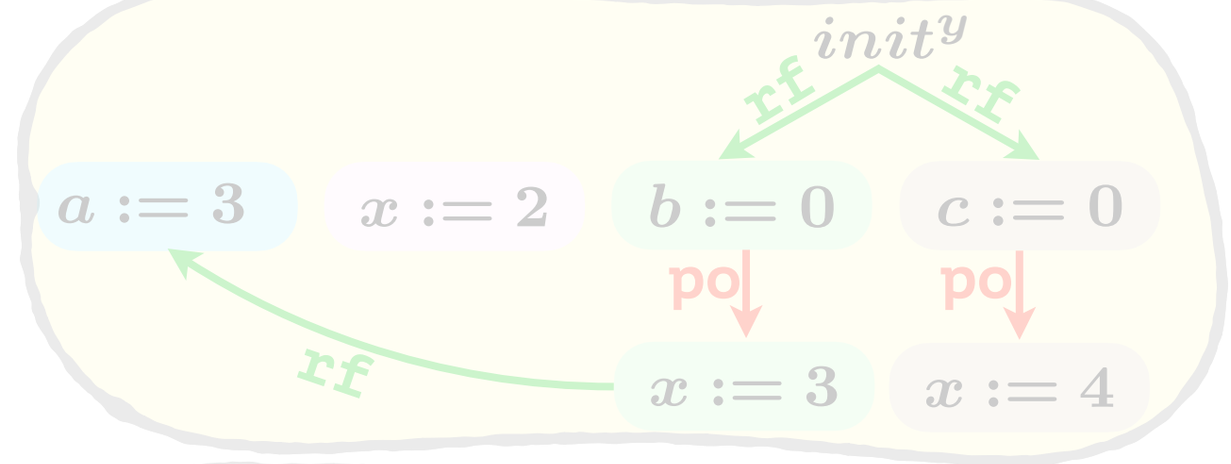

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

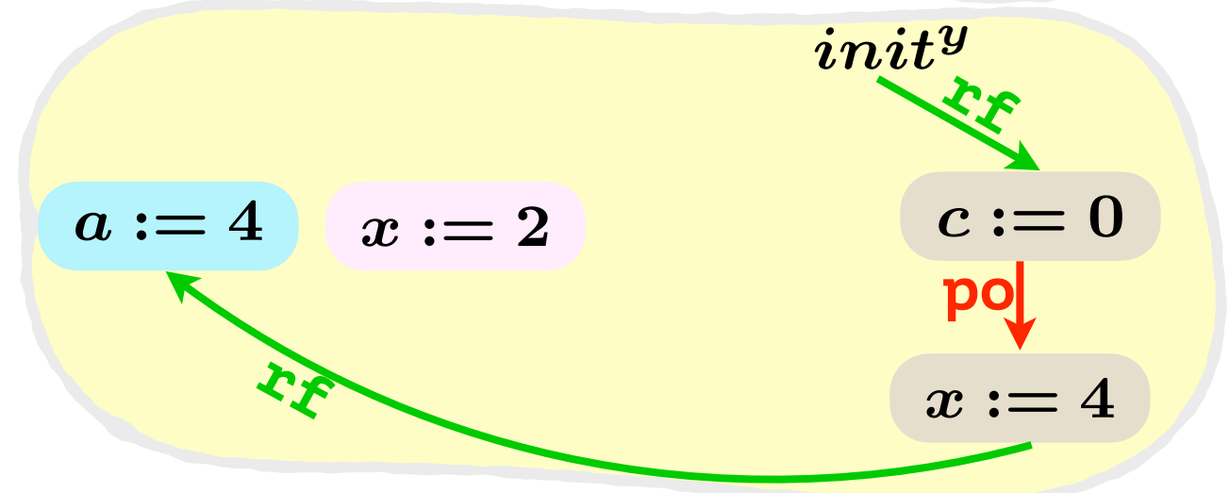

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$


$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$x := 4$
$x := 3$
$x := 4$





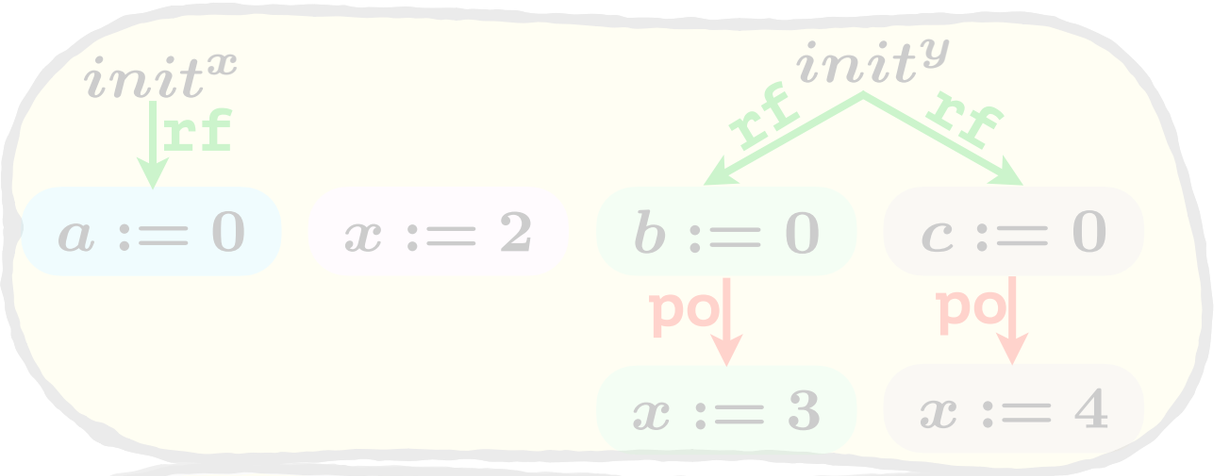


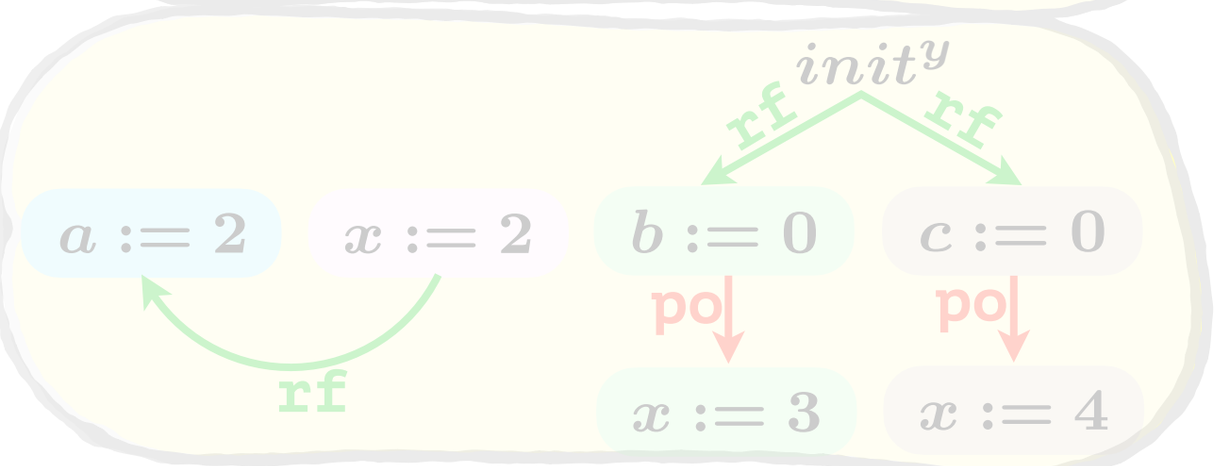


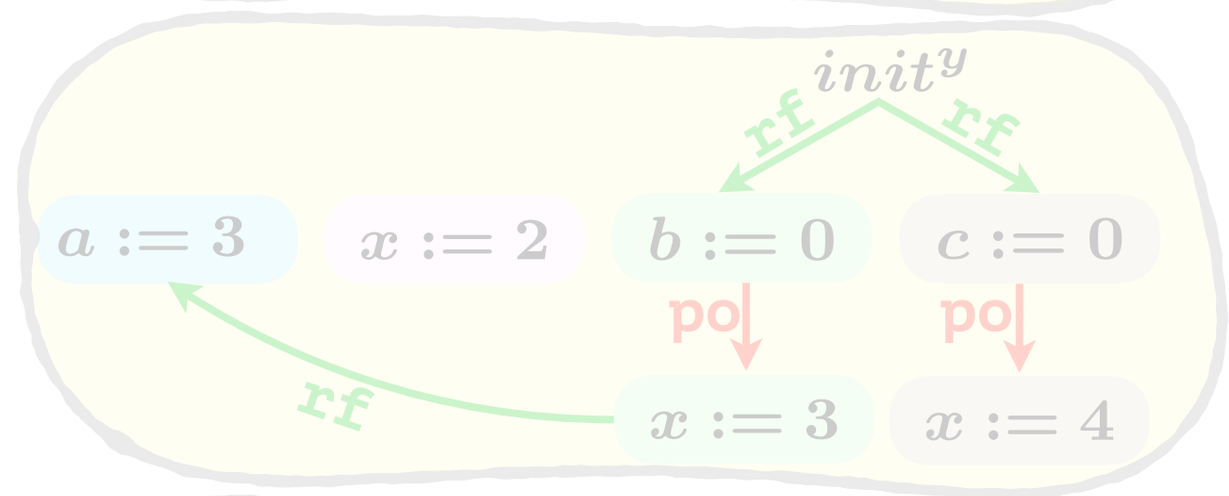
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

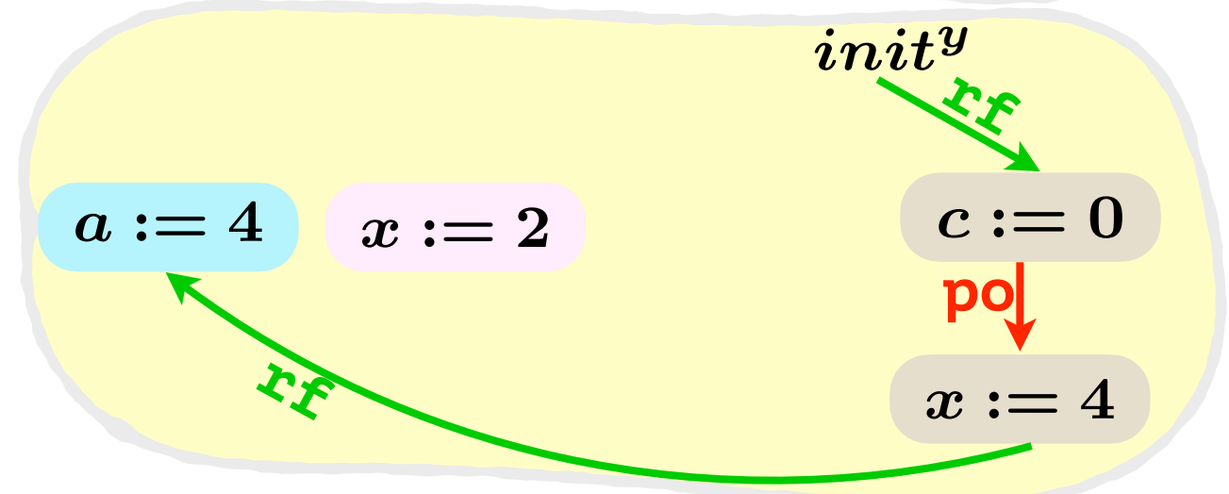
$x := 2$

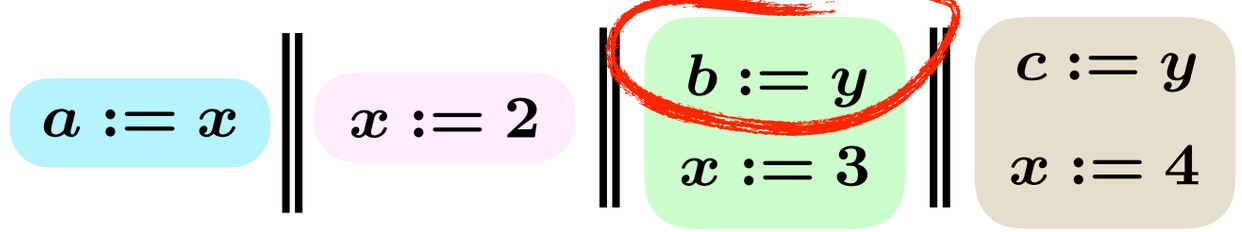
$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$	$x := 4$	

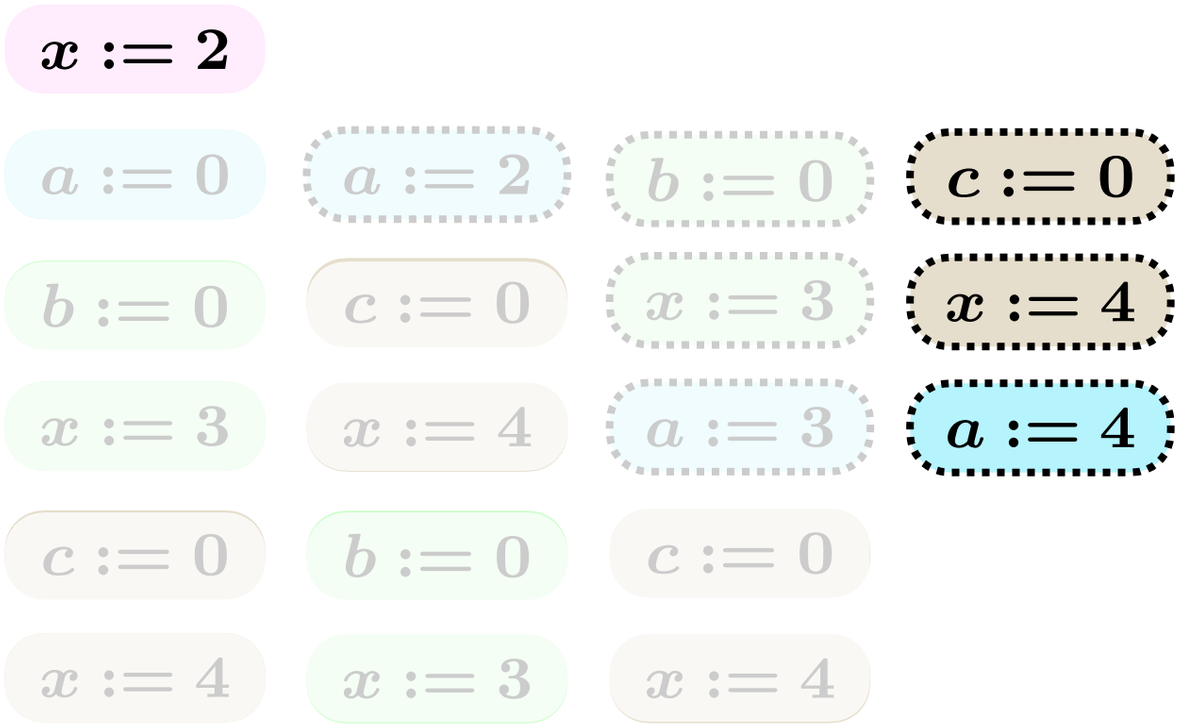

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

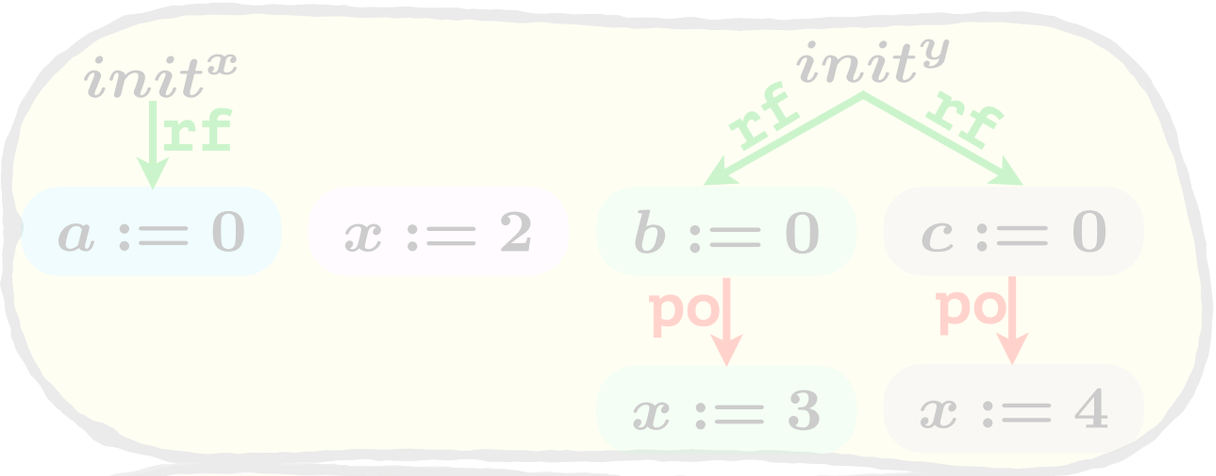

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

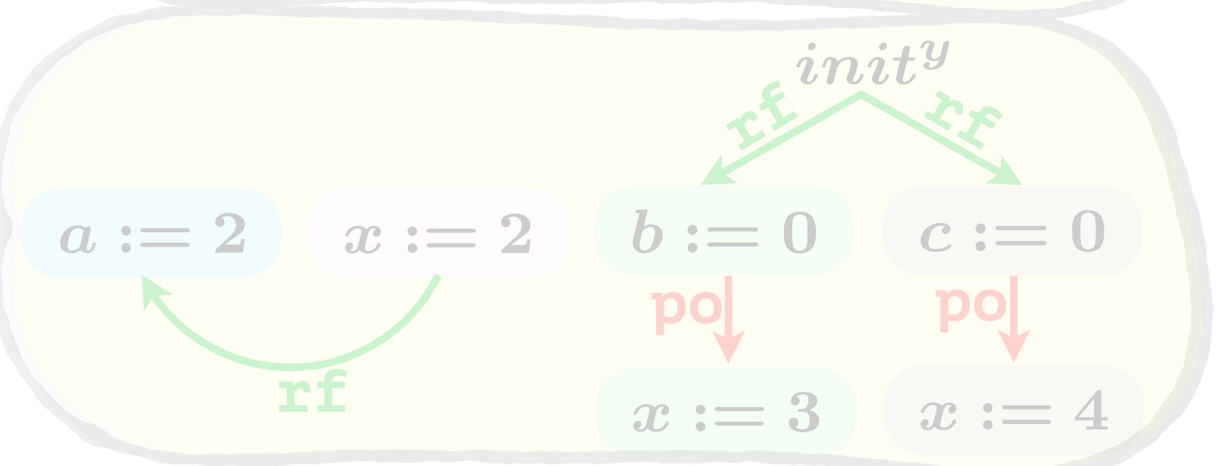

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

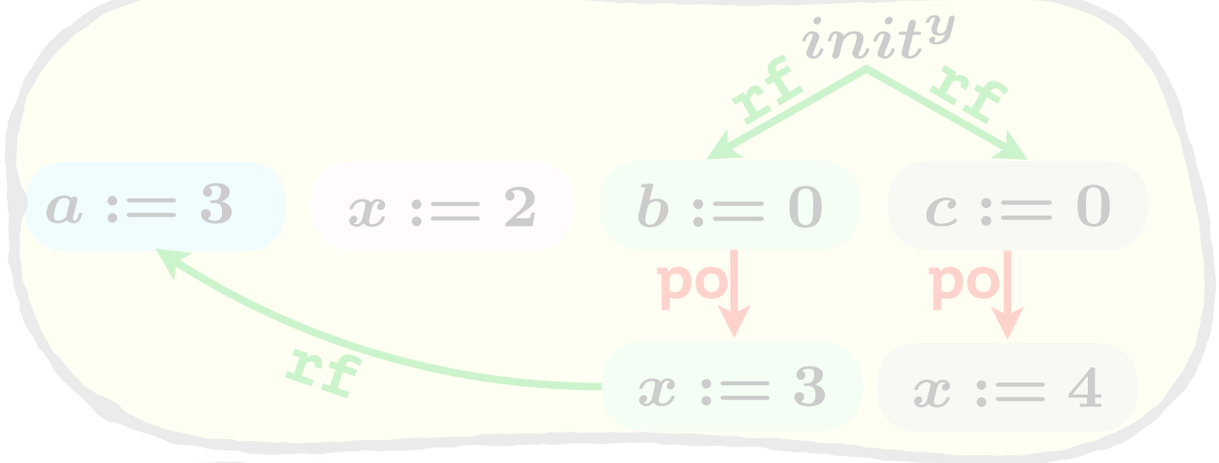

$init^y$
rf
$a := 4$
$x := 2$
$c := 0$
po
rf
$x := 4$

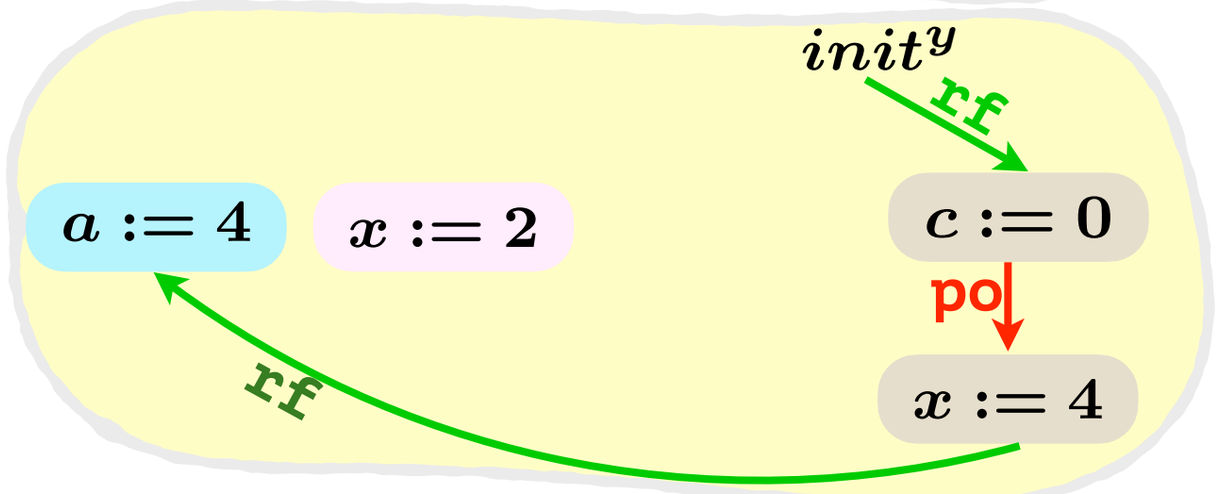

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

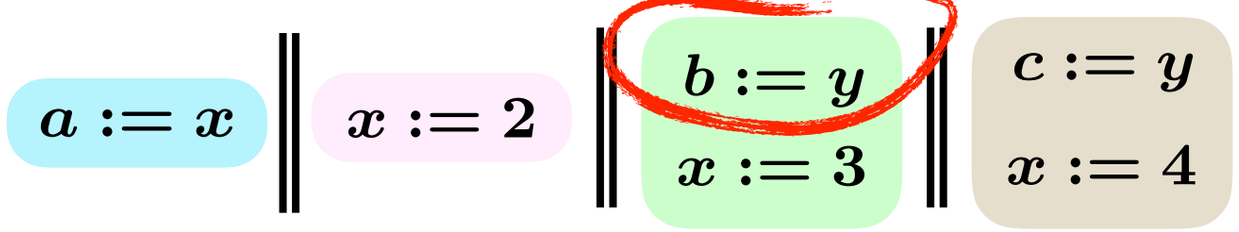

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$x := 4$
$x := 3$
$x := 4$

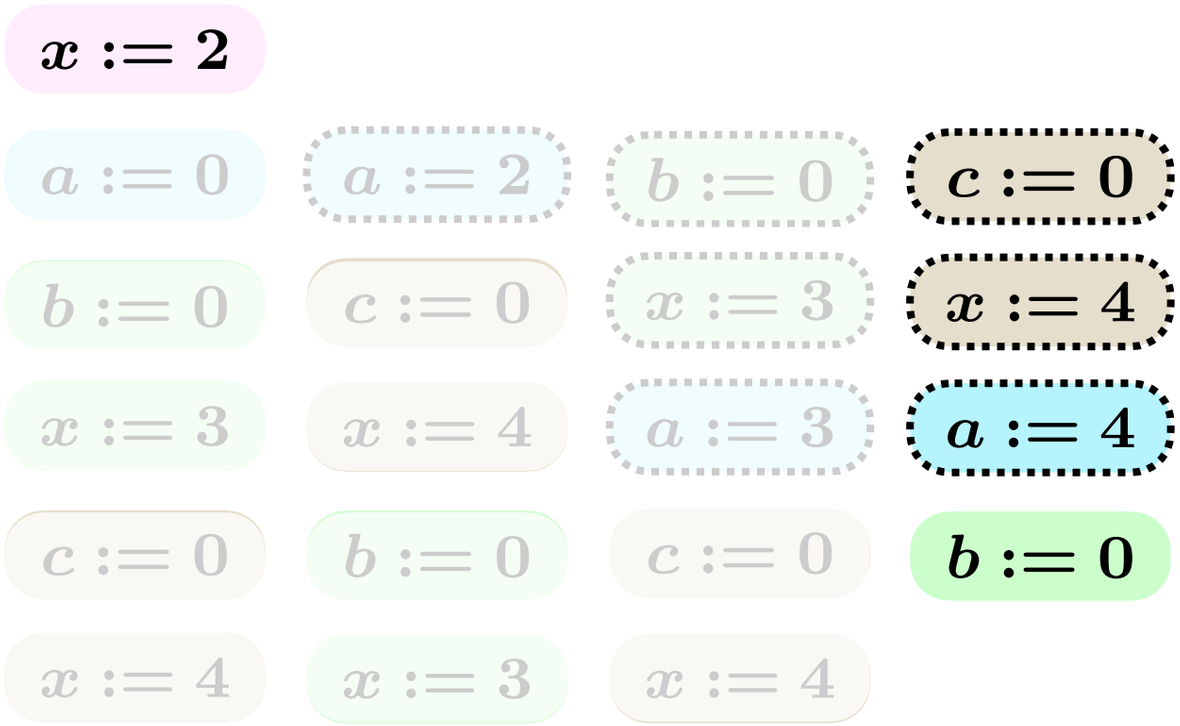

$init^x$
rf
$a := 0$
$x := 2$
$init^y$
rf
rf
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

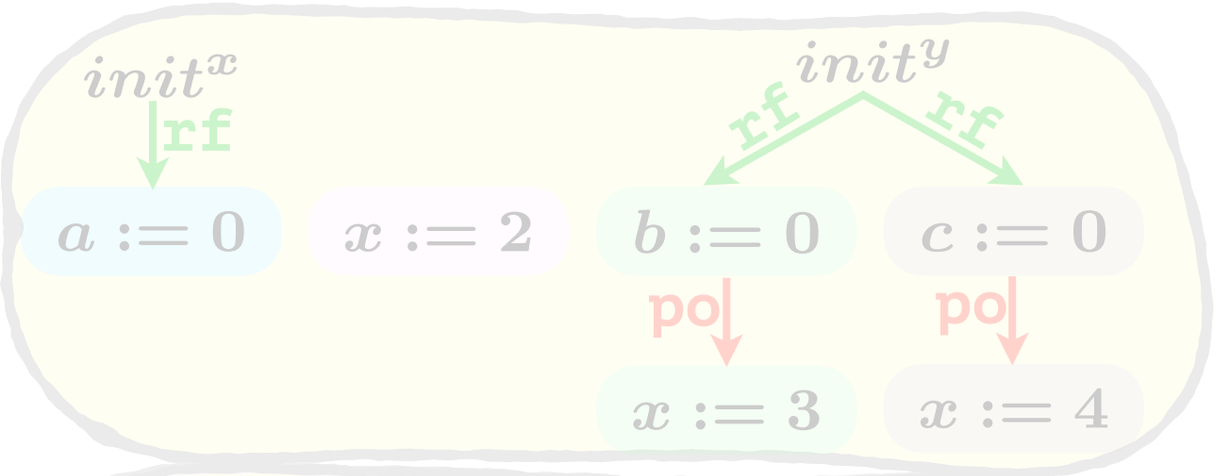

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

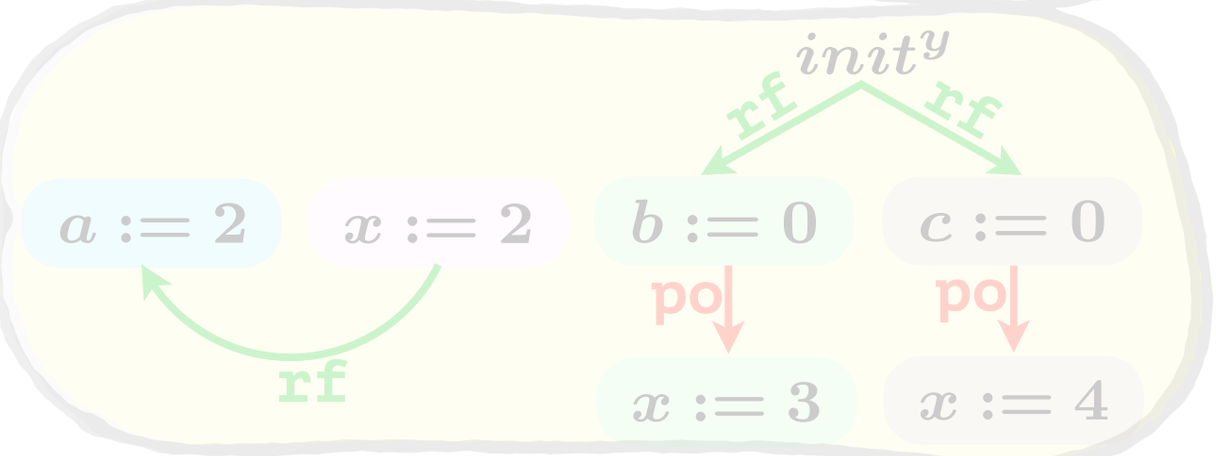

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

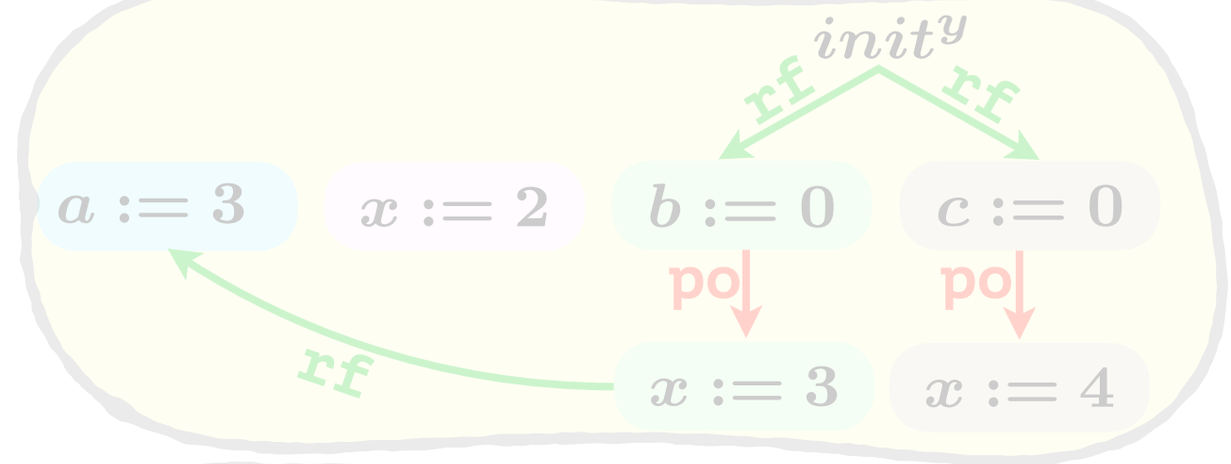

$init^y$
rf
$a := 4$
$x := 2$
$c := 0$
po
rf
$x := 4$

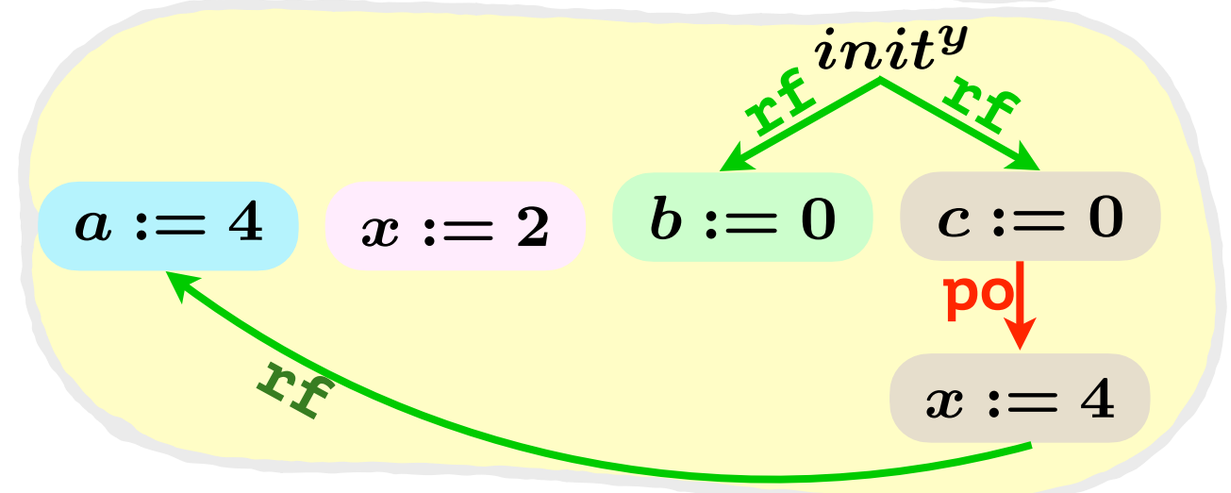

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

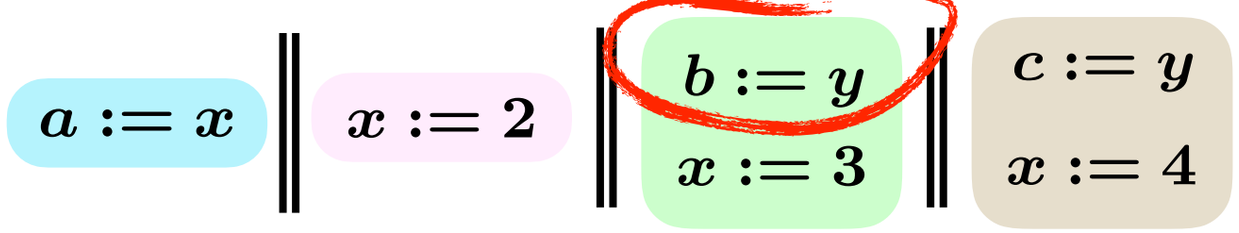

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$

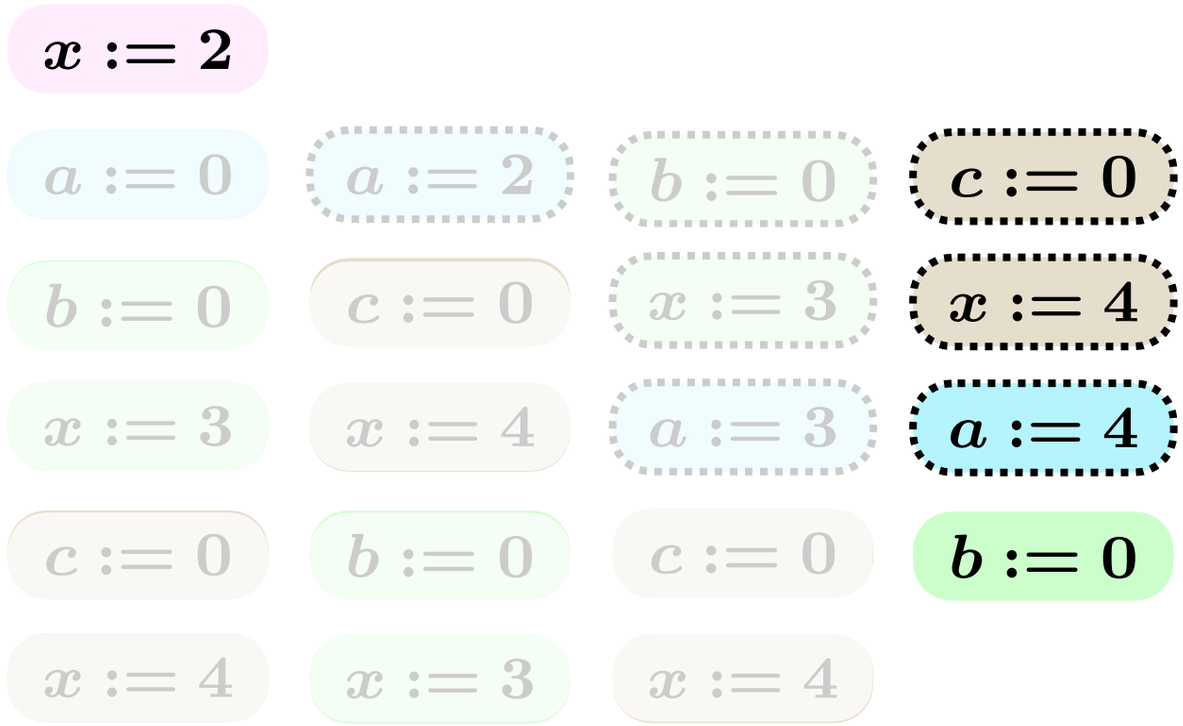

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

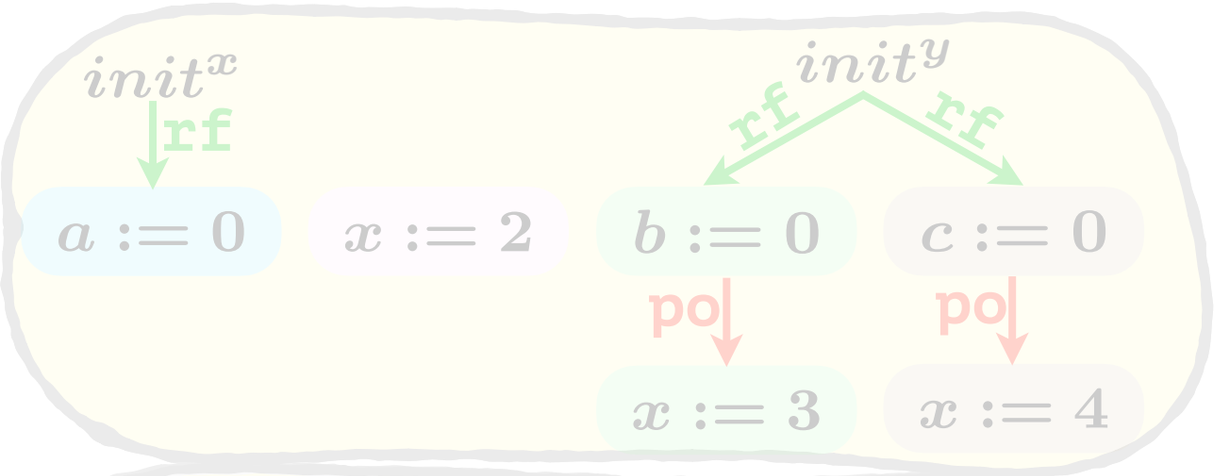

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

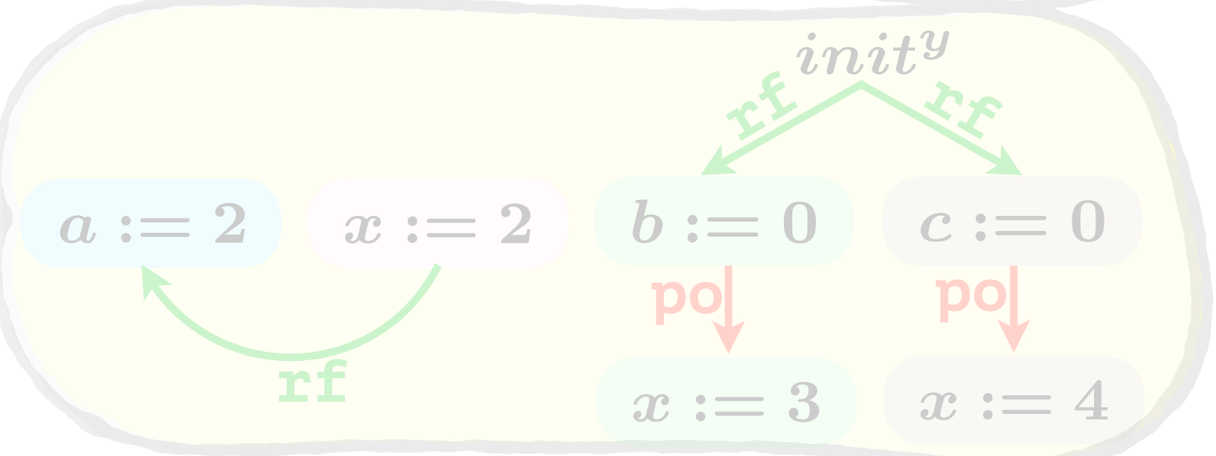

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

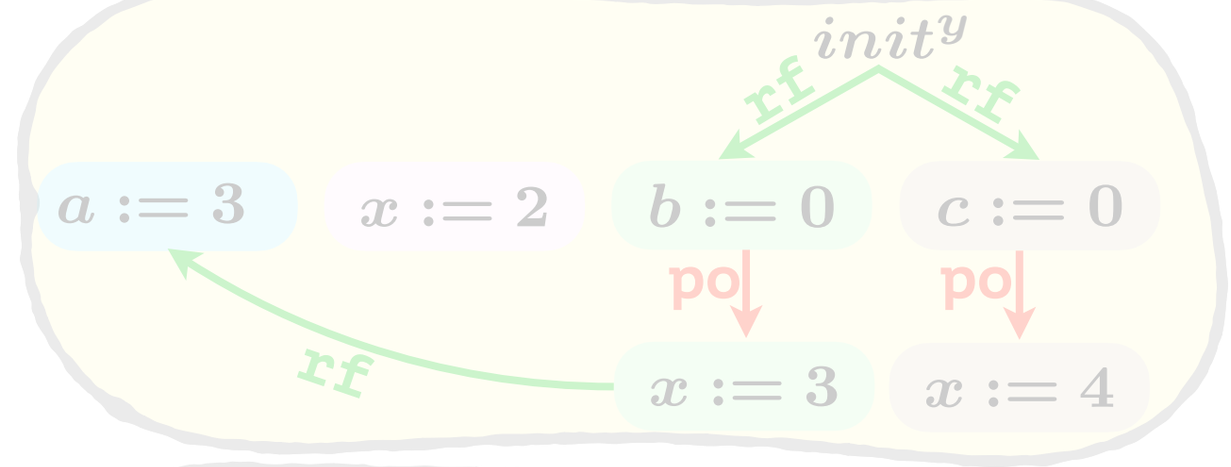

$init^y$
rf
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 4$

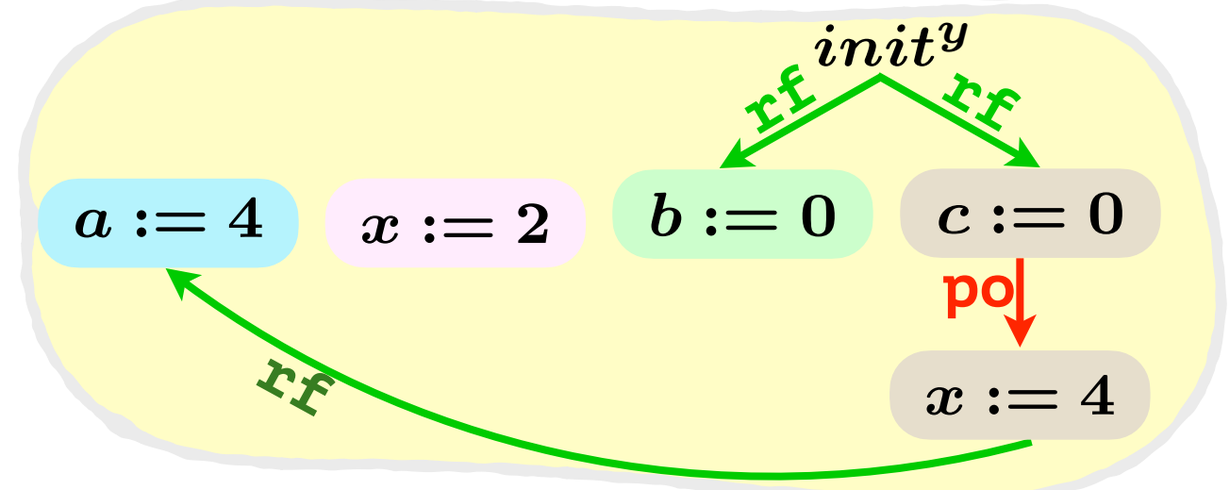

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$


$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$





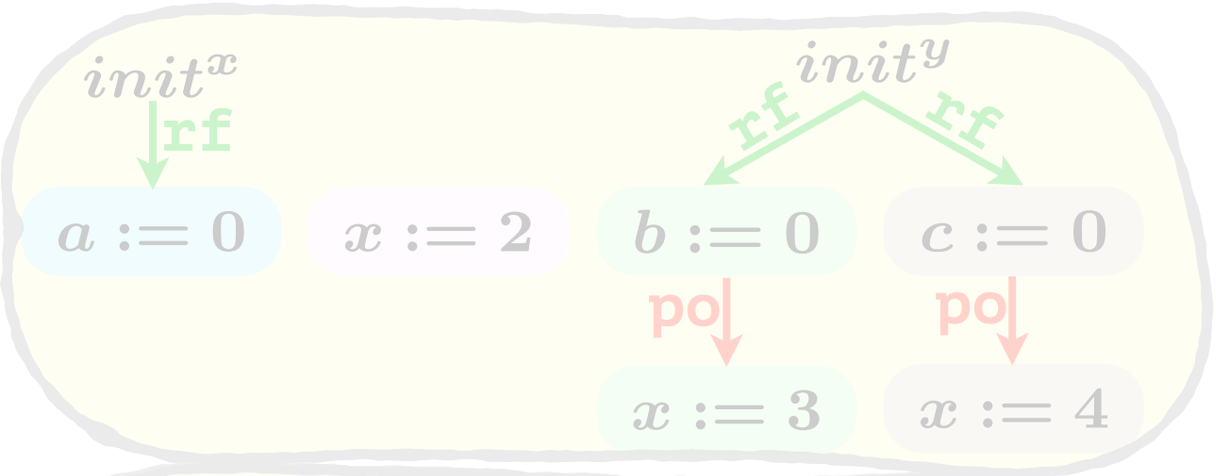


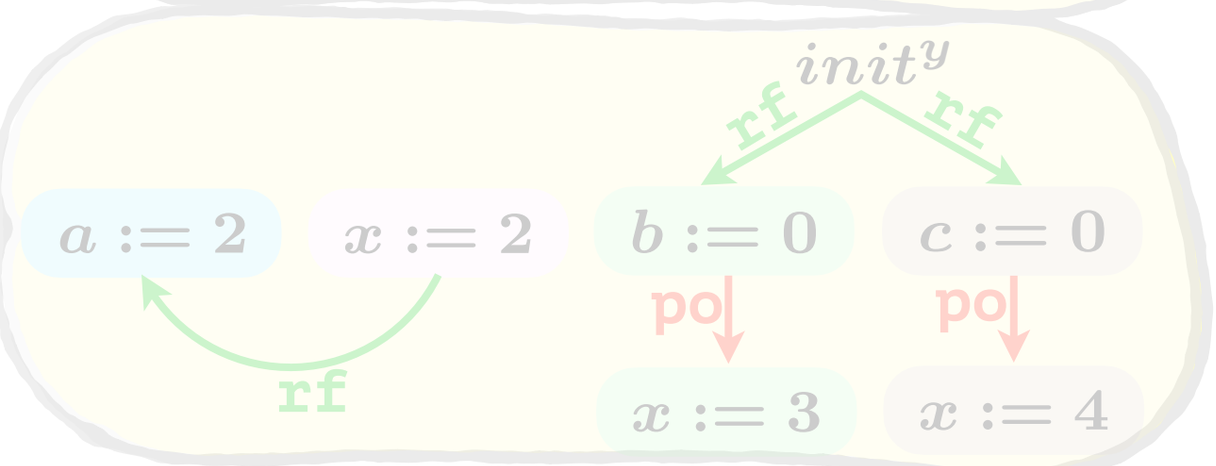


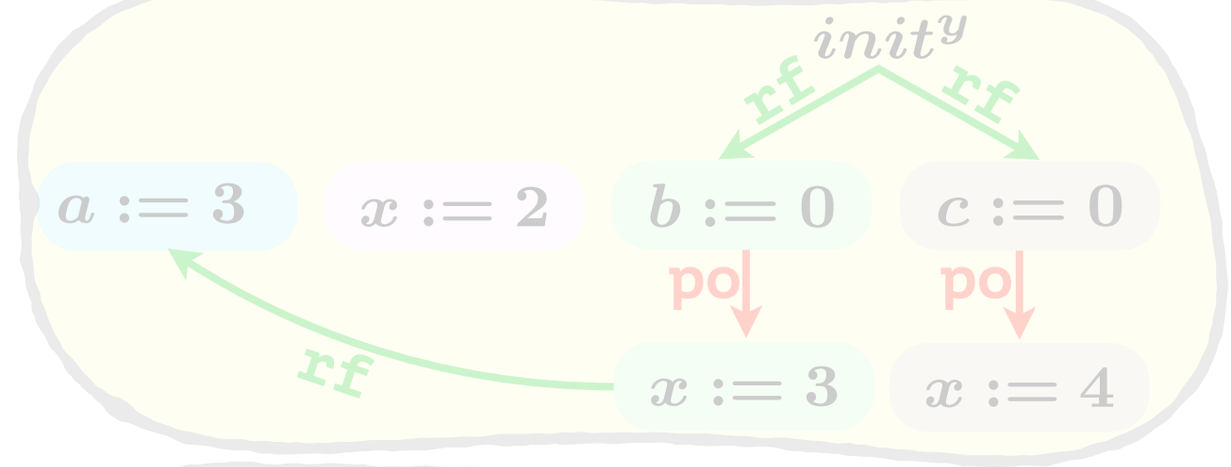
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

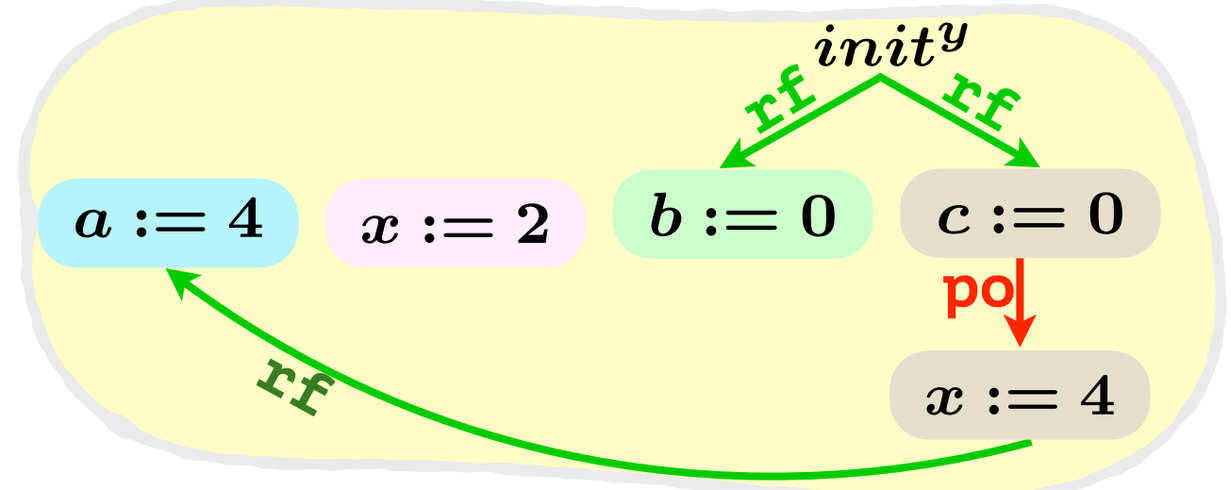
$x := 2$

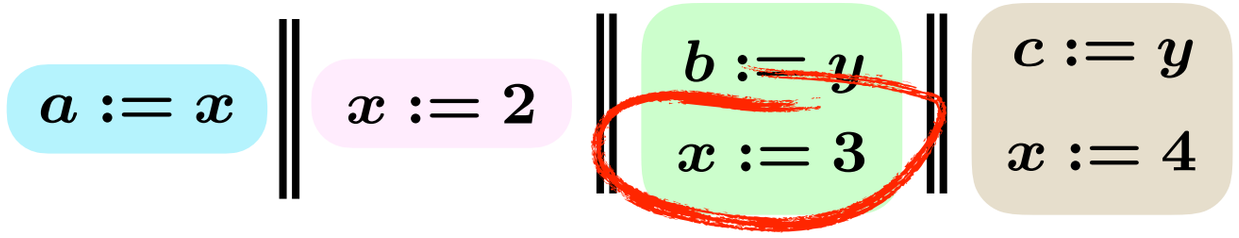
$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	$b := 0$
$x := 4$	$x := 3$	$x := 4$	

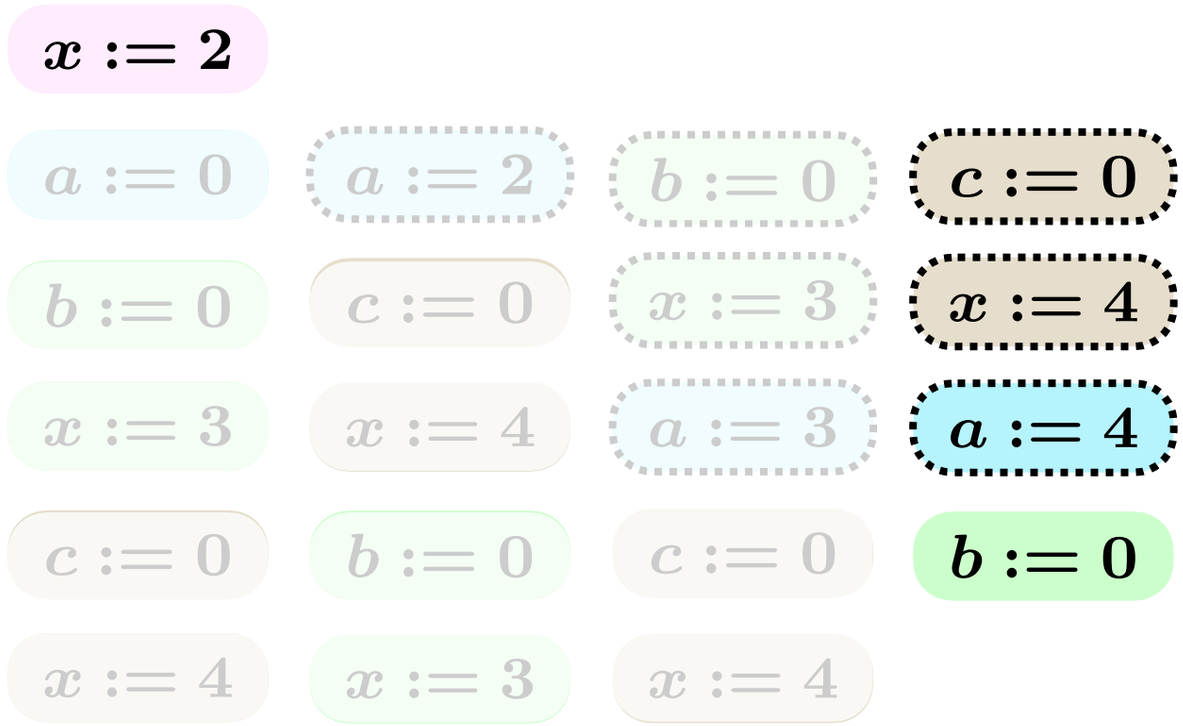

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

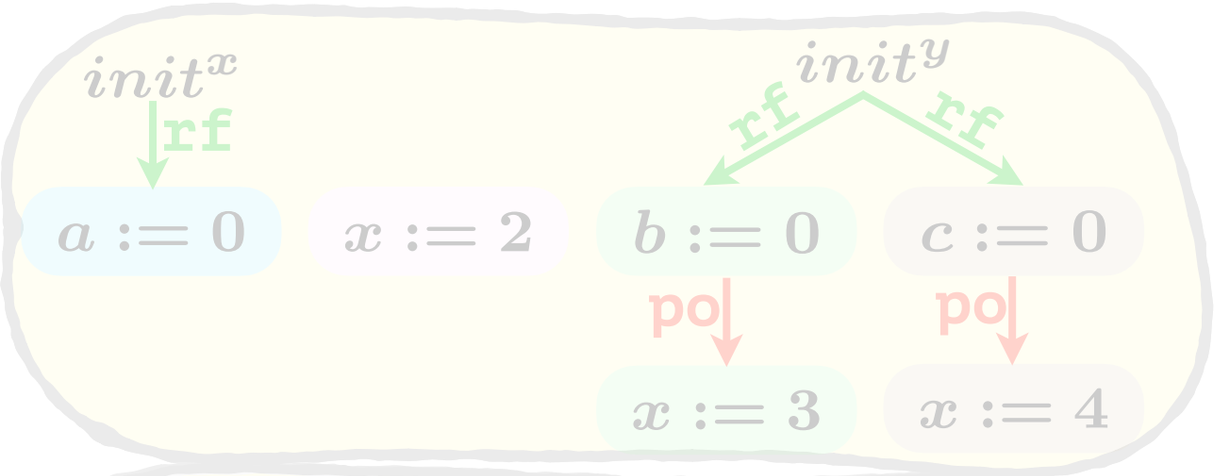

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

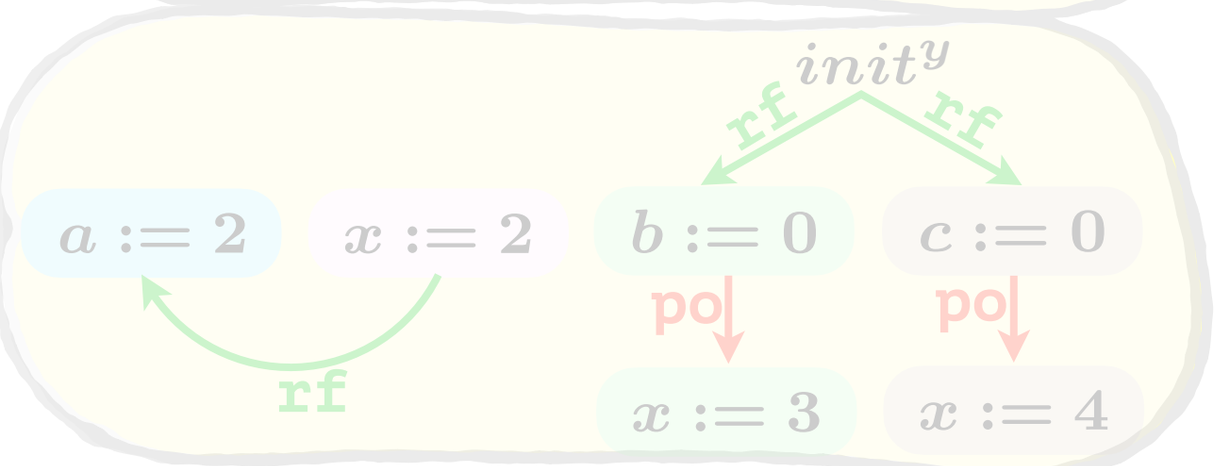

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

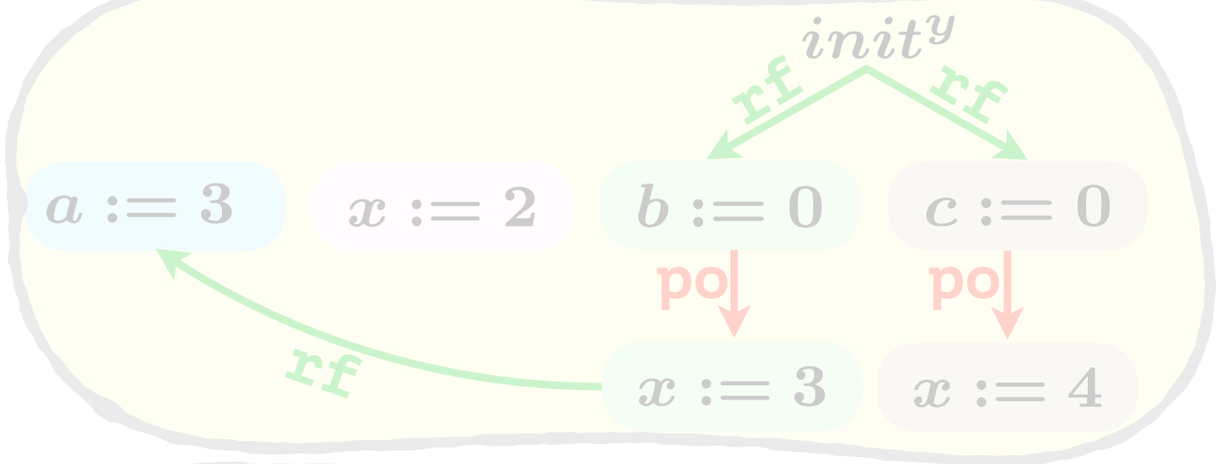

$init^y$
rf
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 4$

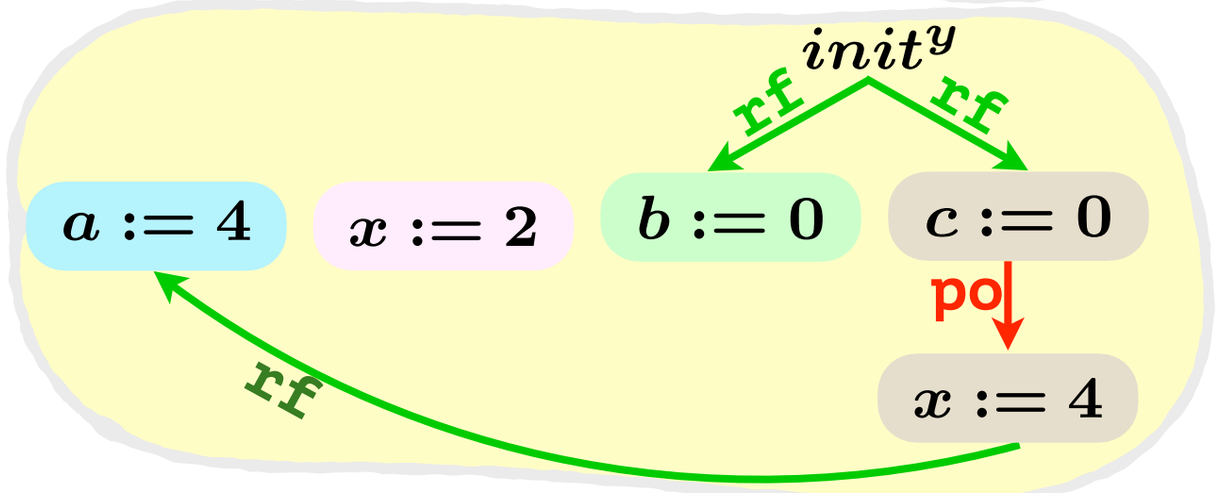

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

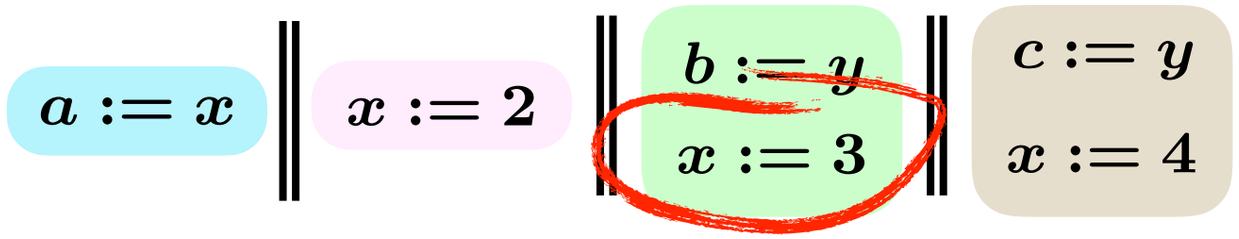

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$x := 3$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$

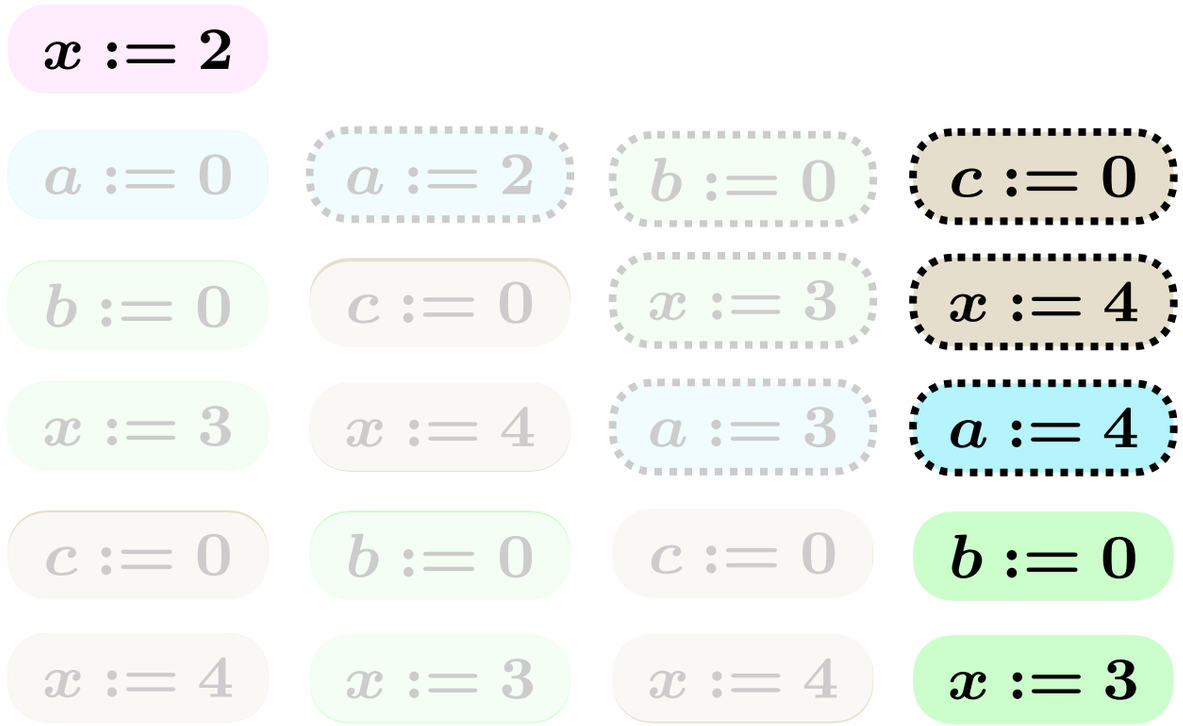

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

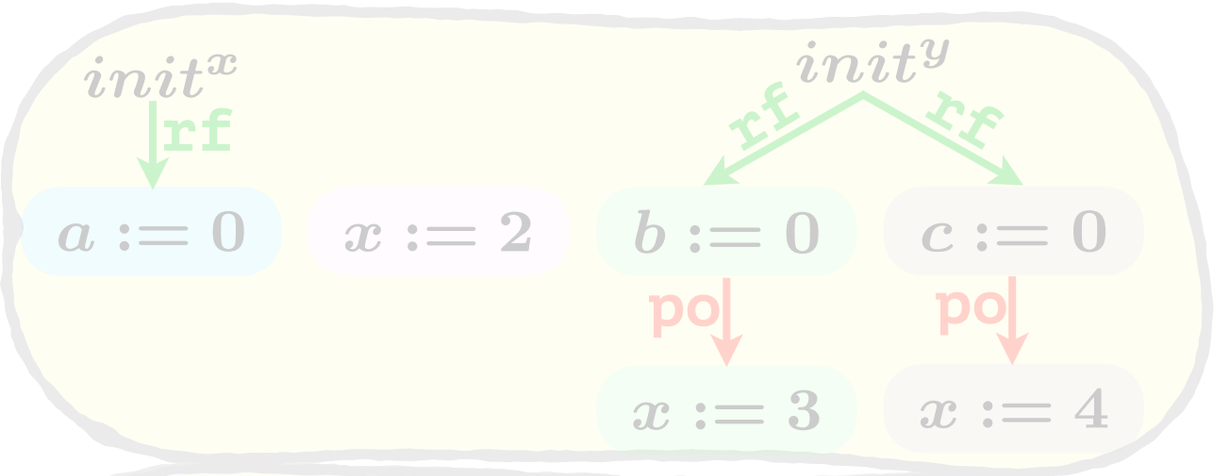

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

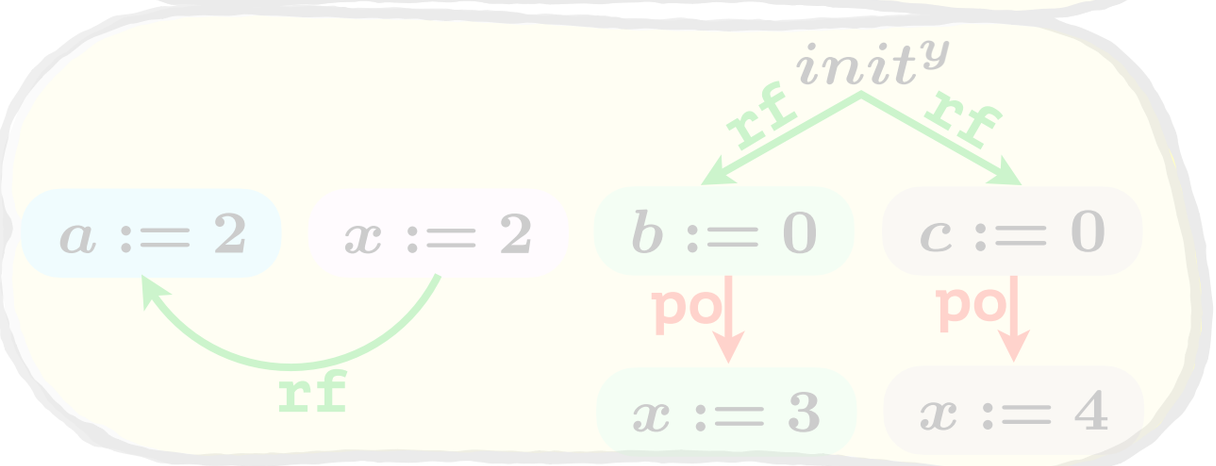

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

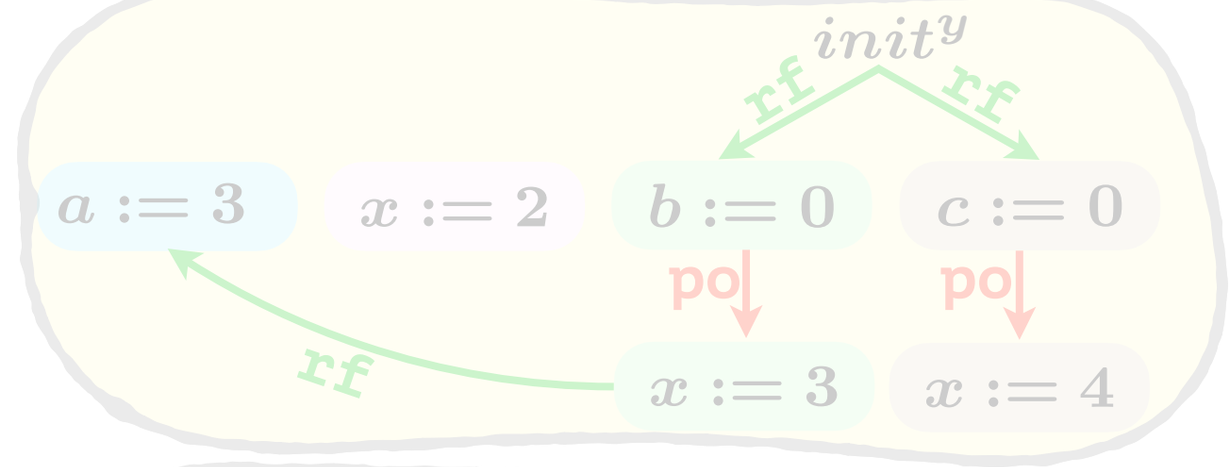

$init^y$
rf
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 4$

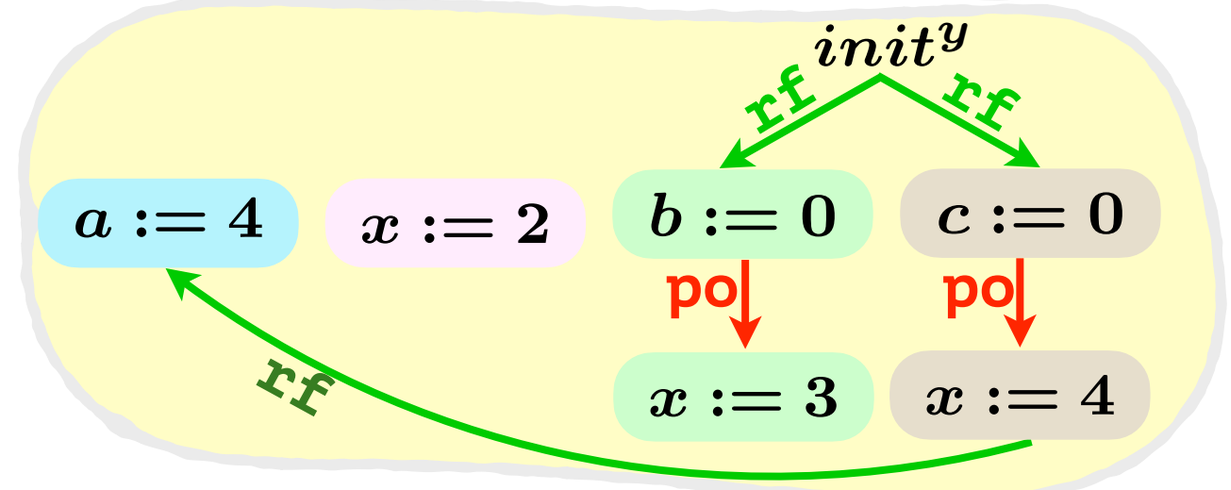

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

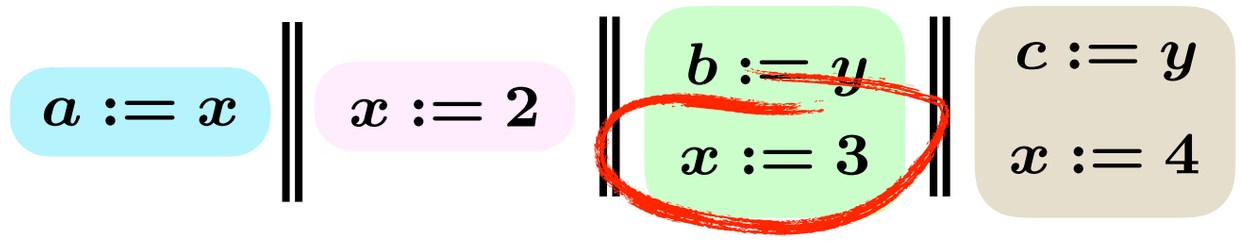

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$
$x := 3$

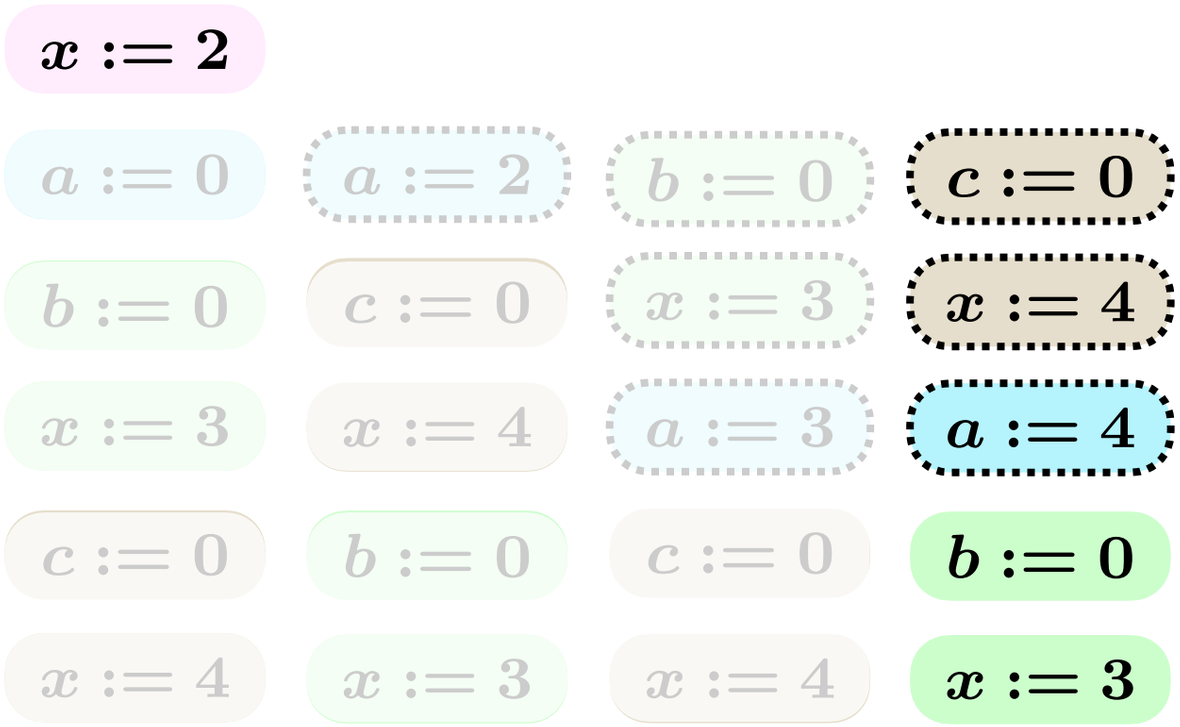

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

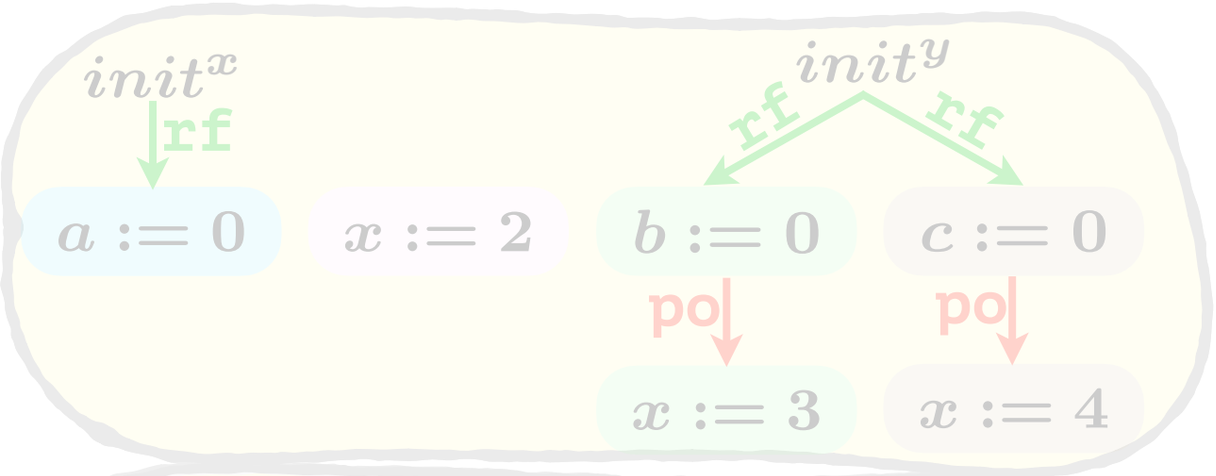

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

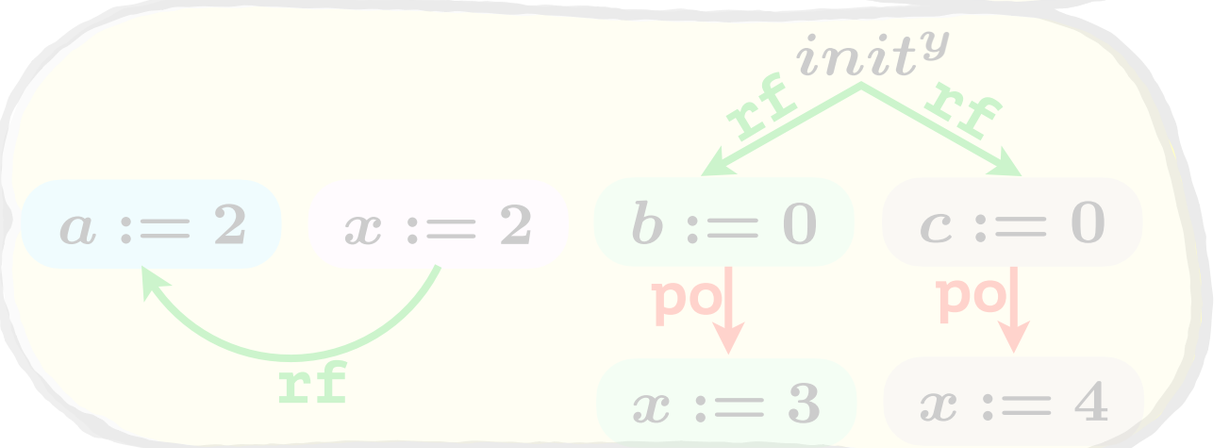

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

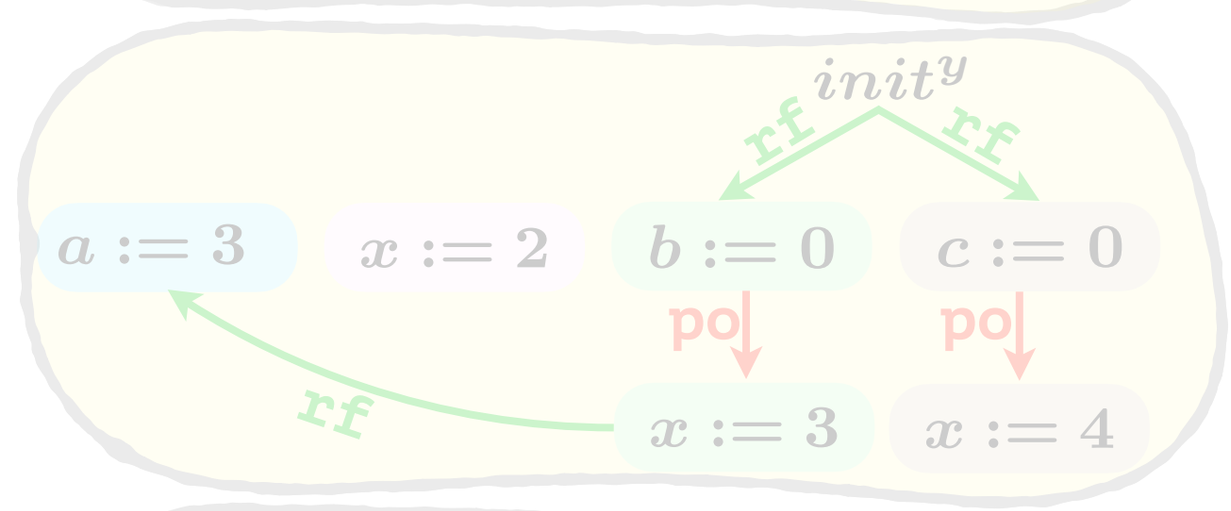

$init^y$
rf
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

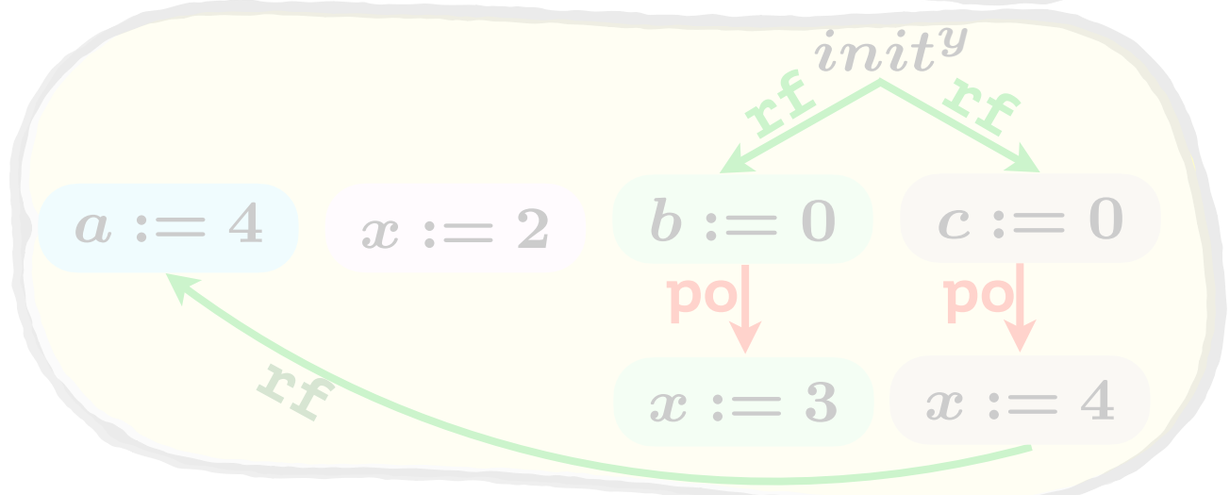

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

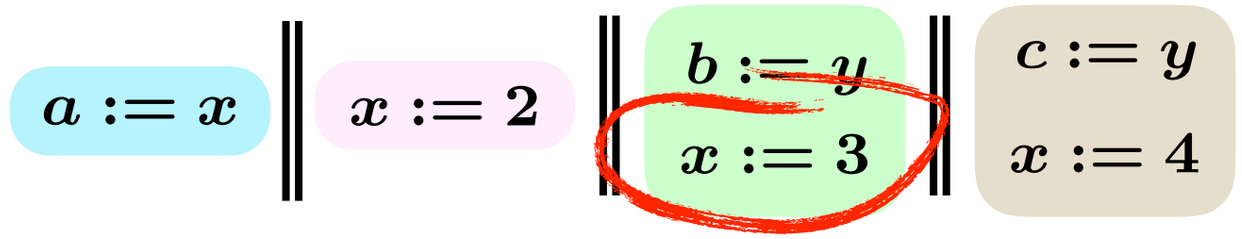

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$
$x := 3$

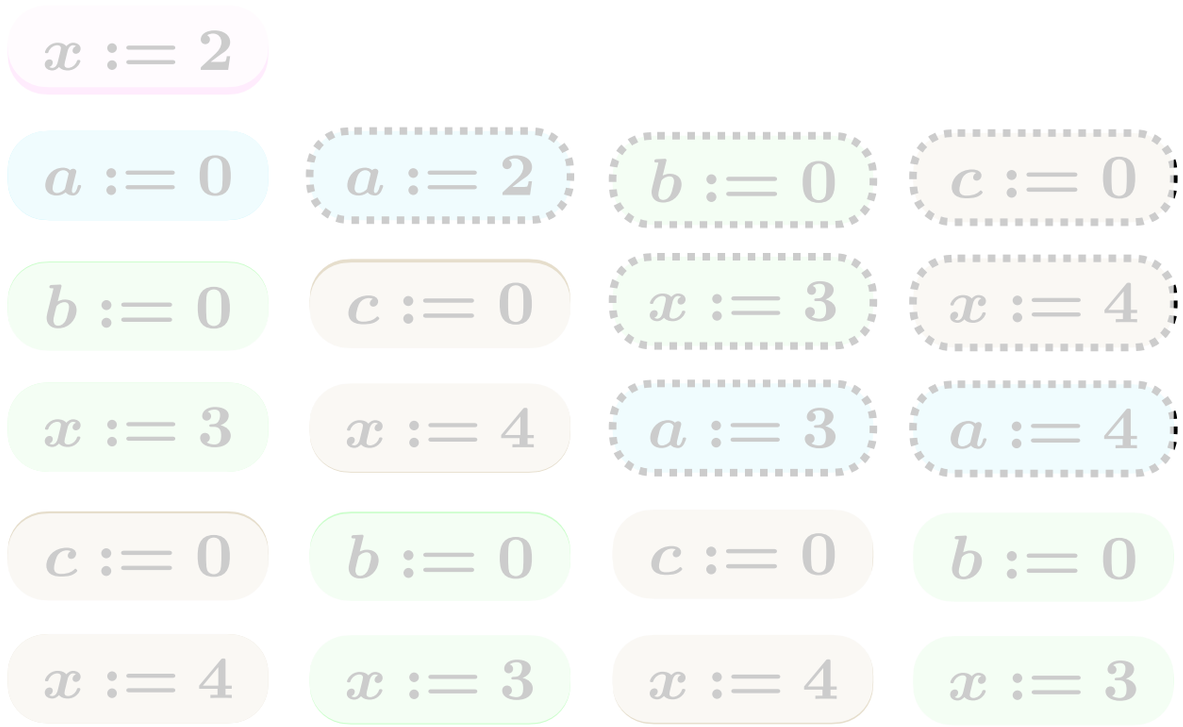

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$


$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

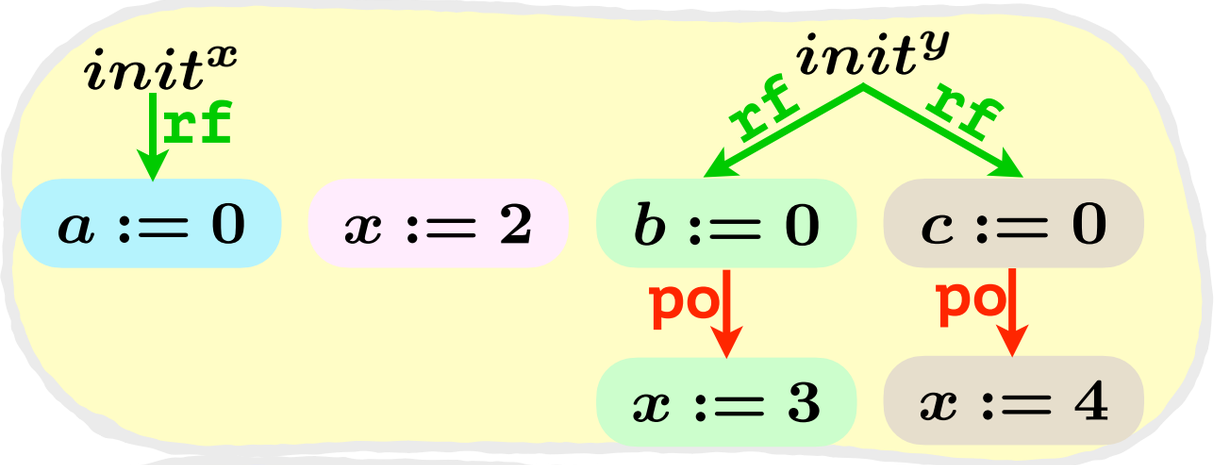

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

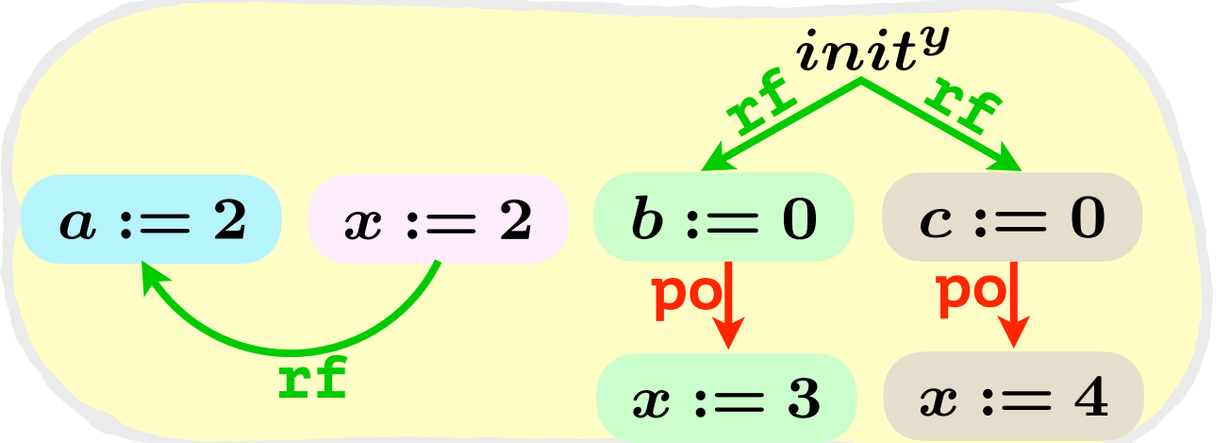

$init^y$
rf
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

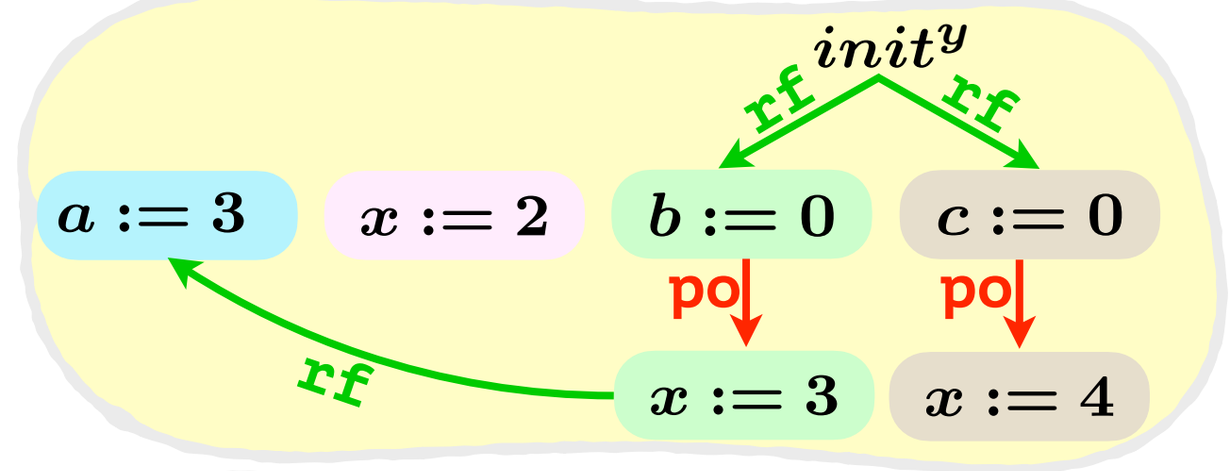

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

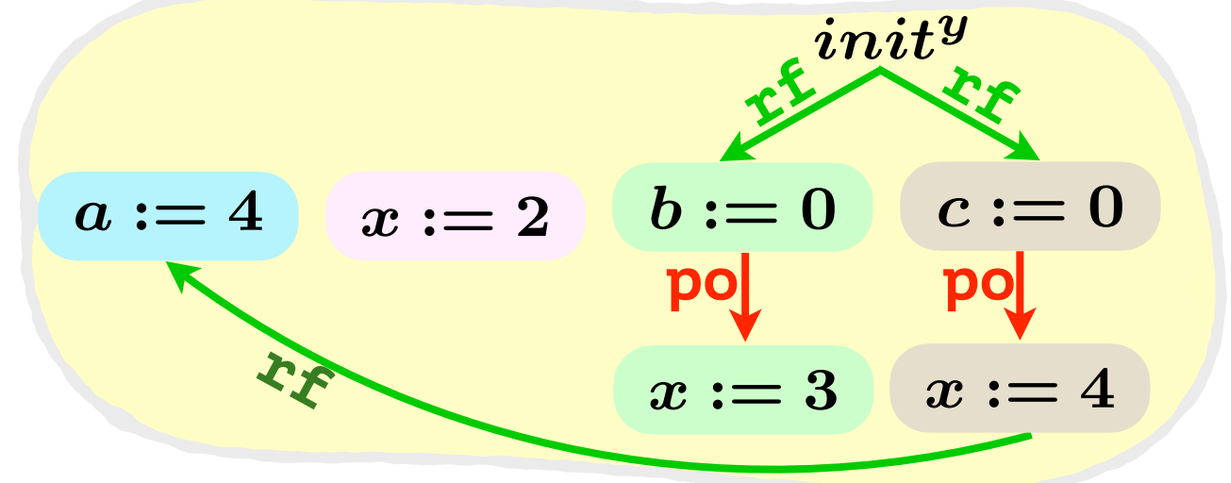

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$
$x := 3$

$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$



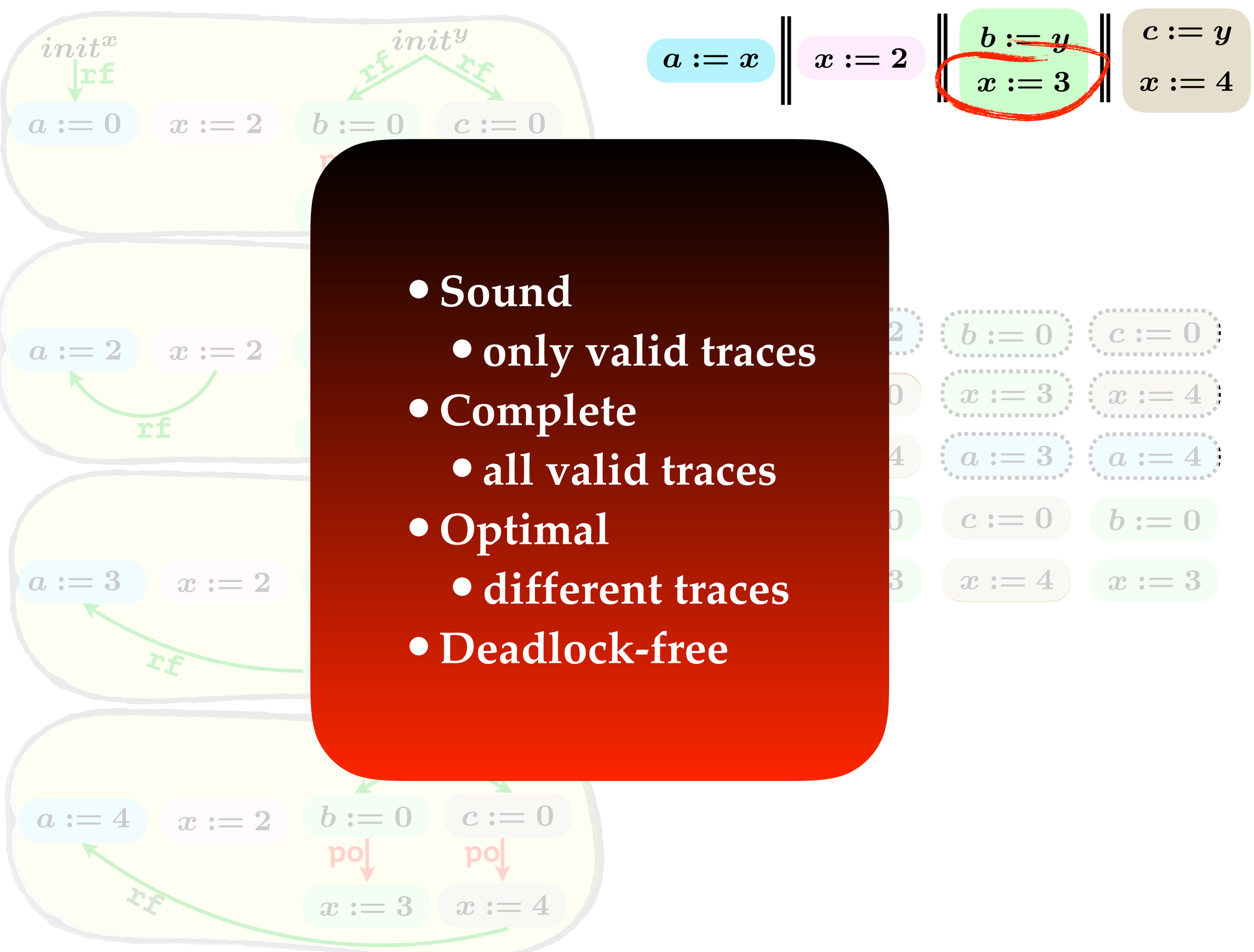






$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	$b := 0$
$x := 4$	$x := 3$	$x := 4$	$x := 3$


$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$
$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
$a := 2$
$x := 2$
rf
$a := 3$
$x := 2$
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$
rf
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
• Sound
• only valid traces
• Complete
• all valid traces
• Optimal
• different traces
• Deadlock-free

Tool	Wrong answer	Execs	Time	Average Execs (*Execs/Nums of tests*)	Average Time (*Time/Execs*)
CDSChecker	0	2 383 704	6m28s	295.59	**0.16ms**
Rcmc/Rc11	0	727 942	14m00s	90.27	1.15ms
Rcmc/Wrc11	81	668 574	12m15s	82.91	1.10ms
Tracer	0	**521 288**	**3m03s**	**64.64**	0.35ms

Litmus Tests

Program	LB	Executions Explored			Total Running Time		
		CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
Indexer(13)	20	190	**64**	**64**	0.15s	0.62s	**0.04s**
Indexer(14)	20	3 075	**512**	**512**	2.72s	11.68s	**0.38s**
Indexer(15)	20	48 261	**4 096**	**4 096**	45.32s	4m22s	**2.98s**
Indexer(16)	20	740 889	*t/o*	**32 768**	12m21s	*t/o*	**25.78s**
Filesystem(16)	20	27	**8**	**8**	0.04s	0.05s	**0.01s**
Filesystem(19)	20	728	**64**	**64**	0.52s	2.02s	**0.03s**
Filesystem(22)	20	19 678	**512**	**512**	13.53s	2m29s	**0.33s**
Filesystem(25)	20	531 415	*t/o*	**4096**	7m04s	*t/o*	**3.12s**

Standard Benchmarks

Program	LB	Executions Explored			Total Running Time		
		CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
Pthread_demo	10	184 758	**184 756**	**184 756**	24.96s	1m03s	**18.00s**
Gcd	8	8 814 044	**1 162 333**	**1 162 333**	38m20s	7m07s	**2m48s**
Fibonacci	6	*t/o*	**525 630**	**525 630**	*t/o*	**31.06s**	57.95s
Szymanski	6	*t/o*	26 037 490	**12 209 410**	*t/o*	44m52s	**14m54s**
Dekker	10	7 306 447	**3 121 870**	**3 121 870**	15m25s	5m12s	**4m52s**
Lamport	8	*t/o*	6 580 870	**3 372 868**	*t/o*	14m40s	**6m58s**
Sigma(5)	5	1 279	945	**120**	0.09s	0.16s	**0.01s**
Peterson	6	*t/o*	**1 897 228**	**1 897 228**	*t/o*	3m16s	**3m15s**
Stack_true	12	2 704 157	**2 704 156**	**2 704 156**	19m03s	54m25s	**10m12s**
Queue_ok	12	581 790	*t/o*	**362 880**	33m27s	*t/o*	**12m52s**

SV-COMP

Program	Executions Explored			Total Running Time		
	CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
N_writers_a_reader(7)	**8**	40 320	**8**	0.01s	0.46s	**0.00s**
N_writers_a_reader(8)	**9**	362 880	**9**	0.01s	4.19s	**0.00s**
N_writers_a_reader(9)	**10**	3 628 800	**10**	0.01s	46.13s	**0.00s**
N_writers_a_reader(10)	**11**	39 916 800	**11**	0.01s	9m35s	**0.00s**
Redundant_co(5)	581	16 632	**91**	0.03s	0.39s	**0.01s**
Redundant_co(10)	10 631	42 678 636	**331**	0.64s	23m56s	**0.02s**
Redundant_co(15)	59 056	*t/o*	**721**	4.57s	*t/o*	**0.06s**
Redundant_co(20)	197 231	*t/o*	**1 261**	20.27s	*t/o*	**0.14s**

Synthetic Programs

Program	LB	Executions Explored			Total Running Time		
		CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
Sigma(6)	6	25 357	10 395	**720**	2.20s	1.96s	**0.04s**
Sigma(7)	7	605 714	135 135	**5 040**	1m02s	29.06s	**0.40s**
Sigma(8)	8	16 667 637	2 027 025	**40 320**	33m18s	8m02s	**3.28s**
Sigma(9)	9	*t/o*	*t/o*	**362 880**	*t/o*	*t/o*	**33.59s**
Control_flow(6)	0	896	55 440	**77**	0.09s	1.97s	**0.01s**
Control_flow(8)	0	4 608	11 007 360	**273**	0.53s	7m58s	**0.03s**
Control_flow(10)	0	22 528	*t/o*	**1 045**	3.27s	*t/o*	**0.16s**
Control_flow(12)	0	106 496	*t/o*	**4 121**	19.10s	*t/o*	**0.79s**
Exponential_bug(6)	6	983 386	1 203 446	**15 601**	1m18s	56.75s	**0.96s**
Exponential_bug(7)	7	2 250 290	2 833 112	**22 841**	3m13s	2m26s	**1.46s**
Exponential_bug(8)	8	4 844 378	6 158 718	**32 313**	7m15s	5m28s	**2.23s**
Exponential_bug(9)	9	9 896 954	12 526 576	**44 428**	15m29s	11m48s	**3.22s**

Parameterized Programs

Program	LB	Executions Explored			Total Running Time		
		CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
Linux_locks(2)	6	47	**21**	**21**	0.03s	0.02s	**0.01s**
Linux_locks(3)	6	14 187 799	**412 814**	**412 814**	16m01s	36.36s	**33.21s**
Ticket_mutex_c11(3)	10	8 054	**4 026**	**4 026**	0.69s	50.76s	**0.25s**
Correia_Ramalhete(3)	5	**5 355**	**5 355**	**5 355**	1.10s	**0.77s**	0.92s

Concurrent Data Structures

Program	LB	Executions Explored		Total Running Time	
		CDSChecker	Tracer	CDSChecker	Tracer
Barrier(3)	10	62 649	**31 944**	5.45s	**1.88s**
Seqlock(3)	5	17 792	**14 864**	1.17s	**0.77s**
MPMC_queue(3)	5	621 882	**239 254**	2m11s	**44.50s**
MCS_lock(2)	5	92 210	**70 072**	8.91s	**5.19s**
Cliffc_hashtable(4)	0	9 520	**4 576**	2.68s	**0.94s**
Concurrent_hashmap(4)	0	110	**42**	0.01s	**0.00s**
Chase_Lev_deque(2) (**deadlock**)	0	162 306	**20 852**	20.65s	**2.10s**
SPSC_queue(2) (**deadlock**)	3	754	**57**	0.18s	**0.01s**

Concurrent Data Structures

Program	LB	Executions Explored		Total Running Time	
		CDSChecker	Tracer	CDSChecker	Tracer
Tidex(3)	5	4 676	**748**	0.37s	**0.04s**
Tidex_nps(2)	5	10 257	**10 254**	6m15s	**22.58s**
CLH_c11(3)	10	2 562	**732**	0.22s	**0.06s**
CLH_rwlock_c11(3)	10	20	**6**	0.01s	**0.00s**
MPSC_c11(3)	10	12 937	**4 824**	1.20s	**0.36s**
Correia_Ramalhete_turn(3) (**mutex broken**)	2	441 494	**96 184**	1m17s	**12.78s**

Concurrent Data Structures

Conclusions

- (Optimal) DPOR
- Release-Acquire Semantics
- Experiments

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Future Work

- Other Memory Models
  - Full C11, SC, TSO, POWER, ARM, Causal Consistency, …
- Weaker Notions of Traces
- Other Properties:
  - Race Detection
  - Liveness: Progress, Termination, …