**Isoperimetry, Littlewood functions, and unitarisability of groups**

Let $\Gamma$ be a discrete group. A group $\Gamma$ is called *unitarisable* if for any Hilbert space $H$ and any uniformly bounded representation $\pi : \Gamma \to B(H)$ of $\Gamma$ on $H$ there exists an operator $S : H \to H$ such that $S^{-1}\pi(g)S$ is a unitary representation for any $g \in \Gamma$. It is well known that amenable groups are unitarisable. It has been open ever since whether amenability characterises unitarisability of groups.

**Dixmier**: Are all unitarisable groups amenable?

One of the approaches to study unitarisability is related to the space of Littlewood functions $T_1(\Gamma)$. We define the **Littlewood exponent** $\mathrm{Lit}(\Gamma)$ of a group $\Gamma$:

$$\mathrm{Lit}(\Gamma) = \inf \left\{ \mathrm{p} : \mathrm{T}_1(\Gamma) \subseteq \ell^{\mathrm{p}}(\Gamma) \right\}.$$

We will show that, on the one hand, $\mathrm{Lit}(\Gamma)$ is related to unitarisability and amenability and, on the other hand, it is related to some geometry of $\Gamma$.
We believe that the most natural way to study groups with $\mathrm{Lit}(\Gamma) \leq p$ is to consider not only the actions on Hilbert spaces, but also the actions on a wider classes of spaces, for example, on $p$-spaces. We will define $p$-isometrisability of groups and discuss some results and open questions.