Certified Reduced-Order Model Predictive Control for Linear Switched Evolution Equations

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In this talk, we are concerned with the solution of infinite-horizon optimal control problems of the form

$$\min_{u\in L^2((0,\infty),\mathbb{R}^\rho)} \mathcal{J}(u) = \int_0^\infty \tfrac{1}{2}\|y(t)-y_\mathrm{d}(t)\|^2_{\mathbb{R}^p} + \tfrac{\lambda}{2}\|u(t)\|^2_{\mathscr{U}_n} + \mu\|u(t)\|_{L^1(\Omega)}\,\mathrm{d}t, \tag{1}$$

subject to (u, y) solving the following linear switched input-output system

$$\begin{cases} \mathcal{M}_{\sigma(t)}\frac{\mathrm{d}}{\mathrm{d}t}\theta(t) + \mathcal{A}_{\sigma(t)}\theta(t) = \mathcal{B}_{\sigma(t)}u(t) & t \geq 0 \\ y(t) = \mathcal{C}_{\sigma(t)}\theta(t) & t \geq 0 \\ \theta(0) = \theta_\circ. \end{cases} \tag{2}$$

and control constraints $u \in \mathscr{U}_{\mathsf{ad}}$. Here $\sigma : [0,\infty) \to \{1,\ldots,L\}$ is a switching signal, that switches through different system operators $\mathcal{M}_i, \mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i$ for $i = 1,\ldots,L$.
To approximate the solution of (1), we apply Model Predictive Control (MPC): the optimal control problem is solved over smaller, receding time intervals $(t_n, t_n + T)$ for some prediction horizon $T > 0$ and the solutions are concatenated in the sampling interval (t_n, t_{n+1}) for $0 < t_{n+1} < t_n + T$. First, we derive optimality conditions for these small-horizon problems and discuss their suboptimality w.r.t. (1). The difficulty here is that the cost functional $\mathcal{J}$ is not differentiable in the classical sense, due to the presence of the L^1-regularization. Second, the repeated solution of small-horizon optimal control problems motivates model reduction: we consider (Petrov-)Galerkin reduced-order models for (2) to speed up the MPC process. To quantify the error, we do a full a posteriori error analysis for the optimal control, optimal state, and optimal value function of the small-horizon problems, which allows us to control the evolving error through the MPC iterations. These estimates are then used to construct two certified ROM-MPC algorithms for the solution of (1), that are up to 10 times faster than the MPC relying on the full-order model. This is joint work with Stefan Volkwein (U. Konstanz), Mattia Manucci, and Benjamin Unger (U. Stuttgart).