Space for Traffic Manoeuvres

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The Challenge

Prove safety (collision freedom) of
traffic manoeuvres on different types of roads.

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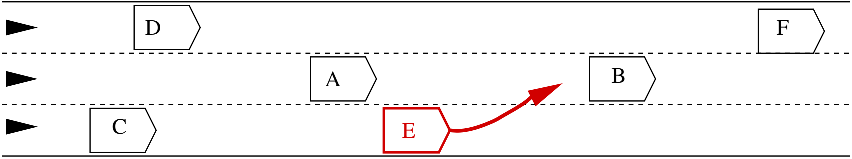
motorways [HLOR11]:



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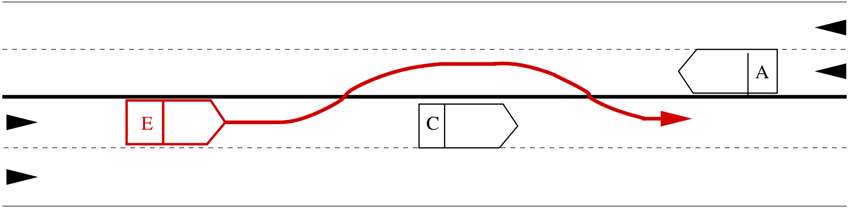
country roads [HLO13]:



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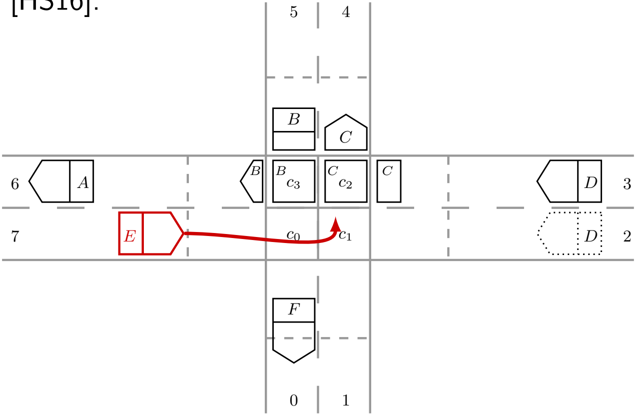
crossings [HS16]:



Our Approach [HLOR11]

Safety is hybrid system verification problem:

$$\text{car dynamics} + \text{car controllers} + \text{assumptions} \models \text{safety}$$

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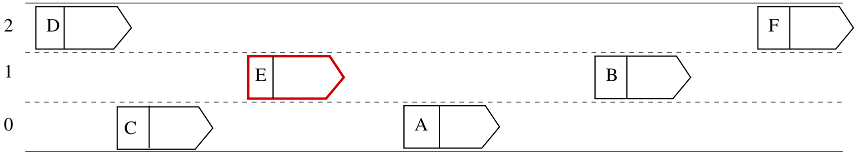
Dedicated Multi-Lane Spatial Logic inspired by work in ProCoS:

- Moszkowski's interval temporal logic
- Zhou, Hoare and Ravn's Duration Calculus

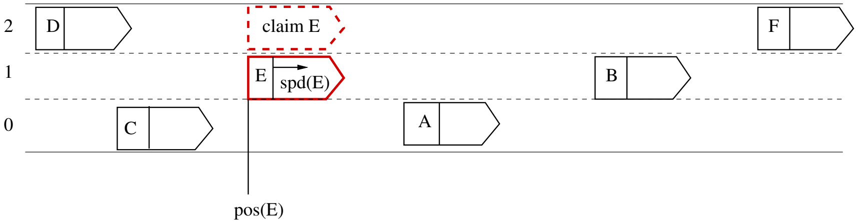
Model



Preliminaries:

- Car identifiers globally unique: $A, B, \ldots$
  Set of all car identifiers: $\mathbb{I}$
- Infinite road ($\mathbb{R}$)
- Lanes: $\mathbb{L} = \{0, \ldots, N\}$

Model



A traffic snapshot is a structure $\mathcal{T} = (pos.spd, res, clm)$, where

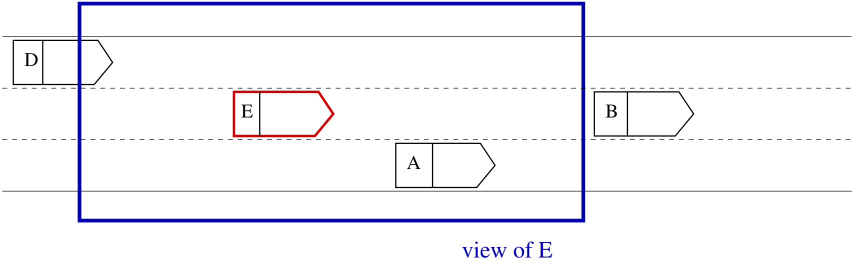
- $pos : \mathbb{I} \to \mathbb{R}$ car positions,
- $spd : \mathbb{I} \to \mathbb{R}$ current speeds,
- $res : \mathbb{I} \to \mathcal{P}(\mathbb{L})$ reserved lanes,
- $clm : \mathbb{I} \to \mathcal{P}(\mathbb{L})$ claimed lanes.

Transitions

$\mathcal{T} \xrightarrow{\alpha} \mathcal{T}'$ for an action α of the following type:

$\mathcal{T} \xrightarrow{t} \mathcal{T}'$	time passes
$\mathcal{T} \xrightarrow{\mathsf{c}(C,n)} \mathcal{T}'$	claim
$\mathcal{T} \xrightarrow{\mathsf{wd\_c}(C)} \mathcal{T}'$	withdraw claim
$\mathcal{T} \xrightarrow{\mathsf{r}(C)} \mathcal{T}'$	reserve
$\mathcal{T} \xrightarrow{\mathsf{wd\_r}(C,n)} \mathcal{T}'$	withdraw reservation

Local View



View $V = (L, X, E)$, where

- L subinterval of $\mathbb{L}$,
- X subinterval of $\mathbb{R}$,
- $E \in \mathbb{I}$ identifier of car under consideration.

MLSL: Syntax

Multi-Lane Spatial Logic (basic form)

Car variables: c, d, special variable ego

Formulae ϕ

$$\phi ::= \mathit{true} \mid c = d \mid \mathit{free} \mid \mathit{re}(c) \mid \mathit{cl}(c) \qquad (\mathit{Atoms})$$

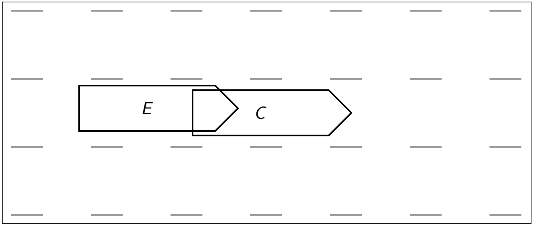
$$\mid \phi_1 \wedge \phi_2 \mid \neg \phi_1 \mid \exists c \colon \phi_1 \qquad (\mathit{FOL})$$

$$\mid \phi_1 \frown \phi_2 \mid \begin{matrix} \phi_2 \\ \phi_1 \end{matrix} \qquad (\mathit{Spatial})$$

MLSL: Semantics

Somewhere: $$\langle \phi \rangle \equiv \mathit{true} \frown \begin{pmatrix} \mathit{true} \\ \phi \\ \mathit{true} \end{pmatrix} \frown \mathit{true}$$

Example: Collision check



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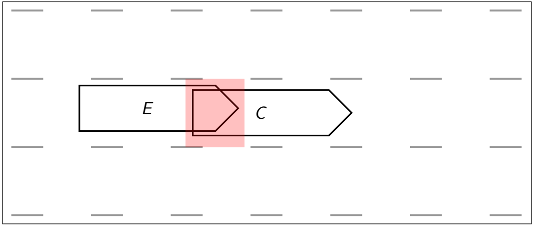

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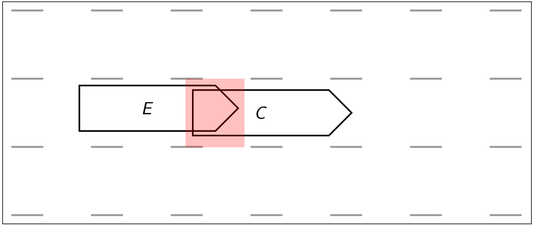

$$\langle re(\text{ego}) \land re(c) \rangle$$

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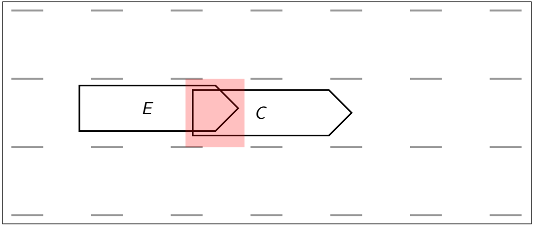

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$$cc \equiv \exists c : c \neq \text{ego} \land \langle re(\text{ego}) \land re(c) \rangle$$

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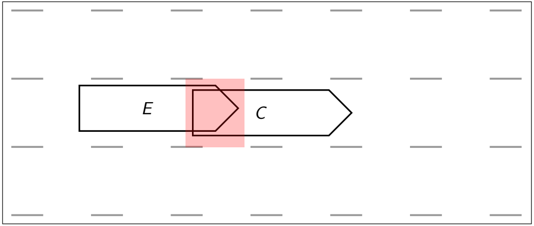


$$cc \equiv \exists c : c \neq \mathrm{ego} \land \langle re(\mathrm{ego}) \land re(c) \rangle$$

Safety from ego's perspective: $\neg cc$

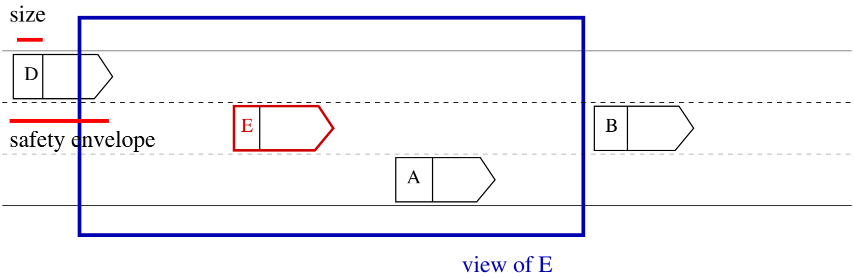
Controller

- Automotive Controlling Timed Automata (ACTA) with data variables:
  - guards and invariants:

    MLSL formulae and clock/data constraints,
  - actions:

    transitions of cars, clock/data updates.

Controller: Sensor Function

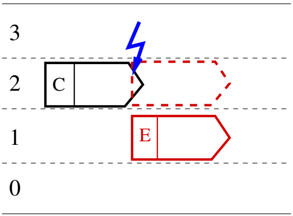


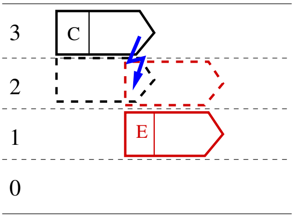
Sensor function describes what a car E can see of other cars.

We assume perfect knowledge: E sees the full safety envelope.

Controller LCP: Lane Change Perfect Knowledge

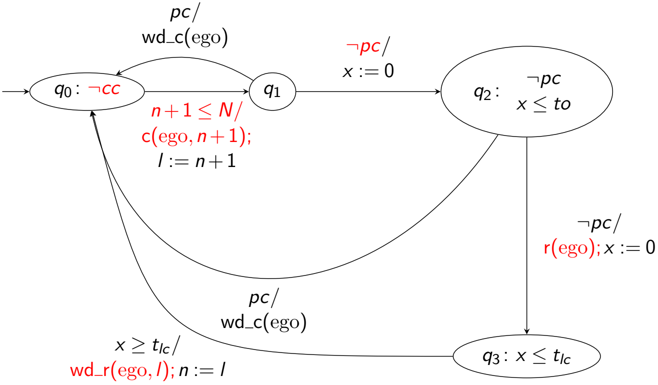
Potential collision: $pc \equiv \exists c : c \neq \mathrm{ego} \land \langle cl(\mathrm{ego}) \land (re(c) \lor cl(c))\rangle$





Controller LCP: Lane Change Perfect Knowledge

- q_0: driving: no collision
- q_1: claiming new lane
- q_2: checking for potential collisions
- q_3: reserving new lane and changing lanes
- q_0: withdrawing reservation of old lane



Safety of LCP

A traffic snapshot safe if it satisfies

$$Safe \equiv \forall c, d : c \neq d \Rightarrow \neg \langle re(c) \land re(d) \rangle .$$

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Assumptions:

A1. There is an initial safe traffic snapshot.

A2. Every car E has a distance controller DC keeping

$$\neg cc \equiv \neg \exists c : c \neq \mathrm{ego} \wedge \langle re(\mathrm{ego}) \wedge re(c) \rangle$$

invariant under time transitions

A3. Every car E is equipped with the controller LCP.

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Theorem

Under the assumptions A1 to A3,
every reachable traffic snapshot is safe.

Linking Spatial and Dynamic Model [ORW17]

- Spatial model using MLSL formulae built up from atoms like

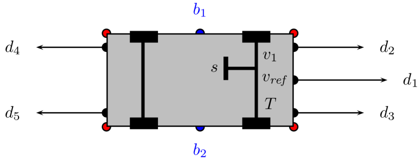
$$free, re(c), cl(c)$$

- Dynamic model built up from

  differential equations for car dynamics

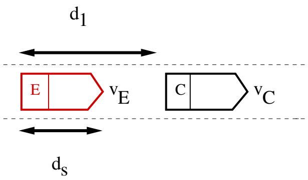
  and

  sensors and actuators of the cars:



Concrete Dynamic Model

Car E follows car C:



Differential equations of the motion of car E:

$$\dot{d}_1(t) = v_C(t) - v_E(t)$$
$$\dot{v}_E(t) = -a(d_1(t), v_C(t))\, v_E(t)^2 + u(t),$$

where $u(t) \in [\underline{u}, \overline{u}]$ and a is an auxiliary function.

Safety distance d_s of car E with initial velocity v_E^0 can be calculated from these equations.

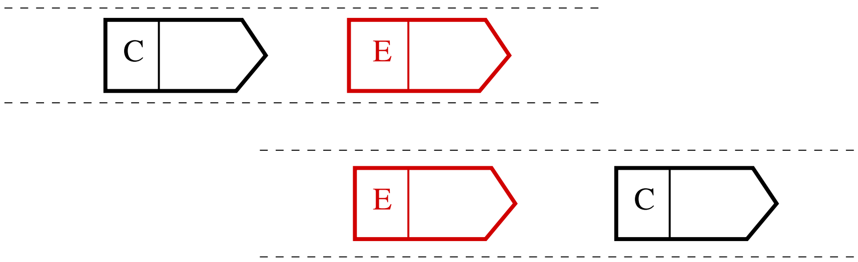
Linking: Distance Controller DC

DC keeps “no collision”

$$\neg cc \;\equiv\; \neg\exists c : c \neq \mathrm{ego} \wedge \langle re(\mathrm{ego}) \wedge re(c)\rangle$$

invariant under time transitions.

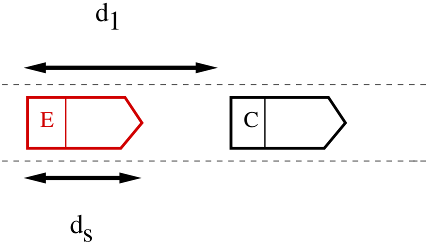
“No collision” is symmetric:



Linking: Distance Controller DC

"No collision forward":

$$\neg ccf \equiv \neg\exists c : c \neq \mathrm{ego} \land \langle re(\mathrm{ego}) \land re(c)\rangle \land \langle c \ ahead \ \mathrm{ego}\rangle$$



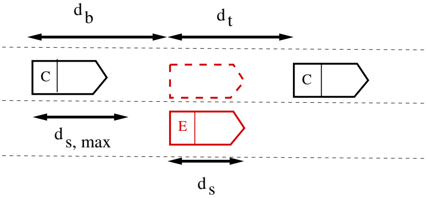
Linking predicate:

$$\neg ccf \Leftarrow d_s < d_1.$$

Linking: Lane-Change Controller LPC

"No potential collision": $\neg\exists c : c \neq \text{ego} \wedge \langle cl(\text{ego}) \wedge (re(c) \vee cl(c))\rangle$

Case 1 : $\phi_{re} \equiv \neg\exists c : c \neq \text{ego} \wedge \langle cl(\text{ego}) \wedge re(c)\rangle$



Linking predicate:

$$\phi_{re} \Leftarrow d_s < d_t \wedge d_{s,max} < d_b.$$

Linking: Lane-Change Controller LPC

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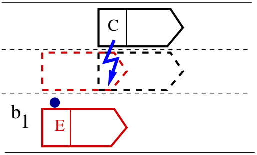
Case 2 : $\phi_{cl} \equiv \neg\exists c : c \neq \mathrm{ego} \wedge \langle cl(\mathrm{ego}) \wedge cl(c)\rangle$



Linking predicate:

$\phi_{cl} \Leftarrow \neg b_t$ holds.

Search for Tool Support

- Satisfiability Problem:

  Given: MLSL formula ϕ

  Question: $\exists M = (\mathcal{T}, V, \nu): \; M \models \phi$?

- Undecidability Result 1 [LH15, Lin15]:

  Halting Problem of two-counter machines

  $\leq$ Satisfiability Problem for MLSL + length ℓ

  Inspired by undecidability proof for the satisfiability problem of the Duration Calculus by Zhou, Hansen and Sestoft.

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- Undecidability Result 2 [Ody15]:

   Empty Intersection Problem for context-free languages
  $\leq$ Satisfiability Problem for MLSL without length

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  abstract view of controllers and checked safety proof

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- Checking MLSL formulas on specific traffic snapshots:
  - translation into QdL [BSc: Bis16]
    (Quantified differential Dynamic Logic) of A. Platzer
  - translation into QLIRA [FHO15]
    (Quantified Linear Integer-Real Aritmetic)

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- Controller verification:
  translation into and use of UPPAAL [OS17]

EMLSL with Modalities

- Sven Linker,
  Proofs for Traffic Safety: Combining Diagrams and Logics.
  PhD thesis, 2015.

- MLSL extended with modalities:

$\Box_{\mathsf{c}(d)}$	
$\Box_{\mathsf{r}(d)}$	after all reservations of d
$\Box_{\mathsf{wd\_c}(d)}$	
$\Box_{\mathsf{wd\_r}(d)}$	
$\Box_{\tau}$	after all time transitions
$\mathbf{G}$	globally, i.e. after all sequences of transitions

Formal Safety Specification

- Safe of a car e :

$$safe(e) \equiv \forall c : c \neq e \wedge \neg \langle re(c) \wedge re(e) \rangle$$

- Global Safety:

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- Distance Controller:

$$\mathrm{DC} \equiv \mathbf{G}\ \forall c, d : c \neq d \rightarrow (\neg \langle re(c) \land re(d) \rangle \rightarrow \Box_\tau \neg \langle re(c) \land re(d) \rangle)$$

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$$pc(c, d) \equiv c \neq d \wedge \langle cl(d) \wedge (re(c) \vee cl(c)) \rangle$$

- Lane Change property:

$$\mathrm{LC} \equiv \mathbf{G}\ \forall d : (\exists c : pc(c, d) \rightarrow \Box_{r(d)} \bot)$$

Formal Safety Proofs

- [Lin15]: using a system of labelled natural deduction for EMLSL:

$$\{ts, v : \mathrm{DC},\ ts, v : \mathrm{LC},\ ts, v : \forall e : safe(e)\}$$
$$\vdash \quad ts, v : \forall e : \mathbf{G}\ safe(e)$$

- [Lin17]: using a formalisation of the semantics of EMLSL

  in Isabelle/HOL

Future Work

- Imperfect knowledge: communication [HLOR11] [BSc: Lam17]
- more on automatisation and tool support

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