Oberseminar Mathematische Stochastik

Mittwoch, 21. Oktober 2015, 17:00 Uhr, M 6

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Spatial random permutations

Abstract:
Spatial random permutations are an interesting, but largely unexplored model of statistical mechanics. The basic idea is to consider a probability measure on the set of permutations mapping a locally finite subset X of R^d to itself (X could be a regular lattice, or even just Z^d), and to give higher probability to permutations where points from X are mapped to nearby points. In finite volume, this can for example be done via an energy function that is proportional to the sum of all distances between points in X and their images under the permutation.

The most interesting (and largely unsolved) questions are about long cycles in the infinite volume limit. A typical (unsolved) question is: fix a point x in X, does the expected length of the cycle containing x diverge when the system size becomes infinite? In some sense, this is a very peculiar sort of percolation problem.

In the first part of the talk we will introduce some variants of the model and present the few things that are known about them. In the second part, we will show numerical simulations in two dimensions that lead to conjectures about the fractal dimension of the trace of long cycles, a Kosterlitz-Thouless phase transition, and possible connections to Schramm-Löwner curves.