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„The classifying space of the one-dimensional bordism category and a cobordism model for TC."

Abstract:
The study of bordism categories and their classifying spaces has proven extremely useful, for example in the study of moduli spaces of manifolds. Usually, this approach uses a topologically enriched bordism category Bordd (aka the ∞-category of bordisms), but in this talk I will be looking at the much simpler homotopy category h(Bordd) where diffeomorphic bordisms are identified. I will begin by recalling both notions and how they differ.

Using a new fibre sequence for bordism categories I will compute the classifying space of h(Bord1) in terms of the spectrum MTSO2. This spectrum, also known as CP∞-1, is related to both Topological Cyclic Homology (TC) and the moduli space of complex curves. Both relations are reflected within the 1-dimensional bordism category. To see this, I will construct a `reduced' variant Bord1red(X) of Bord1 whose classifying space is essentially the TC of the suspension spectrum of the loop space of X. If time permits, I will also sketch how h(Bord1) is related to Connes' cyclic category Λ and use this to describe cocycles on h(Bord1) representing the Miller-Morita-Mumford classes κ_i. (All of this is based on the recent paper arXiv:2004.14902)originally proved by Madsen and Weiss.