



Chapter 0

Organization and Introduction

Variational Image Processing
Summer School on Inverse Problems 2015

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Inverse problems and imaging?

Ill-Posed Problems



forward
problem



inverse
problem



ill-posed





Inverse Problem

$$f = Au$$

Measure data f , linear operator A , desired solution u



Inverse Problem

$$f = Au$$

Measure data f , linear operator A , desired solution u

Well-posedness

- A solution exists
- The solution is unique
- The solution depends continuously on the data

The majority of practically relevant problems is ill-posed!

Why are we interested in variational methods?



Data from: *Microsoft Research GeoLife GPS Trajectories*

Time	'12:44:12'	'12:44:13'	'12:44:15'
Latitude	39.974408918	39.974397078	39.973982524
Longitude	116.30352210	116.30352693	116.30362184

How fast did this person go?



Why are we interested in variational methods?

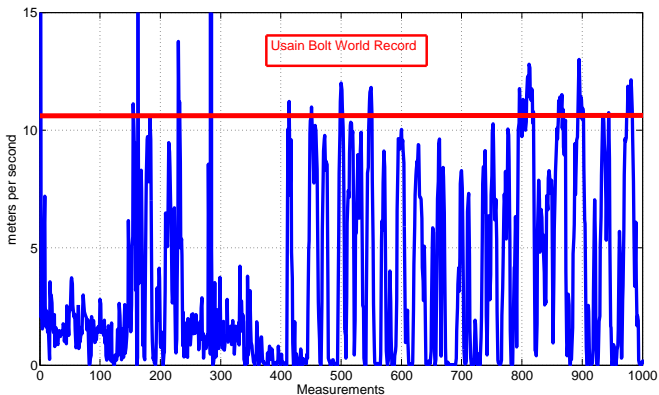
$$v(t_i) = \frac{x(t_i) - x(t_{i-1})}{t_i - t_{i-1}} \approx \partial_t x(t_i)$$



Why are we interested in variational methods?



$$v(t_i) = \frac{x(t_i) - x(t_{i-1})}{t_i - t_{i-1}} \approx \partial_t x(t_i)$$



New world record? Top speed of 161.78 km/h?

Why are we interested in variational methods?



Great! Safari!



Motivation

Organization

Why are we interested in variational methods?



Bad! Nervous focal setting! $f = A * u$



Why are we interested in variational methods?



Blurry image $f = A * u \Rightarrow \mathcal{F}(f) = \mathcal{F}(A) \cdot \mathcal{F}(u)$



Why are we interested in variational methods?



Reconstructed image $u = \mathcal{F}^{-1}(\mathcal{F}(f)/\mathcal{F}(A))$?



Why are we interested in variational methods?



Blurry image $f = A * u \Rightarrow \mathcal{F}(f) = \mathcal{F}(A) \cdot \mathcal{F}(u)$



Why are we interested in variational methods?



Blurry noisy image $f = A * u + n, \Rightarrow \mathcal{F}(f) \approx \mathcal{F}(A) \cdot \mathcal{F}(u)$



Why are we interested in variational methods?



Reconstruction by $\mathcal{F}^{-1}(\mathcal{F}(f)/\mathcal{F}(A))$



① **Theory** - Why do we need variational methods?

② **Applications** - What can we use them for?

③ **Optimization** - How can you implement them yourself?