Statistical Filtering of Indirect Illumination for Computer Graphics

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PIXAR
ANIMATION STUDIOS

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RIPS 2007

Outline

- Introduction
  - Background
  - PCA
- Motion detection
  - Global
  - Local
- Denoising
  - Error Threshold Method
  - Moving Basis
  - Anisotropic Diffusion
  - Hybrid Method
  - Regression
- Artifact detection

Background

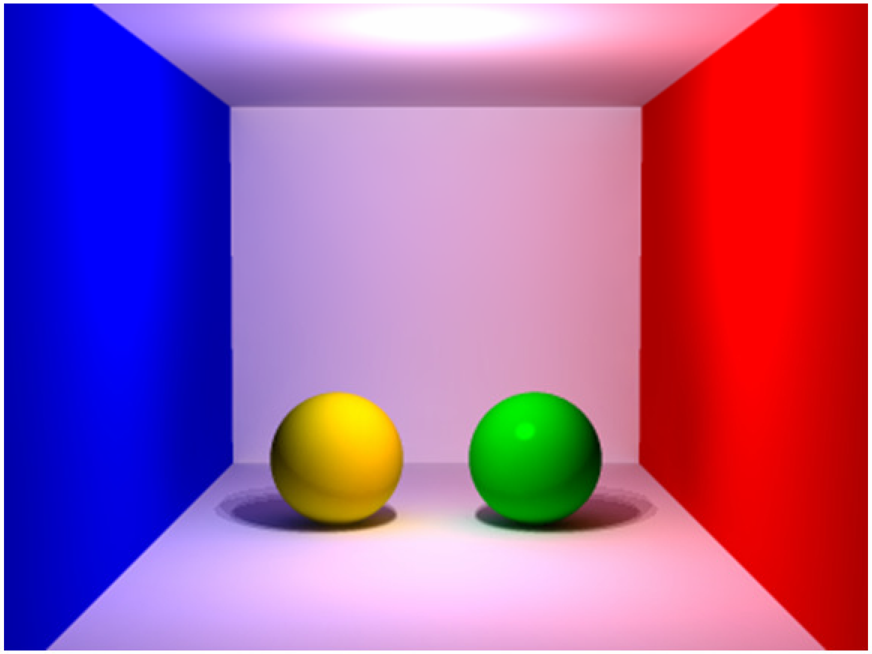
Direct Illumination (D)

Global Illumination (G)

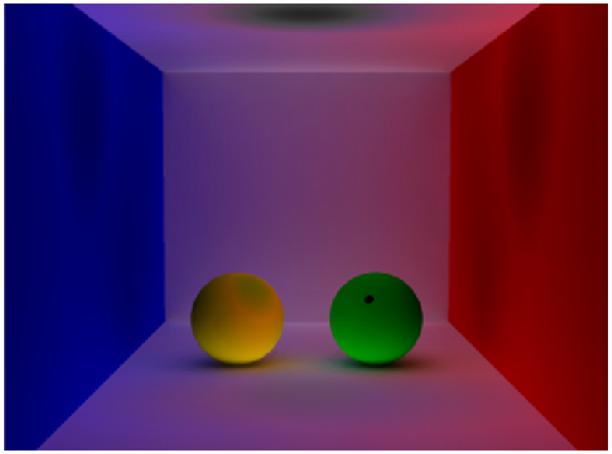
Indirect Illumination

Global Illumination = Direct + Indirect

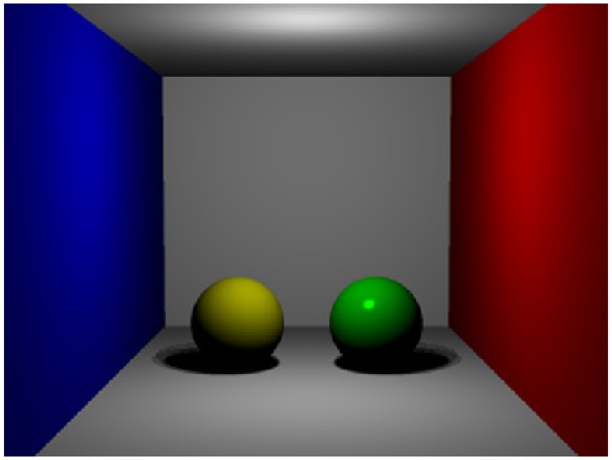
=> **G = D+I**



Global Illumination (G)

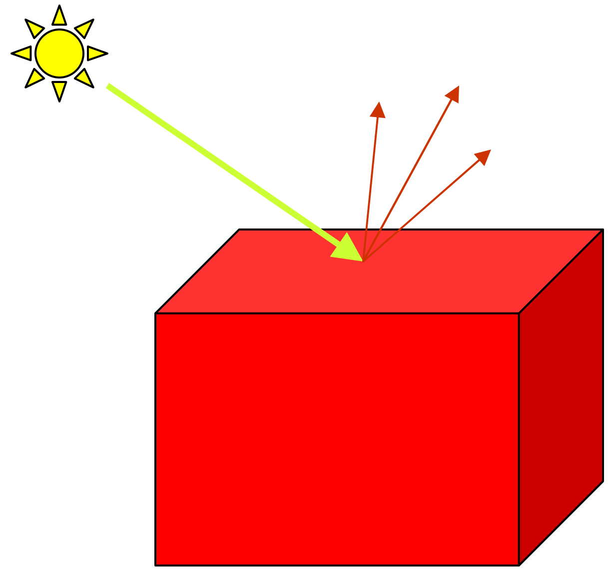


Indirect Illumination (I)

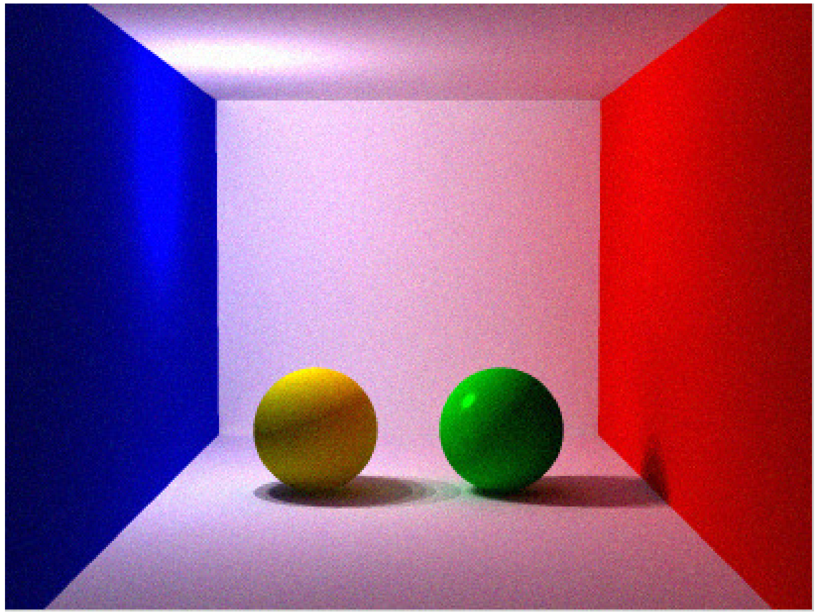


Direct Illumination (D)

Photon Mapping

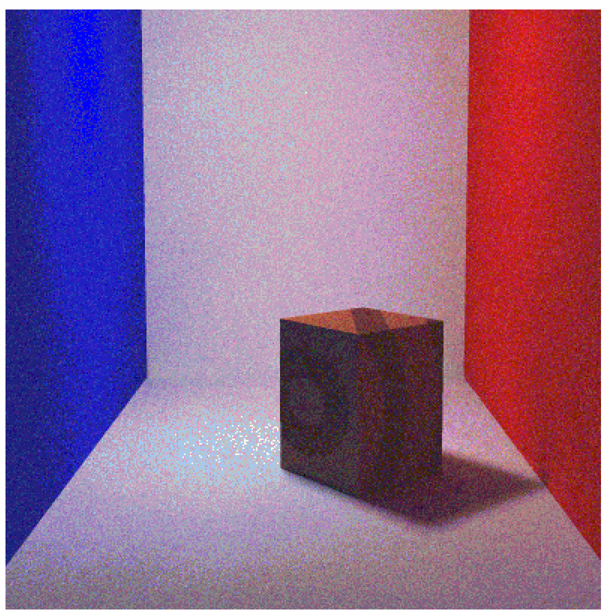


- Used for indirect illumination
- Method that Pixar wishes to use
- Algorithm is computationally expensive
- Lower sampling rates generates noisy images

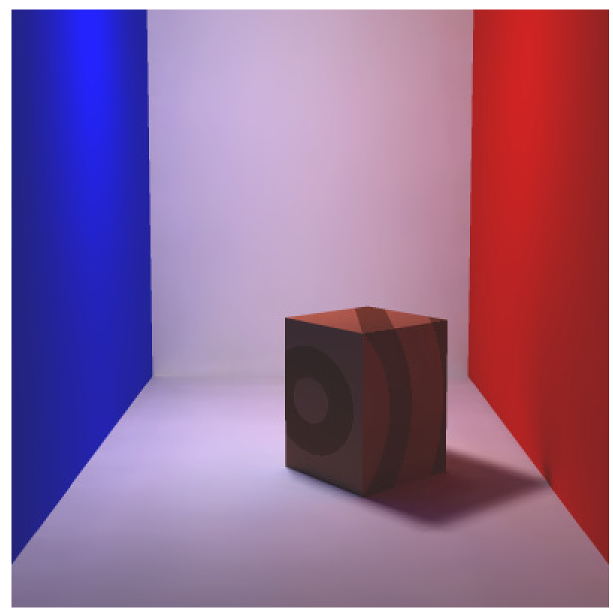


Our Problem

- Remove noise generated by low sample photon mapping
- Detect and replace problematic pixels
- Re-render as few pixels as possible

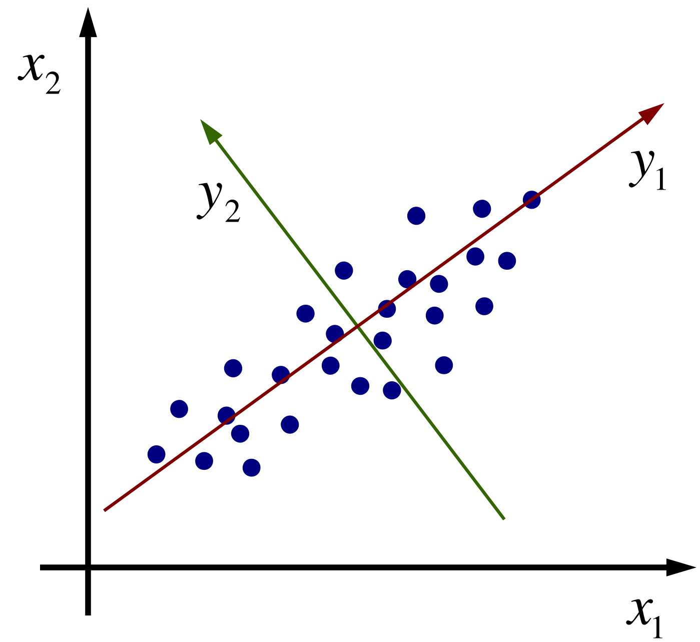


=>



PCA: Basis

- PCA creates a variance-ordered basis
- Basis vectors point in direction of highest successive variance
- Noise-free pixels and noise are represented by different directions in the basis



Transformation of Basis using PCA

PCA: Choosing the Basis

- Noisy animations are represented using PCA:

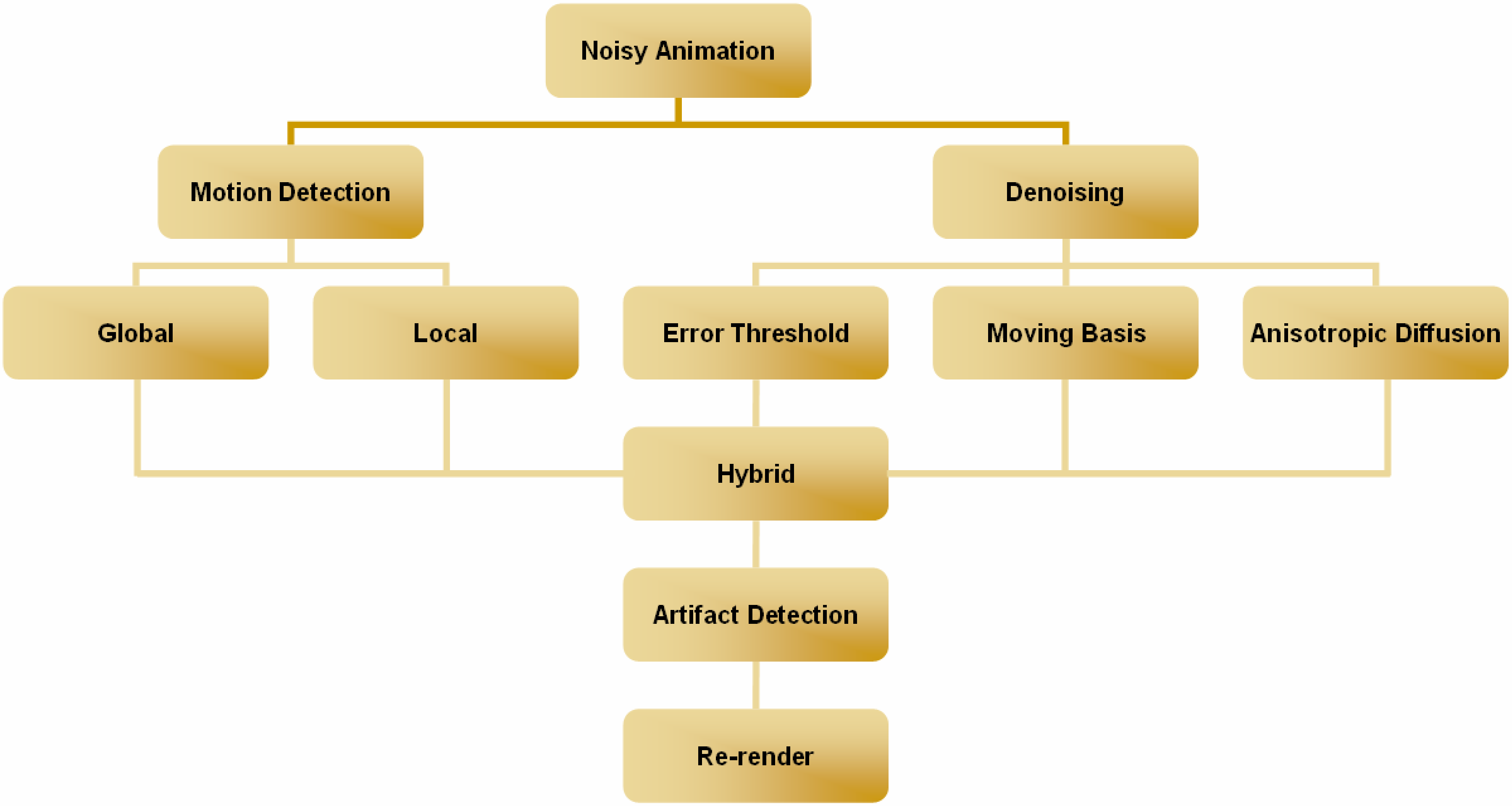
$$I(\vec{x},t) = \sum_{i=1}^{N} w_i(t) B_i(\vec{x})$$

$I(\vec{x},t)$ = Image sequence
N = Number of frames
$B_i(x)$ = Basis Vectors
$w_i(t)$ = Observation Coefficients

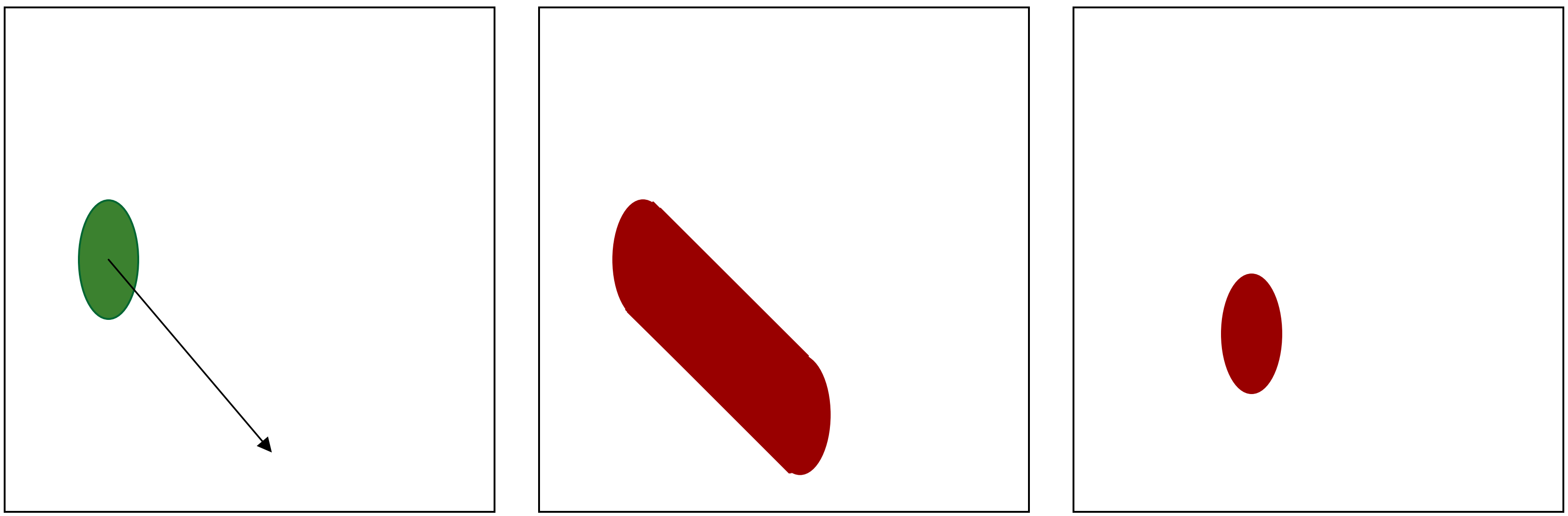
- The first vectors of the basis capture the noise-free indirect illumination
- The last vectors of the basis describe the noise
- Noise-free $I(x,t)$ is determined with a truncated PCA basis:

$$I(\vec{x},t) \approx I_k(\vec{x},t) = \sum_{i=1}^{k} w_i(t) B_i(\vec{x})$$

Grand Scheme



Global vs. Local Motion

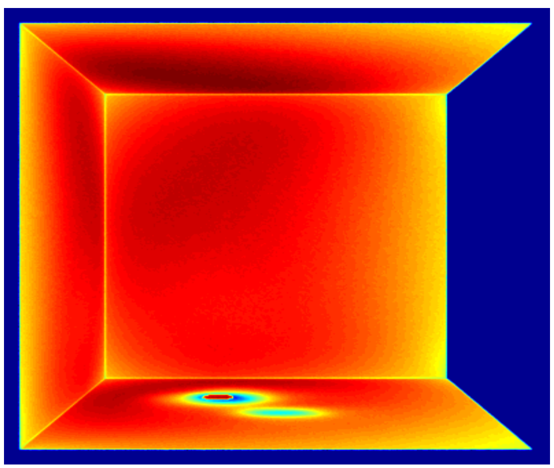


Original Animation | Global motion | Local motion

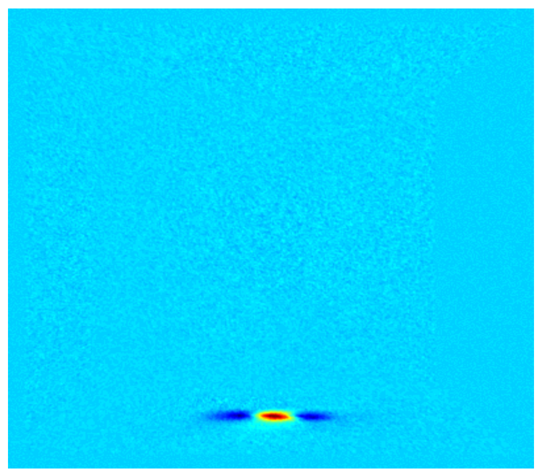
Global Motion Detection

$$I(\vec{x},t)=\sum_{i=1}^{F} w_i(t)\,B_i(\vec{x})$$

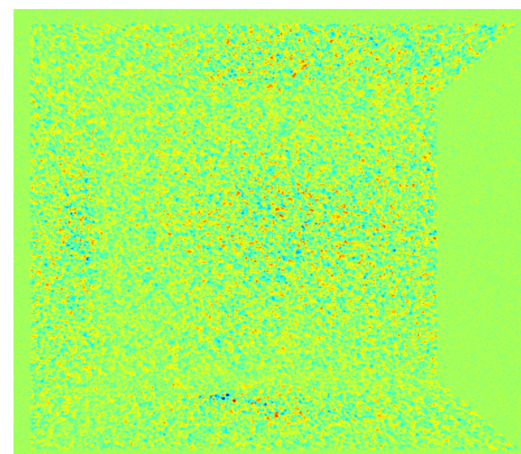
- PCA separates the animation sequence $I(\vec{x},t)$ into spatial and temporal bases
- The spatial basis vectors can be used for detecting global motion



B_1



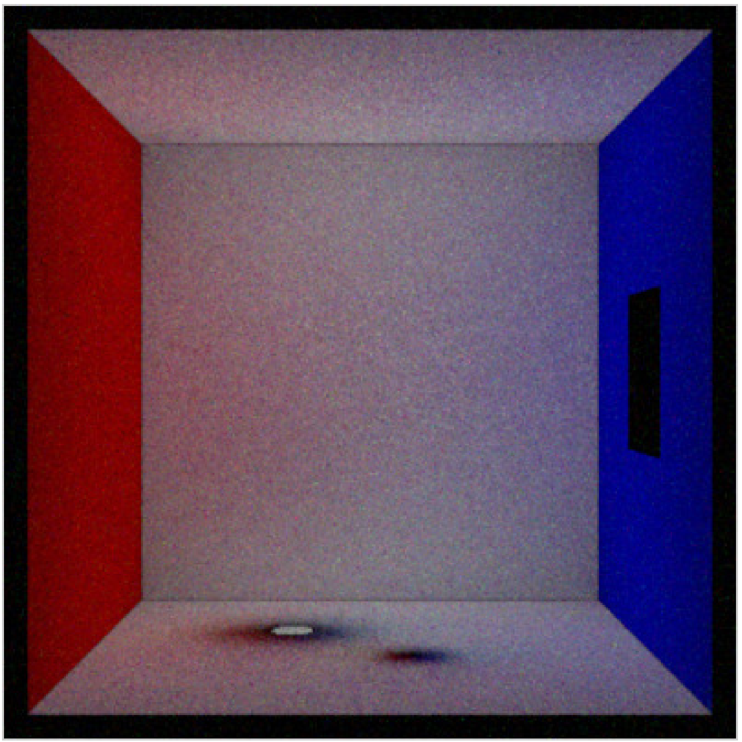
B_3

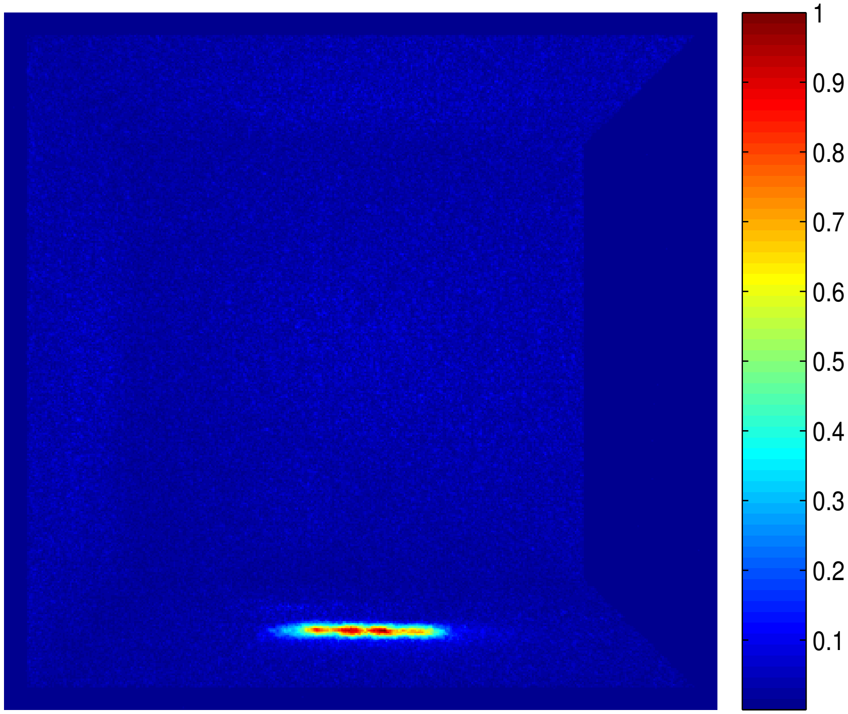


B_9

Global Motion Detection

- The first basis vectors contain information of area of motion
- The last basis vectors contain only noise
- Adding up the absolute values of B_1 to B_5 results in

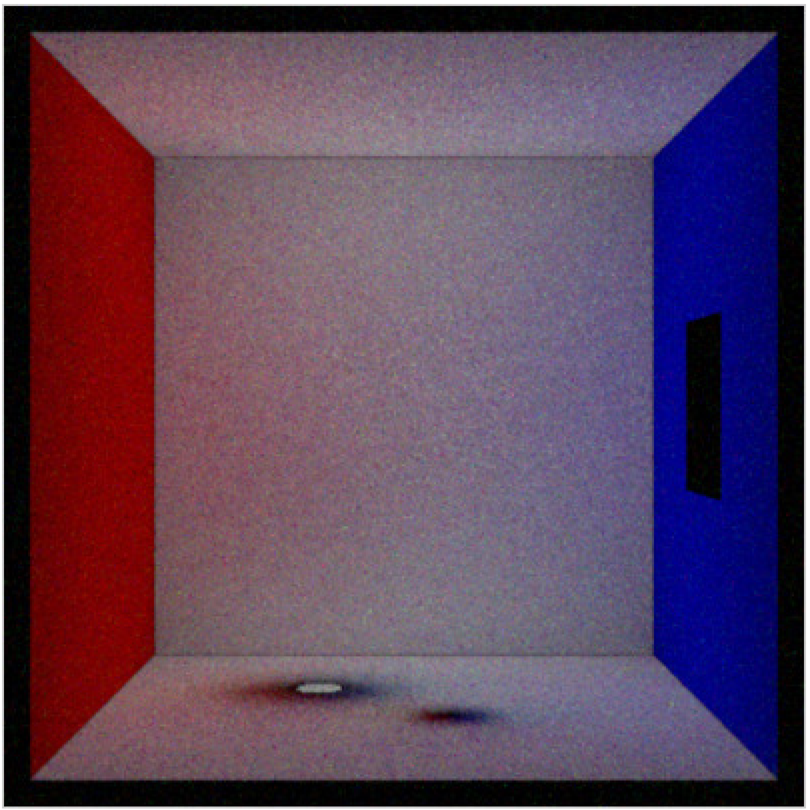




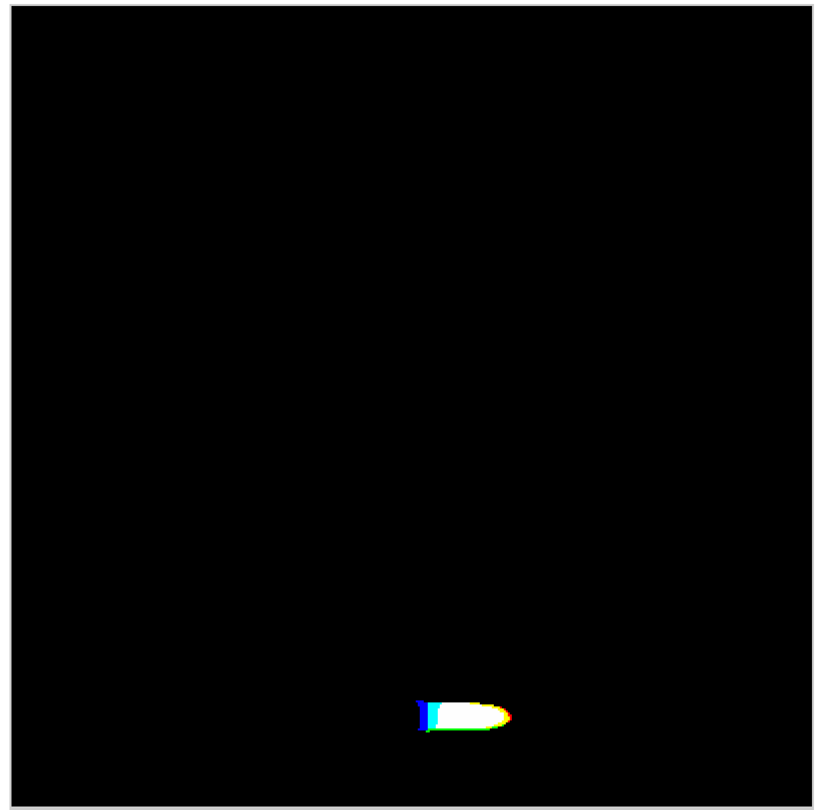
$$\alpha(\vec{x}) = \sum_{i=1}^{5} \left| B_i(\vec{x}) \right|$$

Local Motion Detection using z-scores

$$z(\vec{x},t) = \frac{I(\vec{x},t) - \mu(\vec{x})}{\sigma(\vec{x})}$$



Original animation



Threshold z-scores

Denoising using PCA

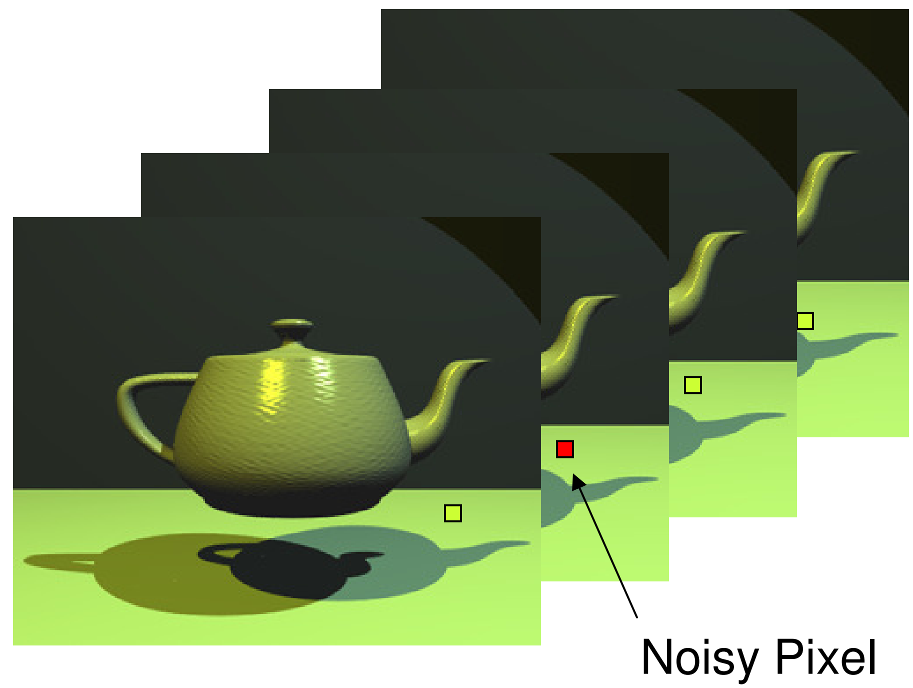
- It represents data with a variance ordered basis

$$I(\vec{x}, t) = \sum_{i=1}^{F} w_i(t) B_i(\vec{x})$$

- First basis vectors contain the noise-free indirect illumination
- Last basis vectors contain noise and motion
- Noise in animation sequence can be filtered using truncated temporal PCA basis

PCA: Motivation

- Most methods denoise individual frames
- Our approach uses temporal correlation of pixel values
- PCA finds a new basis which separates meaningful pixels from noisy ones
- PCA is fast and inexpensive



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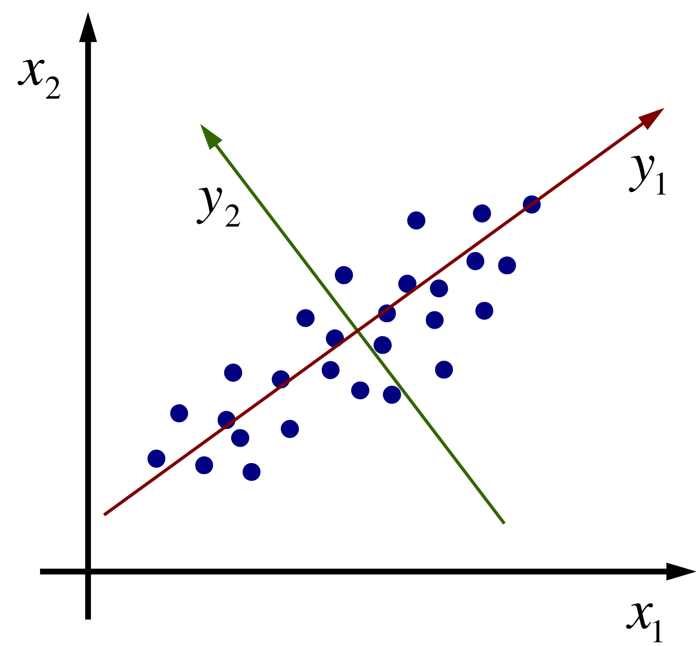

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$$I(\vec{x},t)\approx I_k(\vec{x},t)=\sum_{i=1}^{k} w_i(t)\,B_i(\vec{x})$$

Image Sequence Reconstruction

Noisy animation sequence

Reconstruction with $k = 2$

Reconstruction with $k = 10$

Error Threshold Method*

- Compute the PCA basis
- Create reconstructions for each truncation T_k

$$I_k(\vec{x},t)=\sum_{i=1}^{k} w_i(t)B_i(\vec{x})$$

- Calculate the error (between the reconstruction and the noisy image)

$$error(k,\vec{x})=[I(\vec{x},t)-I_k(\vec{x},t)]^2$$

- Calculate the difference of the error

$$\Delta(error(k,\vec{x}))=error(k,\vec{x})-error(k+1,\vec{x})$$

* MARK MEYER, JOHN ANDERSON, 2007, Statistical Acceleration for Animated Global Illumination

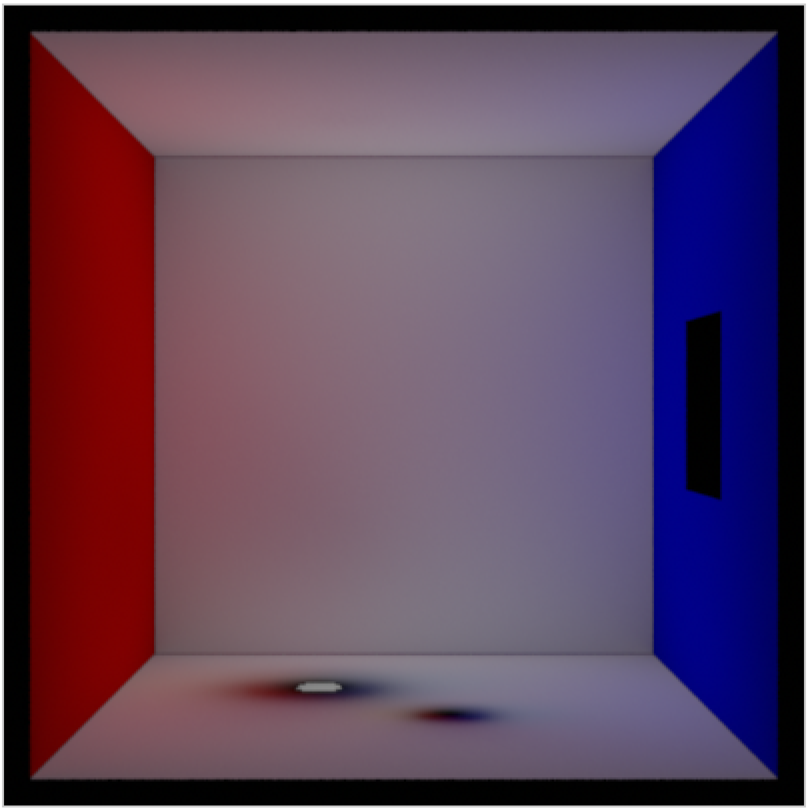
Error Threshold Method

Noisy Animation

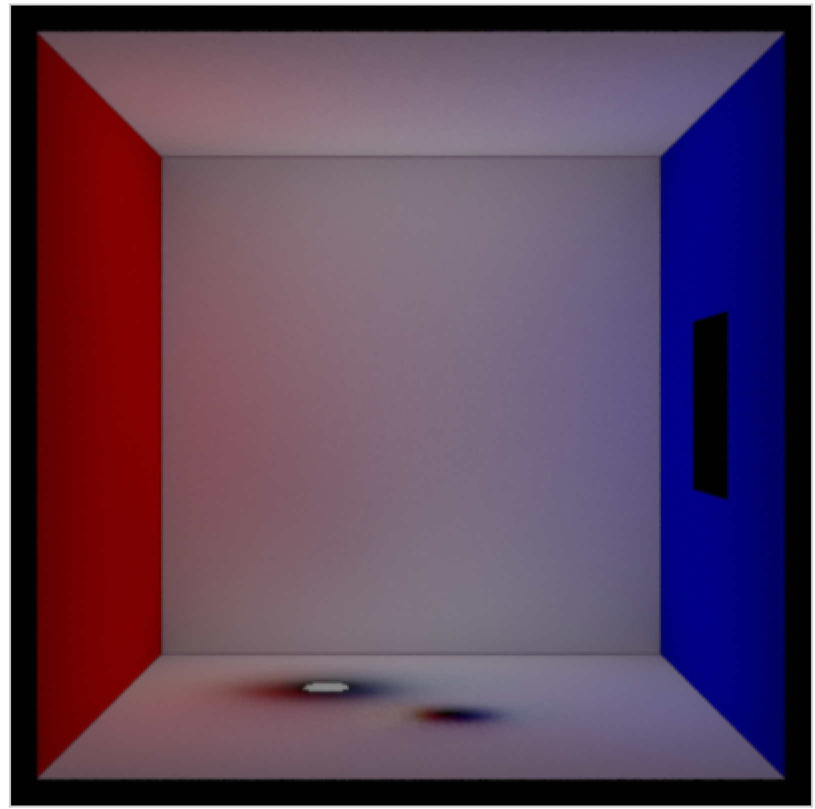
Error Threshold Method

$$I_{PCA}(\vec{x}, t)$$

Error Threshold Method



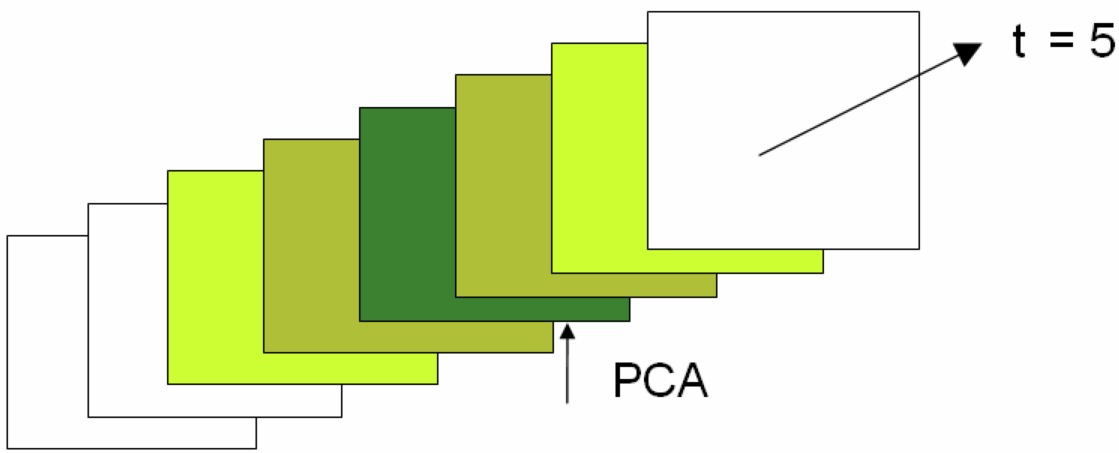
Noise-free Animation
Noisy Animation



Error Threshold Method

Moving Basis

- Pick a single frame
- Select former and following *P* frames and apply Gaussian weights
- Compute PCA for selected frames
- Pick out the middle frame of the reconstruction
- Move on to next frame and repeat procedure



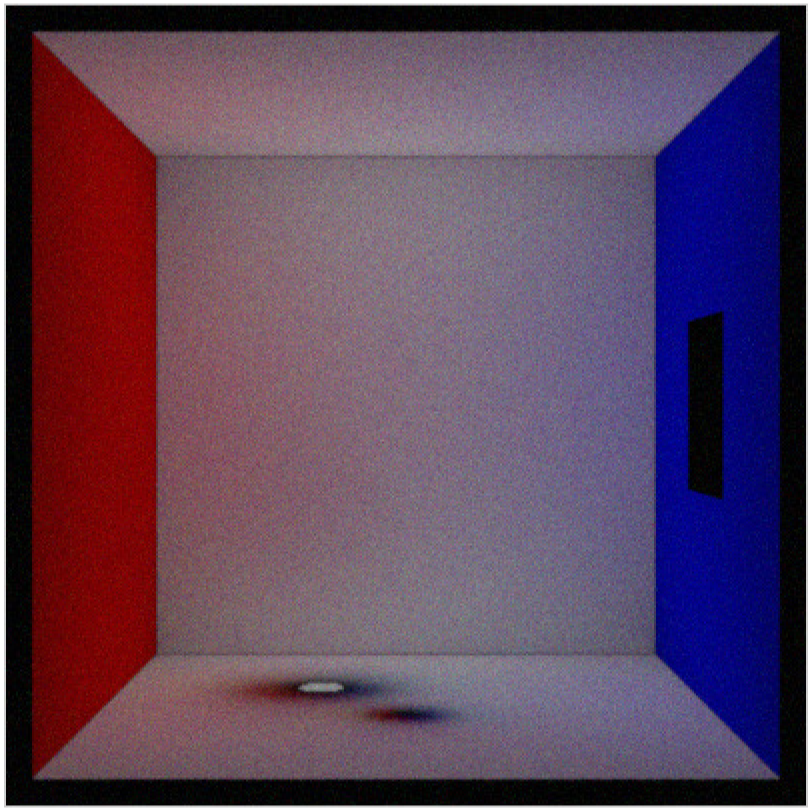
Moving Basis Reconstruction

Noisy animation sequence

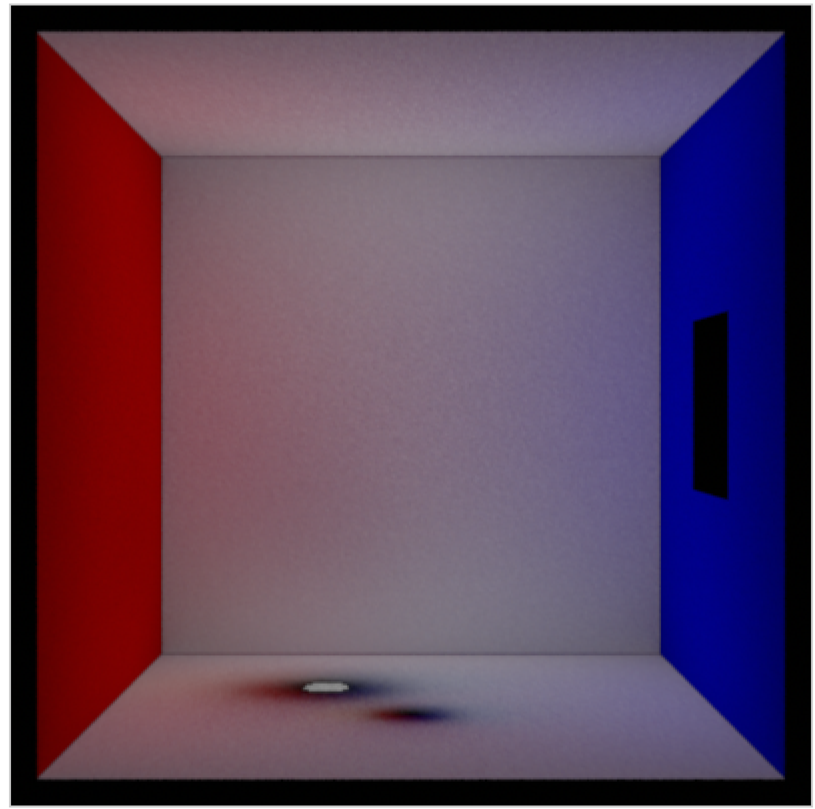
Moving Basis

$$I_{MB}(\vec{x},t)$$

Moving Basis



Noisy animation sequence



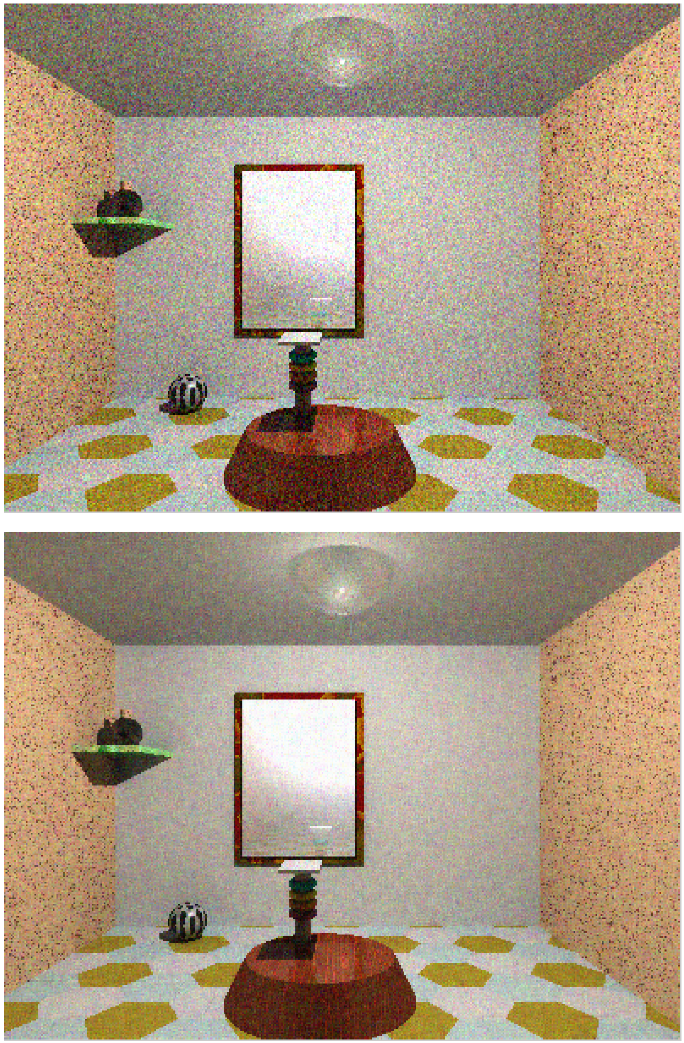
Reconstruction using Moving Basis

Single Frame Denoising*

- Each frame of the image sequence is denoised using anisotropic diffusion:

$$u_t = \nabla \bullet \left(\frac{1}{\sqrt{1+|\nabla u|^2}} \nabla u \right)$$

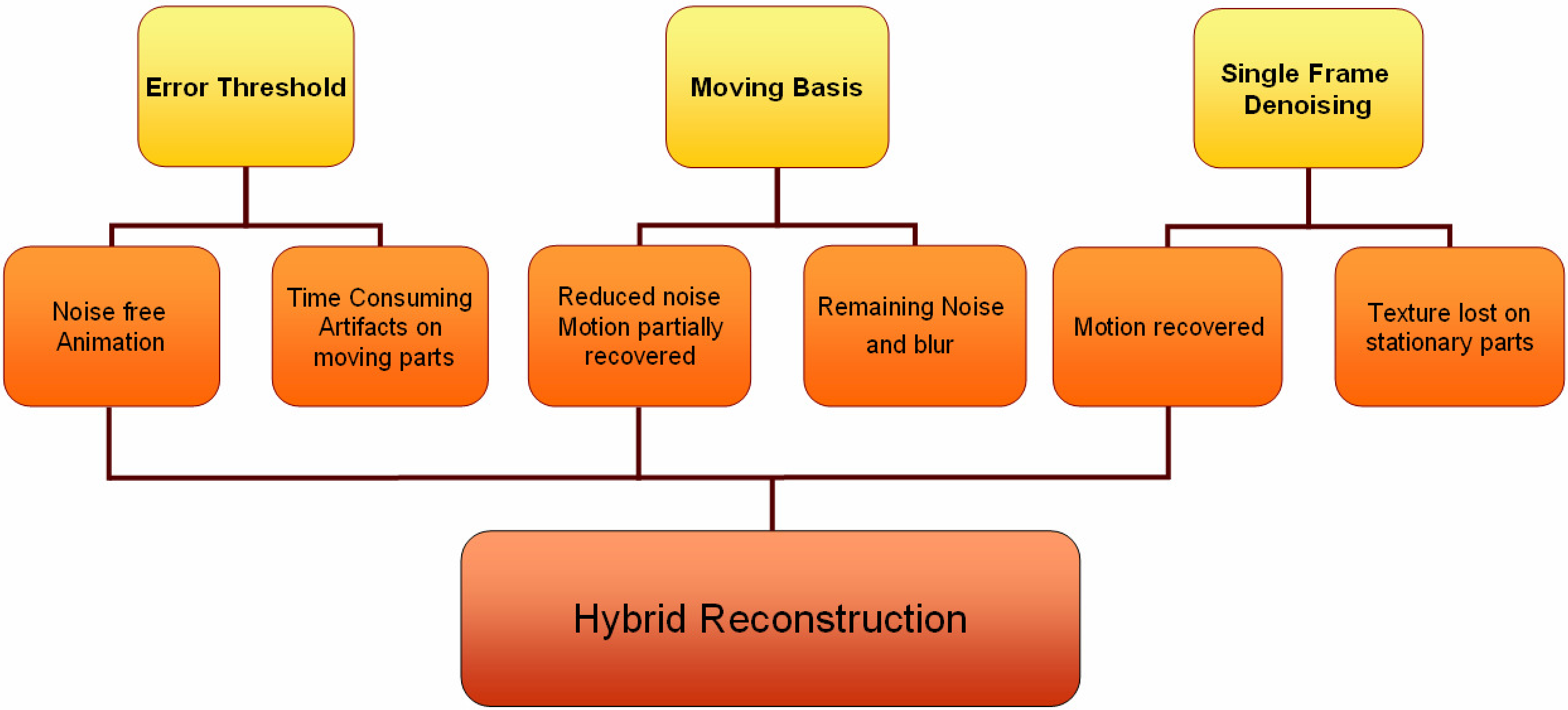
- Motion is preserved
- Little noise is removed



$I_{SF}(\vec{x},t)$

* P. Perona, J. Malik, "Scale-Space and Edge Detection Using Anisotropic Diffusion," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 12, no. 7, pp. 629-639, Jul., 1990

Denoising Methods



Hybrid Method

- Combination of:
  - Denoising methods
  - Motion detection
- Formulation

Global Motion:

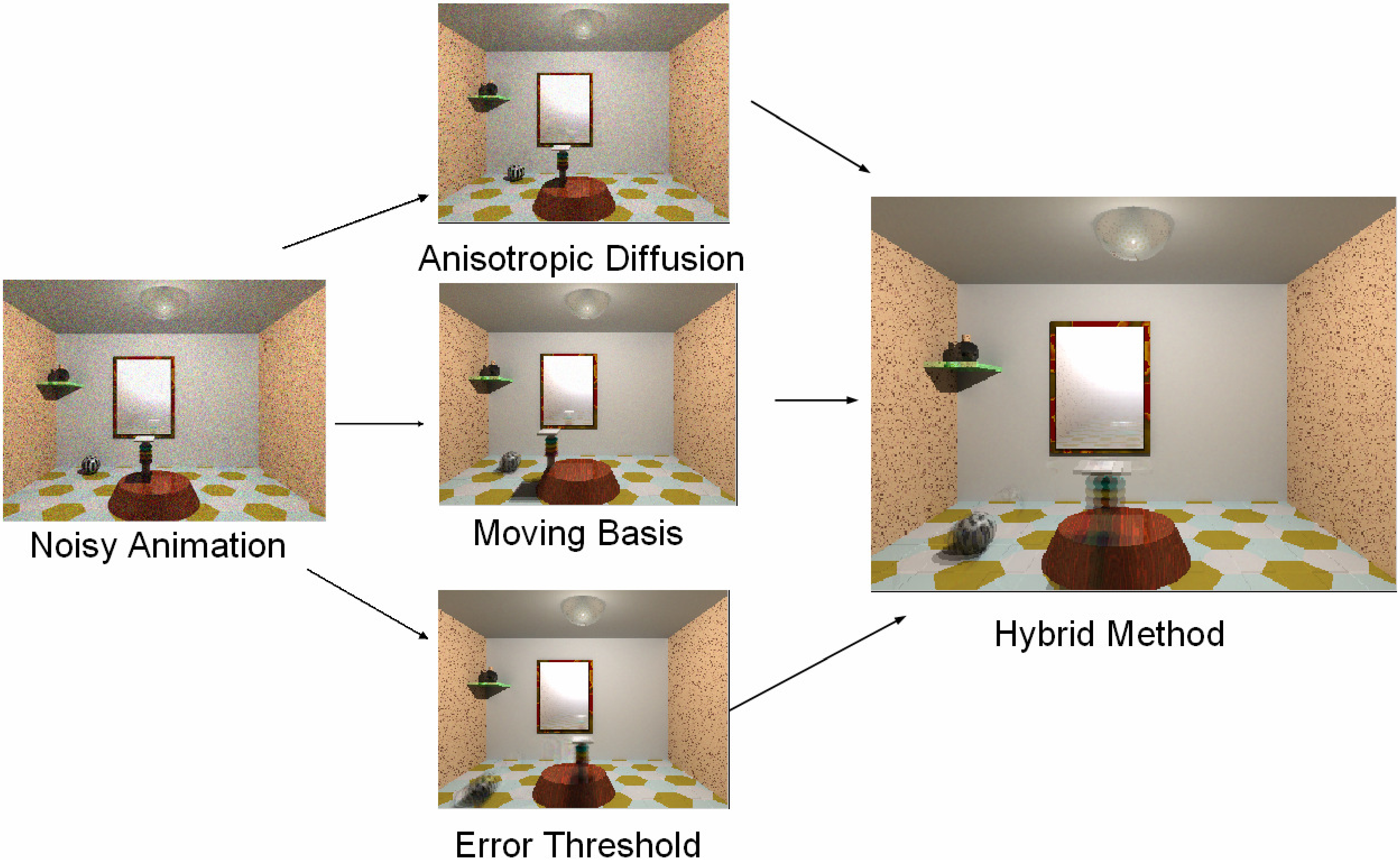
$$\alpha(\vec{x}) = \begin{cases} 0, \text{ motion} \\ 0 < \sigma < 1, \text{motion at some point} \\ 1, \text{ stationary} \end{cases}$$

Local Motion:

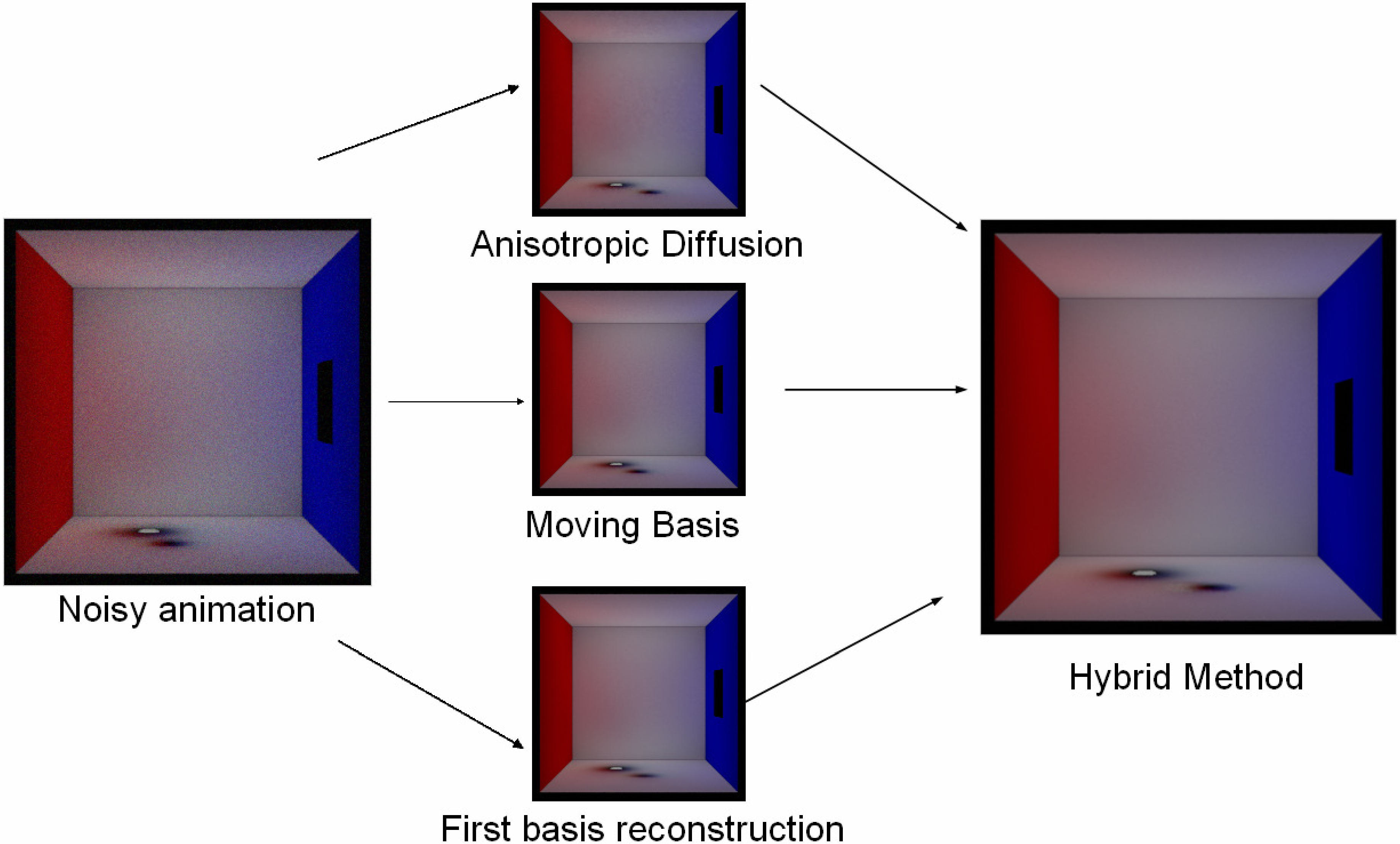
$$\beta(\vec{x},t) = \begin{cases} 0, \text{ currently no motion} \\ 0 < \sigma < 1, \text{ motion at some time} \\ 1, \text{ motion} \end{cases}$$

$$\begin{aligned} I_{Hybrid}(\vec{x},t) = I_{PCA}(\vec{x},t)\alpha(\vec{x}) \\ + I_{MB}(\vec{x},t)(1-\alpha(\vec{x}))\beta(\vec{x},t) \\ + I_{SF}(\vec{x},t)(1-\alpha(\vec{x}))(1-\beta(\vec{x},t)) \end{aligned}$$

Hybrid Method



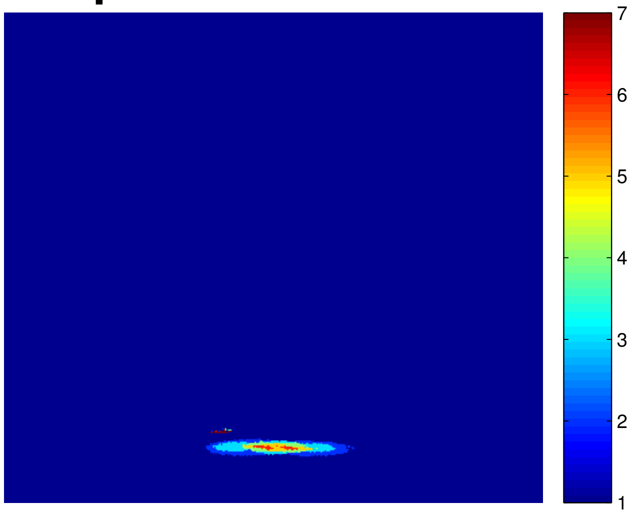
Hybrid Method



Artifact Detection using Regression

- Artifact – any pixel that looks bad ☹
- Regression methods used to detect pixels that have to be re-rendered
- Polynomial regression on all pixels over time
- Degree is determined by a threshold

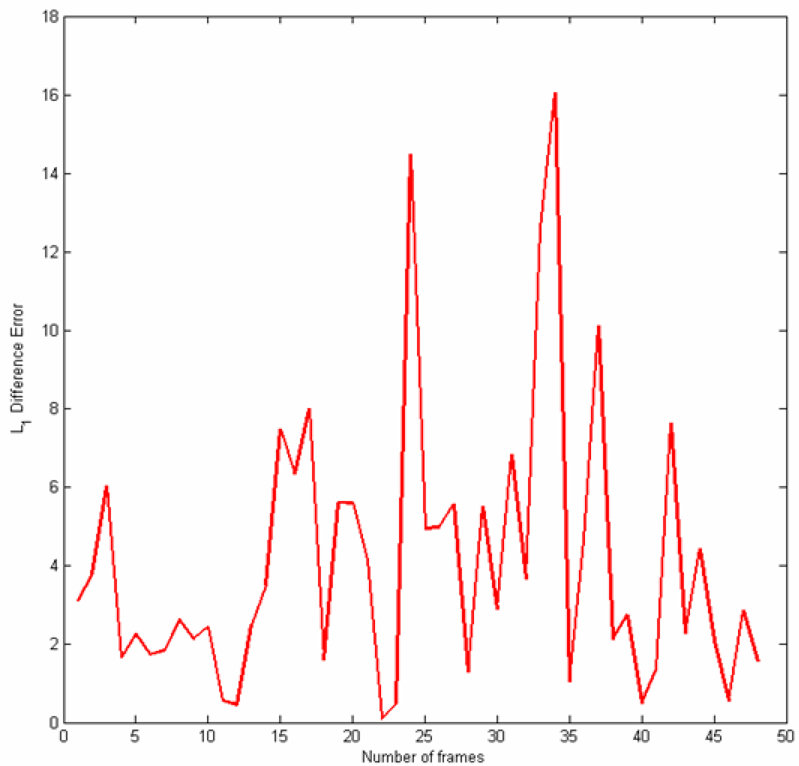
$$n = \arg\min_{i}\left(\frac{1}{F}\left|I(\vec{x},t) - R_{i,\vec{x}}(t)\right| < \delta\right)$$



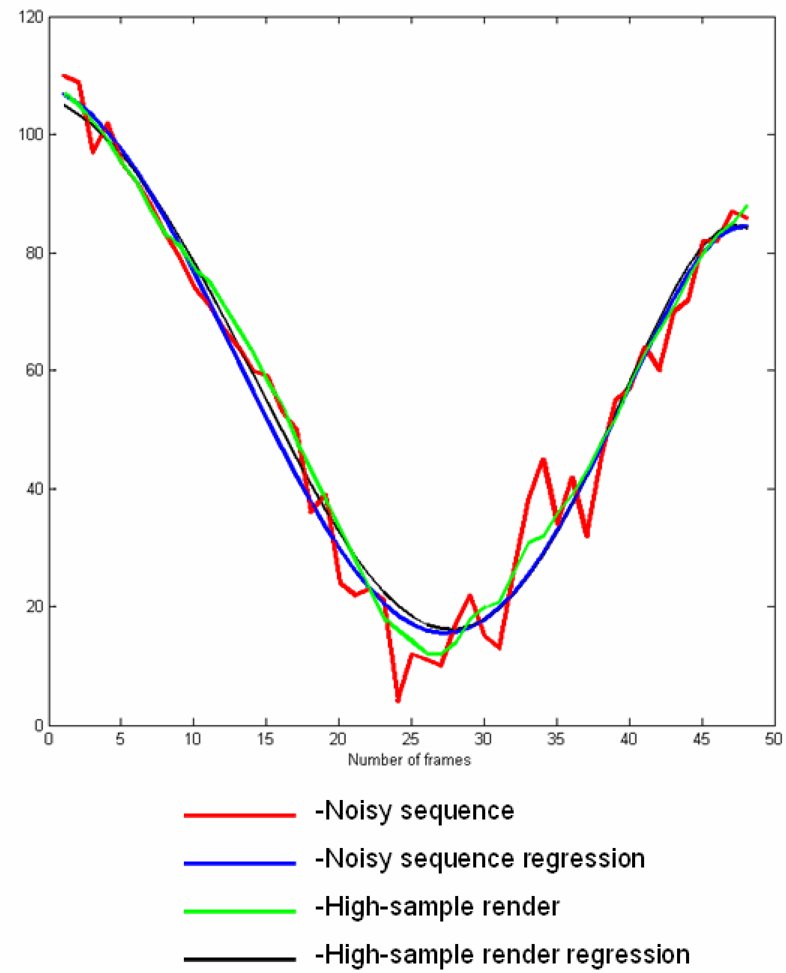
Degree map

Artifact Detection Using Regression

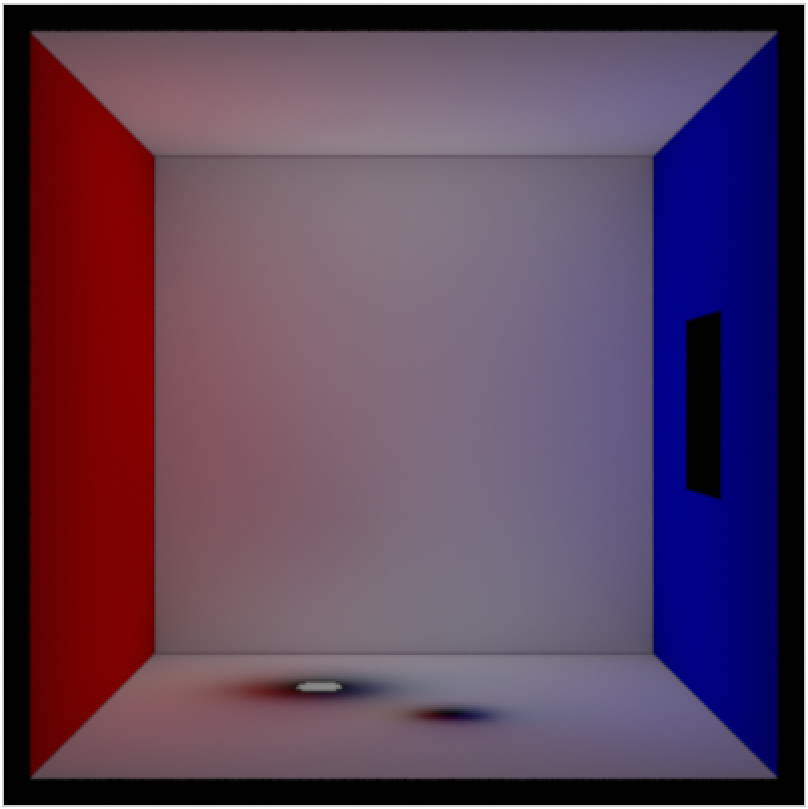
Noisy – noisy regression



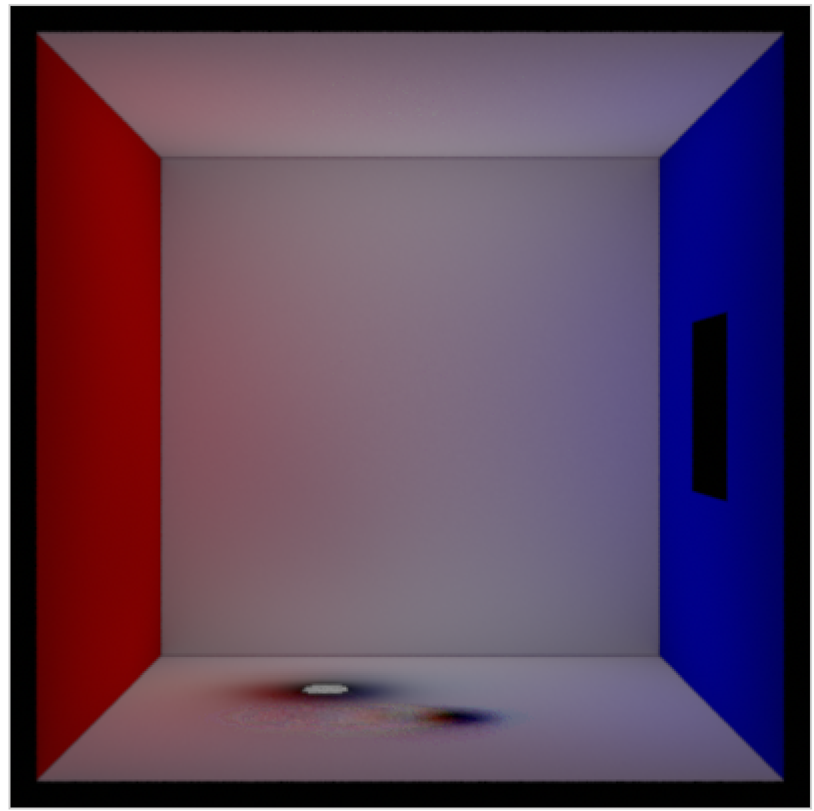
Pixel over time



Regression Results



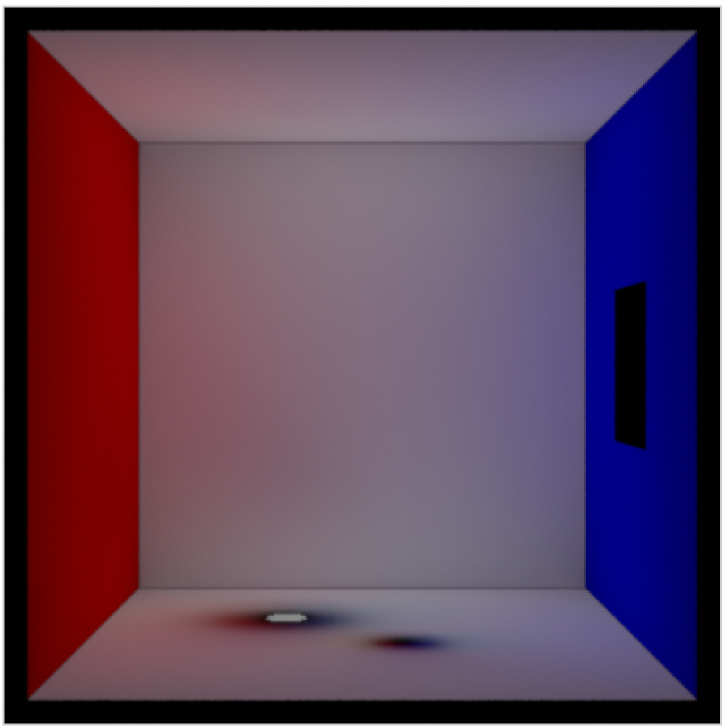
Noise-free Animation



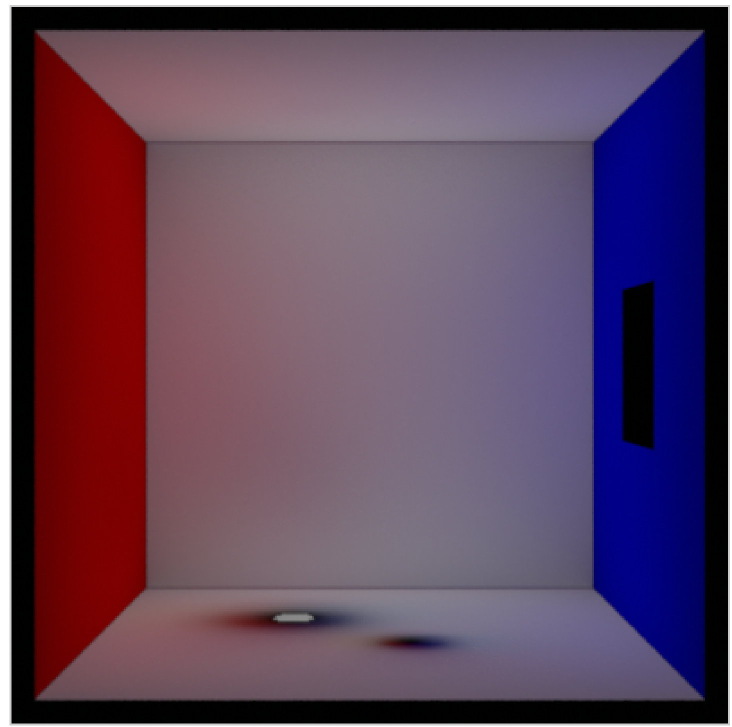
Polynomial Regression

Summary

- Detect motion using spatial PCA basis vectors and z-scores
- Denoise image sequence with hybrid method
- Detection artifacts using regression
- Re-render artifacts



High sample data



Reconstructed data

Future work

- Artifacts detection
  - Improvement of regression
  - Difference between hybrid and regression denoising
- Motion detection
  - Localization of motion using VARIMAX

Acknowledgements

- Todd Wittman, IPAM
- John Anderson, PIXAR
- PIXAR Animation Studios
- IPAM
- RIPS Students 2007