Bubbble convergence, Energy identity, Geodesics and Index estimates for sequences of α-harmonic maps
joint work with T. Lamm

We consider sequences of α-harmonic maps from a closed two-dimensional Riemann surface (M, g) into a closed Riemannian manifold (N, h). They are critical points of the α-energy

$$E_\alpha(u) = \int_\Sigma \frac{1}{2\alpha}\Big((1 + e^{-2\lambda}|\nabla u|^2)^\alpha - 1\Big)e^{2\lambda}\, dv_g.$$

We are interested in the behaviour of the spectrum along a sequence of critical points $\alpha \downarrow 1$ with uniformly bounded energies.

The main difficulty arises from the fact that the sequence can develop finitely many bubbles in the limit. To understand the convergence, a detailed understanding of the neck regions is relevant.

In my talk I would like to emphasise the novelty of our approach:

First, a geometric approximation by geodesics together with a new version of a Hopf differential simplifies the convergence of a sequence on a long cylinder. This leads to the study of what we call the essential length of a α-harmonic map.

Second, an estimate of stable regions for the Schrödinger operator of a α-harmonic map helps us to obtain sufficient control to study the convergence of eigenfunctions.