Towards efficient model checking for variants of ATL under different semantics

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a joint work with
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Outline

- Introduction to specification of strategic abilities in ATL*,
- Model checking multi-valued version of ATL*,
- Partial order reductions for sATL*,
- Simpler strategies for Timed ATL (if time permits).

Specification and Verification of Strategic Ability

- Many important properties are based on **strategic ability**
- Functionality ≈ ability of authorized users to complete some tasks
- Security ≈ inability of unauthorized users to complete certain tasks
- One can try to formalize such properties in modal logics of strategic ability, such as ATL or Strategy Logic
- ...and verify them by model checking

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Motivation: VoteVerif

- New project has just began between the Polish Academy of Sciences and University of Luxembourg
- **VoteVerif**: Verification of Voter-Verifiable Voting Protocols
- Example properties: ballot confidentiality, coercion-resistance, end-to-end voter-verifiability
- Underpinned by existence (or nonexistence) of a suitable strategy for the voter and/or the coercer

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Papers introducing ATL* and TATL

- Alternating-time temporal logic [Alur et al. 1997-2002]
- Timed alternating-time temporal logic [Henzinger and Prabhu, LAMAS 2006]
- Model checking timed ATL for durational concurrent game structures [Laroussinie, Markey, Oreiby, LAMAS 2006]

ATL: What Agents Can Achieve

- ATL: Alternating-time Temporal Logic
- Temporal logic meets game theory
- Main idea: cooperation modalities

$\langle\langle A \rangle\rangle \phi$: coalition A has a collective strategy to enforce ϕ

$\leadsto$ ϕ can include temporal operators: X (next), F (sometime in the future), G (always in the future), U (strong until)

Semantic Variants of ATL

- Basic semantics of ATL assumes perfect information - not very realistic
- Semantic variants for more realistic cases defined in (Jamroga 2003), (Jonker 2003), (Schobbens 2004), (Jamroga & van der Hoek 2004), (Agotnes 2004), ...
- Encapsulate different assumptions about agents and abilities

Semantic Variants of ATL*

Memory of agents:

- Perfect Recall (R) vs. imperfect recall strategies (r)

Available information:

- Perfect Information (I) vs. imperfect information strategies (i)

ATL: What Agents Can Achieve

Example formulae:

- $\bigwedge_{i \in Candidates} \langle\langle v \rangle\rangle \mathrm{F}\ \mathrm{voted}_{\mathrm{v,i}}$:
  "The voter can cast her vote in an arbitrary way"
- $\neg\langle\langle c, v \rangle\rangle \mathrm{F} \bigvee_{i \in Candidates} K_c \mathrm{voted}_{\mathrm{v,i}}$:
  "The coercer cannot learn how the voter voted even if the voter cooperates with the coercer" (in ATL + K)

So, **let's specify and model-check**!

Not that easy...

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Caveat: there are serious **complexity obstacles**:

- Model checking agent logics for agents with perfect information ranges from **P-complete** to **EXPTIME-compl.**,
- Model checking agent logics for agents with imperfect information ranges from **NP-complete** to **undecidable**, depending on the exact syntax, semantics, and representation of models.
- Model checking ATL under imperfect information and imperfect recall is Δ_2^P**-complete** (in the size of a model and a formula).

Not That Easy...

These manifest in:

- State-space explosion,
- Transition-space explosion,
- Invalidity of fixpoint equivalences for ATL under imperfect information (see N. Bulling, C. Dima, V. Goranko, W. Jamroga, ...).

What to do ?

Possible ways out:...

- Symbolic model checking - BDD-based (Lomuscio, Raimondi), SAT-based Unbounded Model Checking for ATL (Kacprzak, Lomuscio, Penczek)
- Abstractions - **multi-valued model checking** over abstract models for variants of ATL(K) (Belardinelli, Lomuscio, Michaliszyn)
- Bisimulation-based reductions - for ATL_{ir} (Belardinelli, Condurache, Dima, ...)
- Upper and lower approximations - for ATL_{ir} (Jamroga, Knapik, Kurpiewski)
- **Partial order reductions** - model checking over smaller models for LTLK-X, CTLK-X, sATL* (Lomuscio, Penczek, Qu, Jamroga, ...)
- **Simpler strategies** - counting strategies for TATL (Andre, Jamroga, Knapik, Penczek, Petrucci)

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Motivation: Multi-Valued Abstraction

State abstraction:

- Cluster similar states into new abstract states
- Model checking over new abstract models

Possible problems:

- Even the values of some basic properties can be hard to compute in some states ⇝ undefined truth values
- Clustered states may disagree on some basic properties ⇝ inconsistent truth values

This leads to **multi-valued verification**

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Syntax

ATL* syntax in Negation Normal Form, augmented with constants for logical values L, and operator $\preccurlyeq$ for comparing truth values:

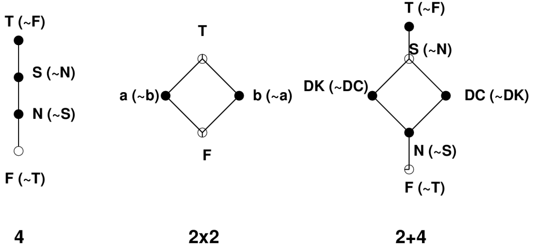
$$\phi ::= \mathrm{c} \mid \mathrm{p} \mid \neg \mathrm{p} \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle\langle A \rangle\rangle \gamma \mid \overline{\langle\langle A \rangle\rangle}\, \gamma \mid \phi \preccurlyeq \phi,$$

$$\gamma ::= \phi \mid \gamma \wedge \gamma \mid \gamma \vee \gamma \mid \mathrm{X}\,\gamma \mid \gamma\,\mathrm{U}\,\gamma \mid \gamma R \gamma,$$

where $c \in L$ and $p \in \mathcal{AP}$.

Models

ATL models with atomic propositions are interpreted in a distributive quasi-Boolean algebra (DM algebra) of truth values



Every element *x* in a DM algebra can be represented by the join of the join-irreducible elements smaller or equal than x.

Models - synchronous semantics

A Concurrent Game Structure is a 7 –tuple
$\mathcal{A} = (Agents, \Sigma, \mathcal{Q}, \mathcal{AP}, \mathcal{V}, protocol, trans)$, where:

- *Agents* is a finite set of all the agents,
- Σ is a finite set of actions,
- $\mathcal{Q}$ is a finite set of global locations,
- $\mathcal{AP}$ is a set of atomic propositions,
- $\mathcal{V} \colon \mathcal{Q} \times \mathcal{AP} \to \{\bot, \top\}$ is a valuation function,
- $protocol \colon Agents \times \mathcal{Q} \to \mathcal{P}(\Sigma) \setminus \{\emptyset\}$ is a protocol function,
- $trans \colon \mathcal{Q} \times \Sigma^{|Agents|} \to \mathcal{Q}$ is a transition function consistent with *protocol* for each agent of *Agents*.

Models - synchronous semantics

A MV-Concurrent Game Structure is a 7 –tuple $\mathcal{A} = (Agents, \Sigma, \mathcal{Q}, \mathcal{AP}, \mathcal{V}, protocol, trans)$, where:

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Models - synchronous semantics

A Tight Durational Concurrent Game Structure is a 7 –tuple $\mathcal{A} = (Agents, \Sigma, \mathcal{Q}, \mathcal{AP}, \mathcal{V}, protocol, trans)$, where:

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- $trans : \mathcal{Q} \times \Sigma^{|Agents|} \to \mathcal{Q} \times \mathbb{N}_+$ is a transition function consistent with *protocol* for each agent of *Agents*.

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Models - synchronous semantics

A Concurrent Game Structure is a 8–tuple $\mathcal{A} = (Agents, \Sigma, \mathcal{Q}, \mathcal{AP}, \mathcal{V}, protocol, trans, \{\sim_a \mid a \in Agents\})$, where:

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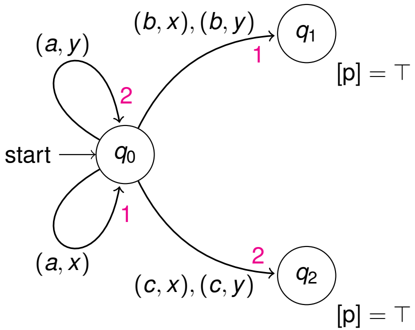
- $\sim_a \subseteq \mathcal{Q} \times \mathcal{Q}$, for each $a \in Agents$, is an indistinguishability relation.

Example of a Model



Perfect Information Strategies - *I*

Let $a \in Agents$:

Perfect recall (R), perfect information strategies (I) ($\Sigma_{R,I}$)

Functions $\sigma_a \colon \mathcal{Q}^+ \to \Sigma$ s.t., $\forall_{\pi \in \mathcal{Q}^+} \sigma_a(\pi) \in protocol(a, \pi_F)$.

(Intuition: no constraints, apart from the protocol of agent *a*)

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(Intuition: agent *a* selects an action based on the final location)

π_F: the final global location of π

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Joint Strategies

- A joint strategy σ_A for agents $A \subseteq Agents$ is a tuple of strategies, one per agent $a \in A$.
- The outcome of σ_A in location $q \in \mathcal{Q}$ is the set $out(q, \sigma_A) \subseteq \mathcal{Q}^\omega$ s.t. $\pi \in out(q, \sigma_A)$ iff $\pi(0) = q$ and for each $i \in \mathbb{N}$: $\pi(i) \xrightarrow{\text{act}'} \pi(i+1)$ for some $\text{act}' \in \Sigma$ s.t. $\text{act}'|_A = \sigma_A(\pi_i)$ and $\text{act}'|_{\overline{A}} \in protocol_{\overline{A}}(\pi(i))$.

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Joint Strategies

- A joint strategy σ_A for agents $A \subseteq Agents$ is a tuple of strategies, one per agent $a \in A$.
- The outcome of σ_A in location $q \in \mathcal{Q}$ is the set $out(q, \sigma_A) \subseteq \mathcal{Q}^\omega$ s.t. $\pi \in out(q, \sigma_A)$ iff $\pi(0) = q$ and for each $i \in \mathbb{N}$: $\pi(i) \xrightarrow{act'} \pi(i+1)$ for some $act' \in \Sigma$ s.t. $act'|_A = \sigma_A(\pi_i)$ and $act'|_{\overline{A}} \in protocol_{\overline{A}}(\pi(i))$.

Intuition: when coalition A follows σ_A, then in every global location, coalition A selects actions according to the joint strategy while the remaining agents $\overline{A}$ can choose any actions.

Semantics

We use **denotational semantics** that interprets Boolean and modal operators as either maximizers or minimizers

$\bigcup$, $\bigcap$ - the least upper bound, the greatest lower bound.

Σ_A - a set of joint strategies for A (variants: IR, iR, Ir, or ir)

$$[X\,\gamma]_{M,\pi} = [\gamma]_{M,\pi[1..\infty]};$$

.....

$$[\langle\langle A\rangle\rangle\gamma]_{M,q} = \bigcup_{\sigma_A\in\Sigma_A}\bigcap_{\pi\in out(q,\sigma_A)}\{[\gamma]_{M,\pi}\};$$

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$[\varphi_1 \preccurlyeq \varphi_2]_{M,q} = \top$ if $[\varphi_1]_{M,q} \leq [\varphi_2]_{M,q}$ and $\bot$ otherwise.

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Multi-Valued ATL* Extends 2-Valued ATL*

Theorem

The logic mv-$ATL^*_{\preccurlyeq}$ is a **conservative extension** *of ATL*, i.e.:*

for every 2-valued model M, ATL formula φ, and state (path) ι:*

$$[\varphi]_{M,\iota} = \top \quad \text{iff} \quad M, \iota \models_{ATL*} \varphi.$$
$$[\varphi]_{M,\iota} = \bot \quad \text{iff} \quad M, \iota \not\models_{ATL*} \varphi.$$

Translation to Simpler Lattices

Theorem

Let $f : L \to L'$ be a mapping that preserves bounds, i.e.,

$$f\Big(\bigcap_{i \in I} x_i\Big) = \bigcap_{i \in I} f(x_i), \qquad \textit{and} \qquad f\Big(\bigcup_{i \in I} x_i\Big) = \bigcup_{i \in I} f(x_i).$$

Then, for any mv-ATL formula φ and any state (resp. path) ι:*

$$[\varphi]_{\mathbf{f(M)},\iota} = \mathbf{x} \qquad \textit{iff} \qquad [\varphi]_{\mathbf{M},\iota} \in \mathbf{f}^{-1}(\mathbf{x})$$

Translation to 2-valued Lattices

Corollary

There exists a simple translation of checking whether $[\varphi]_{M,\iota} = x$ in mv-ATL* to several instances of 2-valued model checking of φ in ATL.*

$[\varphi]_{M,\iota} = \bigcup\{j \in \text{Join-irreducible(L)} \mid [\varphi]_{f_j(M),\iota} = \top\}$

$f_j(M)$ - the model M translated by $f_j : \mathrm{L} \longrightarrow \{\bot, \top\}$:

$f_j(\uparrow j) = \top, \quad f_j(\mathrm{L} \setminus \uparrow j) = \bot.$

Complexity of Multi-Valued ATL* Model Checking: Perfect Information

Theorem

Multi-valued verification of ATL* incurs only **polynomial increase** *in the complexity compared to the 2-valued case.*

Specifically, model checking mv-ATL$_{Ir\preccurlyeq}$ is **P***-complete, and model checking mv-ATL$^*_{Ir\preccurlyeq}$ is* **2EXPTIME***-complete in the size of the model and the formula, and the number of logical values.*

Imperfect Information

The method does not depend on the actual definition of strategy sets Σ_A!

Thus, we have:

Theorem

Model checking mv-ATL$_{ir\preccurlyeq}$ is Δ_2^P-complete, and model checking mv-ATL$^*_{ir\preccurlyeq}$ is **PSPACE***-complete in the size of the model and the formula, and the number of logical values.*

Imperfect Information

Theorem

Model checking $mv\text{-}ATL^*_{iR\preccurlyeq}$ *and* $mv\text{-}ATL_{iR\preccurlyeq}$ *is undecidable in general.*

For the fragment of $mv\text{-}ATL_{iR\preccurlyeq}$ *with singleton coalitions only, model checking is* **EXPTIME***-complete in the size of the model and the formula, and the number of logical values.*

Making model checking more efficient

- Abstraction - **multi-valued model checking** over smaller models,
- **Partial order reductions** - model checking over smaller models
- **Simpler strategies** - counting strategies for TATL

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What to do ?

Idea behind POR

POR is a method of generating reduced state spaces, preserving some temporal formula ψ, that exploits:

- Independency of actions, restricted to the pairs of actions such that one of them is invisible, i.e., does not change valuations of the atomic propositions used in ψ,
- Infinite sequences of global locations that differ in the ordering of independent actions only are called ψ-equivalent,
- ψ does not distinguish between ψ-equivalent sequences,

A reduced state space contains for each infinite sequence at least one ψ-equivalent, but as few as possible.

Networks of automata - asynchronous semantics

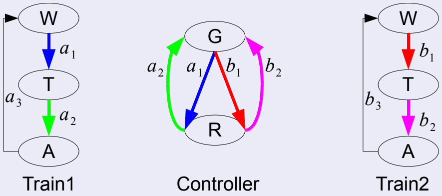

Figure: TC composed of two trains and the controler

Experimental Results - Trains and controler (TC)

Property: if the train 1 is in the tunnel, then no other train is in the tunnel at the same time: $AG(\text{in\_tunnel}_1 \rightarrow \bigwedge_{i=2}^{n} \neg\text{in\_tunnel}_i)$,

State spaces for n trains

$F(n)$ - the size of the full state space.
$R(n)$ - the size of the reduced state space.

- $F(n) = c_n \times 2^{n+1}$, for some $c_n > 1$,
- $R(n) = 2n + 1$.

The reduced state space is *exponentially smaller* than the original one, for both LTL-X and CTL-X.

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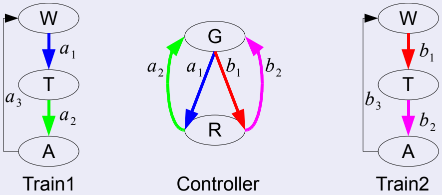

Figure: TC composed of two trains and the controler

Interleaved Interpreted Systems - asynchronous semantics

Assume we have n agents.

Definition

- $Act = A_1 \cup \ldots \cup A_n$ - a set of the actions,
- $\mathcal{Q} = L_1 \times \ldots \times L_n$ - a set of the global locations,
- $t_i : L_i \times A_i \rightarrow L_i$ for $i = 1, \ldots, n$ - an i-local evolution function,
- $Inttrans : \mathcal{Q} \times Act \rightarrow \mathcal{Q}$ - an interleaved evolution function:
  $Inttrans((q_1, \ldots, q_n), \mathrm{act}) = (q'_1, \ldots, q'_n)$ iff
  $t_i(q_i, \mathrm{act}) = q'_i$ if $\mathrm{act} \in A_i$ and $q_i = q'_i$ if $\mathrm{act} \notin A_i$,
- $q \sim_i q'$ iff $q_i = q'_i$ for $i = 1, \ldots, n$ - the indistinguishabilty relations.

sATL* over interleaved models

Restrictions of ATL*

- sATL* (simple ATL*) - ATL* without the next state operator and without nested strategic operators,
- $sATL_{ir}$, $sATL^*_{ir}$, $sATL_{Ir}$, $sATL^*_{Ir}$
- Model checking $sATL_{ir}$ and $sATL^*_{ir}$ is PSPACE-complete in the size of the model representation and the length of a formula.

Theorem

Partial order reductions preserving LTL-X preserve also $sATL^*_{ir}$.

Remark: the theorem does not hold for $sATL^*_{Ir}$.
Partial order reduction methods for LTL-X can be used for $sATL^*_{ir}$.

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What to do ?

Syntax of TATL

Timed Alternating-Time Temporal Logic (TATL)

The language of TATL is defined by the following grammar:

$$\phi ::= \mathsf{p} \mid \neg\phi \mid \phi \vee \phi \mid \langle\langle A\rangle\rangle X\phi \mid \langle\langle A\rangle\rangle \phi U_{\sim\eta}\phi \mid \langle\langle A\rangle\rangle \phi R_{\sim\eta}\phi,$$

where $\mathsf{p} \in \mathcal{AP}$, $A \subseteq Agents$, $\sim \in \{\leq, =, \geq\}$, and $\eta \in \mathbb{N}$.

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TATL, cont'd

$\text{TATL}_{\leq,\geq}$: a subset of TATL with only $\leq, \geq$ allowed,
e.g., $\langle\langle A\rangle\rangle G_{\geq 42}\text{safe} \in \text{TATL}_{\leq,\geq}$, $\langle\langle A\rangle\rangle F_{=13}\text{finish} \notin \text{TATL}_{\leq,\geq}$.

Examples of properties:

- $\langle\langle A\rangle\rangle G_{\geq 42}\text{safe}$: "Coalition *A* **has a strategy to enforce** that safe holds always after reaching 42 time units".
- $\langle\langle A\rangle\rangle F_{=13}\text{finish}$: "Coalition *A* **has a strategy to enforce** that finish is reached in exactly 13 time units".

Counting Strategies: perfect information

Counting strategies ($\Sigma_{\#}$)

Strategies $\sigma_a \in \Sigma_T$ s.t. for each $\pi, \pi' \in \mathcal{S}^+$, if $loc(\pi_F) = loc(\pi'_F)$ and $\#_F(\pi) = \#_F(\pi')$, then $\sigma_a(\pi) = \sigma_a(\pi')$.

(Intuition: action selection depends on the number of visits to the location of π_F)

Alternative notation

A counting strategy is a function $\sigma_a^{\#} : \mathcal{Q} \times \mathbb{N} \to \Sigma$ s.t. $\sigma_a^{\#}(q, k) := \sigma_a(\pi)$ if $q = loc(\pi_F)$ and $k = \#_F(\pi)$.

$\#_F(\pi)$: the number of states of π whose location is $loc(\pi_F)$.

Counting Strategies: perfect information

Threshold strategies ($\Sigma_{\#_n}$)

A counting strategy $\sigma_a^\# \in \Sigma_\#$ is called *n*–**threshold** for some $n \in \mathbb{N}_+$ iff for each location $q \in \mathcal{Q}$ there exist:

- actions $\text{act}_1, \ldots, \text{act}_{n+1} \in \Sigma$, and
- integer intervals $I_1 = [1, i_1), I_2 = [i_1, i_2), \ldots, I_{n+1} = [i_n, \infty)$

s.t. for all $1 \leq j \leq n+1$: $\sigma_a^\#(q, k) = \text{act}_j$ if $k \in I_j$.

Example: a counting strategy is 2–threshold if for any location $q \in \mathcal{Q}$ there are **three** actions $\text{act}_1, \text{act}_2, \text{act}_3$ s.t. first only act_1 is used when q is visited, then only act_2, and finally only act_3, ad infinitum.

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Threshold

Theorem. Threshold for $\mathrm{TATL}_{\leq,\geq}$ is 2

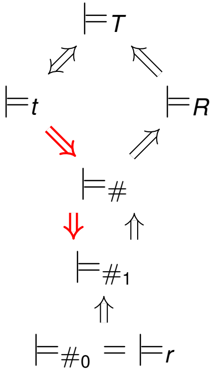
For each $q \in \mathcal{Q}$ and $\phi \in \mathrm{TATL}_{\leq,\geq}$, if $q \models_{I,T} \phi$, then $q \models_{\#_1} \phi$.

This may help to alleviate the explosion of strategies.

Theorem

There is no threshold for TATL.

Hierarchy of satisfaction relations (for I)



The Red implications hold only for $\text{TATL}_{\leq,\geq}$.

Conclusions

Alleviating state/transition/strategy explosions:

- Model checking for ATL^*_{Ir}, ATL^*_{ir}, and $TATL_{\leq,\geq}$ is difficult, but:
- In practical applications one can successfully use:

  Multi-valued model checking over abstract models,
  Partial order reduction methods,
  Counting strategies rather than timed ones.

Lecture based on the papers:

- Partial Order Reductions for Model Checking Temporal-epistemic Logics over Interleaved Multi-agent Systems [A. Lomuscio, W. Penczek, H. Qu: Fundamenta Informaticae, 2010]
- Specification and Verification of Multi-Agent Systems [W. Jamroga, W. Penczek: ESSLLI, 2011]
- Multi-Valued Verification of Strategic Ability [W. Jamroga, B. Konikowska, W. Penczek: AAMAS, 2016]
- Timed ATL: Forget Memory, Just Count [E. Andre, L. Petrucci, W. Jamroga, M. Knapik, W.Penczek, AAMAS, 2017]
- Towards Partial Order Reductions for Fragments of Alternating-Time Temporal Logic [P. Dembiński, W. Jamroga, A. Mazurkiewicz, W. Penczek, ICS PAS Report 1036, 2017]

Thank you!