WESTFÄLISCHE Wilhelms-Universität Münster

Opening Colloquium Mathematics Münster: Dynamics – Geometry – Structure

Book Of Abstracts





The new Cluster of Excellence "Mathematics Münster: Dynamics - Geometry - Structure" celebrates its start. Just like the Cluster interconnects all of Münster's mathematics from pure to applied, the opening colloquium will cover a wide range of mathematics with excellent speakers. We are delighted to welcome you in Münster on this event.

In case of questions, we are happy to assist you. You find contact persons at the registration desk during the breaks.

12:00 - 12:50	Registration
12:50 - 13:00	Welcome Address
13:00 - 13:45	Brendle, Simon: Ricci Flow
14:00 - 14:45	Woodin, W. Hugh: The Continuum Hypothesis
14:45 - 15:30	Coffee Break
15:30 - 16:15	Gee, Toby: Potential modularity.
16:30 - 17:15	Staffilani, Gigliola : The beautiful mathematics involved in the study of dispersive equations.
17:15 - 18:00	Coffee Break
18:00 - 18:45	Nikeghbali, Ashkan : Some problems on the distribution of the values of the Riemann zeta function and related probabilistic models
19:30 - 22:00	Conference Dinner

Thursday, June 20, 2019

Friday, June 21, 2019

9:00 - 9:10	Welcome by the Rector
9:10 - 9:55	De Philippis, Guido : (Boundary) Regularity for Mass Minimising currents
9:55 - 10:15	Coffee Break
10:20 - 11:05	Wahl, Nathalie: Homological stability: what is that for?
11:15 - 12:00	Miller, Jason: Equivalence of Liouville quantum gravity and the Brownian map
12:00 - 13:00	Lunch Break
13:00 - 13:45	Naber, Aaron: Analysis of Geometric Nonlinear Partial Differential Equations.
14:00 - 14:45	Breuillard, Emmanuel: Random walks on finite fields and random polynomials
15:00 - 15:45	Thom, Andreas: Asymptotics of Cheeger constants and unitarisability of groups
17:00 - 22:00	open-air Rock- and Metal Concert "Das Schloss rockt"

Saturday, June 22, 2019

9:00 - 9:45	Cohen, Albert: Optimal sampling and reconstruction in high dimension
10:00 - 10:45	Vaes, Stefaan: Classification problems in operator algebras
10:45 - 11:10	Coffee Break
11:10 - 11:55	Williamson, Geordie: Geometric modular representation theory
12:10 - 12:55	Scholze, Peter: Condensed Mathematics

Venue

Schloss of Münster Lecture Hall S10 Schlossplatz 2 48149 Münster (see map on page 20)

Abstracts

Brendle, Simon
Breuillard, Emmanuel
Cohen, Albert
De Philippis, Guido
Gee, Toby
Miller, Jason
Naber, Aaron
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Williamson, Geordie
Woodin, W. Hugh

Ricci Flow

Simon Brendle

Columbia University

The Ricci flow is a natural evolution equation for Riemannian metrics on a given manifold. From a PDE perspective, the Ricci flow is a system of linear parabolic equations, which can be viewed as the heat equation analogue of the Einstein equations in general relativity. The central problem in the field is to understand singularity formation. In other words, what does the geometry look like at points where the curvature is large? In his spectacular 2002 breakthrough, Perelman achieved a qualitative understanding of singularity formation in dimension 3; this is sufficient for topological conclusions.

In this lecture, we will discuss recent work which gives a complete classification of all the singularity models in dimension 3. It turns out that the only possible singularity models are shrinking spheres and cylinders (or quotients thereof), and the rotationally symmetric Bryant soliton, thereby confirming a conjecture of Perelman. We will also discuss how one can prove results of a similar nature in higher dimensions for initial data with positive isotropic curvature; this connects with Gromov's concept of macroscopic dimension.

Random walks on finite fields and random polynomials

Emmanuel Breuillard

Univ. of Cambridge

Given a non-zero element a in a finite field \mathbb{F}_p we study the mixing properties of the random process obtained by iterating $x \mapsto ax \pm 1$ on \mathbb{F}_p , where ± 1 are independent unbiased random signs. Work of Konyagin on the Waring problem mod p implies that mixing occurs in $O((\log p)^{2+o(1)})$ steps provided the multiplicative order of a is large enough. We show that for most values of the parameter a mixing occurs in $O((\log p)^{1+o(1)})$ steps. We use this information to establish, conditionally on the Riemann hypothesis for Dedekind zeta functions of number fields, a conjecture of Odlyzko and Poonen on irreducibility of random integer polynomials of large degree. In turn this allows us to improve the mixing bounds for most values of a and establish (conditionally on GRH) that mixing occurs in $O(\log p)$ steps and even that a sharp phase transition to equidistribution (cut-off phenomenon) occurs at the mixing time. We will also point at connections to the Lehmer conjecture. Joint work with Péter Varjú.

Optimal sampling and reconstruction in high dimension

Albert Cohen

Sorbonne Université

Motivated by non-intrusive approaches for high-dimensional parametric PDEs, we consider the general problem of approximating an unknown arbirary function in any dimension from the data of point samples. The approximants are picked from given or adaptively chosen finite dimensional spaces. One principal objective is to obtain an approximation which performs as good as the best possible using a sampling budget that is linear in the dimension of the approximating space. We will show that this objectif can is met by taking a random sample distributed according to a well chosen probability measure, and reconstructing by appropriate least-squares or pseudo-spectral methods.

(Boundary) Regularity for Mass Minimising currents

Guido De Philippis

SISSA Trieste

Plateau problem consists in looking for a surface of minimal area among the ones spanning a given curve. It is among the oldest problem in the calculus of variations and its study lead to wonderful development in mathematics.

Federer and Fleming integral currents provide a suitably weak solution to the Plateau problem in arbitrary Riemannian manifolds, in any dimension and co-dimension. Once this week solution has been found a natural question consists in understanding whether it is classical one. i.e. a smooth minimal surface. This is the topic of the regularity theory, which naturally splits into interior regularity and boundary regularity. After the monumental work of Almgren, revised by De Lellis and Spadaro, interior regularity is by now well understood. Boundary regularity is instead less clear and some new phenomena appear.

Aim of the talk is to give an overview of the problem and to present some boundary regularity results we have obtained in the last years.

This is based on joint works with C. De Lellis, J. Hirsch and A. Massaccesi.

Potential modularity.

Toby Gee

Imperial College London

I will discuss some recent results relating Galois representations and automorphic forms, focussing in particular on the case of abelian surfaces.

Equivalence of Liouville quantum gravity and the Brownian map

Jason Miller

Univ. of Cambridge

Over the past several decades, two natural random surface models have emerged within physics and mathematics. The first is Liouville quantum gravity, which has roots in string theory and conformal field theory from the 1980s and 1990s. The second is the Brownian map, which has roots in planar map combinatorics from the 1960s.

We show that the Brownian map is equivalent to Liouville quantum gravity with parameter $\gamma = \sqrt{8/3}$.

Based on joint work with Scott Sheffield.

Analysis of Geometric Nonlinear Partial Differential Equations.

Aaron Naber

Northwestern Univ.

It is well known that solutions of a linear elliptic equation, for instance a harmonic function $\Delta u = 0$, are automatically smooth. Indeed, one has many ways to quantify this and understand precise estimates on solutions. In the context of nonlinear equations, for instance nonlinear harmonic maps, Yang-Mills, Einstein manifolds, etc...,this need not be the case and one may be forced to deal with a singular set Sing(u). Hausdorff dimension estimates on the singular set have been understood since the days of Federer in the 60's, but more refined structure and the ability to produce effective estimates on u have remained elusive.

We will discuss in this talk a new series of techniques for analyzing nonlinear differential equations. The talk will focus on nonlinear harmonic maps, however the techniques have found applications in many equations. We will see that the singular set Sing(u) of such nonlinear equations have manifold structure, or more precisely a rectifiable structure, and how to use these ideas to produce sharp estimates on solutions. Further refinements of these ideas have been used to solve a variety of open conjectures, including the Energy Identity conjecture for Yang-Mills and the n-4-finiteness conjecture for Einstein manifolds.

Some problems on the distribution of the values of the Riemann zeta function and related probabilistic models

Ashkan Nikeghbali

Univ. of Zurich

In this talk we shall review a few well known problems related to the distribution of the values of the Riemann zeta function on the critical line and see how one can use probabilistic models and ideas to hopefully gain better insights into such problems. The main focus of the talk will be on the GUE conjecture and the convergence of some random analytic functions (coming from random matrix theory).

Condensed Mathematics

Peter Scholze

Univ. of Bonn

(joint with Dustin Clausen) Putting a topology onto algebraic objects usually destroys their nice algebraic properties: For example, topological abelian groups do not define an abelian category. We will explain a way to resolve these issues.

The beautiful mathematics involved in the study of dispersive equations.

Gigliola Staffilani

MIT

In recent years great progress has been made in the study of dispersive and wave equations. The toolbox used in order to attack highly nontrivial problems related to these equations has developed to include a variety of techniques from Fourier and harmonic analysis, analytic number theory, math physics, dynamical systems, probability and symplectic geometry. In this talk I will introduce a variety of problems connected with dispersive, such as the derivation of a certain nonlinear Schrodinger equations from a quantum many-particles system, periodic Strichartz estimates, the concept of energy transfer, the invariance of a Gibbs measure associated to an infinite dimension Hamiltonian system, and non-squeezing theorems for such systems when they also enjoy a symplectic structure.

Asymptotics of Cheeger constants and unitarisability of groups

Andreas Thom

 $TU \ Dresden$

Given a group Γ , we establish a connection between the unitarisability of its uniformly bounded representations and the asymptotic behaviour of the isoperimetric constants of Cayley graphs of Γ for increasingly large generating sets.

The connection hinges on an analytic invariant $\operatorname{Lit}(\Gamma) \in [0, \infty]$ which we call the *Littlewood exponent*. Finiteness, amenability, unitarisability and the existence of free subgroups are related respectively to the thresholds 0, 1, 2 and ∞ for $\operatorname{Lit}(\Gamma)$. Using graphical small cancellation theory, we prove that there exist groups Γ for which $1 < \operatorname{Lit}(\Gamma) < \infty$. Further applications, examples and problems are discussed.

Classification problems in operator algebras

Stefaan Vaes

 $KU\ Leuven$

The talk starts by an elementary introduction to the theory of operator algebras: C*-algebras and von Neumann algebras. I will review the fundamental and classical classification results for amenable von Neumann algebras due to Connes and Haagerup. The second part of the talk will focus on similar classification problems for von Neumann algebras given by Bernoulli actions of discrete groups, both in the measure preserving context and in the non measure preserving case.

Homological stability: what is that for?

Nathalie Wahl

Univ. of Copenhagen

Homological stability is a topological property shared by many configuration spaces and groups of matrices, or, more generally, groups of symmetries. In recent years, it has turned into a powerful computational tool. The talk will give an overview of the subject.

Geometric modular representation theory

Geordie Williamson

Univ. of Sydney

This will be a broad survey talk on interactions between geometry and representation theory, with a focus on representations in positive characteristic ("modular representation theory"). I will outline several basic questions (e.g. for modular representations of the symmetric group) which appear very difficult, and have resisted direct algebraic approaches for over a hundred years. Over the last two decades, a new approach has emerged via geometric representation theory. It turns out that subtle questions concerning torsion in cohomology control these problems, and this allows some progress to be made. Many open questions remain, but at this point there can be no doubt that a fascinating and deep theory awaits us.

The Continuum Hypothesis

Hugh W. Woodin

Harvard Univ.

Set Theory, which is the mathematical study of infinity, began in the period 1879-84 with a series of papers by Cantor that defined the fundamental framework of the subject. Within 40 years the key ZFC axioms for Set Theory were in place and the stage was set for the detailed development of transfinite mathematics, or so it seemed.

However, in a completely unexpected development, Cohen showed in 1963 that even the most basic problem of Set Theory, that of Cantor's Continuum Hypothesis (CH), was not solvable on the basis of the ZFC axioms.

The 50 years since Cohen's work has seen a vast development of Cohen's method and the realization that the occurrence of unsolvable problems is ubiquitous in Set Theory.

In fact the entire development in Set Theory seems to have created a fundamental dilemma. On the one hand, the discovery, also over the last 50 years, of a rich hierarchy axioms of infinity seems to argue that Cantor's conception is fundamentally sound and therefore that CH must have an answer.

However on the other hand, the developments of Cohen's method over this same period seem to strongly suggest there can be no preferred extension of the ZFC axioms to a system of axioms that can escape the ramifications of Cohen's method and resolve the problem of CH.

But this dilemma was itself based on a misconception and the key problem has emerged: The Ultimate L Conjecture.

Conference WLAN

Wireless network access for guests is provided within the building. Connect your device to the SSID GuestOnCampus and start your web browser (e.g. Chrome or Firefox). When you start the browser, you will be automatically redirected to the login page (hotsplots). After you have checked the terms of use, you can log on by clicking on the "Log in for free" button. Please note that this is an open wireless network and therefore it is unencrypted. Alternatively, if you are part of the eduroam community, you may connect to the network "eduroam" as usual.

Conference Dinner

The conference dinner will take place at the restaurant "Schlossgarten" (Schlossgarten 4, 48149 Münster, https://www.schlossgarten.com/). A fee of 20 Euros is charged for participation in the conference dinner to cover part of its cost. The fee is collected in cash on the premises. Please bring your workshop badge as identification to the dinner. Additional dinner-participants can be announced and registered in advance at the workshop desk. You can easily reach the the restaurant on your own, see the map on page 20.

Lunch on Friday

On Friday lunch is provided by "Le Feu". Le Feu's specialty, the Flammkuchen, will be prepared freshly on site. The 'pizza-like' Flammkuchen comes in an array of tantalizing toppings, such as creamy leek and bacon, or vegetarian Flammkuchen such as cream and cheese champignon. Lunch is provided in the entrance hall.

Open-air Concert "Das Schloss rockt"

Everyone is invited to attend the open-air rock concert "Das Schloss rockt", co-organized by cluster spokesperson Prof. Christopher Deninger. It starts at 5 pm directly outside the building in which the workshop takes place.

Further information

For further information during the workshop please contact: Carolin Gietz +49 1520 6559998 carolin.gietz@uni-muenster.de



Notes