Kinetic models with non-strict conservations – a probabilistic picture of the long-time behavior

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Abstract:
Homogeneous kinetic equations model the redistribution of energy and momentum in an ensemble of particles which interact in binary collisions. In the original Boltzmann equation, these collisions are fully elastic, i.e., they conserve the total energy and momentum of the two interacting particles precisely. It is then a consequence of Boltzmann's celebrated H-theorem that the ensemble's velocity distribution eventually becomes Gaussian. In this talk, we discuss a class of generalized models, in which the energy is conserved on the macroscopic (ensemble-averaged) level, but not on the microscopic (particle) level. We show that under certain conditions, solutions still equilibrate as time goes to infinity. But the variety of stationary velocity distributions becomes much richer: now steady states are stochastic mixtures of Gaussians and may possess fat energy tails. The key element for the proof of convergence to equilibrium is a suitable representation of transient solutions as the law of a sum of iid random vectors with random weights. Indeed, solutions to the considered models constitute a ttime-continuousïteration of a smoothing transformation. Convergence is then obtained by application of the central limit theorem.
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